Exact partition functions for the effective confining string in gauge theories

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Foreword

- This talk is based on
  - M. Billó and M. Caselle, "Polyakov loop correlators from D0-brane interactions in bosonic string theory", JHEP **0507** (2005) 038 [arXiv:hep-th/0505201].

  also outlined in the LATTICE 2005 talk of M. Caselle:
  - M. Billo, M. Caselle, M. Hasenbusch and M. Panero, "QCD string from D0 branes," PoS (LAT2005) 309 [arXiv:hep-lat/0511008].
- and on
  - M. Billo, M. Caselle and L. Ferro, "The partition function of interfaces from the Nambu-Goto effective string theory," JHEP **0602** (2006) 070 [arXiv:hep-th/0601191].
- plus work in progress with L. Ferro and I. Pesando.

Plan of the talk

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1. The main ideas
2. Polyakov loop correlators
3. Interface partition function
4. Wilson loops
5. Conclusions

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The main ideas

String theory and (lattice) gauge theories

- A description of strongly coupled gauge theories in terms of strings has long been suspected
- These strings should describe the fluctuations of the color flux tube in the confining regime

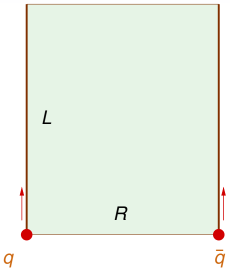
- Potential $V(R)$ between two external, massive quark and anti-quark sources from Wilson loops Back

$$\langle W(L,R)\rangle \sim \mathrm{e}^{-LV(R)} \quad \text{(large } R\text{)}$$

- Area law $\leftrightarrow$ linear potential

$$V(R) = \sigma R + \dots$$

σ is the string tension



Quantum corrections and effective models

- Leading correction for large R

$$V(R) = \sigma R - \frac{\pi}{24}\frac{d-2}{R} + O\left(\frac{1}{R^2}\right) .$$

from quantum fluctuations of $d-2$ massless modes: transverse fluctuations of the string [Lüscher, Symanzik and Weisz]

- Simplest effective description via the two-dimensional conformal field theory of $d-2$ free bosons
  - Higher order interactions among these fields distinguish the various effective theories
  - The underlying string model should determine a specific form of the effective theory, and an expression of the potential $V(R)$ that extends to finite values of R.

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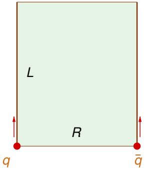
Various observables with an effective string description

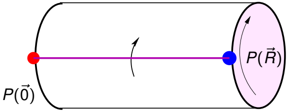
Three typical observables with a geometrically simple effective string picture

- Wilson loop



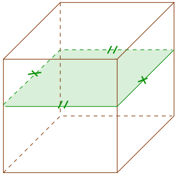
disk

- Correlator of Polyakov loops



cylinder

- interfaces or 't Hooft loops



torus

Various models of effective strings

- "Free" theory: $d - 2$ bosonic fields living on the surface spanned by the string, describing its transverse fluctuations
- Standard bosonic string theory: Nambu-Goto action $\propto$ area of the world-sheet surface
  - Possible first-order formulation á la Polyakov (we'll use this)
  - In $d \neq 26$, bosonic string is ill-defined (conformal invariance broken by quantum effects). This is manifest at short distances in the description of LGT observables.
- Attempts to a consistent string theory description: Polchinski-Strominger, Polyakov, AdS/CFT
  - This is the aim, of course. However, we'll not touch the subject in this talk...

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The Nambu-Goto approach

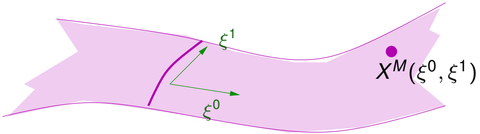
- Action $\sim$ area of the surface spanned by the string in its motion:

$$S = -\sigma \int d\xi^0 d\xi^1 \sqrt{\det g_{\alpha\beta}}$$

where $g_{\alpha\beta}$ is the metric “induced” on the w.s. by the embedding:

$$g_{\alpha\beta} = \frac{\partial X^M}{\partial \xi^\alpha}\frac{\partial X^N}{\partial \xi^\beta} G_{MN}$$

ξ^α = world-sheet coords. (ξ^0 = proper time, ξ^1 spans the extension of the string)



The nambu-Goto approach: perturbative approach

- One can use the world-sheet re-parametrization invariance of the NG action to choose a "physical gauge":
  - The w.s. coordinates ξ^0, ξ^1 are identified with two target space coordinates x^0, x^1
- One can study the 2d QFT for the $d-2$ transverse bosonic fields with the gauge-fixed NG action

$$Z = \int DX^i \mathrm{e}^{-\sigma \int dx^0 dx^1 \sqrt{1+(\partial_0 \vec{X})^2+(\partial_1 \vec{X})^2+(\partial_0 \vec{X}\wedge\partial_1 \vec{X})^2}}$$

$$= \int DX^i \mathrm{e}^{-\sigma \int dx^0 dx^1 \{1+(\partial_0 \vec{X})^2+(\partial_1 \vec{X})^2+\text{int.s}\}}$$

perturbatively, the loop expansion parameter being $1/(\sigma A)$

- [Dietz-Filk, 1982]: up to 2 loop for the 3 geometries (disk, cylinder, torus)

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The first order approach

- The NG goto action can be given a 1st order formulation (no awkward square roots)

$$S = -\sigma \int d\xi^0 d\xi^1 \sqrt{h} h^{\alpha\beta} \partial_\alpha X^M \partial_\beta X^M$$

with $h_{\alpha\beta}$ = independent w.s metric

- Use re-parametrization and Weyl invariance to set $h_{\alpha\beta} \to \eta_{\alpha\beta}$
  - Actually, Weyl invariance is broken by quantum effects in $d \neq 26$
- Remain with a free action but
  - Virasoro constraints $T_{\alpha\beta} = 0$ from $h^{\alpha\beta}$ e.o.m.
  - residual conformal invariance

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Physical gauge vs. covariant quantization

- The residual conformal invariance can be used to fix a light-cone (physical) type of gauge: w.s. coordinates identified with two target space ones (non-covariant choice)
  - One explicitly solves the Virasoro constraints and remains with the $d-2$ transverse directions as the only independent d.o.f.
  - The quantum anomaly for $d \neq 26$ manifests itself as a failure in Lorentz algebra
- In a covariant quantization, the Virasoro constraints are imposed on physical states á la BRST
  - All d directions are treated on the same footing
  - Introduction of ghosts
  - For $d \neq 26$, anomaly in the conformal algebra
  - This is the framework we will use

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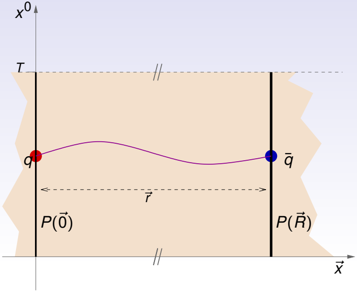
Polyakov loop correlators

The set-up



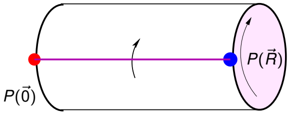
- Finite temperature geometry + static external sources (quarks)
- Polyakov loop = trace of the temporal Wilson line

$$\langle P(\vec{R})\rangle = \mathrm{e}^{-F} \neq 0 \rightarrow \text{de-confinement}$$

- On the lattice, the correlator

$$\langle P(\vec{0})P(\vec{R})\rangle_{\mathrm{c}} \ .$$

  can be measured with great accuracy.
- In the string picture, the correlation is due to the strings connecting the two external sources: cylindric world-sheet



Nambu-Goto description of the correlator

- P.L. correlator = partition function of an open string with
  - Nambu-Goto action
  - Dirichlet boundary conditions (end-points attached to the Polyakov loops)
- Functional integral result (Dietz and Filk):
  - Loop expansion. Expansion parameter $1/(\sigma L R)$
  - Two-loop result [set $\hat{\tau} = \mathrm{i}L/(2R)$, $d = 3$]: Back

$$Z = \mathrm{e}^{-\sigma L R} \frac{1}{\eta(\hat{\tau})} \left(1 - \frac{\pi^2 L}{1152 \sigma R^3} \left[2E_4(\hat{\tau}) - E_2^2(\hat{\tau}) \right] + \dots \right)$$

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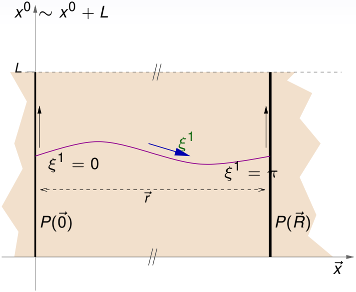
First order formulation

- Action (in conformal gauge)

$$S = \frac{1}{4\pi\alpha'} \int d\xi^0 \int_0^\pi d\xi^1 \left[(\partial_0 X^M)^2 + (\partial_1 X^M)^2 \right] + S_{\text{gh.}}$$



- World-sheet parametrized by
  - $\xi^1 \in [0, \pi]$ (open string)
  - ξ^0 (proper time)

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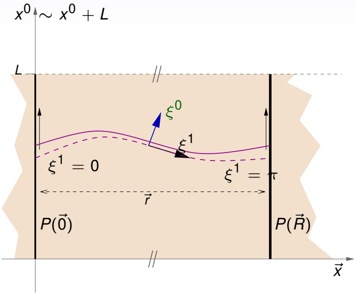

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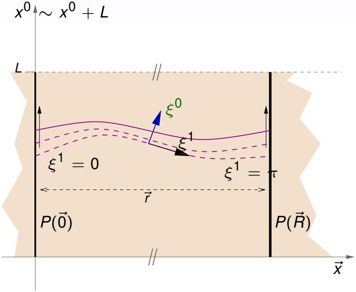

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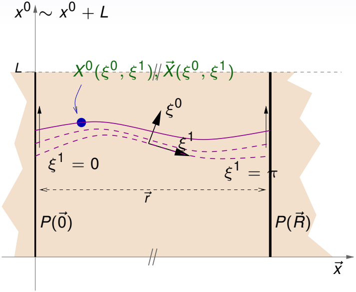

- The field X^M $(M = 0, \ldots, d-1)$ describe the embedding of the world-sheet in the target space

First order formulation

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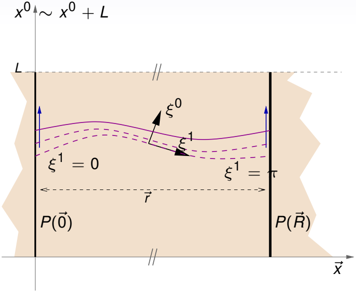


- Boundary conditions:
  - Neumann in "time" direction:

$$\partial_0 X^0(\xi^0, \xi^1)\big|_{\xi^1=0,\pi} = 0$$

  - Dirichlet in spatial directions:

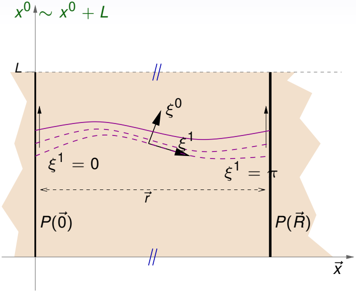
$$\vec{X}(\xi^0, 0) = 0 \,, \qquad \vec{X}(\xi^0, \pi) = \vec{R} \,.$$

"open string between D0-branes"

First order formulation

- Action (in conformal gauge)

$$S = \frac{1}{4\pi\alpha'} \int d\xi^0 \int_0^\pi d\xi^1 \left[(\partial_0 X^M)^2 + (\partial_1 X^M)^2 \right] + S_{\rm gh.}$$



- The string fields have thus the expansion

$$X^0 = \hat{x}^0 + \frac{\hat{p}^0}{\pi\sigma} + \frac{\mathrm{i}}{\sqrt{\pi\sigma}} \sum_{n\neq 0} \frac{\alpha^0}{n} \mathrm{e}^{-\mathrm{i}n\xi^0} \cos n\xi^1$$

$$\vec{X} = \frac{\vec{R}}{\pi}\xi^1 - \frac{1}{\sqrt{\pi\sigma}} \sum_{n\neq 0} \frac{\vec{\alpha}}{n} \mathrm{e}^{-\mathrm{i}n\xi^0} \sin n\xi^1$$

  - $\left[\alpha^M_m, \alpha^N_n\right] = m\,\delta_{m+n,0}\,\delta^{MN}$

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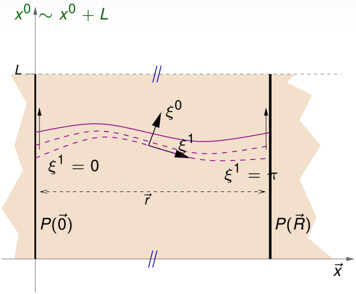


- The target space has finite temperature:

$$x^0 \sim x^0 + L$$

  - The 0-th component of the momentum is therefore discrete:

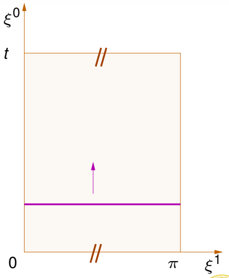
$$p^0 \to \frac{2\pi n}{L}$$

The free energy

- Interaction between the two Polyakov loops (the D0-branes) $\leftrightarrow$ free energy of the open string

$$\mathcal{F} = L \int_0^\infty \frac{dt}{2t} \mathrm{Tr} q^{L_0}$$

- $q = \mathrm{e}^{-2\pi t}$, and t is the only parameter of the world-sheet cylinder (one loop of the open string)



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- L is the "world-volume" of the D0-brane, i.e. the volume of the only direction along which the excitations propagate, the Euclidean time

The free energy

- Interaction between the two Polyakov loops (the D0-branes) $\leftrightarrow$ free energy of the open string

$$\mathcal{F} = L \int_0^\infty \frac{dt}{2t} \mathrm{Tr} q^{L_0}$$

- Virasoro generator L_0 (Hamiltonian)

$$L_0 = \frac{(\hat{p}^0)^2}{2\pi\sigma} + \frac{\sigma R^2}{2\pi} + \sum_{n=1}^{\infty} N_n^{(d-2)} - \frac{d-2}{24}$$

  - $N_n^{(d-2)}$ is the total occupation number for the oscillators appearing in $d-2$ bosonic fields (the -2 is due to the ghosts)

The free energy

- Interaction between the two Polyakov loops (the D0-branes) $\leftrightarrow$ free energy of the open string

$$\mathcal{F} = L \int_0^\infty \frac{dt}{2t} \, \mathrm{Tr} q^{L_0}$$

- Tracing over the oscillators and the discrete zero-mode eigenvalues $p^0 = 2\pi n/L$ yields finally

$$\mathcal{F} = \int_0^\infty \frac{dt}{2t} \sum_{n=-\infty}^{\infty} \mathrm{e}^{-2\pi t \left(\frac{2\pi n^2}{\sigma L^2} + \frac{\sigma R^2}{2\pi} \right)} \left(\frac{1}{\eta(\mathrm{i}t)} \right)^{d-2}$$

Topological sectors

- Poisson resum over the integer n getting

$$\mathcal{F} = \mathcal{F}^{(0)} + 2\sum_{m=1}^{\infty} \mathcal{F}^{(m)}$$

with ▸ Back

$$\mathcal{F}^{(m)} = \sqrt{\frac{\sigma L^2}{4\pi}} \int_0^\infty \frac{dt}{2t^{\frac{3}{2}}} \mathrm{e}^{-\frac{\sigma L^2\, m^2}{4t} - \sigma R^2 t} \left(\frac{1}{\eta(\mathrm{i}t)}\right)^{d-2}$$

- The integer m is the # of times the open string wraps the compact time in its one loop evolution.
- Each topological sector $\mathcal{F}^{(m)}$ describes the fluctuations around an "open world-wheet instanton"

$$X^0(\xi^0 + t, \xi^1) = X^0(\xi^0, \xi^1) + mL$$

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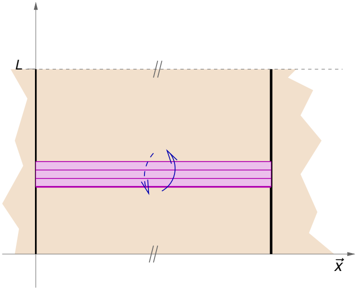

- An example with $m = 0$ (N.B. The classical solution degenerates to a line)

Topological sectors

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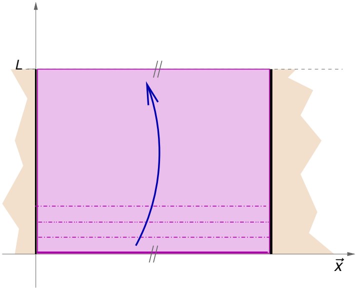


- The case $m = 1$. The world-sheet exactly maps to the cylinder connecting the two Polyakov loops.

The case $m = 1$ and the NG result

- The sector with $m = 1$ of our free energy should correspond to the effective NG partition function
- Expand in series the Dedekind functions:

$$\left(\prod_{r=1}^{\infty} \frac{1}{1 - q^r}\right)^{d-2} = \sum_{k=0}^{\infty} c_k q^k$$

- Plug this into $\mathcal{F}^{(m)}$ Recall and integrate over t using

$$\int_0^{\infty} \frac{dt}{t^{\frac{3}{2}}} \mathrm{e}^{-\frac{\alpha^2}{t} - \beta^2 t} = \frac{\sqrt{\pi}}{|\alpha|} \mathrm{e}^{-2|\alpha|\,|\beta|}$$

The case $m = 1$ and the NG result

- The sector with $m = 1$ of our free energy should correspond to the effective NG partition function
- The result is

$$\mathcal{F}^{(m)} = \frac{1}{2|m|} \sum_k c_k \mathrm{e}^{-|m| L E_k(R)} \,, \qquad (m \neq 0)$$

with

$$E_k(R) = \frac{R}{4\pi\alpha'} \sqrt{1 + \frac{4\pi^2\alpha'}{R^2}\left(k - \frac{d-2}{24}\right)}$$

- This spectrum was derived long ago by Arvis by (formal) quantization.

Recovering the perturbative result

- The case $m = 1$ gives the NG partition function: Back

$$Z = 2\mathcal{F}^{(1)} = \sum_k c_k \mathrm{e}^{-LE_k(R)} .$$

Expanding in inverse powers of the minimal area $A = LR$:

$$Z = \mathrm{e}^{-\sigma LR} \sum_n c_n \mathrm{e}^{-\pi \frac{L}{R}\left(n - \frac{d-2}{24}\right) + \ldots} = \mathrm{e}^{-\sigma LR} \eta(\mathrm{i}\frac{L}{2R})\,(1 + \ldots)$$

one reproduces the functional integral perturbative result (Eisentein series and all ...) [Caselle et al] Recall

Closed string interpretation

- Our first-order formulation is well-suited to give the direct closed string channel description of the correlator:

$$\mathcal{F} = \langle B; \vec{0}\,|\mathcal{D}|B; \vec{R}\,\rangle = \frac{1}{4\sigma}\int_0^\infty ds\,\langle B; \vec{0}\,|\mathrm{e}^{-2\pi s(L_0+L_0^{\mathrm{gh.}})}|B; \vec{R}\,\rangle$$

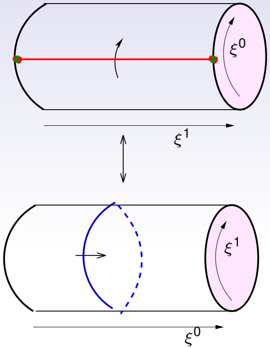
  - $\mathcal{D}$ is the closed string propagator
  - The boundary states enforce on the closed string fields the b.c.'s corresponding to the D-branes (the Polyakov loops)

$$\partial_0 X^0(\xi^0,\xi^1)\big|_{\xi^0=0}\,|B; \vec{R}\rangle = 0\ , \qquad \left(X^i(\xi^0,\xi^1) - R^i\right)\big|_{\xi^0=0}\,|B; \vec{R}\rangle = 0$$

The closed channel expression

- The closed string channel tree level exchange between boundary states corresponds to the modular transformation $t \to 1/t$ of the open string channel 1-loop free energy
- The result of the transformation is

$$\mathcal{F}^{(m)} = L\frac{T_0^2}{4}\sum_k c_k\, G\left(R; M(m,k)\right)$$



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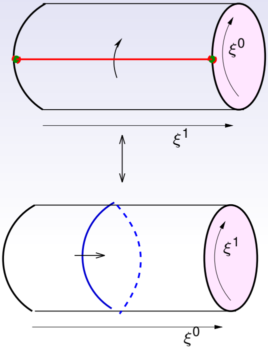

- $G\,(R; M)$ = propagator of a scalar field of mass M over the spatial distance $\vec{R}$ between the two D0-branes:

$$G(R;M) = \int \frac{d^{d-1}p}{(2\pi)^{d-1}}\frac{\mathrm{e}^{\mathrm{i}\vec{p}\cdot\vec{R}}}{p^2+M^2} = \frac{1}{2\pi}\left(\frac{M}{2\pi R}\right)^{\frac{d-3}{2}} K_{\frac{d-3}{2}}(MR)$$

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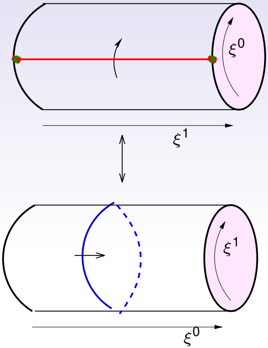

- The mass $M(m,k)$ is that of a closed string state with k representing the total oscillator number, and m the wrapping number of the string around the compact time direction

$$M^2(m,k) = (m\sigma L)^2\left[1 + \frac{8\pi}{\sigma L^2 m^2}\left(k - \frac{d-2}{24}\right)\right]$$

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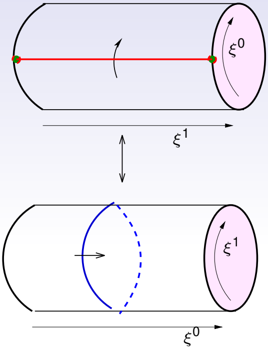
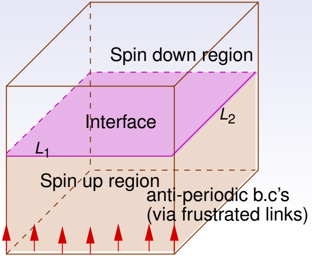


- T_0 = usual D0-brane tension in bosonic string theory:

$$T_0^2 = 8\pi\left(\frac{\pi}{\sigma}\right)^{\frac{d}{2}-2}$$

Interface partition function

Interfaces



Back

- An interface separating regions with different magnetization is observed in simulations of spin models (Ising, etc.), and its fluctuations are measured
- A similar situation can be engineered and studied in LGT, by considering the so-called 't Hooft loops
- It is rather natural to try to describe the fluctuating interface by means of some effective string theory
  - Some string predictions (in particular, the universale effect of the quantum fluctuations of the $d-2$ transverse free fields) have already been considered [De Forcrand, 2004]

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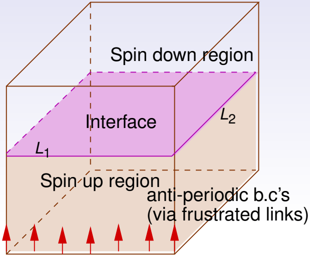

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The Nambu Goto model for interfaces

- In the “physical gauge” approach, we consider a string whose world-sheet is identified with the minimal interface, which has the topology of a torus T_2, of sides L_1 and L_2, i.e., area $A = L_1 L_2$ and modulus $u = L_2/L_1$ Recall
- We are thus dealing with the one-loop partition function $\mathcal{Z}$ of a closed string.
- The functional integral approach [Dietz-Filk, 1982] gives the result up to two loops:

$$\mathcal{Z} \propto \mathrm{e}^{-\sigma A} \frac{1}{[\eta(\mathrm{i}u)]^{2d-4}} \Big\{ 1 + \frac{(d-2)^2}{2\sigma A} \Big[\frac{\pi^2}{36} u^2 E_2^2(\mathrm{i}u) - \frac{\pi}{6} u E_2(\mathrm{i}u) + \frac{d}{8(d-2)} \Big] \Big\}$$

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The NG partition function?

- The partition function for the NG interface string in the operatorial formulation was not avaliable (to our knowledge) in the literature
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  - ▸ It is not too difficult to propose the analogue of Arvis formula for the spectrum E_k based on canonical quantization [Drummond,Kuti,...]

$$E^2_{n,N+\tilde{N}} = \sigma^2 L_1^2 \left\{ 1 + \frac{4\pi}{\sigma L_1^2}\left(N + \tilde{N} - \frac{d-2}{12}\right) + \frac{4\pi^2}{\sigma^2 L_1^4} n^2 + \vec{p}_T^{\,2}) \right\}$$

where $N, \tilde{N}$ = occupation #'s of left (right)-moving oscillators, n the discretized momentum in the direction x^1, $\vec{p}_T$ the transverse momentum

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- This would play the same rôle of the partition function for the Polyakov loop correlators based on Arvis' spectrum Recall and would resum the loop expansion.
    - However, the "naive" form of a partition function based on this spectrum:

$$\sum_{N,\tilde{N},n} \delta(N-\tilde{N}+n) c_N c_{\tilde{N}} \, \mathrm{e}^{-L_2 E_{N+\tilde{N},n}}$$

(where c_N, $c_{\tilde{N}}$ = multiplicities of left- and right-moving oscillator states) does not reproduce the functional integral 2-loop result

The first order approach

- We start from the Polyakov action in the conformal gauge, and do not impose any physical gauge identifying world-sheet and target space coordinates
- We consider the closed string one loop partition function, and we have thus a toroidal world-sheet
- This world-sheet can be mapped in many topologically distinct ways on the target space torus T_d

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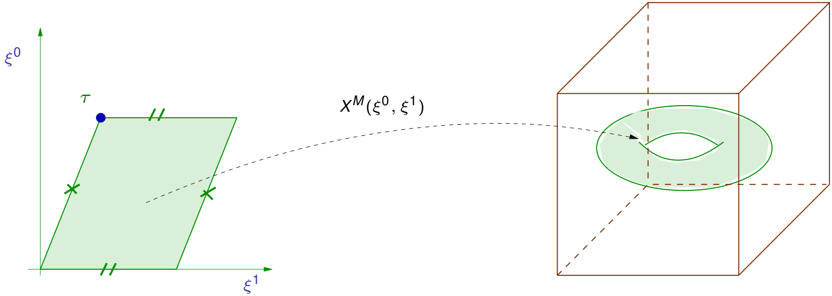

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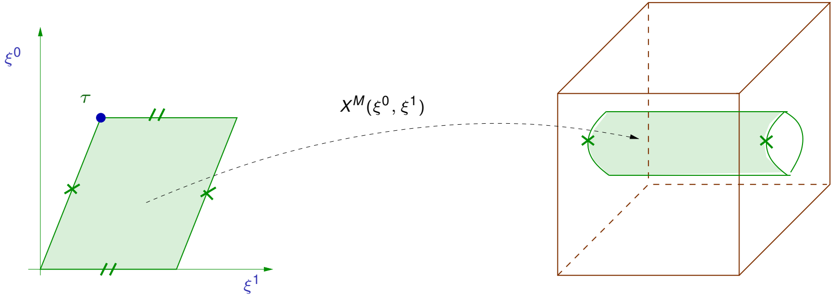

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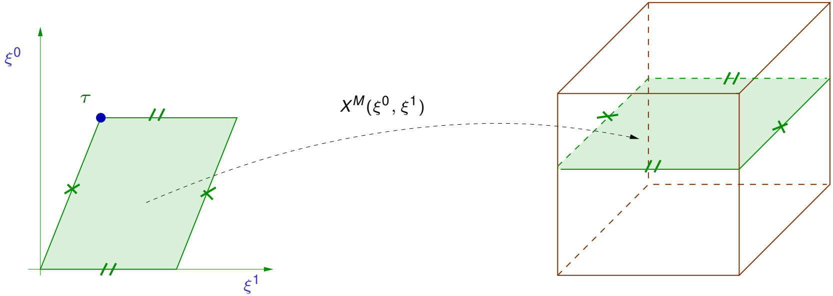

String partition function

- In the Polyakov formulation, the partition function includes an integration over the modular parameter $\tau = \tau_1 + \mathrm{i}\tau_2$:

$$\mathcal{I}^{(d)} = \int \frac{d^2\tau}{\tau_2} Z^{(d)}(q,\bar{q})\, Z^{\mathrm{gh}}(q,\bar{q})$$

  - $Z^{(d)}(q,\bar{q})$ is the CFT partition function of d compact bosons:

$$Z^{(d)}(q,\bar{q}) = \mathrm{Tr}\, q^{L_0 - \frac{d}{24}}\, \bar{q}^{\tilde{L}_0 - \frac{d}{24}}$$

    where $q = \exp 2\pi\mathrm{i}\tau$, $\bar{q} = \exp(-2\pi\mathrm{i}\bar{\tau})$.
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CFT partition function of a compact boson

- Consider a compact boson field

$$X(\xi^0,\xi^1) \sim X(\xi^0,\xi^1) + L$$

- In the operatorial formulation, we find

$$Z(q,\bar{q}) = \sum_{n,w\in\mathbb{Z}} q^{\frac{1}{8\pi\sigma}\left(\frac{2\pi n}{L}+\sigma w L\right)^2} \bar{q}^{\frac{1}{8\pi\sigma}\left(\frac{2\pi n}{L}-\sigma w L\right)^2} \frac{1}{\eta(q)}\,\frac{1}{\eta(\bar{q})}$$

  - The Dedekind functions encode the non-zero mode contributions
  - The 0-mode n denotes the discretized momentum $p = 2\pi n/L$
  - The integer w is the winding around the compact target space: X must be periodic in ξ^1, but we can have

$$X(\xi^0,\xi^1+2\pi) = X(\xi^0,\xi^1) + wL$$

CFT partition function of a compact boson

- Consider a compact boson field

$$X(\xi^0,\xi^1) \sim X(\xi^0,\xi^1) + L$$

- Upon Poisson resummation over the momentum n,

$$Z(q,\bar{q}) = \sigma L \sum_{m,w\in\mathbb{Z}} \mathrm{e}^{-\frac{\sigma L^2}{2\tau_2}|m-\tau w|^2} \frac{1}{\sqrt{\tau_2}\eta(q)\eta(\bar{q})}$$

  - Sum over “world-sheet instantons”: classical solutions of the field X with wrappings w (along ξ^1) and m (along ξ^0, loop geometry):

$$X(\xi^0,\xi^1+2\pi) = X(\xi^0,\xi^1) + wL$$
$$X(\xi^0+2\pi\tau_2,\xi^1+2\pi\tau_1) = X(\xi^0,\xi^1) + mL\,.$$

The interface sector

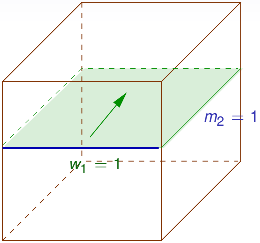
- The partition function includes $Z^{(d)}(q, \bar{q})$, the product of partition functions for the d compact bosons X^M $\rightarrow$ contains the sum over windings w^M and discrete momenta n^M

- We can select the topological sector corresponding to an interface in the x^1, x^2 plane
  - considering a string winding once in the x^1 direction:

$$w_1 = 1 \, , \quad w_2 = w_3 = \ldots = w_d = 0$$

  - Poisson resumming over $n^2, \ldots, n^d$ and then choosing

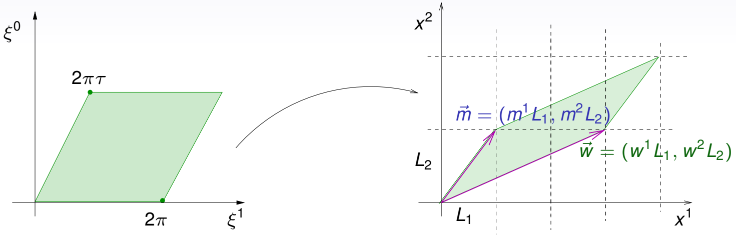
$$m_2 = 1 \, , \quad m_3 = m_4 = \ldots = m_d = 0$$



The issue of modular invariance

- There are many choices of winding numbers m^1, m^2, w^1, w^2 that describe toroidal interfaces aligned along the x^1, x^2-plane in target space.

- The corresponding area is $L_1 L_2(w^1 m^2 - m^1 w^2)$:



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- The wrapping numbers w, m in each direction transform under the modular group of the world-sheet torus:

$$S: \qquad \tau \to -\frac{1}{\tau}\,, \qquad \begin{pmatrix} m \\ w \end{pmatrix} \to \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} m \\ w \end{pmatrix}\,,$$

$$T: \qquad \tau \to \tau + 1\,, \qquad \begin{pmatrix} m \\ w \end{pmatrix} \to \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} m \\ w \end{pmatrix}\,.$$

- The possible values are arranged in modular orbits.
- For the non-trivial wrappings along x^1, x^2, the area is preserved under the modular action.

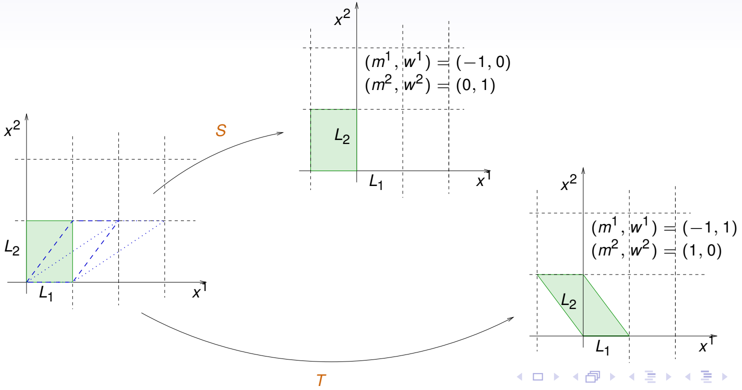
The issue of modular invariance

- There are many choices of winding numbers m^1, m^2, w^1, w^2 that describe toroidal interfaces aligned along the x^1, x^2-plane in target space.
- We are interested in the mappings with minimal area $L_1 L_2$, such as the ones chosen before

$$m^1, \quad m^2 = 1, \quad w^1 = 1, \quad w^2 = 0 \ .$$

The issue of modular invariance

- There are many choices of winding numbers m^1, m^2, w^1, w^2 that describe toroidal interfaces aligned along the x^1, x^2-plane in target space.
- The numbers for area $L_1 L_2$ belong to modular orbits:



The issue of modular invariance

- There are many choices of winding numbers m^1, m^2, w^1, w^2 that describe toroidal interfaces aligned along the x^1, x^2-plane in target space.

- In the partition function we can
  - sum over all the equivalent m^i, w^i and integrate over the fundamental modular cell for τ;
  - or sum over the particular choice $m^1, m^2 = 1, w^1 = 1, w^2 = 0$ and integrate τ over the entire upper half plane.
- The second choice is convenient, as it allows to perform easily the integration.

The interface partition function

- The expression for the partition function of the interface in the first-order, covariant, bosonic string theory, is thus

$$\mathcal{I}^{(d)} = \prod_{i=2}^{d} \left(\sqrt{\frac{\sigma}{2\pi}} L_i \right) \sum_{N,\tilde{N}=0}^{\infty} \sum_{n_1 \in \mathbb{Z}} c_N c_{\tilde{N}} \int_{-\infty}^{\infty} d\tau_1 \mathrm{e}^{2\pi\mathrm{i}(N-\tilde{N}+n_1)} \int_0^{\infty} \frac{d\tau_2}{(\tau_2)^{\frac{d+1}{2}}}$$
$$\times \exp\left\{ -\tau_2 \left[\frac{\sigma L_1^2}{2} + \frac{2\pi^2 n_1^2}{\sigma L_1^2} + 2\pi(N + \tilde{N} - \frac{d-2}{12}) \right] - \frac{1}{\tau_2} \left[\frac{\sigma L_2^2}{2} \right] \right\}$$

The result

- The integration over the parameters τ_1, τ_2 of the world-sheet torus can be performed in terms of Bessel functions of the $K_\nu(z)$ type.
- The final result depends only on the geometry of the target space, in particular on the area $A = L_1 L_2$ and the modulus $u = L_2/L_1$ of the interface plane: ▸ Back

$$\mathcal{I}^{(d)} = 2\left(\frac{\sigma}{2\pi}\right)^{\frac{d-2}{2}} V_T \sum_{m=0}^{\infty}\sum_{k=0}^{m} c_k c_{m-k} \left(\frac{\mathcal{E}}{u}\right)^{\frac{d-1}{2}} K_{\frac{d-1}{2}}(\sigma A \mathcal{E})$$

with V_T the transverse volume and

$$\mathcal{E} = \sqrt{1 + \frac{4\pi u}{\sigma A}(m - \frac{d-2}{12}) + \frac{4\pi u^2 (2k-m)^2}{\xi^2 A^2}}$$

- This expression resums the loop expansion of the functional integral

Check of the result (and new findings)

- Expanding in powers of $1/(\sigma A)$ we get

$$\mathcal{I}^{(d)} \propto \frac{\mathrm{e}^{-\sigma A}}{\eta^{2d-4}(\mathrm{i}u)} \cdot \cdot \left\{ 1 + \frac{(d-2)^2}{2\sigma A} \left[\frac{\pi^2}{36} u^2 E_2^2(\mathrm{i}u) - \frac{\pi}{6} u E_2(\mathrm{i}u) + \frac{d}{8(d-2)} \right] + \ldots \right\}$$

- Not too difficult to go to higher loops. For instance, the 3-rd loop is reported in the paper.

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  - One-loop, universal quantum fluctuations of the $d-2$ transverse directions

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  - Two-loop correction: agrees with Dietz-Filk!

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Comparison with existing simulations

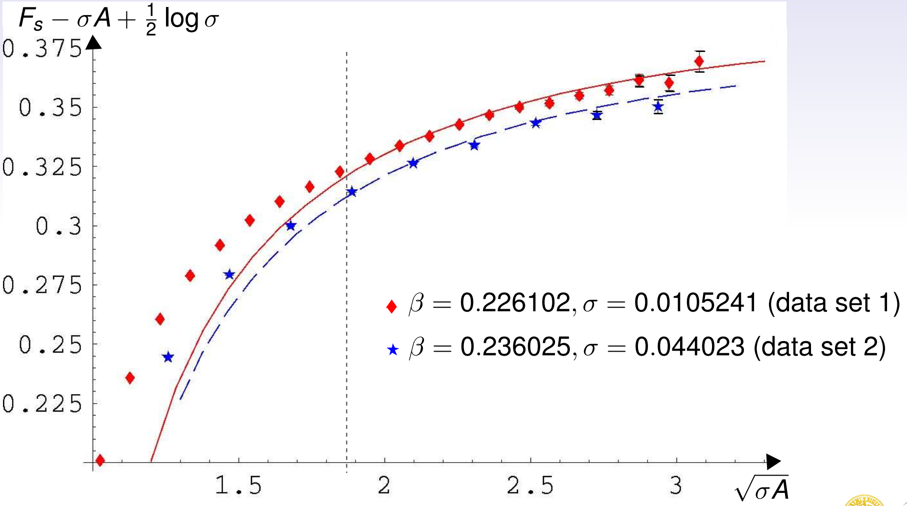
- There are very accurate (and very recent) MC data about the free energy F_s of interfaces in the 3d Ising model

  Caselle et al., 2006
- Previous work has shown that (in certain ranges of parameters) the 3d Ising model indeed has an effective string description.
  - The string tension σ corresponding to certain specific Ising coupling β is known with great accuracy
- We can compare the F_s MC data with the free energy F obtained from our partition function in $d = 3$:

$$F = -\log\left(\frac{\mathcal{I}^{(3)}}{V_T}\right) + \mathcal{N}\ .$$

The constant $\mathcal{N}$ is the only free parameter to be fitted.

Fit to the Monte Carlo data (square lattices)



Fit to the Monte Carlo data (square lattices)

$L_{\min}$	$(\sqrt{\sigma A})_{\min}$	$\mathcal{N}$	$\chi^2/(\mathrm{d.o.f})$
		Data set 1	
19	1.949	0.91957(18)	4.22
20	2.051	0.91891(22)	1.84
21	2.154	0.91836(27)	0.63
22	2.257	0.91829(33)	0.70
23	2.359	0.91797(45)	0.63
		Data set 2	
9	1.888	0.91052(21)	7.22
10	2.098	0.90924(33)	2.71
11	2.308	0.90820(51)	1.12

- The fit of our expression to the two best MC data set avaliable.
  - In each row, only the data corresponding to lattice sizes $L \geq L_{\min}$, i.e., to $\sqrt{\sigma A} \geq (\sqrt{\sigma A})_{\min}$ are used
  - The reduced χ^2 becomes of order unity for $(\sqrt{\sigma A})_{\min} \gtrsim 2$.

Comparison to MC data for rectangular lattices

- In the quoted reference also some data regarding rectangular lattices ($u \neq 1$) are presented.
- Our expression agrees with such data within the (small) error bars:

L_1	L_2	$\sqrt{\sigma A}$	u	F_s	diff ($N = 100$)
10	12	2.29843	6/5	7.1670(6)	0.0016
10	15	2.56972	3/2	8.4449(12)	−0.0004
10	18	2.81498	9/5	9.6976(17)	−0.0009
10	20	2.96725	2	10.5235(25)	−0.0012
10	22	3.11208	11/5	11.3466(36)	0.0017

- No fitted parameters (the normaliz. $\mathcal{N}$ was already fixed by previous fit).

Some remarks

- Any “naive” treatment of bosonic string in $d \neq 26$ suffers from the breaking of conformal invariance (heavily used to solve the model) at the quantum level. This applies to the 1st order treatment we used as well.
  - This manifests itself more and more as the area decreases
  - Our explicit expression of the NG partition function should allow to study the amount and the onset of the discrepancy of the NG model with the “real” (= simulated) interfaces
- There have been some recent attempts in the literature [see Kuti, Lattice 2005] to the inferface partition function using the Polchinski-Strominger string
  - No problems with quantum conformal invariance
  - But non-local terms in the action
  - Apparently (computations are not so detailed) it should agree with NG up to two loops. Discrepancies should appear from then on. Further study of such model is required.

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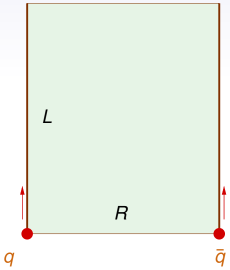
Wilson loops

Rectangular Wilson Loops

- Let us reconsider the Wilson loop (which is the typical test ground for confinement) ▸ Recall

- The effective string partition function for the Wilson loop
  - must be invariant under $L \leftrightarrow R$;
  - must exhibit the area law;
  - must contain the (1-loop universal) transverse bosonic fluctuations responsible of the Lüscher term



Rectangular Wilson Loops

- Let us reconsider the Wilson loop (which is the typical test ground for confinement) ▸ Recall
- Its loop expansion starts as ▸ Back

Dietz-Filk, 1982

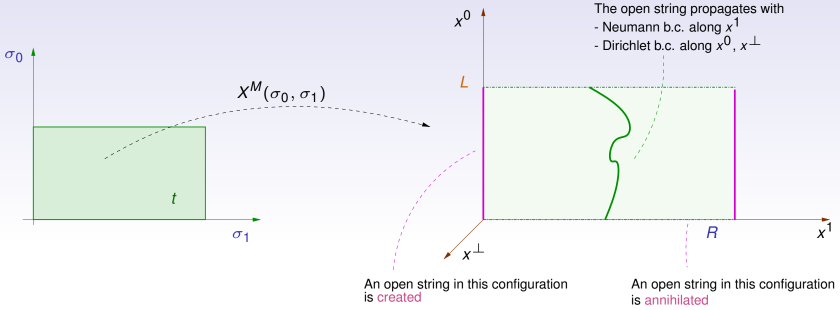
$$\mathcal{Z} \propto \mathrm{e}^{-\sigma A} \frac{1}{[\eta(\mathrm{i}u)]^{\frac{d-2}{2}}} \Big\{ 1 + \frac{1}{\sigma A} \frac{\pi^2}{576} \Big[-5u^2 E_4(\mathrm{i}u) + (d-7) E_2(\mathrm{i}u) E_2(\frac{\mathrm{i}}{u}) \Big] + O\left(\frac{1}{(\sigma A)^2} \right) \Big\}$$

with $A = LR$, $u = L/R$.

First-order formulation for the Wilson loop

- We are working on the first-order, operatorial derivation of the Wilson loop partition function.
  - Analogously to the Polyakov loop and interface cases, we should be able to get the exact expression resumming the loop expansion
- At the moment we are facing some problems and we are unsure about the result. Still, some ideas involved in this computation are interesting.
- Let's sketch some points.

Operatorial description



$$\langle B_{\rm op}(0)| \int \frac{dt}{t^{\omega}} \mathrm{e}^{-2\pi t L_0} |B_{\rm op}(R)\rangle$$

- $|B_{\rm op}\rangle$ is an open string boundary state: [Imamura et al, 2005]

$$X^0(\xi^0,0)|B_{\rm op}(0)\rangle = \frac{\xi^0 L}{\pi}|B_{\rm op}(0)\rangle~, \quad X^1(\xi^0,0)|B_{\rm op}\rangle = 0~.$$

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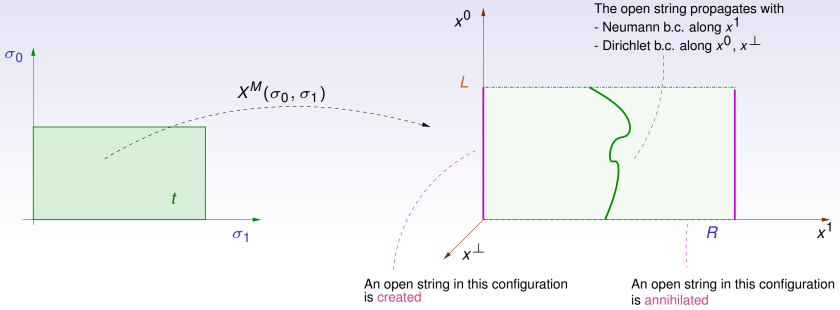

$$\langle B_{\rm op}(0)| \int \frac{dt}{t^\omega} e^{-2\pi t L_0} |B_{\rm op}(R)\rangle$$

- For $\omega = 0$, usual parametriz. of the open string propagator; however a different value has to be assumed for the result to be invariant under $L \leftrightarrow R$. (Puzzling)

A puzzling result (for the moment)

- The open boundary states can be given an explicit expression in terms of oscillators (they correspond to states of definite position or momentum for each harmonic oscillator a_n, $a_n^\dagger$)
- The matrix elements and the integration over t can be carried out.
- The result can be expressed in terms of Bessel functions, similarly to the cases already considered.
- Expanding the result for large A
  - we recover the functional integral result up to 1 loop; ▸ Recall
  - the second loop has the same form, but with slightly different coefficients...
- It is not clear if this is due to errors in our computation, or to some deeper reason like some difference between the different approaches to the effective string which manifest themselves with the Wilson loop bc's only.

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- The covariant quantization in 1st order formalism of the NG action is a convenient way to derive partition functions of the string with different b.c.s, related to different LGT observables
  - It reproduces the partition function based on Arvis spectrum for the Polyakov loop correlator case ∼ D0-brane interaction with compact time
  - It yields the partition function for the interfaces ∼ appropriate sector of one loop closed strings
  - The situation for the Wilson loops is not yet clear; in any case, ∼ propagation of open strings between suitable open boundary states.
- Our approach is “naïve”, as it neglects the breaking of conformal invariance in $d < 26$. This is more and more manifest as the minimal area A decreases.

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Outlook

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  - The most immediate:
    - understand fully the Wilson loop case.
  - Investigate if these techniques can be useful for considering so-called k-strings instead of Polyakov loops.
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