(D)-instanton effects in magnetized brane worlds

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CONSTITUENTS, FUNDAMENTAL FORCES AND SYMMETRIES ...
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Disclaimer

This talk is mostly based on

- M. Billo, M. Frau, I. Pesando, P. Di Vecchia, A. Lerda and R. Marotta, “Instanton effects in N=1 brane models and the Kahler metric of twisted matter,” arXiv:0709.0245 [hep-th].

It also uses a bit

- M. Billo, M. Frau, I. Pesando, P. Di Vecchia, A. Lerda and R. Marotta, “Instantons in N=2 magnetized D-brane worlds,” arXiv:0708.3806 [hep-th].

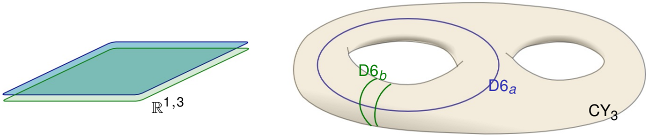
and, of course, builds over a vast literature. The few references scattered on the slides are by no means intended to be exhaustive. I apologize for the many relevant ones which will be missing.

Plan of the talk

Introduction

Wrapped brane scenarios

- Type IIB: magnetized D9 branes
- Type IIA (T-dual): intersecting D6 (easier to visualize)



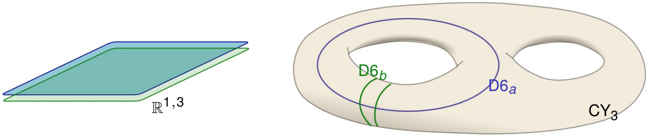
Supersymmetric gauge theories on $\mathbf{R}^{1,3}$ with chiral matter and interesting phenomenology

[recall Lüst lectures]

- families from multiple intersections, tuning different coupling constants, ...

Wrapped brane scenarios

- Type IIB: magnetized D9 branes
- Type IIA (T-dual): intersecting D6 (easier to visualize)

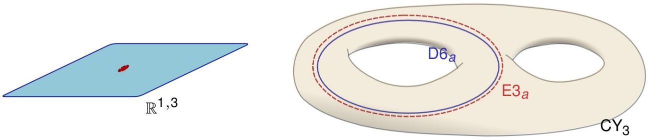


- low energy described by SUGRA with vector and matter multiplets
- can be derived directly from string amplitudes (with different field normaliz.s)
- novel stringy effects (pert. and non-pert.) in the eff. action?

Euclidean branes and instantons

Ordinary instantons

E3 branes wrapped on the same cycle as some D6 branes are point-like in $\mathbf{R}^{1,3}$ and correspond to instantonic config.s of the gauge theory on the D6



Analogous to the D3/D(-1) system:

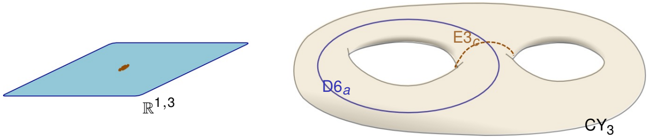
- ADHM from strings attached to the instantonic branes
  Witten, 1995; Douglas, 1995-1996; ...
- non-trivial instanton profile of the gauge field
  Billo et al, 2001

N.B. In type IIB, use D9/E5 branes

Euclidean branes and instantons

Exotic instantons

E3 branes wrapped differently from the D6 branes are still point-like in $\mathbf{R}^{1,3}$ but do not correspond to ordinary instantons config.s.



Still they can,in certain cases, give important non-pert, stringy contributions to the effective action, .e.g. Majorana masses for neutrinos, moduli stabilizing terms, . . .

Blumenhagen et al, 2006; Ibanez and Uranga, 2006; ...

- Potentially crucial for string phenomenology

Perspective of this work

Clarify some aspects of the "stringy instanton calculus", i.e., of computing the contributions of Euclidean branes

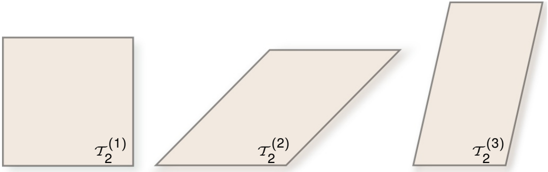
- Focus on ordinary instantons, but should be useful for exotic instantons as well
- Choose a toroidal compactification where string theory is calculable.
- Realize (locally) $\mathcal{N} = 1$ gauge SQCD on a system of D9-branes and discuss contributions of E5 branes to the superpotential
- Analyze the rôle of annuli bounded by E5 and D9 branes in giving these terms suitable holomorphicity properties

The set-up

The background geometry

Internal space:

$$\frac{\mathcal{T}_2^{(1)} \times \mathcal{T}_2^{(2)} \times \mathcal{T}_2^{(3)}}{\mathbb{Z}_2 \times \mathbb{Z}_2}$$



The Kähler param.s and complex structures determine the string frame metric and the B field.

- String fields: $X^M \to (X^\mu, Z^i)$ and $\psi^M \to (\psi^\mu, \Psi^i)$, with

$$Z^i = \sqrt{\frac{T_2^{(i)}}{2U_2^{(i)}}}(X^{2i+2} + U^{(i)} X^{2i+3})$$

- Spin fields: $S^{\dot{A}} \to (S_\alpha S_{---}, S_\alpha S_{-++}, \ldots, S^{\dot\alpha} S^{+++}, \ldots)$

The background geometry

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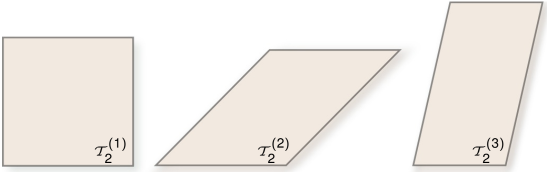


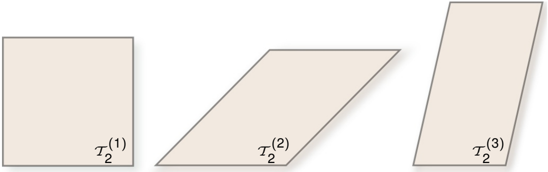
- Action of the orbifold group elements:

$$\begin{aligned}
h_1 &: \ (Z^1, Z^2, Z^3) \to (Z^1, -Z^2, -Z^3) \ , \\
h_2 &: \ (Z^1, Z^2, Z^3) \to (-Z^1, Z^2, -Z^3) \ , \\
h_3 &: \ (Z^1, Z^2, Z^3) \to (-Z^1, -Z^2, Z^3) \ ,
\end{aligned}$$

The background geometry

Internal space:

$$\frac{\mathcal{T}_2^{(1)} \times \mathcal{T}_2^{(2)} \times \mathcal{T}_2^{(3)}}{\mathbb{Z}_2 \times \mathbb{Z}_2}$$



- Supergravity basis: $s, t^{(i)}, u^{(i)}$, with Back

Lüst et al, 2004; ...

$$\mathrm{Im}(s) \equiv s_2 = \frac{1}{4\pi}\,\mathrm{e}^{-\phi_{10}}\; T_2^{(1)} T_2^{(2)} T_2^{(3)} \;,$$

$$\mathrm{Im}(t^{(i)}) \equiv t_2^{(i)} = \mathrm{e}^{-\phi_{10}}\, T_2^{(i)} \;, \quad u^{(i)} = u_1^{(i)} + \mathrm{i}\, u_2^{(i)} = U^{(i)} \;,$$

- $\mathcal{N} = 1$ bulk Kähler potential:

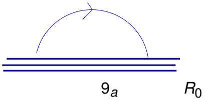
Antoniadis et al, 1996

$$K = -\log(s_2) - \sum_{i=1} \log(t_2^{(i)}) - \sum_{i=1} \log(u_2^{(i)})$$

$\mathcal{N} = 1$ from magnetized branes

The gauge sector

Place a stack of N_a fractional D9 branes ("color branes" $9a$).



- Massless spectrum of $9a/9a$ strings gives rise, in $\mathbf{R}^{1,3}$, to the $\mathcal{N} = 1$ vector multiplets for the gauge group $\mathrm{U}(N_a)$
- The gauge coupling constant is given by

$$\frac{1}{g_a^2} = \frac{1}{4\pi}\,\mathrm{e}^{-\phi_{10}}\,T_2^{(1)}\,T_2^{(2)}\,T_2^{(3)} = s_2$$

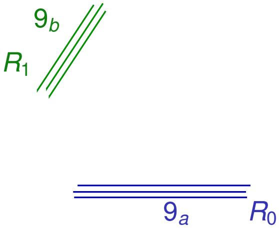
$\mathcal{N} = 1$ from magnetized branes

Adding flavors

Add D9-branes (“flavor branes” $9b$)
with quantized magnetic fluxes

$$f_b^{(i)} = \frac{m_b^{(i)}}{n_b^{(i)}}$$

and in a different orbifold rep.



- (Bulk) susy requires $\nu_b^{(1)} - \nu_b^{(2)} - \nu_b^{(3)} = 0$, where

$$f_b^{(i)} / T_2^{(i)} = \tan \pi \nu_b^{(i)} \quad \text{with} \quad 0 \leq \nu_b^{(i)} < 1 \ ,$$

$\mathcal{N} = 1$ from magnetized branes

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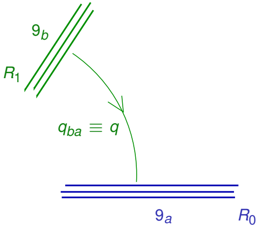

- $9a/9b$ strings are twisted by the relative angles ▸ Back

$$\nu_{ba}^{(i)} = \nu_b^{(i)} - \nu_a^{(i)}$$

- If $\nu_{ba}^{(1)} - \nu_{ba}^{(2)} - \nu_{ba}^{(3)} = 0$, this sector is supersymmetric: massless modes fill up a chiral multiplet q_{ba} in the anti-fundamental rep $\bar{N}_a$ of the color group

$\mathcal{N} = 1$ from magnetized branes

Add D9-branes ("flavor branes" $9b$) with quantized magnetic fluxes

$$f_b^{(i)} = \frac{m_b^{(i)}}{n_b^{(i)}}$$

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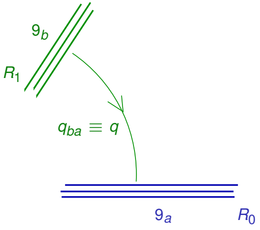
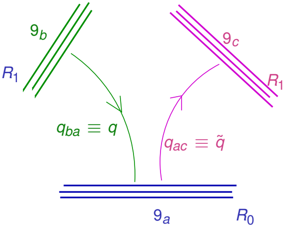


- The degeneracy of this chiral multiplet is $N_b|I_{ab}|$, where I_{ab} is the # of Landau levels for the (a, b) "intersection"

$$I_{ab} = \prod_{i=1} \left(m_a^{(i)} n_b^{(i)} - m_b^{(i)} n_a^{(i)}\right)$$

Introduce a third stack of $9c$ branes such that we get a chiral mult. q_{ac} in the fundamental rep N_a and that

$$N_b|I_{ab}| = N_c|I_{ac}| \equiv N_F$$



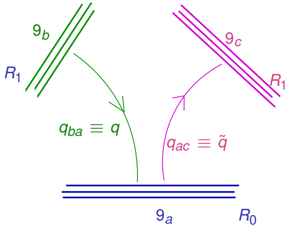
- This gives a (local) realization of $\mathcal{N} = 1$ SQCD: same number N_F of fundamental and anti-fundamental chiral multiplets, resp. denoted by q_f and $\tilde{q}^f$

$\mathcal{N} = 1$ from magnetized branes

Engineering $\mathcal{N} = 1$ SQCD

Introduce a third stack of $9c$ branes such that we get a chiral mult. q_{ac} in the fundamental rep N_a and that

$$N_b |I_{ab}| = N_c |I_{ac}| \equiv N_F$$



Kinetic terms of chiral mult. scalars from disk amplitudes

$$\sum_{f=1}^{N_F} \left\{ D_\mu q^{\dagger f} \, D^\mu q_f + D_\mu \tilde{q}^f \, D^\mu \tilde{q}_f^\dagger \right\}$$

Sugra Lagrangian: different field normaliz. s

$$\sum_{f=1}^{N_F} \left\{ K_Q \, D_\mu Q^{\dagger f} \, D^\mu Q_f + K_{\tilde{Q}} \, D_\mu \tilde{Q}^f \, D^\mu \tilde{Q}_f^\dagger \right\}$$

- Related via the Kähler metrics: $q = \sqrt{K_Q}\, Q \,, \; \tilde{q} = \sqrt{K_{\tilde{Q}}}\, \tilde{Q}$

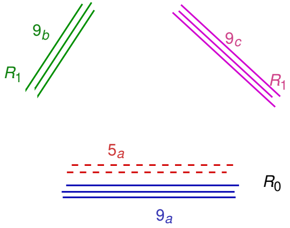
Back

Non-perturbative sectors from $E5$

Adding "ordinary" instantons

Add a stack of k E5 branes whose internal part coincides with the D9a:

- ordinary instantons for the D9a gauge theory
- would be exotic for the D9b, c gauge theories



- New types of open strings: $E5_a/E5_a$ (neutral sector), $D9_a/E5_a$ (charged sector), $D9_b/E5_a$ or $E5_a/D9_c$ (flavored sectors, twisted)
- These states carry no momentum in space-time: moduli, not fields. [Collective name: $\mathcal{M}_k$]
- charged or neutral moduli can have KK momentum

Non-perturbative sectors from $E5$

The spectrum of moduli

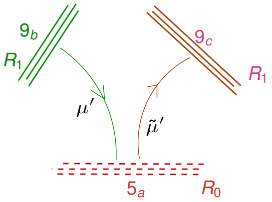
Sector		ADHM	Meaning	Chan-Paton	Dimension
$5_a/5_a$	NS	a_μ	centers	adj. $\mathrm{U}(k)$	(length)
		D_c	Lagrange mult.	$\vdots$	$(\text{length})^{-2}$
	R	M^α	partners	$\vdots$	$(\text{length})^{\frac{1}{2}}$
		$\lambda_{\dot\alpha}$	Lagrange mult.	$\vdots$	$(\text{length})^{-\frac{3}{2}}$
$9_a/5_a$	NS	$w_{\dot\alpha}$	sizes	$N_a \times \overline{k}$	(length)
$5_a/9_a$		$\bar{w}_{\dot\alpha}$	$\vdots$	$k \times \overline{N}_a$	$\vdots$
$9_a/5_a$	R	μ	partners	$N_a \times \overline{k}$	$(\text{length})^{\frac{1}{2}}$
$5_a/9_a$		$\bar{\mu}$	$\vdots$	$k \times \overline{N}_a$	$\vdots$
$9_b/5_a$	R	μ'	flavored	$N_F \times \overline{k}$	$(\text{length})^{\frac{1}{2}}$
$5_a/9_c$		$\tilde{\mu}'$	$\vdots$	$k \times \overline{N}_F$	$\vdots$

Non-perturbative sectors from $E5$

Some observations

- Among the neutral moduli we have the center of mass position x_0^μ and its fermionic partner θ^α (related to susy broken by the E5a): Back

$$a^\mu = x_0^\mu \, 1\!\!1_{k\times k} + y_c^\mu \, T^c \quad , \qquad M^\alpha = \theta^\alpha \, 1\!\!1_{k\times k} + \zeta_c^\alpha \, T^c \ ,$$



- In the flavored sectors only fermionic zero-modes:
  - μ'_f (D9_b/E5_a sector)
  - $\tilde{\mu}'^f$ (E5_a/D9_c sector)

The stringy instanton calculus

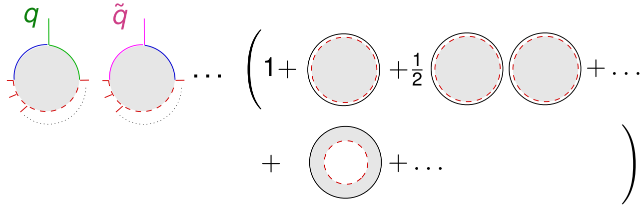
Instantonic correlators

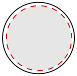
The stringy way

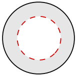
In presence of Euclidean branes, dominant contribution to correlators of gauge/matter fields from one-point functions.

Polchinski, 1994; Green and Gutperle, 1997-2000; Billo et al, 2002; Blumenhagen et al, 2006

E.g., a correlator of chiral fields $\langle q\tilde{q}\ldots\rangle$ is given by



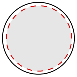
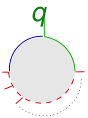
Disks:  $= -\frac{8\pi^2}{g_a^2}\,k + S_{\rm mod}(\mathcal{M}_k)$ (with moduli insertions)

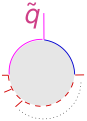
Annuli:  $\equiv \mathcal{A}_{5_a}$ (no moduli insert.s, otherwise suppressed)

The effective action

in an instantonic sector

The various instantonic correlators can be obtained by "shifting" the moduli action by terms dependent on the gauge/matter fields. In the case at hand,

$S_{\mathrm{mod}}(q, \tilde{q}; \mathcal{M}_k) =$  $+$ 

$+$ 

$+ \dots$

$$\begin{aligned}
&= \mathrm{tr}_k \Big\{ \mathrm{i} D_c \Big(\bar{w}_{\dot\alpha} (\tau^c)^{\dot\alpha}_{\ \dot\beta} w^{\dot\beta} + \mathrm{i} \bar{\eta}^c_{\mu\nu} \big[a^\mu, a^\nu \big] \Big) \\
&- \mathrm{i} \lambda^{\dot\alpha} \Big(\bar{\mu} w_{\dot\alpha} + \bar{w}_{\dot\alpha} \mu + \big[a_\mu, M^\alpha \big] \sigma^\mu_{\alpha\dot\alpha} \Big) \Big\} \\
&+ \mathrm{tr}_k \sum_f \Big\{ \bar{w}_{\dot\alpha} \big[q^{\dagger f} q_f + \tilde{q}^f \tilde{q}_f^\dagger \big] w^{\dot\alpha} - \frac{\mathrm{i}}{2} \bar{\mu}\, q^{\dagger f} \mu'_f + \frac{\mathrm{i}}{2} \tilde{\mu}'^f \tilde{q}_f^\dagger\, \mu \Big\} \,.
\end{aligned}$$

The effective action

in an instantonic sector

- There are other relevant diagrams involving the superpartners of q and $\tilde{q}$, related to the above by susy Ward identities. Complete result:

$$q(x_0)\,,\ \tilde{q}(x_0)\ \rightarrow\ q(x_0,\theta)\,,\ \tilde{q}(x_0,\theta)$$

  in $S_{\mathrm{mod}}(q,\tilde{q};\mathcal{M}_k)$.
- The moduli have to be integrated over

The instanton partition function

as an integral over moduli space

Summarizing, the effective action has the form (Higgs branch)

$$S_k = \mathcal{C}_k \, \mathrm{e}^{-\frac{8\pi^2}{g_a^2}\,k} \, \mathrm{e}^{\mathcal{A}'_{5a}} \int d\mathcal{M}_k \, \mathrm{e}^{-S_{\mathrm{mod}}(q,\tilde{q};\mathcal{M}_k)}$$

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- In $\mathcal{A}'_{5_a}$ the contribution of zero-modes running in the loop is suppressed because they're already explicitly integrated over

Blumenhagen et al, 2006

The instanton partition function

as an integral over moduli space

Summarizing, the effective action has the form (Higgs branch)

$$S_k = \mathcal{C}_k \, \mathrm{e}^{-\frac{8\pi^2}{g_a^2} k} \, \mathrm{e}^{\mathcal{A}'_{5a}} \int d\mathcal{M}_k \, \mathrm{e}^{-S_{\mathrm{mod}}(q,\tilde{q};\mathcal{M}_k)}$$

- $\mathcal{C}_k$ is a normalization factor, determined (up to numerical constants) counting the dimensions of the moduli $\mathcal{M}_k$:

$$\mathcal{C}_k = \left(\sqrt{\alpha'}\right)^{-(3N_a - N_F)k} \left(g_a\right)^{-2N_a k} \ .$$

The β-function coeff. b_1 appears, and one can write

$$\mathcal{C}_k \, \mathrm{e}^{-\frac{8\pi^2}{g_a^2} k} = \left(\Lambda_{\mathrm{PV}}^{b_1} \prod_f Z_f\right)^k$$

Instanton induced superpotential

In $S_{\rm mod}(q,\tilde{q};\mathcal{M}_k)$, the superspace coordinates x_0^μ and θ^α appear only through superfields $q(x_0,\theta),\tilde{q}(x_0,\theta),\ldots$ Recall

- We can separate x,θ from the other moduli $\widehat{\mathcal{M}}_k$ writing

$$S_k = \int d^4x_0\, d^2\theta\; W_k(q,\tilde{q})\;,$$

with the effective superpotential

$$W_k(q,\tilde{q}) = \Big(\Lambda_{\rm PV}^{b_1}\prod_{f=1}^{N_F} Z_f\Big)^k\, \mathrm{e}^{\mathcal{A}'_{5a}} \int d\widehat{\mathcal{M}}_k\; \mathrm{e}^{-S_{\rm mod}(q,\tilde{q};\widehat{\mathcal{M}}_k)}$$

Issues of holomorphicity

A superpotential is expected to be holomorphic. We found

$$W_k(q,\tilde{q}) = \left(\Lambda_{\mathrm{PV}}^{b_1}\prod_{f=1}^{N_F} Z_f\right)^k \mathrm{e}^{\mathcal{A}'_{5a}} \int d\widehat{\mathcal{M}}_k\, \mathrm{e}^{-S_{\mathrm{mod}}(q,\tilde{q};\widehat{\mathcal{M}}_k)}$$

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- $S_{\mathrm{mod}}(q,\tilde{q};\widehat{\mathcal{M}}_k)$ explicitly depends on $q^\dagger$ and $\tilde{q}^\dagger$. This dependence disappears upon integrating over $\widehat{\mathcal{M}}_k$ as a consequence of the cohomology properties of the integration measure.
- However, we have to re-express the result in terms of the SUGRA fields Q and $\tilde{Q}$ ▸ Recall

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- The Pauli-Villars scale Λ_{PV} has to be replaced by the holomorphic scale Λ_{hol}, obtained by integrating the Wilsonian β-function of the $\mathcal{N} = 1$ SQCD, with

Novikov et al, 1983; Dorey et al, 2002; ...

$$\Lambda_{\mathrm{hol}}^{b_1} = g_a^{2N_a} \Lambda_{\mathrm{PV}}^{b_1} \prod_f Z_f \,.$$

Issues of holomorphicity

A superpotential is expected to be holomorphic. We found

$$W_k(q,\tilde{q}) = \left(\Lambda_{\mathrm{PV}}^{b_1} \prod_{f=1}^{N_F} Z_f\right)^k \mathrm{e}^{\mathcal{A}'_{5_a}} \int d\widehat{\mathcal{M}}_k \, \mathrm{e}^{-S_{\mathrm{mod}}(q,\tilde{q};\widehat{\mathcal{M}}_k)}$$

- $\mathcal{A}'_{5_a}$ can introduce a non-holomorphic dependence on the complex and Kähler structure moduli of the compactification space. Back
- Our aim is to consider the interplay of all these observations. For this we need the explicit expression of the mixed annuli term $\mathcal{A}'_{5_a}$

The ADS/TVY superpotential

To be concrete, let's focus on the single instanton case, $k = 1$. In this case, the integral over the moduli can be carried out explicitly.

- Balancing the fermionic zero-modes requires $N_F = N_a - 1$
- The end result is

Dorey et al, 2002; Akerblom et al, 2006; Argurio et al, 2007

$$W_{k=1}(q, \tilde{q}) = \mathrm{e}^{\mathcal{A}'_{5a}} \left(\Lambda_{\mathrm{PV}}^{2N_a+1} \prod_{f=1}^{N_a-1} Z_f \right) \frac{1}{\det\left(\tilde{q} q\right)}$$

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Dorey et al, 2002; Akerblom et al, 2006; Argurio et al, 2007

$$W_{k=1}(q, \tilde{q}) = \mathrm{e}^{\mathcal{A}'_{5a}} \left(\Lambda_{\mathrm{PV}}^{2N_a+1} \prod_{f=1}^{N_a-1} Z_f \right) \frac{1}{\det\left(\tilde{q}q\right)}$$

- Same form as the ADS/TVY superpotential

Affleck et al, 1984; Taylor et al, 1983;

The ADS/TVY superpotential

To be concrete, let's focus on the single instanton case, $k = 1$. In this case, the integral over the moduli can be carried out explicitly.

- Balancing the fermionic zero-modes requires $N_F = N_a - 1$
- The end result is

Dorey et al, 2002; Akerblom et al, 2006; Argurio et al, 2007

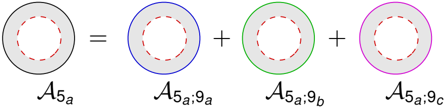
$$W_{k=1}(q, \tilde{q}) = \mathrm{e}^{\mathcal{A}'_{5a}} \left(\Lambda_{\mathrm{PV}}^{2N_a+1} \prod_{f=1}^{N_a-1} Z_f \right) \frac{1}{\det\left((\tilde{q}q\right)}$$

- We'll see how these factors conspire to give an holomorphic expression in the sugra variables Q and $\tilde{Q}$

Instanton annuli and threshold corrections

The mixed annuli

The amplitude $\mathcal{A}_{5_a}$ is a sum of cylinder amplitudes with a boundary on the E5a (both orientations)



The mixed annuli

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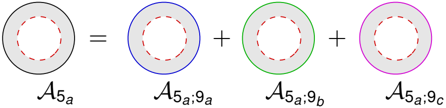
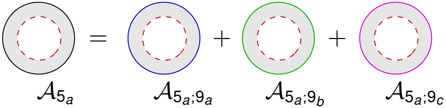


- Both UV and IR divergent. The UV divergences (IR in the closed string channel) cancel if tadpole cancellation assumed. Regulate the IR with a scale μ

The mixed annuli

The amplitude $\mathcal{A}_{5_a}$ is a sum of cylinder amplitudes with a boundary on the E5a (both orientations)



- There is a relation between these instantonic annuli and the running gauge coupling constant Back

Abel and Goodsell, 2006; Akerblom et al, 2006

$$\mathcal{A}_{5_a} = -\frac{8\pi^2 k}{g_a^2(\mu)}\bigg|_{1-\text{loop}} .$$

- Indeed, in susy theories, mixed annuli compute the running coupling by expanding around the instanton bkg

Billo et al, 2007

The explicit computation of the annuli confirms the relation of these annuli to the running coupling. Imposing the appropriate b.c.'s and GSO one starts from

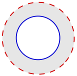
$$\int_0^\infty \frac{d\tau}{2\tau}\left[\mathrm{Tr}_{\mathrm{NS}}\left(P_{\mathrm{GSO}}\,P_{\mathrm{orb.}}\,q^{L_0}\right)-\mathrm{Tr}_{\mathrm{R}}\left(P_{\mathrm{GSO}}\,P_{\mathrm{orb.}}\,q^{L_0}\right)\right]\;.$$

- For $\mathcal{A}_{5_a;9_a}$, KK copies of zero-modes on internal tori $\mathcal{T}_2^{(i)}$ give a (non-holomorphic) dependence on the Kähler and complex moduli Lüst and Stieerger, 2003

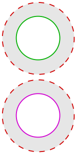
  .
- For $\mathcal{A}_{5_a;9_b}$ and $\mathcal{A}'_{5_a;9_c}$, the modes are twisted and the result depends from the angles $\nu_{ba}^{(i)}$ and $\nu_{ac}^{(i)}$ ▸ Recall

Expression of the annuli

Explicit result



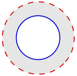
$$\mathcal{A}_{5_a;9_a} = -8\pi^2 k\left[\frac{3N_a}{16\pi^2}\log(\alpha'\mu^2) + \frac{N_a}{16\pi^2}\sum_i \log\left(U_2^{(i)} T_2^{(i)} (\eta(U^{(i)})^4\right)\right] ,$$



$$\mathcal{A}_{5_a;9_b} + \mathcal{A}_{5_a;9_c} = 8\pi^2 k\left(\frac{N_F}{16\pi^2}\log(\alpha'\mu^2) + \frac{N_F}{32\pi^2}\log\left(\mathbf{\Gamma}_{ba}\,\mathbf{\Gamma}_{ac}\right)\right) ,$$

Expression of the annuli

Explicit result



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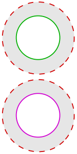
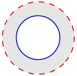


$$\mathcal{A}_{5_a;9_b} + \mathcal{A}_{5_a;9_c} = 8\pi^2 k\left(\frac{N_F}{16\pi^2}\log(\alpha'\mu^2) + \frac{N_F}{32\pi^2}\log\left(\boldsymbol{\Gamma}_{ba}\boldsymbol{\Gamma}_{ac}\right)\right),$$

- β-function coefficient of SQCD: $3N_a - N_F$

Expression of the annuli

Explicit result



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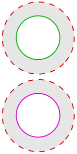

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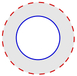
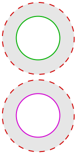
- Non-holomorphic threshold corrections

Expression of the annuli

Explicit result



$$\mathcal{A}_{5_a;9_a} = -8\pi^2 k \Bigg[\frac{3N_a}{16\pi^2} \log(\alpha' \mu^2) + \frac{N_a}{16\pi^2} \sum_i \log \left(U_2^{(i)} T_2^{(i)} (\eta(U^{(i)})^4 \right) \Bigg] ,$$



$$\mathcal{A}_{5_a;9_b} + \mathcal{A}_{5_a;9_c} = 8\pi^2 k \left(\frac{N_F}{16\pi^2} \log(\alpha' \mu^2) + \frac{N_F}{32\pi^2} \log \left(\mathbf{\Gamma}_{ba} \mathbf{\Gamma}_{ac} \right) \right) ,$$

- $\mathbf{\Gamma}_{ba} = \dfrac{\Gamma(1-\nu_{ba}^{(1)})}{\Gamma(\nu_{ba}^{(1)})} \dfrac{\Gamma(\nu_{ba}^{(2)})}{\Gamma(1-\nu_{ba}^{(2)})} \dfrac{\Gamma(\nu_{ba}^{(3)})}{\Gamma(1-\nu_{ba}^{(3)})}$

Lüst and Stieberger, 2003

Akerblom et al, 2007

Holomorphicity properties

The "primed" annuli

The instanton-induced correlators involve the primed part $\mathcal{A}'_{5a}$ of the mixed annuli ▸ Recall

- We must subtract the contrib. of the zero-modes running in the loop, which are responsible for the IR divergences
- To this aim, we use the natural UV cut-off of the low-energy theory, the Plack mass

$$M_P^2 = \frac{1}{\alpha'}\,\mathrm{e}^{-\phi_{10}}\,s_2$$

We write then

$$\mathcal{A}_{5a} = -k\frac{b_1}{2}\log\frac{\mu^2}{M_P^2} + \mathcal{A}'_{5a}$$

The "primed" annuli

The instanton-induced correlators involve the primed part $\mathcal{A}'_{5a}$ of the mixed annuli ▸ Recall

- With some algebra, and recalling the definition of the sugra variables, we find ▸ Recall ▸ Back

$$\mathcal{A}'_{5_a} = -N_a \sum_{i=1}^{3} \log\left(\eta(u^{(i)})^2\right) + N_a \log g_a^2 + \frac{N_a - N_F}{2} K + \frac{N_F}{2} \log(K_{ba} K_{ac})$$

with (similarly for K_{ac})

$$K_{ba} = (4\pi s_2)^{-\frac{1}{4}} \left(t_2^{(1)} t_2^{(2)} t_2^{(3)}\right)^{-\frac{1}{4}} \left(u_2^{(1)} u_2^{(2)} u_2^{(3)}\right)^{-\frac{1}{2}} \left(\boldsymbol{\Gamma}_{ba}\right)^{\frac{1}{2}}$$

Back to the ADS/VTY superpotential

Getting holomorphicity

We found (recall that $N_F = N_a - 1$ in this case)

$$W_{k=1}(q, \tilde{q}) = \mathrm{e}^{\mathcal{A}'_{5a}} \left(\Lambda_{\mathrm{PV}}^{2N_a+1} \prod_{f=1}^{N_a-1} Z_f\right) \frac{1}{\det\left(\tilde{q}q\right)}$$

- Insert the expression of the annuli

Back to the ADS/VTY superpotential

Getting holomorphicity

We found (recall that $N_F = N_a - 1$ in this case)

$$W_{k=1}(q, \tilde{q}) = \mathrm{e}^{K/2} \prod_{i=1}^{3} \left(\eta(u^{(i)})^{-2N_a}\right) \left(g_a^{2N_a} \Lambda_{\mathrm{PV}}^{2N_a+1} \prod_{f=1}^{N_a-1} Z_f\right)$$

$$\times \left(K_{ba} K_{ac}\right)^{\frac{N_a-1}{2}} \frac{1}{\det(\tilde{q} q)}$$

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$$\times \left(K_{ba}K_{ac}\right)^{\frac{N_a-1}{2}} \frac{1}{\det(\tilde{q}q)}$$

- Rewrite in terms of the holomorphic scale Λ_{hol}

Back to the ADS/VTY superpotential

Getting holomorphicity

We found (recall that $N_F = N_a - 1$ in this case)

$$W_{k=1}(q, \tilde{q}) = \mathrm{e}^{K/2} \prod_{i=1}^{3} \left(\eta(u^{(i)})^{-2N_a} \right) \Lambda_{\mathrm{hol}}^{2N_a+1}$$

$$\times \left(K_{ba} K_{ac}\right)^{\frac{N_a-1}{2}} \frac{1}{\det(\tilde{q} q)}$$

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$$\times \left(K_{ba}K_{ac}\right)^{\frac{N_a-1}{2}} \frac{1}{\det(\tilde{q}q)}$$

- Make an holomorphic redefinition of the scale Λ_{hol} into $\widehat{\Lambda}_{\mathrm{hol}}$

Back to the ADS/VTY superpotential

Getting holomorphicity

We found (recall that $N_F = N_a - 1$ in this case)

$$W_{k=1}(q, \tilde{q}) = \mathrm{e}^{K/2}\ \widehat{\Lambda}_{\text{hol}}^{2N_a+1}$$

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$$\times\ (K_{ba}K_{ac})^{\frac{N_a-1}{2}}\ \frac{1}{\det(\tilde{q}q)}$$

- Rescale the chiral multiplet to their sugra counterparts

Back to the ADS/VTY superpotential

Getting holomorphicity

We found (recall that $N_F = N_a - 1$ in this case)

$$W_{k=1}(Q, \tilde{Q}) = \mathrm{e}^{K/2}\ \widehat{\Lambda}_{\mathrm{hol}}^{2N_a+1}$$

$$\times \left(\frac{K_{ba}K_{ac}}{K_Q K_{\tilde{Q}}}\right)^{\frac{N_a-1}{2}} \frac{1}{\det(\tilde{Q}Q)}$$

Back to the ADS/VTY superpotential

Getting holomorphicity

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$$\times \left(\frac{K_{ba}K_{ac}}{K_Q K_{\tilde{Q}}}\right)^{\frac{N_a-1}{2}} \frac{1}{\det(\tilde{Q}Q)}$$

- If we assume that the Kähler metrics for the chiral multiplets are given by

$$K_Q = K_{ba}\ , \quad K_{\tilde{Q}} = K_{ac}$$

we finally obtain an expression which fits perfectly in the low energy lagrangian

Back to the ADS/VTY superpotential

Getting holomorphicity

$$W_{k=1}(Q, \tilde{Q}) = \mathrm{e}^{K/2} \; \widehat{\Lambda}_{\mathrm{hol}}^{2N_a+1} \frac{1}{\det(\tilde{Q}Q)}$$

Back to the ADS/VTY superpotential

Getting holomorphicity

$$W_{k=1}(Q, \tilde{Q}) = \mathrm{e}^{K/2}\ \widehat{\Lambda}_{\text{hol}}^{2N_a+1} \frac{1}{\det(\tilde{Q}Q)}$$

- A part from the prefactor $\mathrm{e}^{\frac{K}{2}}$, the rest is a holomorphic expression in the variables of the Wilsonian scheme.

The Kähler metric for twisted matter

The holomorphicity properties of the instanton-induced superpotential suggest that the Kähler metric of chiral multiplets Q arising from twisted $D9_a/D9_b$ strings is given by ▸ Back

$$K_Q = (4\pi\, s_2)^{-\frac{1}{4}} \left(t_2^{(1)} t_2^{(2)} t_2^{(3)}\right)^{-\frac{1}{4}} \left(u_2^{(1)} u_2^{(2)} u_2^{(3)}\right)^{-\frac{1}{2}} \left(\mathbf{\Gamma}_{ba}\right)^{\frac{1}{2}}$$

with

$$\mathbf{\Gamma}_{ba} = \frac{\Gamma(1-\nu_{ba}^{(1)})}{\Gamma(\nu_{ba}^{(1)})} \frac{\Gamma(\nu_{ba}^{(2)})}{\Gamma(1-\nu_{ba}^{(2)})} \frac{\Gamma(\nu_{ba}^{(3)})}{\Gamma(1-\nu_{ba}^{(3)})}$$

This is very interesting because:

- for twisted fields, the Kähler metric cannot be derived from compactification of DBI

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This is very interesting because:

- the part dependent on the twists, namely Γ_{ba}, is reproduced by a direct string computation

  Lüst et al, 2004; Bertolini et al, 2005

- the prefactors, depending on the geometric moduli, are more difficult to get directly: the present suggestion is welcome!

The Kähler metric for twisted matter

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with

$$\Gamma_{ba} = \frac{\Gamma(1-\nu_{ba}^{(1)})}{\Gamma(\nu_{ba}^{(1)})} \frac{\Gamma(\nu_{ba}^{(2)})}{\Gamma(1-\nu_{ba}^{(2)})} \frac{\Gamma(\nu_{ba}^{(3)})}{\Gamma(1-\nu_{ba}^{(3)})}$$

This is very interesting because:

- We have checked this expression against the known results for Yukawa couplings of magnetized branes: perfect consistency!

Cremades et al, 2004

More on holomorphicity

The perturbative side

We have seen the relation between the instanton annuli and the running gauge coupling ▸ Recall

- There is a general relation of the 1-loop corrections to the gauge coupling to the Wilsonian gauge coupling f

Dixon et al, 1991; Kaplunovsky and Louis, 1994-95; ...

$$\frac{1}{g^2(\mu)} = \frac{1}{8\pi^2}\left[\frac{b}{2}\log\frac{\mu^2}{M_P^2} - f - \frac{c}{2}K + T(G)\log\frac{1}{g^2} - \sum_r n_r T(r)\log K_r\right]$$

where (T_A = generators of the gauge group, n_r = # chiral mult. in rep. r)

$$T(r)\,\delta_{AB} = \mathrm{Tr}_r\big(T_A T_B\big) \quad , \quad T(G) = T(\mathrm{adj})$$

$$b = 3\,T(G) - \sum_r n_r\,T(r) \quad , \quad c = T(G) - \sum_r n_r\,T(r)\,,$$

More on holomorphicity

The perturbative side

We have seen the relation between the instanton annuli and the running gauge coupling ▸ Recall

- There is a general relation of the 1-loop coupling, given by ordinary annuli, to the 1-loop corrections to the Wilsonian gauge coupling f

Dixon et al, 1991; Kaplunovski and Louis,

$$\frac{1}{g^2(\mu)} = \frac{1}{8\pi^2}\left[\frac{b}{2}\log\frac{\mu^2}{M_P^2} - f - \frac{c}{2}K + T(G)\log\frac{1}{g^2} - \sum_r n_r T(r)\log K_r\right]$$

- This gives an interpretation for the non-holomorphic terms appearing in the running coupling based on perturbative considerations.

More on holomorphicity

Consistency

In the case of SQCD, one has N_F chiral multiplets in the N_a and in the $\bar{N}_a$ rep. Matching the DKL formula with the 1-loop result for $1/g_A^2(\mu)$ ▸ Recall one identifies the Kähler metrics K_Q and $K_{\tilde{Q}}$ of the chiral multiplets.

- This determination, based on the holomorphicity of perturbative contributions to the eff. action, is in full agreement with the expression given before ▸ Recall, derived from the holomorphicity of instanton contributions .

Remarks and conclusions

- Also in $\mathcal{N} = 2$ toroidal models the instanton-induced superpotential is in fact holomorphic in the appropriate sugra variables if one includes the mixed annuli in the stringy instanton calculus

Akerblom et al, 2007; Billo et al, 2007

- W.r.t. to the “color” $D9_a$ branes, the $E5_a$ branes are ordinary instantons. For the gauge theories on the $D9_b$ or the $D9_c$, they would be exotic (less clear from the field theory viewpoint)
- The study of the mixed annuli and their relatio to holomorphicity can be relevant for exotic, new stringy effects as well.