Bosonic string theory for LGT observables

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Foreword

- This talk is based on
  - M. Billó and M. Caselle, "Polyakov loop correlators from D0-brane interactions in bosonic string JHEP **0507** (2005) 038 [arXiv:hep-th/0505201].

  also outlined in the LATTICE 2005 talk of M. Caselle:
  - M. Billo, M. Caselle, M. Hasenbusch and M. Panero, "QCD string from D0 branes," PoS (LAT2005) 309 [arXiv:hep-lat/0511008].
- and on a paper in preparation:
  - M. Billó, L. Ferro and M. Caselle, "The partition function for the effective string theory of interfaces", to appear (soon!).

Plan of the talk

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1. The main ideas
2. Polyakov loop correlators
3. Interface partition function

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The main ideas

String theory and (lattice) gauge theories

- A description of strongly coupled gauge theories in terms of strings has long been suspected
- These strings should describe the fluctuations of the color flux tube in the confining regime

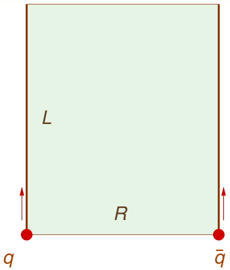
- Potential $V(R)$ between two external, massive quark and anti-quark sources from Wilson loops

$$\langle W(L,R)\rangle \sim \mathrm{e}^{-LV(R)} \quad \text{(large } R\text{)}$$

- Area law $\leftrightarrow$ linear potential

$$V(R) = \sigma R + \ldots$$

σ is the string tension



Quantum corrections and effective models

- Leading correction for large R

$$V(R) = \sigma R - \frac{\pi}{24}\frac{d-2}{R} + O\left(\frac{1}{R^2}\right) .$$

from quantum fluctuations of $d-2$ massless modes: transverse fluctuations of the string

Lüscher, Symanzik and Weisz

- Simplest effective description via the $c = d - 2$ two-dimensional conformal field theory of free bosons
  - Higher order interactions among these fields distinguish the various effective theories
  - The underlying string model should determine a specific form of the effective theory, and an expression of the potential $V(R)$ that extends to finite values of R.

Various models of effective strings

- "Free" theory: the $d-2$ bosonic fields living on the surface spanned by the string, describing its transverse fluctuations
- Standard bosonic string theory. Nambu-Goto action $\propto$ area of the world-sheet surface
  - Possible first-order formulation á la Polyakov (we'll use this)
  - In $d \neq 26$, bosonic string is ill-defined (conformal invariance broken by quantum effects). This is manifest at short distances in the description of LGT observables.
- Attempts to a consistent string theory description: Polchinski-Strominger, Polyakov, AdS/CFT

The Nambu-Goto approach

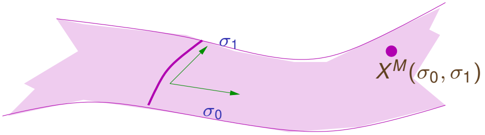
- Action $\sim$ area of the surface spanned by the string in its motion:

$$S = -\sigma \int d\sigma_0 d\sigma_1 \sqrt{\det g_{\alpha\beta}}$$

where $g_{\alpha\beta}$ is the metric "induced" on the w.s. by the embedding:

$$g_{\alpha\beta} = \frac{\partial X^M}{\partial \sigma_\alpha} \frac{\partial X^N}{\partial \sigma_\beta} G_{MN}$$

σ_α = world-sheet coords. (σ_0 = proper time, $\sigma = 1$ spans the extension of the string)



The nambu-Goto approach (cont.ed)

- One can use the world-sheet re-parametrization invariance of the NG action to choose a “physical gauge”:
  - The w.s. coordinates σ^0, σ^1 are identified with two target space coordinates x^0, x^1
- One can study the 2d QFT for the $d-2$ transverse bosonic fields with the gauge-fixed NG action

$$Z = \int DX^i \mathrm{e}^{-\sigma \int dx^0 dx^1 \sqrt{1+(\partial_0 \vec{X})^2+(\partial_1 \vec{X})^2+(\partial_0 \vec{X} \wedge \partial_1 \vec{X})^2}}$$
$$= \int DX^i \mathrm{e}^{-\sigma \int dx^0 dx^1 \left\{1+(\partial_0 \vec{X})^2+(\partial_1 \vec{X})^2+\text{int.s}\right\}}$$

perturbatively, the loop expansion parameter being $1/(\sigma A)$ [e.g., Dietz-Filk, 1982]

The first order approach

- The NG goto action can be given a 1st order formulation (no awkward square roots)

$$S = -\sigma \int d\sigma_0 d\sigma_1 \sqrt{h} h^{\alpha\beta} \partial_\alpha X^M \partial_\beta X^M$$

  with $h_{\alpha\beta}$ = independent w.s metric
- Use re-parametrization and Weyl invariance to set $h_{\alpha\beta} \to \eta_{\alpha\beta}$
  - Actually, Weyl invariance is broken by quantum effects in $d \neq 26$
- Remain with a free action but
  - Virasoro constraints $T_{\alpha\beta} = 0$ from $h^{\alpha\beta}$ e.o.m.
  - residual conformal invariance

Physical gauge vs. covariant quantization

- The residual conformal invariance can be used to fix a light-cone (physical) type of gauge: w.s. coordinates identified with two target space ones (non-covariant choice)
  - One explicitly solves the Virasoro constraints and remains with the $d - 2$ transverse directions as the only independent d.o.f.
  - The quantum anomaly for $d \neq 26$ manifests as a failure in Lorentz algebra
- In a covariant quantization, the Virasoro constraints are imposed on physical states á la BRST
  - All d directions are treated on the same footing
  - Introduction of ghosts
  - For $d \neq 26$, anomaly in the conformal algebra
  - This is the framework we will use

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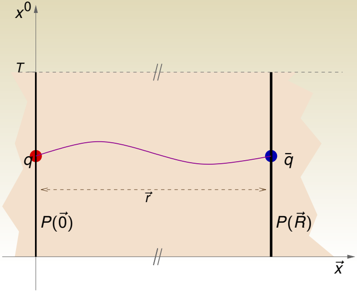
Polyakov loop correlators

The set-up



- Finite temperature geometry + static external sources (quarks)
- Polyakov loop = trace of the temporal Wilson line

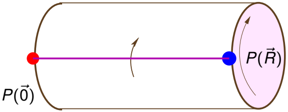
$$\langle P(\vec{R})\rangle = \mathrm{e}^{-F} \neq 0 \rightarrow \text{de-confinement}$$

- On the lattice, the correlator

$$\langle P(\vec{0})P(\vec{R})\rangle_{\mathrm{c}} \ .$$

can be measured with great accuracy.

- In the string picture, the correlation is due to the strings connecting the two external sources: cylindric world-sheet



Nambu-Goto description of the correlator (1)

- P.L. correlator = partition function of an open string with
  - Nambu-Goto action
  - Dirichlet boundary conditions (end-points attached to the Polyakov loops)
- Operatorial formulation:
  - Spectrum obtained via formal quantization by Arvis:

$$E_n(R) = \sigma R \sqrt{1 + \frac{2\pi}{\sigma R^2}\left(n - \frac{d-2}{24}\right)}\ .$$

  - Partition function: Back

$$Z = \sum_n w_n e^{-L E_n(R)}$$

w_n = multiplicities of the bosonic string: $\eta(q) = \sum_n w_n q^{n - \frac{1}{24}}$

Nambu-Goto description of the correlator (1)

- P.L. correlator = partition function of an open string with
  - Nambu-Goto action
  - Dirichlet boundary conditions (end-points attached to the Polyakov loops)
- Operatorial formulation:
  - Expansion of the energy levels:

$$E_n = \sigma R + \frac{\pi}{R}\left(n - \frac{d-2}{24}\right) + \dots$$

  - Expansion of the partition function

$$Z = \mathrm{e}^{-\sigma LR} \sum_n w_n \mathrm{e}^{-\pi \frac{L}{R}\left(n - \frac{d-2}{24}\right) + \dots} = \mathrm{e}^{-\sigma LR} \eta(\mathrm{i}\frac{L}{2R})\,(1 + \dots)$$

Nambu-Goto description of the correlator (2)

- Functional integral result (Dietz and Filk):
  - Loop expansion. Expansion parameter $1/(\sigma LR)$
  - Two-loop result [set $\hat{\tau} = \mathrm{i}L/(2R)$, $d = 3$]:

$$Z = \mathrm{e}^{-\sigma LR} \frac{1}{\eta(\hat{\tau})} \left(1 - \frac{\pi^2 L}{1152 \sigma R^3} \left[2E_4(\hat{\tau}) - E_2^2(\hat{\tau})\right] + \ldots\right)$$

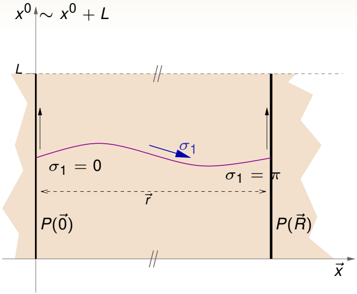
- This is reproduced by the partition function of the operatorial formulation, upon expanding the energy levels E_n

Caselle et al

First order formulation

- Action (in conformal gauge)

$$S = \frac{1}{4\pi\alpha'} \int d\sigma_0 \int_0^{\pi} d\sigma_1 \left[(\partial_\tau X^M)^2 + (\partial_\sigma X^M)^2 \right] + S_{\text{gh.}}$$



- World-sheet parametrized by
  - $\sigma_1 \in [0, \pi]$ (open string)
  - σ_0 (proper time)

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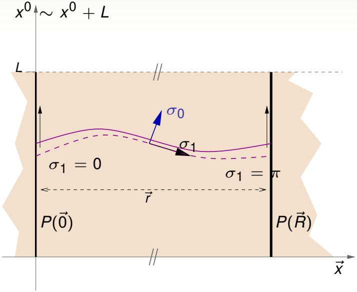

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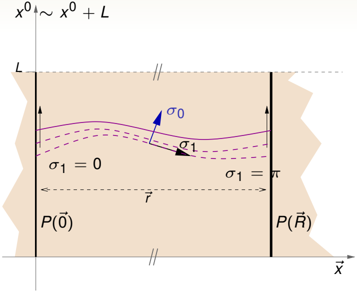

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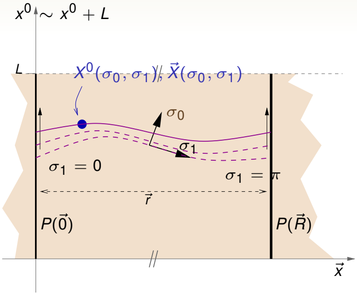

- The field X^M ($M = 0, \ldots, d-1$) describe the embedding of the world-sheet in the target space

First order formulation

- Action (in conformal gauge)

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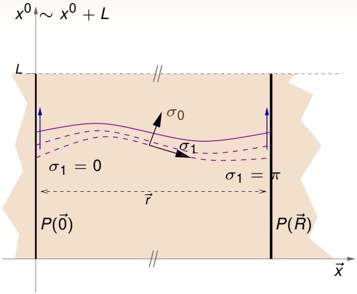


- Boundary conditions:
  - Neumann in "time" direction:

$$\partial_\sigma X^0(\tau, \sigma)\big|_{\sigma=0,\pi} = 0$$

  - Dirichlet in spatial directions:

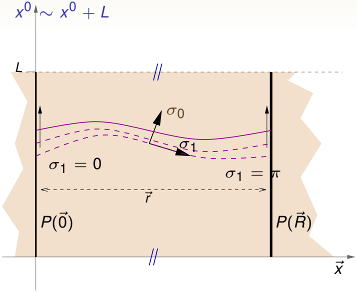
$$\vec{X}(\tau, 0) = 0 \, , \qquad \vec{X}(\tau, \pi) = \vec{R}$$

"open string suspended between two D0-branes"

First order formulation

- Action (in conformal gauge)

$$S = \frac{1}{4\pi\alpha'} \int d\sigma_0 \int_0^\pi d\sigma_1 \left[(\partial_\tau X^M)^2 + (\partial_\sigma X^M)^2 \right] + S_{\rm gh.}$$



- The string fields have thus the expansion

$$X^0 = \hat{x}^0 + \frac{\hat{p}^0}{\pi\sigma} + \frac{\mathrm{i}}{\sqrt{\pi\sigma}} \sum_{n\neq 0} \frac{\alpha^0}{n} \mathrm{e}^{-\mathrm{i}n\sigma_0} \cos n\sigma_1$$

$$\vec{X} = \frac{\vec{R}}{\pi}\sigma_1 - \frac{1}{\sqrt{\pi\sigma}} \sum_{n\neq 0} \frac{\vec{\alpha}}{n} \mathrm{e}^{-\mathrm{i}n\sigma_0} \sin n\sigma_1$$

- Canonical quantization leads to

$$\left[\alpha_m^M, \alpha_n^N \right] = m\, \delta_{m+n,0}\, \delta^{MN} .$$

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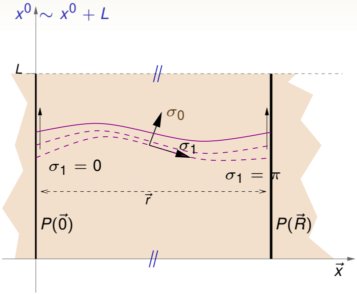


- The target space has finite temperature:

$$x^0 \sim x^0 + L$$

  - The 0-th component of the momentum is therefore discrete:

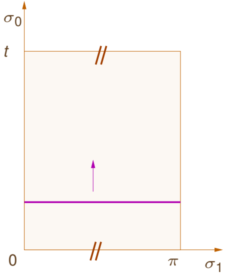
$$p^0 \to \frac{2\pi n}{L}$$

The free energy

- Interaction between the two Polyakov loops (the D0-branes) $\leftrightarrow$ free energy of the open string

$$\mathcal{F} = L \int_0^\infty \frac{dt}{2t} \,\mathrm{Tr} q^{L_0}$$

- $q = \mathrm{e}^{-2\pi t}$, and t is the only parameter of the world-sheet cylinder (one loop of the open string)



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- L is the “world-volume” of the D0-brane, i.e. the volume of the only direction along which the excitations propagate, the Euclidean time

The free energy

- Interaction between the two Polyakov loops (the D0-branes) $\leftrightarrow$ free energy of the open string

$$\mathcal{F} = L \int_0^\infty \frac{dt}{2t} \mathrm{Tr} q^{L_0}$$

- Virasoro generator L_0 (Hamiltonian)

$$L_0 = \frac{(\hat{p}^0)^2}{2\pi\sigma} + \frac{\sigma R^2}{2\pi} + \sum_{n=1}^{\infty} N_n^{(d-2)} - \frac{d-2}{24}$$

  - $N_n^{(d-2)}$ is the total occupation number for the oscillators appearing in $d-2$ bosonic fields (the -2 is due to the ghosts)

The free energy

- Interaction between the two Polyakov loops (the D0-branes) $\leftrightarrow$ free energy of the open string

$$\mathcal{F} = L \int_0^\infty \frac{dt}{2t} \mathrm{Tr} q^{L_0}$$

- The trace over the oscillators yields, for each bosonic direction,

$$q^{-\frac{1}{24}} \prod_{r=1}^{\infty} \frac{1}{1-q^r} = \frac{1}{\eta(\mathrm{i}t)}$$

The free energy

- Interaction between the two Polyakov loops (the D0-branes) $\leftrightarrow$ free energy of the open string

$$\mathcal{F} = L \int_0^\infty \frac{dt}{2t} \mathrm{Tr} q^{L_0}$$

- We must trace also over the discrete zero-mode eigenvalues $p^0 = 2\pi n/L$. Altogether,

$$\mathcal{F} = \int_0^\infty \frac{dt}{2t} \sum_{n=-\infty}^{\infty} \mathrm{e}^{-2\pi t \left(\frac{2\pi n^2}{\sigma L^2} + \frac{\sigma R^2}{2\pi} \right)} \left(\frac{1}{\eta(\mathrm{i}t)} \right)^{d-2}$$

Topological sectors

- Poisson resum over the integer n getting

$$\mathcal{F} = \mathcal{F}^{(0)} + 2\sum_{m=1}^{\infty} \mathcal{F}^{(m)}$$

with ▸ Back

$$\mathcal{F}^{(m)} = \sqrt{\frac{\sigma L^2}{4\pi}} \int_0^\infty \frac{dt}{2t^{\frac{3}{2}}} \mathrm{e}^{-\frac{\sigma L^2 m^2}{4t} - \sigma R^2 t} \left(\frac{1}{\eta(\mathrm{i}t)}\right)^{d-2}$$

- The integer m is the # of times the open string wraps the compact time in its one loop evolution.
- Each topological sector $\mathcal{F}^{(m)}$ describes the fluctuations around an "open world-wheet instanton"

$$X^0(\sigma_0 + t, \sigma_1) = X^0(\sigma_0, \sigma_1) + mL$$

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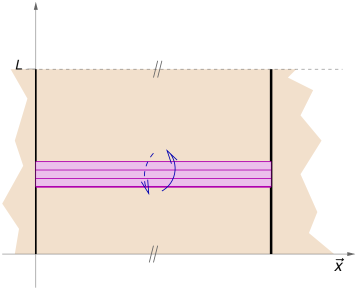

- An example with $m = 0$ (N.B. The classical solution degenerates to a line)

Topological sectors

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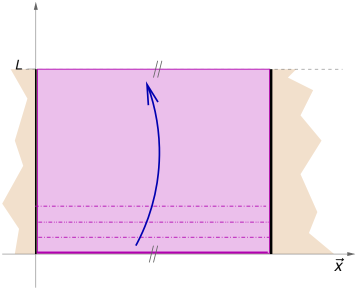


- The case $m = 1$. The world-sheet exactly maps to the cylinder connecting the two Polyakov loops.

The case $m = 1$ and the NG result

- The sector with $m = 1$ of our free energy should correspond to the effective NG partition function
- Expand in series the Dedekind functions:

$$\left(\prod_{r=1}^{\infty} \frac{1}{1 - q^r} \right)^{d-2} = \sum_{k=0}^{\infty} w_k q^k$$

- Plug this into $\mathcal{F}^{(m)}$ Recall and integrate over t using

$$\int_0^{\infty} \frac{dt}{t^{\frac{3}{2}}} \mathrm{e}^{-\frac{\alpha^2}{t} - \beta^2 t} = \frac{\sqrt{\pi}}{|\alpha|} \mathrm{e}^{-2|\alpha|\,|\beta|}$$

The case $m = 1$ and the NG result

- The sector with $m = 1$ of our free energy should correspond to the effective NG partition function
- The result is

$$\mathcal{F}^{(m)} = \frac{1}{2|m|} \sum_k w_k \mathrm{e}^{-|m| L E_k(R)} , \qquad (m \neq 0)$$

with

$$E_k(R) = \frac{R}{4\pi\alpha'} \sqrt{1 + \frac{4\pi^2\alpha'}{R^2} \left(k - \frac{d-2}{24}\right)}$$

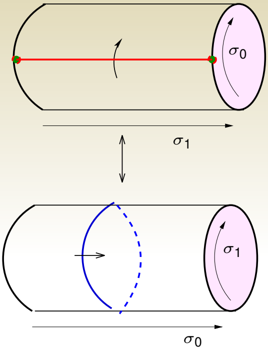
- So, in particular,

$$2\mathcal{F}^{(1)} = Z(R)$$

Transformation to the closed channel

- The modular transformation $t \to 1/t$ maps the open string channel 1-loop free energy to a closed string channel tree level exchange between boundary states
- The result of the transformation is

$$\mathcal{F}^{(m)} = L\frac{T_0^2}{4}\sum_k w_k\, G(R; M(m,k))$$



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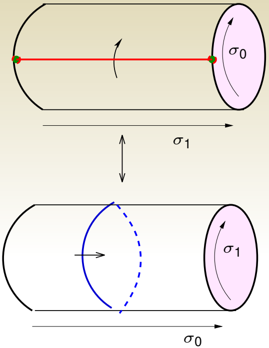

$$\mathcal{F}^{(m)} = L\frac{T_0^2}{4}\sum_k w_k\, G\,(R; M(m,k))$$

- $G\,(R; M)$ = propagator of a scalar field of mass M^2 over the distance $\vec{R}$ between the two D0-branes along the $d-1$ spatial directions:

$$G(R;M) = \int \frac{d^{d-1}p}{(2\pi)^{d-1}}\frac{\mathrm{e}^{\mathrm{i}\vec{p}\cdot\vec{R}}}{p^2+M^2} = \frac{1}{2\pi}\left(\frac{M}{2\pi R}\right)^{\frac{d-3}{2}} K_{\frac{d-3}{2}}(MR)$$

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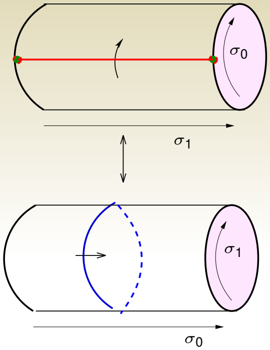

- The result of the transformation is

$$\mathcal{F}^{(m)} = L\frac{T_0^2}{4}\sum_k w_k\, G\,(R; M(m,k))$$

- The mass $M(m,k)$ is that of a closed string state with k representing the total oscillator number, and m the wrapping number of the string around the compact time direction

$$M^2(m,k) = (m\sigma L)^2\left[1 + \frac{8\pi}{\sigma L^2 m^2}\left(k - \frac{d-2}{24}\right)\right]$$

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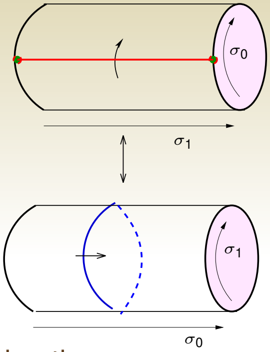

- T_0 = usual D0-brane tension in bosonic string theory:

$$T_0^2 = 8\pi\left(\frac{\pi}{\sigma}\right)^{\frac{d}{2}-2}$$

Closed string interpretation

- Our first-order formulation is well-suited to give the direct closed string channel description of the correlator:

$$\mathcal{F} = \langle B; \vec{0}\,|\mathcal{D}|B; \vec{R}\,\rangle = \frac{1}{4\sigma}\int_0^\infty ds\,\langle B; \vec{0}\,|\mathrm{e}^{-2\pi s(L_0+L_0^{\mathrm{gh.}})}|B; \vec{R}\,\rangle$$

  - $\mathcal{D}$ is the closed string propagator
  - The boundary states enforce on the closed string fields the b.c.'s corresponding to the D-branes (the Polyakov loops)

$$\partial_\tau X^0(\sigma,\tau)\big|_{\tau=0}\,|B; \vec{R}\rangle = 0\;, \qquad \left(X^i(\sigma,\tau) - R^i\right)\big|_{\tau=0}\,|B; \vec{R}\rangle = 0$$

  - The b.s. has a component in each closed string Hilbert space sector corresponding to winding number m
- The modular transformed form of the free energy in indeed exactly retrieved

Recapitulating

- The NG partition function describes the lattice data about Polyakov loop correlators for various gauge theories and dimensions:
  - very well for rather large R, L
  - with deviations stronger and stronger as R, L decrease
- These deviations should be related to the breaking of conformal invariance in $d \neq 26$
- In our first-order approach, we derive this NG partition function with standard bosonic string theory techniques: interaction between two D0-branes á la Polchinski
  - We neglect the effects of the Polyakov mode which arises for $d \neq 26$
  - The deviations at short distances could be attributed to this extra mode (eventually to be taken into account)

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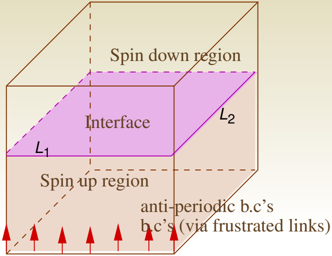
Interface partition function

Interfaces



Back

- An interface separating regions with different magnetization is observed in simulations of spin models (Ising, etc.), and its fluctuations are measured
- A similar situation can be engineered and studied in LGT, by considering the so-called 't Hooft loops

- It is rather natural to try to describe the fluctuating interface by means of some effective string theory
  - Some string predictions (in particular, the universale effect of the quantum fluctuations of the $d-2$ transverse free fields) have already been considered

e.g., De Forcrand, 2004

The Nambu Goto model for interfaces

- In the "physical gauge" approach, we consider a string whose world-sheet is identified with the minimal interface, which has the topology of a torus $T2$, of sides L_1 and L_2, i.e., area $A = L_1 L_2$ and modulus $u = L_2/L_1$ ▸ Recall
- We are thus dealing with the one-loop partition function $\mathcal{Z}$ of a closed string.
- The functional integral approach [Dietz-Filk, 1982]gives the result up to two loops:

$$\mathcal{Z} \propto \mathrm{e}^{-\sigma A} \frac{1}{[\eta(\mathrm{i}u)]^{2d-4}} \Big\{ 1 + \frac{(d-2)^2}{2\sigma A} \Big[\frac{\pi^2}{36} u^2 E_2^2(\mathrm{i}u) - \frac{\pi}{6} u E_2(\mathrm{i}u) + c_d \Big] \Big\}$$

The NG partition function?

- The partition function for the NG interface string in the operatorial formulation is not avaliable (to our knowledge) in the literature
- This would be the analogue of the partition function for the Polyakov loop correlators based on Arvis' spectrum ▸ Recall and would resum the loop expansion.

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  - It is not too difficult to propose the analogue of Arvis formula for the spectrum, based on canonical quantization [Drummond,Kuti,...]

$$E^2_{n,N+\tilde{N}} = \sigma^2 L_1^2 \left\{ 1 + \frac{4\pi}{\sigma L_1^2} \left(N + \tilde{N} - \frac{d-2}{12} \right) + \frac{4\pi^2}{\sigma^2 L_1^4} n^2 + \vec{p}_T^2) \right\}$$

where $N, \tilde{N}$ = occupation #'s of left (right)-moving oscillators, n the discretized momentum in the direction x^1, $\vec{p}_T$ the transverse momentum

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  - However, the "naive" form of a partition function based on this spectrum:

$$\sum_{N,\tilde{N},n} \delta(N - \tilde{N} + n) c_N c_{\tilde{N}} \, \mathrm{e}^{-L_2 E_{N+\tilde{N},n}}$$

    (where c_N, $c_{\tilde{N}}$ = multiplicities of left- and right-moving oscillator states) does not reproduce the functional integral 2-loop result

The first order approach

- We start from the Polyakov action in the conformal gauge, and do not impose any physical gauge identifying world-sheet and target space coordinates
- We consider the closed string one loop partition function, and we have thus a toroidal world-sheet
- This world-sheet can be mapped in many topologically distinct ways on the target space torus T_d

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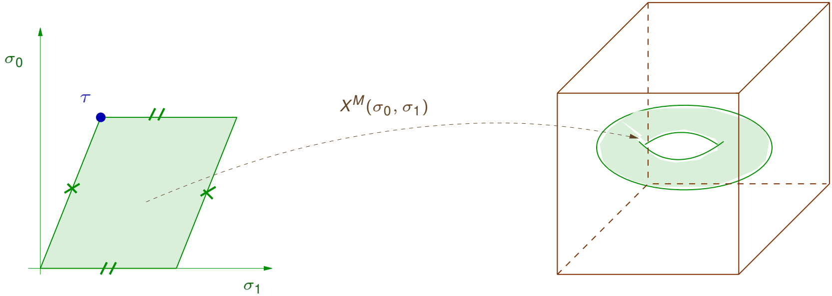

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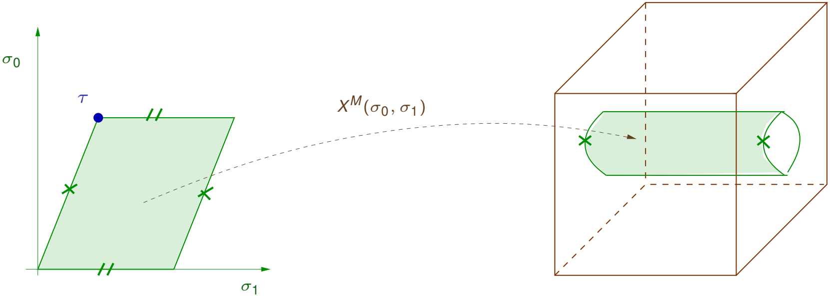

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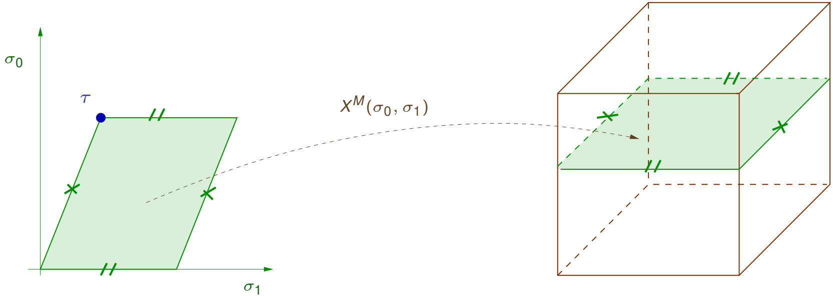

String partition function

- In the Polyakov formulation, the partition function includes an integration over the modular parameter $\tau = \tau_1 + \mathrm{i}\tau_2$:

$$\mathcal{Z} = \int \frac{d^2\tau}{\tau_2} Z^{(d)}(q,\bar{q})\, Z^{\mathrm{gh}}(q,\bar{q})$$

  - $Z^{(d)}(q,\bar{q})$ CFT partition function of d compact bosons:

$$Z^{(d)}(q,\bar{q}) = \mathrm{Tr}\, q^{L_0 - \frac{d}{24}}\, \bar{q}^{\tilde{L}_0 - \frac{d}{24}}$$

    where $q = \exp 2\pi\mathrm{i}\tau$, $\bar{q} = \exp -2\pi\mathrm{i}\bar{\tau}$.
  - The CFT partition function of the ghost system, $Z^{\mathrm{gh}}(q,\bar{q})$ will cancel the (non-zero modes of) two bosons

CFT partition function of a compact boson

- Consider a compact boson field

$$X(\sigma^0,\sigma^1) \sim X(\sigma^0,\sigma^1) + L$$

- In the operatorial formulation, we find

$$Z(q,\bar{q}) = \sum_{n,w\in\mathbb{Z}} q^{\frac{1}{8\pi\sigma}\left(\frac{2\pi n}{L}+\sigma w L\right)^2} \bar{q}^{\frac{1}{8\pi\sigma}\left(\frac{2\pi n}{L}-\sigma w L\right)^2} \frac{1}{\eta(q)}\,\frac{1}{\eta(\bar{q})}$$

  - The Dedekind functions encode the non-zero mode contributions
  - The 0-mode n denotes the discretized momentum $p = 2\pi n/L$
  - The integer w is the winding around the compact target space:: X must be periodic in σ^1, but we can have

$$X(\sigma^0,\sigma^1+2\pi) = X(\sigma^0,\sigma^1) + wL$$

CFT partition function of a compact boson

- Consider a compact boson field

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- Upon Poisson resummation over the momentum n,

$$Z(q,\bar{q}) = \sigma L \sum_{m,w\in\mathbb{Z}} \mathrm{e}^{-\frac{\sigma L^2}{2\tau_2}|m-\tau w|^2} \frac{1}{\sqrt{\tau_2}\eta(q)\eta(\bar{q})}$$

  - This is natural expression from the path-integral formulation
  - Sum over "world-sheet instantons": classical solutions of the field X with wrappings w (along σ_1) and m (along σ_0, loop geometry):

$$X(\sigma^0,\sigma^1+2\pi) = X(\sigma^0,\sigma^1) + wL$$
$$X(\sigma^0+2\pi\tau_2,\sigma^1+2\pi\tau_1) = X(\sigma^0,\sigma^1) + mL\,.$$

The interface sector

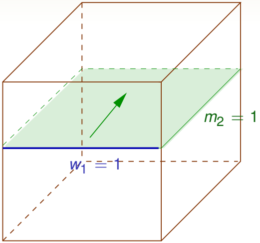
- The partition function includes $Z^{(d)}(q, \bar{q})$, the product of partition functions for the d compact bosons X^M $\rightarrow$ contains the sum over windings w^M and discrete momenta n^M

- We can select the topological sector corresponding to an interface in the x^1, x^2 plane
  - considering a string winding once in the x^1 direction:

$$w_1 = 1\ , \quad w_2 = w_3 = \ldots = w_d = 0$$

  - Poisson resumming over $n^2, \ldots, n^d$ and then choosing

$$m_2 = 1\ , \quad m_3 = m_4 = \ldots = m_d = 0$$



The interface partition function

- The expression for the partition function of the interface in the first-order, covariant, bosonic string theory, is thus

$$\mathcal{Z} = \prod_{i=2}^{d} (\sigma L_i) \sum_{N,\tilde{N}=0}^{\infty} \sum_{n_1 \in \mathbb{Z}} c_N c_{\tilde{N}} \int_{-\infty}^{\infty} d\tau_1 \mathrm{e}^{2\pi \mathrm{i}(N-\tilde{N}+n_1)} \int_0^{\infty} \frac{d\tau_2}{(\tau_2)^{\frac{d+1}{2}}}$$
$$\times \exp\left\{ -\tau_2 \left[\frac{\sigma L_1^2}{2} + \frac{2\pi^2 n_1^2}{\sigma L_1^2} + 2\pi(k + k' - \frac{d-2}{12}) \right] - \frac{1}{\tau_2} \left[\frac{\sigma L_2^2}{2} \right] \right\}$$

The result

- The integration over the parameters τ_1, τ_2 of the world-sheet torus can be performed.
- The final result depends only on the geometry of the target space, in particular on the area $A = L_1 L_2$ and the modulus $u = L_2/L_1$ of the interface plane: Back

$$\mathcal{Z} = 2\prod_{i=2}^{d}(\sigma L_i)\sum_{m=0}^{\infty}\sum_{k=0}^{m} c_k c_{m-k}\left(\frac{X}{u}\right)^{\frac{d-1}{2}} K_{\frac{d-1}{2}}(\sigma A X)$$

with

$$X = \sqrt{1 + \frac{4\pi u}{\sigma A}(m - \frac{d-2}{12}) + \frac{4\pi u^2 (2k-m)^2}{\sigma^2 A^2}}$$

- This is the expression that should resum the loop expansion of the functional integral

Check of the result (and new findings)

- Expanding in powers of $1/(\sigma A)$ we get

$$Z \propto \frac{\mathrm{e}^{-\sigma A}}{\eta^{2d-4}(\mathrm{i}u)} \cdot \left\{1 + \frac{(d-2)^2}{2\sigma A}\left[\frac{\pi^2}{36}u^2 E_2^2(\mathrm{i}u) - \frac{\pi}{6}uE_2(\mathrm{i}u) + c_d\right] + \dots\right\}$$

- Not too difficult to go to higher loops. In particular, we have worked out the 3-rd loop
  - New numerical simulations [Hasembush et al., to appear] are precise enough to be sensible to the 3-rd order corrections and seem to match our prediction.
  - If confirmed, this means that NG would still be a good model for the sizes considered in such simulations
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  - Two-loop correction: agrees with Dietz-Filk!

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- Any “naive” treatment of bosonic string in $d \neq 26$ suffers from the breaking of conformal invariance (heavily used to solve the model) at the quantum level. This applies to the 1st order treatment we used as well.
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- The covariant quantization in 1st order formalism of the NG action is a convenient way to derive partition functions of the string with different b.c.s, related to different LGT observables
  - It reproduces the partition function based on Arvis spectrum for the Polyakov loop correlator case $\sim$ D0-brane interaction with compact time
  - It yields the partition function for the interfaces $\sim$ appropriate sector of one loop closed strings
- Various developments are possible
  - The most pressing task:
    - ⋆ Finish the paper about the interface spectrum!
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