Aspects of the stringy instanton calculus (II)

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Plan of the talk

1. Brane interactions with closed string fluxes
2. An $\mathcal{N} = 1$ local example example
3. Flux effects and stringy instantons

Brane interactions with closed string fluxes

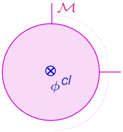
Flexibility of the string view-point

- Realizing instantonic sectors in a stringy way makes it natural to investigate modifications or extensions of non-perturbative contributions in field theory models admitting a string description.
  - Take into account the interactions with closed strings, i.e. with bulk fields (loosely speaking, with the "gravitational" sector)
  - In brane-world models, many different (wrapped) Euclidean branes appear as instantonic objects in the 4d gauge/matter theory. Different non-perturbative constributions may arise, some corresponding to ordinary field-theory instantons, some not

Interactions with closed strings

- Include the effect of closed strings , similarly to what A. Lerda described before for open strings
  - Extra term in the moduli action from disk diagrams coupling moduli to bulk fields

$$S_{mod}(\mathcal{M}) \to S_{mod}(\mathcal{M}, \phi^{cl})$$



  - Integrating over the moduli yields non-perturbative corrections to effective action for the bulk fields

$$\int d\mathcal{M} e^{-S_{mod}(\mathcal{M},\phi^{cl})} = S^{n.p.}_{eff}(\phi^{cl})$$

Among the first lines pursued to find D-instanton induced interactions in the gravitational sector (ex R^4 term in eff. action of type IIB) Green and Gutperle 9701093, ...

Closed string backgrounds

- We may also consider turning on a closed string background
- Mixed open/closed diagrams are now interpreted as describing a deformation of the of the gauge theory action and of the moduli action in the background
  - We can study the non-perturbative sectors of the deformed theory integrating over the moduli with the deformed moduli action
- Well known example: non-commutative field theories from open strings in $B^{\mu\nu}$ background

  Chu and Ho, 9812219; Seiberg and Witten, 9908142; ...

  - D-instantons effects in such theories can be studied using the method outlined above

    Billo et al, 0511036; ...

RR backgrounds

- Also RR backgrounds can be studied (despite the fact that the σ-model description is lacking), by "perturbatively" inserting RR vertices.
  - In the field theory limit $\alpha' \to 0$, only diagrams with few insertions matter
  - Insert the background RR value in the corresponding amplitudes
- Examples of effects of a constant RR background where the method applies:
  - Non-anti-commutative (NAC) field theories

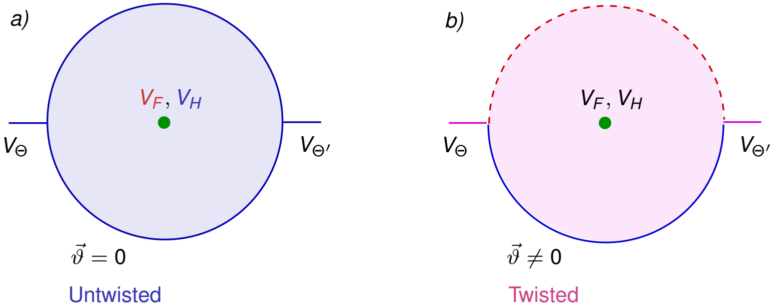
    de Boer et al, 0302078; Ooguri and Vafa, 0302109; . . . ; Billo et al, 0402160; . . .
  - Nekrasov's ϵ-deformations of the instanton moduli space in $\mathcal{N} = 2$ gauge theories

    Nekrasov, 0206161; . . . ; Billo et al, 0606013; . . .

Flux interactions on branes

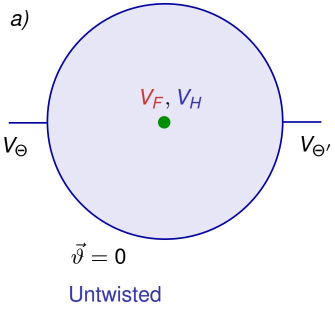
- As a concrete example (useful for the following) let us describe the coupling of open string fermions living on some branes to closed string fluxes, from both the NSNS and RR sector
- It can be described in a general, 10d set-up.
- Here we will be interested in the effect of fluxes on the ordinary and exotic instantons in an $\mathcal{N} = 1$ brane-engineered gauge theory

The disk diagrams



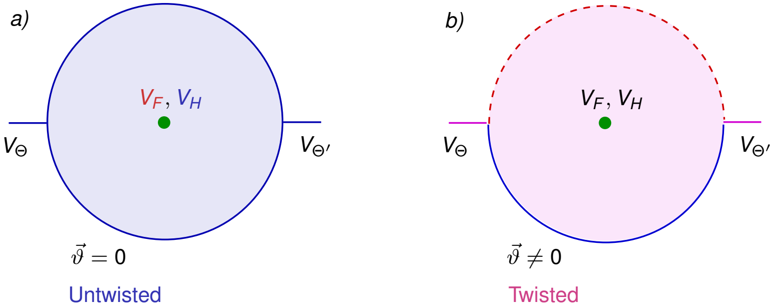
- Θ and Θ' are massless fermions from the R sector of open strings

The disk diagrams



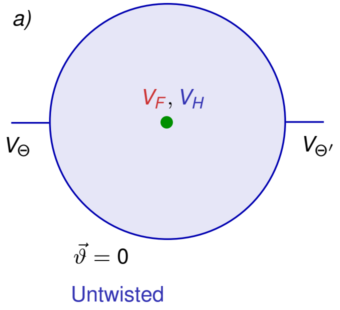
- F (H) is a closed string vertex corresponding to a RR (NS-NS) field strength

The disk diagrams



- We can treat open string with generic b.c., including both the twisted and untwisted case

The disk diagrams



- We work in a flat geometry (non-compact, toroidal or orbifolded directions)

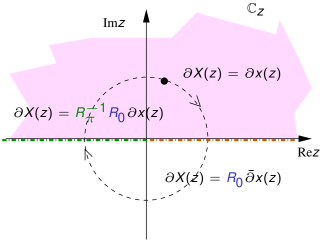
Boundary conditions

- The disk amplitude depends on the $\leftrightarrow$ boundary conditions, imposed by the brane, e.g.

$$\bar\partial x^M\Big|_{\sigma=0,\pi} = (R_\sigma)^M{}_N\,\partial x^N\Big|_{\sigma=0,\pi}\,, \qquad R_\sigma = (1-\mathcal{F}_\sigma)^{-1}(1+\mathcal{F}_\sigma)$$

- For a string stretching between different branes, we get twisted fields:

$$X^M(e^{2\pi i}z) = R^M_N\,X^N(z)\,,\quad R = R_\pi^{-1}R_0$$



- In a suitable complex basis Z^I, $I = 1,\ldots 5$,

$$R = diag\left(e^{2\pi i\vartheta^1}, e^{-2\pi i\vartheta^1},\ldots, e^{2\pi i\vartheta^5}, e^{-2\pi i\vartheta^5}\right)$$

$$\partial Z^I(e^{2\pi i}z) = e^{2\pi i\vartheta^I}\,\partial Z^I(z)$$

Specializing the result

The resulting general form of the amplitude ▸ Details Billo et al, 0807.1666 can be applied to many different situations and generate various types of flux interactions.

- We will concentrate here on toroidal (orbifold) compactifications of IIB to 4d and consider the interactions induced by constant internal fluxes F_3 and H on
  - space-filling branes. In this case we consider untwisted strings
  - instantonic branes. We consider untwisted strings (neutral moduli) but also also twisted ($\theta^4, 5 = 1/2$) ND strings forcharged moduli.

Untwisted case

- The general result reduces to ($m, n \dots$ are internal indices)

$$\mathcal{A} \equiv \mathcal{A}_F + \mathcal{A}_H \sim i\Theta\Gamma^{mnp}\Theta\, T_{mnp}$$

with

$$T_{mnp} = (F\mathcal{R}_0)_{mnp} + \frac{1}{g_s}[(\partial B R_0)_{mnp} + (\partial B R_0)_{npm} + (\partial B R_0)_{pmn}]$$

- The factor of g_s is due to the relative normalizazion of RR and NS-NS vertices to account for their 10d kinetic terms in the Einstein frame

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- For unmagnetized branes,the reflection matrix R_0 is simply +1 for NN and -1 for DD directions
- The spinorial reflection is simply $\mathcal{R}_0 = \prod_{\hat{m} \in DD} \Gamma^{\hat{m}}$

4d notation

- Decomposing the 10d spinors into 4+6-dimensional parts: $\Theta_{\mathcal{A}} \;\longrightarrow\; (\Theta^{\alpha A}, \Theta_{\dot{\alpha} A})$, the flux coupling in 4d notation reads

$$-i\,\Theta^{\alpha A}\Theta_{\alpha}{}^{B}\big(\overline{\Sigma}^{mnp}\big)_{AB}\,T^{\mathrm{IASD}}_{mnp} - i\,\Theta_{\dot{\alpha}A}\Theta^{\dot{\alpha}}{}_{B}\big(\Sigma^{mnp}\big)^{AB}\,T^{\mathrm{ISD}}_{mnp}$$

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  - ISD and IASD tensors are defined as follows:

$$T^{\mathrm{ISD}}_{mnp} = \frac{1}{2}\left(T - i *_6 T\right)_{mnp} \quad , \quad T^{\mathrm{IASD}}_{mnp} = \frac{1}{2}\left(T + i *_6 T\right)_{mnp} ,$$

  - In a complex basis,

$$T^{\mathrm{ISD}} \rightarrow T_{(0,3)} \oplus T_{(2,1)_{\mathrm{P}}} \oplus T_{(1,2)_{\mathrm{NP}}}$$
$$T^{\mathrm{IASD}} \rightarrow T_{(3,0)} \oplus T_{(1,2)_{\mathrm{P}}} \oplus T_{(2,1)_{\mathrm{NP}}}$$

where (N)P stands for (non)-primitive

Coupling on unmagnetized branes

- In this case the fermions $\Theta^{\alpha A}$ are fields (gauginos, ...)
- The coupling T_{mnp} depends on the type of brane: Back

	0-3	4	5	6	7	8	9	T_{mnp}
D3	–	×	×	×	×	×	×	$(*_6 F)_{mnp} - \frac{1}{g_s} H_{mnp}$
D5	–	–	–	×	×	×	×	$\frac{1}{g_s} H_{\hat{m}\hat{n}p}$; $-\frac{1}{2} F_{\hat{m}}{}^{qr} \epsilon_{qrnp}$; $-\frac{1}{g_s} H_{mnp}$
D7	–	–	–	–	–	×	×	$F_{\hat{m}\hat{n}}{}^{q} \epsilon_{qp} + \frac{1}{g_s} H_{\hat{m}\hat{n}p}$
D9	–	–	–	–	–	–	–	$F_{\hat{m}\hat{n}\hat{p}}$

- We neglected the H-components that would be projected out by the appropriate orientifold projections

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- Can be extended to magnetized branes, by taking general reflection matrices R_0, $\mathcal{R}_0$

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D9	–	–	–	–	–	–	–	$F_{\hat{m}\hat{n}\hat{p}}$

- F and H do not appear of the same footing.

Coupling on instantonic branes

- In this case the fermions $\Theta^{\alpha A}$ are neutral moduli
- The coupling T_{mnp} depends on the type of brane: Back

	0-3	4	5	6	7	8	9	T_{mnp}
D(-1)	×	×	×	×	×	×	×	$-iF_{mnp} - \frac{1}{g_s} H_{mnp}$
E1	×	—	—	×	×	×	×	$\frac{1}{g_s} H_{\hat{m}\hat{n}p}$; $-i\epsilon_{\hat{m}\hat{q}} F^{\hat{p}}{}_{np}$; $-\frac{1}{g_s} H_{mnp}$
E3	×	—	—	—	—	×	×	$-\frac{i}{2}\epsilon_{\hat{m}\hat{n}\hat{r}\hat{s}} F^{\hat{r}\hat{s}}{}_{p} + \frac{1}{g_s} H_{\hat{m}\hat{n}p}$
E5	×	—	—	—	—	—	—	$i(*_6 F)_{\hat{m}\hat{n}\hat{p}}$

- We will use this result to discuss the influence of fluxes on the stringy instanton calculus

An $\mathcal{N} = 1$ example

A simple laboratory: $\mathbb{C}^3/(\mathbb{Z}_2 \times \mathbb{Z}_2)$

To analyze the flux effects on the non-perturbative effective action of brane-world gauge theories, it is useful to focalize on a simple (yet non-trivial) example

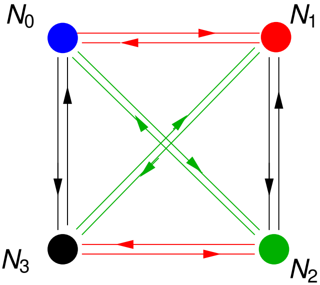
- We consider a local model of an $\mathcal{N} = 1$ compactification given by the orbifold $\mathbb{C}^3/(\mathbb{Z}_2 \times \mathbb{Z}_2)$, generated by

$$
\begin{aligned}
h_1 &: \ (Z^1, Z^2, Z^3) \to (Z^1, -Z^2, -Z^3) \\
h_2 &: \ (Z^1, Z^2, Z^3) \to (-Z^1, Z^2, -Z^3)
\end{aligned}
$$

- The properties of the 4 irreducible representations, and the transformations of the string fields under this group are easily worked out ▸ Details

We consider fractional D3 branes transverse to the orbifold

- 4 types of fD3's: the CP indices of open string endpoints attached to fD3(A) transform in the orbifold irrep R_A
- Given a system of $\{N_A\}$ fD3's, the open string massless spectrum is encoded in a quiver
  - Nodes $\leftrightarrow$ $U(N_A)$ $\mathcal{N} = 1$ vector multiplets
  - Arrows: bifundamental chiral multiplets



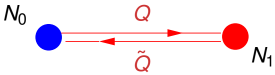
Different instantonic sectors

- 4 types of fD(-1)'s associated to the nodes of the quiver
- W.r.t. the $U(N_A)$ gauge theory on a given node,
  - the D(-1)'s occupying the same node A are found to correspond to ordinary gauge instantons
  - D(-1)'s on a node $B \neq A$ have a different spectrum of moduli, and correspond to "exotic" or "stringy" instantons
- Analogue in smooth compactifications (e.g. in the blown-up orbifold): for the gauge theory on a stack of branes wrapped on a cycle C_A,
  - ordinary instantons arise from Euclidean branes entirely wrapped on C_A
  - "exotic" ones from E-branes wrapped on $C_B \neq C_A$

A realization of SQCD

A system of N_0 (N_1) fD3's of type 0 (1) realizes SQCD

- $U(N_0) \times U(N_1)$ $\mathcal{N} = 1$ gauge theory
- Two chiral multiplets:



$$Q \in N_0 \times \bar{N}_1 \ , \quad \tilde{Q} \in \bar{N}_0 \times N_1$$

- The "quark" multiplets can be grouped into

$$\Phi = \begin{pmatrix} 0 & Q^u{}_f \\ \tilde{Q}^f{}_u & 0 \end{pmatrix}$$

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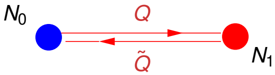

$$Q \in N_0 \times \bar{N}_1 \ , \quad \tilde{Q} \in \bar{N}_0 \times N_1$$

- The diagonal $U(1)$ factor is decoupled, the other $U(1)$ factor is IR free $\rightarrow$ we in fact have an $SU(N_0) \times SU(N_1)$ theory
- We focus on one the gauge factors, so we see a SQCD with

$$N_c = N_0 \ , \quad N_f = N_1$$

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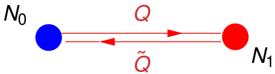


$$Q \in N_0 \times \bar{N}_1 \;, \quad \tilde{Q} \in \bar{N}_0 \times N_1$$

- The massless d.o.f. in the Higgs phase parametrize solutions to the D-flatness eq.s ▸ Back

$$Q^u_f Q^{\dagger f}_v = \tilde{Q}^{\dagger u}_f \tilde{Q}^f_v$$

“Ordinary” D-instantons

- Including k_0 fractional D(-1) of type 0 corresponds to work in the instanton # k_0 sector of the gauge theory

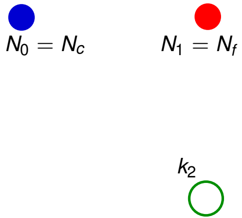
- In SQCD, the $k_0 = 1$ sector is responsible of
  - the ADS/VTY superpotential for $N_f = N_c - 1$
    Affleck et al, 1984; Taylor et al, 1983
  - Beasley-Witten F-terms for $N_f \geq N_c$
    Beasley and Witten, 0409149, 0512039
- In presence of fluxes, other effects (some of stringy nature) arise

Exotic D-instantons

D(-1)'s of type 2 or 3 give "exotic", a.k.a. "stringy" non-perturbative effects

- "Exotic" non-perturbative contributions have attracted much interest recently in brane-world constructions
- Could generate very interesting terms (neutrino masses ...)

  Blumenhagen et al 0609191; Ibanez and Uranga, 0609213; ... ;



- However, severe restrictions from integration over fermionic 0-modes: difficult to get non-vanishing results

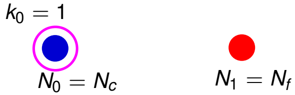
  Argurio et al, 0704.0262; Bianchi et al, 0704.0784; ...

- To this aim, fluxes might come to the rescue!

  Blumenhagen et al, 0708.0403; Petersson, 0711.1837;...

Ordinary instanton: spectrum

Let us focus on a single D(-1) of type 0 in the SQCD set-up



Ordinary instanton: spectrum

Let us focus on a single D(-1) of type 0 in the SQCD set-up

- Neutral moduli: $\{x^\mu, D_c, \theta^\alpha, \lambda_{\dot\alpha}\}$
  - x, θ: position of the instanton + superpartner
  - D_c ($c = 1, 2, 3$): auxiliary fields (see later)

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- Charged moduli: $\{w_{\dot\alpha u}, \mu_u\}$, $\{\bar{w}_{\dot\alpha}^u, \mu^u\}$ from the two orientations.
  - $w_{\dot\alpha}$ bosonic, μ fermionic: effect of ND b.c.'s.
  - u= color index

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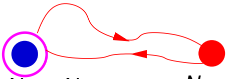

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  - $w_{\dot\alpha}$ bosonic, μ fermionic: effect of ND b.c.'s.
  - u= color index
- Flavored moduli: μ'_f, $\bar\mu'^f$ from the two orientations
  - Fermionic only! D(-1) of type 0, D3 of type 1 can be seen as branes wrapped on non-parallel (exceptional cycles): "exotic" configuration
  - f= flavor index

Ordinary instanton: action

The disks with moduli insertions yield the action

$$S_{mod} = \frac{D_c D^c}{2g_0^2} + iD_c(\bar{w}^u \tau^c w_u) + i\lambda \cdot (\bar{\mu}^u w_u + \bar{w}^u \mu_u)$$

The dimensions of the moduli are chosen as follows:

x_μ	D_c	θ^α	$\lambda_{\dot\alpha}$	$w_{\dot\alpha}$	μ	μ'
M^{-1}	M^2	$M^{-1/2}$	$M^{3/2}$	M^{-1}	$M^{-1/2}$	$M^{-1/2}$

- In the field theory limit $\alpha' \to 0$, D_c and $\lambda_{\dot\alpha}$ are Lagrange multiplier for the bosonic and fermionic constraints of the ADHM construction.
  - Indeed, $1/g_0^2 \propto (2\pi\alpha')^2/g_s$ goes to 0 for g_s fixed, i.e. fixed gauge coupling

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- x^μ, θ^α have the dimensions of supercoordinates
  - They do not enter in the pure moduli action

Ordinary instanton: action

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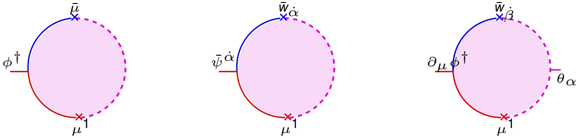
- The $w_{\dot\alpha u}$ are related to the size and orientation of the instanton: $\bar{w}^u \cdot w_u = \rho^2$ once the constraints are solved

Field-dependent terms

New diagrams with insertions of matter fields from D3(0)/D3(1) strings give extra terms in the moduli action ▸ Back



$$S_{mod} + \bar{w}^u_{\dot\alpha}\big(\phi^\dagger(x)\Phi(x,\theta) + \Phi(x,\theta)\phi^\dagger(x)\big)w^{\dot\alpha}_v$$
$$+ \bar\mu^u\phi^{\dagger f}_u(x)\mu'_f - \bar\mu'^f\phi^{\dagger u}_f(x)\mu_u + \bar{w}^{\dot\alpha u}\psi^{\dagger\dot\alpha}_u(x)\mu' - \bar\mu'\psi^{\dagger u}_{\dot\alpha}(x)w^{\dot\alpha}_u$$

- $\Phi^u{}_f$ is a chiral multiplet:

$$\Phi(x,\theta) = \phi(x) + \theta^\alpha\psi_\alpha(x) + \theta^2F(x)$$

  - The moduli x, θ enter in the moduli action only through this expansion

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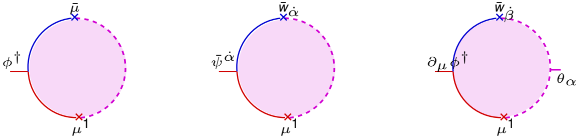
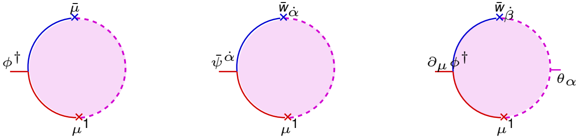


$$S_{mod} + \bar{w}^u_{\dot\alpha}\left(\phi^\dagger(x)\Phi(x,\theta) + \Phi(x,\theta)\phi^\dagger(x)\right) w^{\dot\alpha}_v$$
$$+ \bar{\mu}^u \phi^{\dagger f}_u(x)\mu'_f - \bar{\mu}'^f \phi^{\dagger u}_f(x)\mu_u + \bar{w}^{\dot\alpha u}\psi^{\dagger\dot\alpha}_u(x)\mu' - \bar{\mu}'\psi^{\dagger u}_{\dot\alpha}(x) w^{\dot\alpha}_u$$

- The moduli action is not holomorphic.
- The dependence on $\phi^\dagger(x) = \Phi^\dagger(x, \bar\theta = 0)$, is not extended (in the $\alpha' \to 0$ limit) to anti-chiral multiplets $\Phi^\dagger(x, \bar\theta)$,

Field-dependent terms

New diagrams with insertions of matter fields from D3(0)/D3(1) strings give extra terms in the moduli action ▸ Back



$$S_{mod} + \bar w^u_{\dot\alpha}\big(\phi^\dagger(x)\Phi(x,\theta) + \Phi(x,\theta)\phi^\dagger(x)\big)\, w^{\dot\alpha}_v$$
$$+\, \bar\mu^u \phi^{\dagger f}_u(x)\mu'_f - \bar\mu'^f\phi^{\dagger u}_f(x)\mu_u + \bar w^{\dot\alpha u}\psi^{\dagger\dot\alpha}_u(x)\mu' - \bar\mu'\psi^{\dagger u}_{\dot\alpha}(x) w^{\dot\alpha}_u$$

- These terms involve the "quarks", and can be rewritten in terms of $\bar D_{\dot\alpha}\Phi^\dagger(x,\bar\theta)\big|_{\bar\theta=0}$
  - Responsible for Beasley-Witten multifermion terms in the effective action (see later)

Blumenhagen et al, 0708.0403; Garcia-Extebarria, 0805.0713

Symmetries of the moduli action

- The gauge theory action and the moduli action are invariant under $U(1)^3 \subset SO(6)$ global symmetries surviving the orbifold projection.
- We can assign to the fields/moduli the following charges

	ϕ	ψ^α	$\mu, \bar{\mu}, \theta^\alpha$	$\mu', \bar{\mu}'$	$\lambda_{\dot{\alpha}}$
q	1	$-1/2$	$3/2$	-1	$-3/2$
q'	1	1	0	1	0
q''	1	1	0	1	0

The bosonic moduli $a^\mu, w_{\dot{\alpha}}, \bar{w}_{\dot{\alpha}}, D_c$ are invariant

- These symmetries powerfully constrain the form of the instantonic contributions to the effective action
- They can be extended to $\{k_A\}$ D-instantons in a generic quiver theory with $\{N_A\}$ D3-branes

Non-perturbative F-terms

Low energy effective action in the instanton sector:

$$S_{n.p.} = \int d^4x\, d^2\theta\; \mathrm{e}^{2\pi\tau_{YM}(M_s)} (M_s)^{3N_c - N_f} \int d\widehat{\mathcal{M}}\, \mathrm{e}^{-S_{mod}(\Phi,\Phi^\dagger)}$$

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- The pure disks and annuli attached to the D(-1) give the exponential of the classical instanton action with the 1-loop coupling τ_{YM} evaluated at $M_s = 1/\sqrt{\alpha'}$
- The dimensionality of $d\mathcal{M}$ implies the factor $M_s^{3N_c - N_f}$
- Together, these terms reconstruct the dynamical scale $\Lambda^{3N_c - N_f} = \Lambda^{b_1}$

Form of the F-term corrections

- Write

$$S_{n.p.} = \int d^4x\, d^2\theta\ W_{n.p.}\ , \qquad W_{n.p.} = \Lambda^{b_1} \int d\,\widehat{\mathcal{M}}\ \mathrm{e}^{-S_{mod}(\Phi,\Phi^\dagger)}$$

- Ansatz (due to the form of S_{mod})

$$W_{n.p.} \sim \Lambda^{b_1}\,(\Phi^\dagger)^n\,\Phi^m\left(\bar{D}_{\dot\alpha}\Phi^\dagger\,\bar{D}^{\dot\alpha}\Phi^\dagger\right)^p\Big|_{\bar\theta=0}$$

- Exploit the $U(1)^3$ symmetry requiring that

$$q, q', q''[W_{n.p.}] = q, q', q''[d\mathcal{M}]$$

  This fixes

$$p = -n = 1 - N_c + N_f\ , \qquad m = 1 - N_c - N_f\ .$$

Form of the F-term corrections

- Write

$$S_{n.p.} = \int d^4x\, d^2\theta \;W_{n.p.}\;, \qquad W_{n.p.} = \Lambda^{b_1} \int d\,\widehat{\mathcal{M}}\; \mathrm{e}^{-S_{mod}(\Phi,\Phi^\dagger)}$$

- The form of the induced interactions is thus

$$W_{n.p.} \sim \Lambda^{b_1} \left. \frac{\left(\bar{D}_{\dot\alpha}\Phi^\dagger\, \bar{D}^{\dot\alpha}\Phi^\dagger\right)^p}{(\Phi^\dagger)^p\, \Phi^{\,p+2N_c-2}} \right|_{\bar\theta=0}$$

for $p = 0, 1, \ldots$.

The ADS superpotential

- In the case $p = 0$, i.e. $N_f = N_c - 1$, the structure is $W_{n.p.} \sim \Lambda^{2N_f+3}\Phi^{-2N_f}$
- The integrals over the moduli can be done explicitly
- $W_{n.p.}$ should depend on low-energy fields only. We have to impose the D-flatness condition Recall
- By doing so, in the result of the integration over $d\mathcal{M}$ only the low-energy d.o.f. (meson fields, ...) appear
- We get the ADS superpotential

$$W(M) = \frac{\Lambda^{2N_f+3}}{det M}$$

where M is the meson superfield $(M)_f{}^{f'} = \tilde{Q}_f{}^u Q_u{}^{f'}$

BW multifermion terms

- For $p > 0$, i.e. $N_f \geq N_c$, one gets the multifermion instanton interactions in SQCD of BW Beasley and Witten, 0409149
- For $p = 1$ and $N_f = N_c = 2$, the form of the interaction is

$$W_{n.p.} \sim \Lambda^4 \left. \frac{\bar{D}_{\dot\alpha}\Phi^\dagger\, \bar{D}^{\dot\alpha}\Phi^\dagger}{\Phi^\dagger\, \Phi^3} \right|_{\bar\theta=0}$$

- The moduli integral in this case can be done Matsuo et al, 0803.0798. Result in accordance with the above structure:

$$W_{\text{n.p.}} = \mathcal{C}\,\Lambda^4 \left. \frac{\epsilon_{f_1 f_1'}\, \epsilon^{f_2 f_2'}\, \bar{D}_{\dot\alpha} {M^\dagger}^{f_1}{}_{f_2}\, \bar{D}^{\dot\alpha} {M^\dagger}^{f_1'}{}_{f_2'} + 2\, \bar{D}_{\dot\alpha} B^\dagger\, \bar{D}^{\dot\alpha} \tilde{B}^\dagger}{\left(\mathrm{tr} M^\dagger M + B^\dagger B + \tilde{B}^\dagger \tilde{B}\right)^{3/2}} \right|_{\bar\theta=0}$$

in terms of the $SU(2)$ meson and baryon fields.

BW multifermion terms

- For $p > 0$, i.e. $N_f \geq N_c$, one gets the multifermion instanton interactions in SQCD of BW Beasley and Witten, 0409149
- For $p > 1$, more general multi-fermion terms
- The BW multi-fermion terms are non-holomorphic but are annihilated by the anti-chiral supercharges $\overline{Q}_{\dot{\alpha}}$

Possible multi-instanton corrections

- More general configurations of "ordinary" D-instantons: k_0, k_1 generic (but $k_2 = k_3 = 0$)

  - In this case one can argue that there can be holomorphic non-perturbative corrections of the form

$$W_{n.p.} = \mathcal{C}\, M_s^{(k_0 b_1 + k_1 \beta_1)}\, e^{2\pi i (k_0 \tau_0 + k_1 \tau_1)}\, \phi^{(3 - k_0 b_1 - k_1 \beta_1)}$$

    (here ϕ is the v.e.v. of the chiral multiplet Φ).
    - Can be promoted to depende on the entire multiplet Φ,

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    (here ϕ is the v.e.v. of the chiral multiplet Φ).
    - Can be promoted to depende on the entire multiplet Φ,
  - See Schmidt-Sommerfeld's talk for a general discussion of *Multi D-instanton Effects in String Compactifications*

Flux effects and stringy instantons

Incorporating flux effects

- From the table derived before Recall one sees that D3 fermions couple to the flux combination

$$G = F - \frac{i}{g_s} H$$

- One finds that $G_{3,0}$ gives mass to the gaugino while $G_{0,3}$ corresponds to the GVW bulk superpotential Gukov et al, 9906070

$$W \sim \int G \wedge \Omega \sim G_{0,3}$$

- We want to investigate flux effects in the low energy effective theory for the massless d.o.f. in the Higgs phase
- The fluxes may modify the non-perturbative contributions which in this context are due to (fractional) D(-1) branes

Flux corrections

Applying our results for the flux interactions on D(-1)'s ▸ Recall to the "ordinary" instanton configuration ($k_0 = 1$) one gets extra contributions to the moduli action of the form: ▸ Back

$$S^{(flux)}_{mod} \sim i\alpha'^2 G_{(0,3)} \lambda_{\dot\alpha}\lambda^{\dot\alpha} + i G_{(3,0)} \theta_\alpha \theta^\alpha + i G_{(3,0)} \bar\mu_u \mu^u$$

(The last term corrisponds to couplings with twisted moduli)

- I will now briefly discuss some of the effects that these extra terms induce in the non-perturbative low energy effective action
- For simplicity, from now on $G = G_{(3,0)}$ and $\bar G = G_{(0,3)}$.

One-instanton effects at $G \neq 0$

- If we pull down once the term $G\bar{\mu}_u \mu^u$, we get

$$S_{n.p.}(G) = \int d^4x\, d^2\theta\, W_{n.p.}(G)\ ,$$

$$W_{n.p.}(G) = \Lambda^{b_1} \int d\,\widehat{\mathcal{M}}\ \mathrm{e}^{-S_{mod}(\Phi,\bar{\Phi})}\ (i\, G\bar{\mu}\,\mu)$$

- Ansatz:

$$W_{n.p.}(G) = \mathcal{C}\, G\, \Lambda^{b_1}\, (\Phi^\dagger)^n\, \Phi^m \left(\bar{D}_{\dot{\alpha}}\Phi^\dagger\, \bar{D}^{\dot{\alpha}}\Phi^\dagger\right)^p \Big|_{\bar{\theta}=0}$$

- Exploiting the $U(1)^3$ symmetries (one has $q[G] = -3$), one finds

$$p = -n - 2 = 2 - N_c + N_f\ , \quad m = -N_c - N_f$$

Multifermion terms at $N_f = N_c - 1$

The case $p = 1$ corresponds to an SQCD with $N_f = N_c - 1$

- In presence of G-flux, besides the ADS superpotential, we get a multifermion interaction of the form

$$W_{n.p.}(G) = \mathcal{C}\, G\, \Lambda^{2N_c+1} \left. \frac{\bar{D}_{\dot{\alpha}} \Phi^\dagger\, \bar{D}^{\dot{\alpha}} \Phi^\dagger}{(\Phi^\dagger)^3\, \Phi^{2N_c-1}} \right|_{\bar{\theta}=0}$$

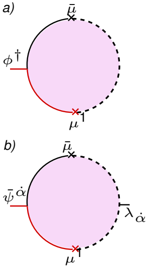
- For $N_c = 2$ the moduli integral can be explicitly done and the result can be expressed in terms of the low energy d.o.f

$$W_{n.p.}(G) = \mathcal{C}\, G\, \Lambda^5 \left. \frac{\bar{D}^2 M^\dagger}{(M^\dagger M)^{3/2}} \right|_{\bar{\theta}=0}$$

- This appears as a non-perturbative effect of the soft supersymmetry breaking due to the G-flux in the microscopic theory.

Stringy effects in ordinary instantons

$G_{(0,3)}$ appears in the moduli action with an α'^2 in front. We must include other terms vanishing in the $\alpha' \to 0$ limit



- From disk diagrams one has extra terms that correspond to

$$\Phi^\dagger(x, \bar{\theta} = 0) \to \bar{\Phi}^\dagger(x, \bar{\theta} = \alpha'\lambda)$$

$$\bar{D}_{\dot{\alpha}}\Phi^\dagger(x, \bar{\theta} = 0) \to \bar{D}_{\dot{\alpha}}\Phi^\dagger(x, \bar{\theta} = \alpha'\lambda)$$

in the moduli action, which becomes ▸ Recall

$$S_{mod} = \frac{2\pi^3\alpha'^2}{g_s} D_c D^c + i\, D_c\left(\bar{w}_{\dot{\alpha}}(\tau^c)^{\dot{\alpha}}{}_{\dot{\beta}} w^{\dot{\beta}}\right) + i\,\frac{\sqrt{2}}{\pi\alpha'}\,\bar{\theta}_{\dot{\alpha}}\left(\bar{\mu}\, w^{\dot{\alpha}} + \bar{w}^{\dot{\alpha}}\,\mu\right)$$
$$+ \frac{1}{2}\,\bar{w}_{\dot{\alpha}}\left(\Phi\,\Phi^\dagger + \Phi^\dagger\,\Phi\right) w^{\dot{\alpha}} + \frac{i}{2}\,\bar{\mu}^1\,\Phi^\dagger\,\mu - \frac{i}{2}\,\bar{\mu}\,\Phi^\dagger\,\mu^1$$
$$+ i\,\bar{w}_{\dot{\alpha}}\left(\bar{D}^{\dot{\alpha}}\Phi^\dagger\right)\mu^1 - i\,\bar{\mu}^1\left(\bar{D}_{\dot{\alpha}}\Phi^\dagger\right) w^{\dot{\alpha}} + \text{flux terms}$$

Stringy effects in ordinary instantons

When the λ-integration is saturated using $\bar{\theta}$-terms in the above superfields

- The fermionic ADHM constraint is not imposed: we lose contact with gauge instanton solutions
- We get explicit α' factors in front of the corresponding contributions, which are D-terms:

$$S_{n.p.} = \int d^4x\, d^2\theta\, d^2\bar{\theta}\, K_{n.p.}\,, \quad K_{n.p.} = \alpha'^2\, \Lambda^{b_1} \int d\,\widehat{\mathcal{M}}'\; \mathrm{e}^{-S'_{mod}(\Phi,\Phi^\dagger)}$$

- The form of is constrained by the $U(1)^3$ symmetries to be

$$K_{n.p.} = \mathcal{C}\,\alpha'^2\, \Lambda^{3N_c - N_f}\, (\Phi^\dagger)^{3+N_c-N_f}\, \Phi^{3-N_c-N_f}\, \left(\bar{D}_{\dot\alpha}\Phi^\dagger\, \bar{D}^{\dot\alpha}\Phi^\dagger\right)^{N_f-N_c} + \ldots\,.$$

- Such terms deserve to be further investigated

Stringy effects in ordinary instantons

with $\bar{G} \neq 0$

- If we perform the $d^2\bar{\theta}$ integration using the $\bar{G}\,\bar{\theta}\bar{\theta}$ interaction Recall we get

$$S_{n.p.}(\bar{G}) = \alpha'^2 \int d^4x\, d^2\theta \;\; W_{n.p.}(\bar{G})$$

$$W_{n.p.}(\bar{G}) = \alpha'^2 \frac{2\pi i}{g_s} \Lambda^{b_1}\, \bar{G} \int d\,\widehat{\mathcal{M}}' \;\, \mathrm{e}^{-\,S_{mod}(\Phi,\Phi^\dagger)|_{\bar{\theta}=0}}$$

- The schematic form of $W_{n.p.}$ can be fixed similarly to previous cases

Non-holomorphic terms at $N_f = N_c$

Let us consider, for instance, the case $N_f = N_c$.

- At $\bar{G} = 0$ we got BW multifermion F-terms
- Now we get also a non-holomorphic contribution of the form

$$W_{n.p.} = \mathcal{C}\, \alpha'^2\, \bar{G}\, \Lambda^{2N_c}\, {\Phi^{\dagger}}^3\, \Phi^{3-2N_c}\Big|_{\bar{\theta}=0}$$

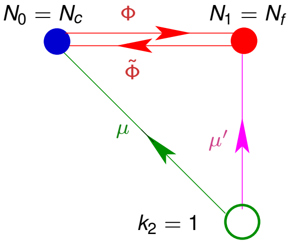
- For $N_c = 2$, the explicit integral over the moduli yields

$$W_{n.p.} = \mathcal{C}\, \alpha'^2\, \bar{G}\, \Lambda^4\, \frac{\det M^{\dagger}}{\left(\mathrm{tr} M^{\dagger} M + B^{\dagger} B + \tilde{B}^{\dagger} \tilde{B}\right)^{1/2}}\Bigg|_{\bar{\theta}=0} ,$$

(M is the meson, B and $\widetilde{B}$ the baryon superfields).

Exotic (stringy) instantons

- Let us consider a set-up in which the instantonic brane does not correspond to a classical instanton for the gauge group
- D(-1)/D3 strings have only fermionic excitations $\mu_u, \bar{\mu}^u$ and $\mu'_f, \bar{\mu}'^f$



- The field-dependent moduli action is simply

$$S_{mod} = (\alpha')^2 D_c D^c + \mu_u \Phi(x,\theta)^u{}_f \bar{\mu}'^f - \mu'_f \tilde{\Phi}(x,\theta)^f{}_u \bar{\mu}^u$$

  Notice that the field-dependent terms are now holomorphic

- The integration over the $\bar{\theta} = \alpha' \lambda$'s kills any contribution to the effective action

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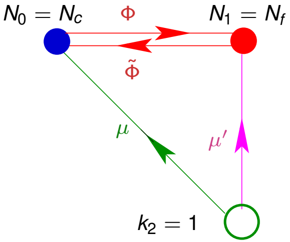

- Including flux corrections, the moduli action becomes

$$S_{mod} = (\alpha')^2 D_c D^c + \mu_u \Phi(x,\theta)^u{}_f \bar{\mu}'^f - \mu'_f \tilde{\Phi}(x,\theta)^f{}_u \bar{\mu}^u + \bar{G}\,\bar{\theta}_{\dot{\alpha}}\bar{\theta}^{\dot{\alpha}} + \ldots$$

- The $\bar{\theta}$ integral can now be saturated with the $\bar{G}\,\bar{\theta}_{\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}$ term
- At linear level, other flux interactions become irrelevant

Exotic but Holomorphic

- We get therefore

$$S_{n.p.} = \int d^4x\, d^2\theta\, W_{n.p.}(\bar{G}) \;,$$

$$W_{n.p.} = \mathcal{C}\, \alpha'^2\, M_s{}^{-(N_c+N_f)}\, \mathrm{e}^{2\pi i \tau_2}\, \bar{G}$$

$$\times \int d^3D\, d^{N_c}\mu^2\, d^{N_c}\bar{\mu}^2\, d^{N_f}\mu^3\, d^{N_f}\bar{\mu}^3\, \mathrm{e}^{-\frac{2\pi^3\alpha'^2}{g_s} D_c D^c + \frac{i}{2}\left(\bar{\mu}^3\Phi\mu^2 - \bar{\mu}^2\Phi\mu^3\right)}$$

- The integration vanishes unless $N_c = N_f$, in which case it is easy and we get an holomorphic superpotential contribution

$$W_{n.p.} = \mathcal{C}\, M_s^{2-2N_c} \mathrm{e}^{2\pi i \tau_2}\, \bar{G}\, \det M \;.$$

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$$W_{n.p.} = \mathcal{C}\, M_s^{2-2N_c} \mathrm{e}^{2\pi i \tau_2}\, \bar{G}\, \det M \; .$$

- M_s does not combine with $\mathrm{e}^{2\pi i \tau_2}$ to give the scale Λ: τ_2 is not the YM coupling on the N_c branes

- The technologies of the so-called "stringy instanton calculus" are an essential tool to devise the structure of non-perturbative contributions to the effective action for gauge theories engineered by brane constructions in a string compactification.

Conclusions

- The technologies of the so-called "stringy instanton calculus" are an essential tool to devise the structure of non-perturbative contributions to the effective action for gauge theories engineered by brane constructions in a string compactification.
- In such a situation
  - Different types of instantonic branes, ordinary (i.e., corresponding to gauge instantons) and exotic
  - Fluxes may be turned on

  and we must be able to follow the pattern through which the l.e.e.a is affected by all this

Conclusions

- The technologies of the so-called "stringy instanton calculus" are an essential tool to devise the structure of non-perturbative contributions to the effective action for gauge theories engineered by brane constructions in a string compactification.
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THANK YOU FOR YOUR ATTENTION!

General result (RR)

Back

$$\mathcal{A}_F = -8c_F\, \Theta' \Gamma^M \Theta \left[F \mathcal{R}_0 (2 I_1 - I_2) \right]_M + \frac{4c_F}{3!} \Theta' \Gamma^{MNP} \Theta \left[F \mathcal{R}_0 I_2 \right]_{MNP}$$

- $\Theta_{\mathcal{A}}$: polarization of the open string R vertex, with $\mathcal{A} = 1, \ldots, 16$ = (antichiral) 10d spinor index labeling $\vec{\epsilon}_{\mathcal{A}} = \frac{1}{2}(\pm, \pm, \pm, \pm, \pm)$

Back

$$\mathcal{A}_F = -8c_F\, \Theta'\Gamma^M\Theta\, \left[F\mathcal{R}_0(2I_1 - I_2)\right]_M + \frac{4c_F}{3!}\Theta'\Gamma^{MNP}\Theta\, \left[F\mathcal{R}_0 I_2\right]_{MNP}$$

- The IIB RR vertex is a bi-spinor containing the fields strengths:

$$F_{\mathcal{AB}} = \sum_{n=1,3,5} \frac{1}{n!} F_{M_1 \dots M_n} \left(\Gamma^{M_1 \dots M_n}\right)_{\mathcal{AB}} ,$$

General result (RR)

Back

$$\mathcal{A}_F = -8c_F\,\Theta'\Gamma^M\Theta\left[F\mathcal{R}_0(2I_1-I_2)\right]_M+\frac{4c_F}{3!}\Theta'\Gamma^{MNP}\Theta\left[F\mathcal{R}_0 I_2\right]_{MNP}$$

- l.m. and r.m. fields identification at the boundary:

$$\widetilde{X}^M(\bar{z}) = (R_0)^M_{\ N}X^N(\bar{z}) \quad , \quad \widetilde{s}_{\vec{\epsilon}_\mathcal{A}}(\bar{z}) = (\mathcal{R}_0)^\mathcal{A}_{\ \mathcal{B}}\,s_{\vec{\epsilon}_\mathcal{B}}(\bar{z})$$

where $\mathcal{R}_0$ is the spinorial reflection matrix. Thus

$$F_{\mathcal{A}\mathcal{B}} \rightarrow (F\mathcal{R}_0)_{\mathcal{A}\mathcal{B}}$$

General result (RR)

▸ Back

$$\mathcal{A}_F = -8c_F\,\Theta'\Gamma^M\Theta\left[F\mathcal{R}_0(2I_1 - I_2)\right]_M + \frac{4c_F}{3!}\Theta'\Gamma^{MNP}\Theta\left[F\mathcal{R}_0 I_2\right]_{MNP}$$

- I_1 and I_2 are $\vec{\vartheta}$-dependent diagonal matrices:

$$(I_1)_{\mathcal{A}_3}^{\mathcal{A}_3} = \frac{1}{2}\,\mathrm{e}^{-\frac{i\pi\alpha' s}{2}}\left(\mathrm{e}^{-2\pi i\left(\alpha' t - \vec{\vartheta}\cdot\vec{\epsilon}_3\right)} - 1\right) B\left(\alpha' s; \alpha' t - \vec{\vartheta}\cdot\vec{\epsilon}_3\right)$$

$$(I_2)_{\mathcal{A}_3}^{\mathcal{A}_3} = \frac{1}{2}\,\mathrm{e}^{-\frac{i\pi\alpha' s}{2}}\left(\mathrm{e}^{-2\pi i\left(\alpha' t - \vec{\vartheta}\cdot\vec{\epsilon}_3\right)} - 1\right) B\left(\alpha' s + 1; \alpha' t - \vec{\vartheta}\cdot\vec{\epsilon}_3\right)$$

where $\vec{\epsilon}_3$ is the spinorial weight of the r.m. part of the RR vertex

that

General result (NS-NS)

$$\mathcal{A}_H = -4c_H\,\Theta'\Gamma^N\Theta\,\delta^{MP}\left[\partial B R_0(2I_1 - I_2)\right]_{[MN]P} + 2c_H\,\Theta'\Gamma^{MNP}\Theta\left[\partial B R_0 I_2\right]_{MNP}$$

- We use an effective NS-NS vertex containing the derivatives of B

$$V_H(z,\bar{z}) = \mathcal{N}_H\left(\partial_M B_{NP}\right)\mathrm{e}^{-i\pi\alpha' k_{\mathrm{L}}\cdot k_{\mathrm{R}}}\left[\psi^M\psi^N\mathrm{e}^{i\,k_{\mathrm{L}}\cdot X}\right](z) \times\left[\widetilde{\psi}^P\,\mathrm{e}^{-\widetilde{\phi}}\,\mathrm{e}^{i\,k_{\mathrm{R}}\cdot\widetilde{X}}\right](\bar{z})$$

General result (NS-NS)

$$\mathcal{A}_H = -4c_H\,\Theta'\Gamma^N\Theta\,\delta^{MP}\left[\partial B R_0(2I_1 - I_2)\right]_{[MN]P} + 2c_H\,\Theta'\Gamma^{MNP}\Theta\left[\partial B R_0 I_2\right]_{MNP}$$

- In presence of D-branes, the left-right identifications leads to

$$(\partial B) \to (\partial B R_0)$$

with the vectorial reflection matrix R_0

General result (NS-NS)

$$\mathcal{A}_H = -4c_H\,\Theta'\Gamma^N\Theta\,\delta^{MP}\left[\partial B R_0(2I_1 - I_2)\right]_{[MN]P}$$
$$+\,2c_H\,\Theta'\Gamma^{MNP}\Theta\left[\partial B R_0 I_2\right]_{MNP}$$

- I_1 and I_2 are again given by:

$$\left(I_1\right)^{\mathcal{A}_3}_{\mathcal{A}_3} = \frac{1}{2}\,\mathrm{e}^{-\frac{i\pi\alpha' s}{2}}\left(\mathrm{e}^{-2\pi i\left(\alpha' t - \vec{\vartheta}\cdot\vec{\epsilon}_3\right)} - 1\right) B(\alpha' s; \alpha' t - \vec{\vartheta}\cdot\vec{\epsilon}_3)$$
$$\left(I_2\right)^{\mathcal{A}_3}_{\mathcal{A}_3} = \frac{1}{2}\,\mathrm{e}^{-\frac{i\pi\alpha' s}{2}}\left(\mathrm{e}^{-2\pi i\left(\alpha' t - \vec{\vartheta}\cdot\vec{\epsilon}_3\right)} - 1\right) B(\alpha' s + 1; \alpha' t - \vec{\vartheta}\cdot\vec{\epsilon}_3)$$

but $\vec{\epsilon}_3$ is now the vectorial weight associated to $\psi^P(z_3)$ in the r.m. part of the NS-NS vertex

Details on the orbifold

Back

- Character table and Clebsh-Gordan series:

	e	h_1	h_2	h_3
R_0	1	1	1	1
R_1	1	1	-1	-1
R_2	1	-1	1	-1
R_3	1	-1	-1	1

$$R_0 \otimes R_A = R_A \ , \quad R_i \otimes R_j = \delta_{ij} R_0 + |\epsilon_{ijk}| R_k$$

- Transformations of massless string fields:

NS fields	irrep
$\partial Z^i, \psi^i$	R_i

,

chiral S^A	anti-chiral S_A	irrep
$S^0 \equiv S^{+++}$	$S_0 \equiv S_{---}$	R_0
$S^1 \equiv S^{+--}$	$S_1 \equiv S_{-++}$	R_1
$S^2 \equiv S^{-+-}$	$S_2 \equiv S_{+-+}$	R_2
$S^3 \equiv S^{--+}$	$S_3 \equiv S_{++-}$	R_3