$\mathcal{N} = \mathbf{1/2}$ *gauge theory and its instantons from open strings in R-R background*

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UNIVERSITÀ DEGLI STUDI DI TORINO

This talk is based on...

- M. Billo, M. Frau, I. Pesando, F. Fucito, A. Lerda and A. Liccardo, "Classical gauge instantons from open strings," JHEP **0302** (2003) 045 [arXiv:hep-th/0211250].
- M. Billo, M. Frau, I. Pesando and A. Lerda, "N = 1/2 gauge theory and its instanton moduli space from open strings in R-R background," arXiv:hep-th/0402160.

Outline

For a general introduction...

... see previous talk!

Gauge theories from String Theory

- String theory (which might well lead us to the T.O.E.) is anyhow, more modestly, a very precious tool to study gauge theories. For instance,
  - perturbative amplitudes (may gluons, ...) via string techniques;
  - AdS/CFT and its extensions;
  - instantonic effects, (see previous talk).
- In the string framework, gauge d.o.f. arise from open strings suspended between D-branes in a well-suited limit

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Gauge theories in closed string backgrounds

- Open strings interact with closed strings. We can turn on a closed string background and still look at the massless open string d.o.f..
- In this way, deformations of the gauge theory are naturally suggested by their string realization. Such deformations are characterized by
  - new geometry in (super)space-time;
  - new mathematical structures;
  - new types of interactions and couplings.

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Non-commutative field theories and NS-NS backgrounds

$B_{\mu\nu}$ background: new geometry

- The most famous example is that of (gauge) field theories in the background of the $B^{\mu\nu}$ field of the NS-NS sector of closed string.
- They are a stringy realization of non-commutative field theories, *i.e.* theories defined on a non commutative space-time:

$$[x^\mu, x^\nu] = \theta^{\mu\nu}(B) \ .$$

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Non-commutative field theories and NS-NS backgrounds

$B_{\mu\nu}$ background: new mathematical structure

- There arises a non-commutative associative algebra: ordinary product $\longrightarrow$ Moyal $\star$ product:

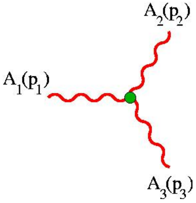
$$f(x)\star g(x) = f(x)\exp\left(\frac{i}{2}\overleftarrow{\frac{\partial}{\partial x^\mu}}\theta^{\mu\nu}\overrightarrow{\frac{\partial}{\partial x^\nu}}\right)g(x)$$

$$= f(x)g(x) + \frac{i}{2}\partial_\mu f(x)\theta^{\mu\nu}\partial_\nu g(x) + \mathcal{O}(\theta^2)$$

Non-commutative field theories and NS-NS backgrounds

$B_{\mu\nu}$ background: new interactions

There are new interactions and couplings.



For instance, a 3-photon coupling in the $\mathrm{U}(1)$ theory

$$\left(A_1 \cdot A_2 \; p_2 \cdot A_3 + \text{cyclic} \;\right) \; \underline{\underline{p_1 \cdot \theta \cdot p_2}} + \mathcal{O}(\theta^3) \; .$$

Non-anticommutative theories and RR backgrounds

$C_{\mu\nu}$ RR background: new geometry

- Another case, recently attracting attention, is that of gauge (and matter) fields in the background of a "graviphoton" field strength $C_{\mu\nu}$ from the Ramond-Ramond sector of closed strings.
- These turn out to be defined on a non-anticommutative superspace, where the, say, anti-chiral fermionic coordinates satisfy

$$\left\{\theta^{\dot\alpha}, \theta^{\dot\beta}\right\} \propto C^{\dot\alpha\dot\beta} \propto (\sigma^{\mu\nu})^{\dot\alpha\dot\beta} C_{\mu\nu}.$$

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$C_{\mu\nu}$ RR background: new structure

- The superspace deformation can be rephrased as a modification of the product among functions, which now becomes

$$f(\theta) \star g(\theta) = f(\theta) \exp\Big(- \frac{1}{2} \frac{\overleftarrow{\partial}}{\partial \theta^{\dot\alpha}} C^{\dot\alpha\dot\beta} \frac{\overrightarrow{\partial}}{\partial \theta^{\dot\beta}} \Big) g(\theta) \ .$$

- There are also new interactions between the gauge and matter fields: see later in the talk.

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The focus of the talk

- We shall analyze a particular deformation of a gauge theory induced by a RR background.
- This is the case where the $\mathcal{N} = 1$ superspace becomes partly non-anticommutative because of the "graviphoton" $C_{\mu\nu}$ background and the pure $\mathcal{N} = 1$ gauge theory is deformed to the so-called $\mathcal{N} = 1/2$ theory of [Seiberg, 2003].
- We shall derive explicitely from string diagrams (in the traditional RNS formulation) the $\mathcal{N} = 1/2$ theory.
- Moreover, along the lines of the previous talk, we will derive from string diagrams the instantonic solutions of this theory and their ADHM moduli space.

The $\mathcal{N} = \mathbf{1/2}$ gauge theory from open strings

We will now proceed as follows.

- Review the set-up to retrieve the action of pure $\mathcal{N} = 1$ gauge theory from open string disk amplitudes.
- Retrieve the action of the so-called $\mathcal{N} = 1/2$ gauge theory [Seiberg, 2003] by inserting closed string vertices for a certain constant RR field strength.

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The set-up

- Type IIB string theory on target space

$$\mathbb{R}^4 \times \frac{\mathbb{R}^6}{\mathbb{Z}_2 \times \mathbb{Z}_2}$$

Decompose $x^M \to (x^\mu, x^a)$, $(\mu = 1, \ldots 4\,, \quad a = 5, \ldots, 10$.

- $\mathbb{Z}_2 \times \mathbb{Z}_2 \subset \mathrm{SO}(6)$ is generated by
  - g_1: a rotation by π in the 7-8 and by $-\pi$ in the 9-10 plane;
  - g_1: a rotation by π in the 5-6 and by $-\pi$ in the 9-10 plane.
- The origin is a fixed point $\Rightarrow$ the orbifold is a singular, non-compact, Calabi-Yau space.

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The set-up: Killing spinors

- Of the 8 spinor weights of $\mathrm{SO}(6)$,

$$\vec{\lambda} = (\pm\frac{1}{2}, \pm\frac{1}{2}, \pm\frac{1}{2})$$

it is easy to see that the only invariant ones w.r.t. the generators $g_{1,2}$ are

$$\vec{\lambda}^{(+)} = (+\frac{1}{2}, +\frac{1}{2}, +\frac{1}{2}) \ , \qquad \vec{\lambda}^{(-)} = (-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2})$$

(resp. chiral and anti-chiral). In this orbifold realization, they describe the $2 (= 8/4)$ Killing spinors of the CY.

- We remain with $8 (= 32/4)$ real susies in the bulk.

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The set-up: internal spin fields

- Bosonizing the $\mathrm{SO}(6)$ current algebra by

$$\mathrm{e}^{\mathrm{i}\varphi_1} = \frac{\psi^5 + \mathrm{i}\psi^6}{\sqrt{2}}\ ,\ \ \mathrm{e}^{\mathrm{i}\varphi_2} = \frac{\psi^7 + \mathrm{i}\psi^8}{\sqrt{2}}\ ,\ \ \mathrm{e}^{\mathrm{i}\varphi_3} = \frac{\psi^9 + \mathrm{i}\psi^{10}}{\sqrt{2}}\ .$$

  (up to cocycles), the spin fields are $S^{\vec{\lambda}} = \mathrm{e}^{\mathrm{i}\lambda^i\varphi_i}$.
- The correlators of spin fields are immediate upon use of

$$\langle\varphi_i(z)\varphi_j(w)\rangle = \delta_{ij}\log(z-w)\ .$$

- Only two of these internal spin fields survive the orbifold projection:

$$S^{(\pm)} = \mathrm{e}^{\mathrm{i}\lambda^{(\pm)i}\varphi_i} = \mathrm{e}^{\pm\frac{\mathrm{i}}{2}(\varphi_1+\varphi_2+\varphi_3)}\ .$$

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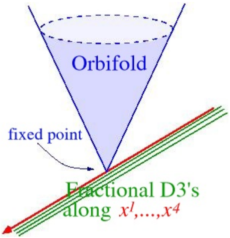
Fractional D3 branes



- Place N fractional D3 branes, localized at the orbifold fixed point. The branes preserve $4 = 8/2$ real supercharges.
- The Chan-Patons of open strings attached to fractional branes transform in an irrep of $\mathbb{Z}_2 \times \mathbb{Z}_2$.
- The fractional branes must sit at the orbifold fixed point *(otherwise would transform in the reducible regular rep)*

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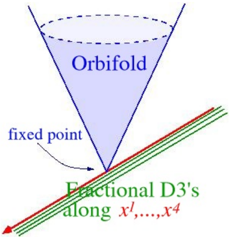

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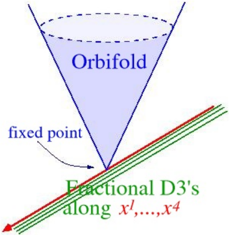

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Fractional D3 branes and pure $\mathcal{N} = 1$ gauge theory

- Spectrum of massless open strings attached to the N fractional D3's corresponds to $\mathcal{N} = 1$ pure $\mathrm{U}(N)$ gauge theory. Schematically,

$$\text{NS:} \begin{cases} \psi^\mu & \rightarrow \quad A_\mu \\ \not{\psi}^a & \quad\ \ \text{no scalars!} \end{cases} \qquad \text{R:} \begin{cases} S^\alpha S^{(+)} & \rightarrow \quad \Lambda_\alpha \\ S^{\dot\alpha} S^{(-)} & \rightarrow \quad \Lambda_{\dot\alpha} \end{cases}$$

- The action is retrieved from disk amplitudes in the $\alpha' \rightarrow 0$ limit, as described in Alberto's talk. One gets indeed

$$S = \frac{1}{g^2_{\mathrm{YM}}} \int d^4x \, \mathrm{Tr}\Big(\frac{1}{2} F^2_{\mu\nu} - 2\bar\Lambda_{\dot\alpha} \bar{\not{D}}^{\dot\alpha\beta} \Lambda_\beta\Big) \, .$$

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Auxiliary fields

- The action can be obtained from cubic diagram only introducing the (anti-selfdual) auxiliary field $H_{\mu\nu} \equiv H_c \bar{\eta}^c_{\mu\nu}$:

$$S' = \frac{1}{g^2_{\rm YM}} \int d^4x \; {\rm Tr}\Big\{ (\partial_\mu A_\nu - \partial_\nu A_\mu)\partial^\mu A^\nu + 2{\rm i}\,\partial_\mu A_\nu \left[A^\mu, A^\nu\right] - 2\bar{\Lambda}_{\dot\alpha} \bar{\not{D}}^{\dot\alpha\beta}\Lambda_\beta + H_c H^c + H_c\,\bar{\eta}^c_{\mu\nu}\left[A^\mu, A^\nu\right] \Big\} \;,$$

- Integrating out H_c gives $H_{\mu\nu} \propto [A_\mu, A_\nu]$ and the usual action
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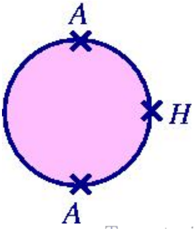
Auxiliary fields in the open string set-up

- The auxiliary field $H_{\mu\nu}$ is associated to the (non-BRST invariant) vertex

$$V_H(y;p) = (2\pi\alpha') \, \frac{H_{\mu\nu}(p)}{2} \, \psi^\nu \psi^\mu(y) \, \mathrm{e}^{i\sqrt{2\pi\alpha'}p\cdot X(y)} \; .$$



We have then, for instance,

$$\frac{1}{2}\langle\!\langle \, V_H \, V_A \, V_A \, \rangle\!\rangle \;\; = \;\; -\frac{1}{g^2_{\rm YM}} \, \mathrm{Tr}\Big(H_{\mu\nu}(p_1) A^\mu(p_2) A^\nu(p_3) \Big)$$
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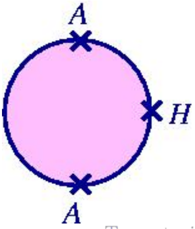

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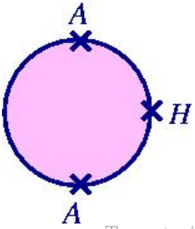

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The graviphoton background

- RR vertex in 10D, in the symmetric superghost picture:

$$\mathcal{F}_{\dot{A}\dot{B}}\, S^{\dot{A}} \mathrm{e}^{-\phi/2}(z)\, \tilde{S}^{\dot{B}} \mathrm{e}^{-\tilde{\phi}/2}(\bar{z}) \ .$$

Bispinor $\mathcal{F}_{\dot{A}\dot{B}} \rightsquigarrow$ 1-, 3- and a.s.d. 5-form field strengths.

- On $\mathbb{R}^4 \times \frac{\mathbb{R}^6}{\mathbb{Z}_2 \times \mathbb{Z}_2}$, a surviving 4D bispinor vertex is

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with $\mathcal{F}_{\dot{\alpha}\dot{\beta}} = \mathcal{F}_{\dot{\beta}\dot{\alpha}}$.

- This $\sim$ decomposing the 5-form along the holom. 3-form of the CY $\rightsquigarrow$ an a.s.d. 2-form in 4D

$$C_{\mu\nu} \propto \mathcal{F}_{\dot{\alpha}\dot{\beta}} (\bar{\sigma}^{\mu\nu})^{\dot{\alpha}\dot{\beta}} \ ,$$

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Bispinor $\mathcal{F}_{\dot{A}\dot{B}} \rightsquigarrow$ 1-, 3- and a.s.d. 5-form field strengths.

- On $\mathbb{R}^4 \times \frac{\mathbb{R}^6}{\mathbb{Z}_2 \times \mathbb{Z}_2}$, a surviving 4D bispinor vertex is

$$\mathcal{F}_{\dot{\alpha}\dot{\beta}}\, S^{\dot{\alpha}} S^{(+)} \mathrm{e}^{-\phi/2}(z)\, \tilde{S}^{\dot{\beta}} \tilde{S}^{(+)} \mathrm{e}^{-\tilde{\phi}/2}(\bar{z}) \ .$$

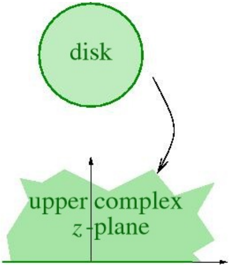
with $\mathcal{F}_{\dot{\alpha}\dot{\beta}} = \mathcal{F}_{\dot{\beta}\dot{\alpha}}$.

- This $\sim$ decomposing the 5-form along the holom. 3-form of the CY $\rightsquigarrow$ an a.s.d. 2-form in 4D

$$C_{\mu\nu} \propto \mathcal{F}_{\dot{\alpha}\dot{\beta}} (\bar{\sigma}^{\mu\nu})^{\dot{\alpha}\dot{\beta}} \ ,$$

the graviphoton f.s. of $\mathcal{N} = 1/2$ theories.

Inserting graviphotons in disk amplitudes



- Conformally mapping the disk to the upper half z-plane, the D3 boundary conditions on spin fields read

$$S^{\dot\alpha} S^{(+)}(z) = \tilde S^{\dot\alpha} \tilde S^{(+)}(\bar z)\Big|_{z=\bar z} .$$

(opposite sign for $\tilde S^{\alpha} \tilde S^{(+)}(\bar z)$).

- When closed string vertices are inserted in a D3 disk,

$$\tilde S^{\dot\alpha} \tilde S^{(+)}(\bar z) \longrightarrow S^{\dot\alpha} S^{(+)}(\bar z) .$$

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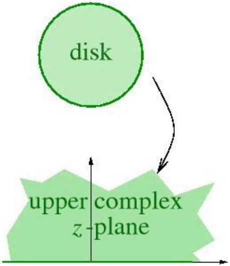

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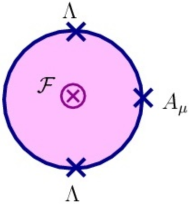
Disk amplitudes with a graviphoton

Start inserting a graviphoton vertex:

$$\langle\!\langle\, V_\Lambda\, V_\Lambda\, V_{A}\, V_{\mathcal{F}} \,\rangle\!\rangle$$



where

$$V_{\mathcal{F}}(z,\bar z) = \mathcal{F}_{\dot\alpha\dot\beta}\, S^{\dot\alpha} S^{(+)} \mathrm{e}^{-\phi/2}(z)\, S^{\dot\beta} S^{(+)} \mathrm{e}^{-\phi/2}(\bar z)\ .$$

$\rightsquigarrow$ we need two $S^{(-)}$ operators to "saturate the charge"

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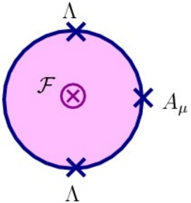

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We insert therefore two chiral gauginos:

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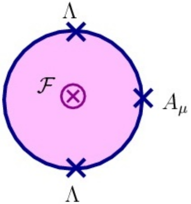

with vertices

$$V_\Lambda(y;p) \;=\; (2\pi\alpha')^{\frac{3}{4}}\, \Lambda^\alpha(p)\, S_\alpha S^{(-)}\, \mathrm{e}^{-\frac{1}{2}\phi(y)}\, \mathrm{e}^{i\sqrt{2\pi\alpha'}p\cdot X(y)} \;.$$

Without other insertions, however,

$$\langle S^{\dot\alpha} S^{\dot\beta} S_\alpha S_\beta \rangle \propto \epsilon^{\dot\alpha\dot\beta}\epsilon_{\alpha\beta}$$

$\rightsquigarrow$ vanishes when contracted with $\mathcal{F}_{\dot\alpha\dot\beta}$.

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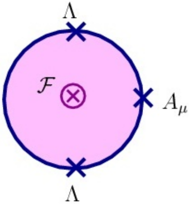

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Disk amplitudes with a graviphoton

To this effect, insert a gauge field vertex:

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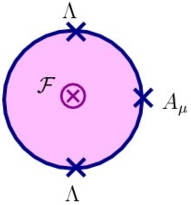

that must be in the 0 picture:

$$V_A(y;p) = 2\mathrm{i}\,(2\pi\alpha')^{\frac{1}{2}}\, A_\mu(p) \Big(\partial X^\mu(y) + \mathrm{i}\,(2\pi\alpha')^{\frac{1}{2}}\, p\cdot\psi\,\psi^\mu(y)\Big) \mathrm{e}^{\mathrm{i}\sqrt{2\pi\alpha'}p\cdot X(y)}$$

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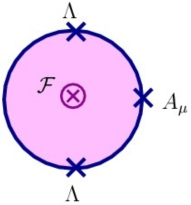

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Evaluation of the amplitude

- We have

$$\langle\!\langle\, V_\Lambda V_\Lambda V_A V_{\mathcal{F}} \,\rangle\!\rangle \;\equiv\; C_4 \int \frac{\prod_i dy_i dz d\bar{z}}{dV_{\rm CKG}} \left\langle V_\Lambda(y_1;p_1) V_\Lambda(y_2;p_2) V_A(y_3;p_3) V_{\mathcal{F}}(z,\bar{z}) \right\rangle$$

where the normalization for a D3 disk is

$$C_4 = \frac{1}{\pi^2 \alpha'^2} \, \frac{1}{g^2_{\rm YM}}$$

and the $\mathrm{SL}(2,\mathbb{R})$-invariant volume is

$$dV_{\rm CGK} = \frac{dy_a \, dy_b \, dy_c}{(y_a - y_b)(y_b - y_c)(y_c - y_a)} \quad .$$

Explicit expression of the amplitude

- Altogether, the explicit expression is

$$\langle\!\langle\, V_\Lambda\, V_\Lambda\, V_A\, V_{\mathcal{F}}\, \rangle\!\rangle = \frac{8}{g^2_{\rm YM}}\,(2\pi\alpha')^{\frac{1}{2}}\,{\rm Tr}\Big(\Lambda^\alpha(p_1)\,\Lambda^\beta(p_2)\,p_3^\nu A^\mu(p_3)\Big)\,\mathcal{F}_{\dot\alpha\dot\beta}$$
$$\times \int \frac{\prod_i dy_i dz d\bar z}{dV_{\rm CKG}}\,\Big\{\big\langle S_\alpha(y_1)\,S_\beta(y_2)\;:\psi^\nu\psi^\mu:(y_3)\,S^{\dot\alpha}(z)\,S^{\dot\beta}(\bar z)\big\rangle$$
$$\times \big\langle S^{(-)}(y_1)\,S^{(-)}(y_2)\,S^{(+)}(z)\,S^{(+)}(\bar z)\big\rangle$$
$$\times \big\langle {\rm e}^{-\frac{1}{2}\phi(y_1)}{\rm e}^{-\frac{1}{2}\phi(y_2)}{\rm e}^{-\frac{1}{2}\phi(z)}{\rm e}^{-\frac{1}{2}\phi(\bar z)}\big\rangle$$
$$\times \big\langle {\rm e}^{{\rm i}\sqrt{2\pi\alpha'}p_1\cdot X(y_1)}{\rm e}^{{\rm i}\sqrt{2\pi\alpha'}p_2\cdot X(y_2)}{\rm e}^{{\rm i}\sqrt{2\pi\alpha'}p_3\cdot X(y_3)}\big\rangle\Big\}\;.$$

Evaluation of the amplitude: correlators

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1. Superghosts

$$\left\langle \mathrm{e}^{-\frac{1}{2}\phi(y_1)}\,\mathrm{e}^{-\frac{1}{2}\phi(y_2)}\,\mathrm{e}^{-\frac{1}{2}\phi(z)}\,\mathrm{e}^{-\frac{1}{2}\phi(\bar{z})}\right\rangle$$

$$= \Big[(y_1 - y_2)\,(y_1 - z)\,(y_1 - \bar{z})\,(y_2 - z)\,(y_2 - \bar{z})\,(z - \bar{z})\Big]^{-\frac{1}{4}} \quad .$$

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2. Internal spin fields

$$\left\langle S^{(-)}(y_1)S^{(-)}(y_2)S^{(+)}(z)S^{(+)}(\bar{z})\right\rangle = (y_1-y_2)^{\frac{3}{4}}(y_1-z)^{-\frac{3}{4}}(y_1-\bar{z})^{-\frac{3}{4}}(y_2-z)^{-\frac{3}{4}}(y_2-\bar{z})^{-\frac{3}{4}} \times (z-\bar{z})^{\frac{3}{4}} \quad .$$

Evaluation of the amplitude: correlators

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3. 4D spin fields

$$\begin{aligned}&\left\langle S_\gamma(y_1)S_\delta(y_2) :\psi^\mu\psi^\nu:(y_3)\, S^{\dot\alpha}(z)S^{\dot\beta}(\bar z)\right\rangle\\ &= \frac{1}{2}\,(y_1-y_2)^{-\frac{1}{2}}(z-\bar z)^{-\frac{1}{2}}\\ &\times\left((\sigma^{\mu\nu})_{\gamma\delta}\,\varepsilon^{\dot\alpha\dot\beta}\,\frac{(y_1-y_2)}{(y_1-y_3)(y_2-y_3)}\right.\\ &\quad\left.+\,\varepsilon_{\gamma\delta}\,(\bar\sigma^{\mu\nu})^{\dot\alpha\dot\beta}\,\frac{(z-\bar z)}{(y_3-z)(y_3-\bar z)}\right)\quad .\end{aligned}$$

Evaluation of the amplitude: correlators

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4. Momentum factors

$$\left\langle e^{i\sqrt{2\pi\alpha'}p_1\cdot X(y_1)}e^{i\sqrt{2\pi\alpha'}p_2\cdot X(y_2)}e^{i\sqrt{2\pi\alpha'}p_3\cdot X(y_3)}\right\rangle \xrightarrow{\text{on shell}} 1 \ .$$

Evaluation of the amplitude: $\mathrm{SL}(2,\mathbb{R})$ *fixing*

- We may, for instance, choose

$$y_1 \to \infty \; , \quad z \to \mathrm{i} \; , \quad \bar{z} \to -\mathrm{i} \; .$$

- The remaining integrations turn out to be

$$\int_{-\infty}^{+\infty} dy_2 \int_{-\infty}^{y_2} dy_3 \, \frac{1}{\left(y_2^2+1\right)\left(y_3^2+1\right)} = \frac{\pi^2}{2} \; .$$

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Final result for the amplitude

- We finally obtain for $\langle\!\langle\, V_\Lambda\, V_\Lambda\, V_A\, V_{\mathcal{F}} \,\rangle\!\rangle$ the result

$$\frac{8\pi^2}{g^2_{\rm YM}}\,(2\pi\alpha')^{\frac{1}{2}}\,{\rm Tr}\Big(\Lambda(p_1)\cdot\Lambda(p_2)\,p_3^\nu A^\mu(p_3)\Big)\,\mathcal{F}_{\dot\alpha\dot\beta}\,(\bar\sigma_{\nu\mu})^{\dot\alpha\dot\beta}\quad .$$

- This result is finite for $\alpha' \to 0$ if we keep constant

$$C_{\mu\nu} \equiv 4\pi^2\,(2\pi\alpha')^{\frac{1}{2}}\,\mathcal{F}_{\dot\alpha\dot\beta}\,(\bar\sigma_{\mu\nu})^{\dot\alpha\dot\beta}$$

- $C_{\mu\nu}$, of dimension (length) will be exactly the one of $\mathcal{N} = 1/2$ theory.
- We get an extra term in the gauge theory action:

$$\frac{i}{g^2_{\rm YM}}\int d^4x\;{\rm Tr}\Big(\Lambda\cdot\Lambda\;(\partial^\mu A^\nu - \partial^\nu A^\mu)\Big)C_{\mu\nu}\;.$$

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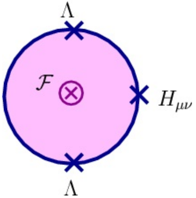
Another contribute

- Another possible diagram with a graviphoton insertion is

$$\langle\!\langle\, V_\Lambda\, V_\Lambda\, V_H\, V_{\mathcal{F}} \,\rangle\!\rangle\,.$$



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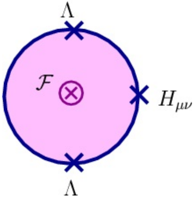

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- Recall that the auxiliary field vertex in the 0 picture is

$$V_H(y;p) = (2\pi\alpha')\frac{H_{\mu\nu}(p)}{2}\psi^\nu\psi^\mu(y)\mathrm{e}^{i\sqrt{2\pi\alpha'}p\cdot X(y)}$$

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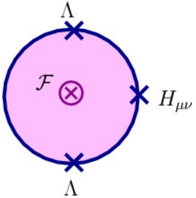

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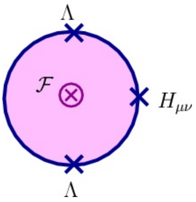
- The evaluation of this amplitude paralles exactly the previous one and contributes to the field theory action the term:

$$\frac{1}{2g_{\rm YM}^2}\int d^4x\;\mathrm{Tr}\left(\Lambda\cdot\Lambda\, H^{\mu\nu}\right)C_{\mu\nu}\;,$$

having introduced $C_{\mu\nu}$ as above.

Another contribute



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- All other amplitudes involving $\mathcal{F}$ vertices either
  - vanish because of their tensor structure;
  - vanish in the $\alpha' \to 0$ limit, with $C_{\mu\nu}$ fixed.

The deformed gauge theory action

- From disk diagrams with RR insertions we obtain, in the field theory limit

$$\boxed{\alpha' \to 0 \quad \text{with } C_{\mu\nu} \text{ fixed}}$$

the action

$$\begin{aligned}\tilde{S}' = \frac{1}{g^2_{\mathrm{YM}}} \int d^4x \, \mathrm{Tr}\Big\{ & \big(\partial_\mu A_\nu - \partial_\nu A_\mu\big)\partial^\mu A^\nu + 2\mathrm{i}\, \partial_\mu A_\nu \big[A^\mu, A^\nu\big] \\ & - 2\bar{\Lambda}_{\dot\alpha} \bar{\not{D}}^{\dot\alpha\beta} \Lambda_\beta + \mathrm{i}\big(\partial^\mu A^\nu - \partial^\nu A^\mu\big)\Lambda\cdot\Lambda\, C_{\mu\nu} \\ & + H_c H^c + H_c\, \bar\eta^c_{\mu\nu}\Big(\big[A^\mu, A^\nu\big] + \frac{1}{2}\Lambda\cdot\Lambda\, C^{\mu\nu}\Big)\Big\} \quad .\end{aligned}$$

The deformed gauge theory action

- Integrating on the auxiliary field H_c, we get

$$\begin{aligned}\tilde{S} = \frac{1}{g_{\mathrm{YM}}^2} \int d^4x \; \mathrm{Tr}\Big\{ &\frac{1}{2} F_{\mu\nu}^2 - 2\bar{\Lambda}_{\dot{\alpha}} \bar{\not{D}}^{\dot{\alpha}\beta} \Lambda_\beta \\ &+ \mathrm{i}\, F^{\mu\nu}\, \Lambda\cdot\Lambda\, C_{\mu\nu} - \frac{1}{4} \Big(\Lambda\cdot\Lambda\, C_{\mu\nu}\Big)^2 \Big\} \\ = \frac{1}{g_{\mathrm{YM}}^2} \int d^4x \; \mathrm{Tr}\Big\{ &\Big(F_{\mu\nu}^{(-)} + \frac{\mathrm{i}}{2} \Lambda\cdot\Lambda\, C_{\mu\nu}\Big)^2 + \frac{1}{2} F_{\mu\nu} \widetilde{F}^{\mu\nu} \\ &- 2\bar{\Lambda}_{\dot{\alpha}} \bar{\not{D}}^{\dot{\alpha}\beta} \Lambda_\beta \Big\}\,,\end{aligned}$$

i.e., exactly the action of Seiberg's $\mathcal{N} = 1/2$ gauge theory.

The deformed gauge theory action

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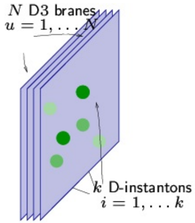
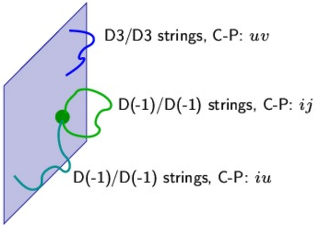
$$= \frac{1}{g^2_{\mathrm{YM}}} \int d^4x \; \mathrm{Tr}\left\{ \boxed{\left(F^{(-)}_{\mu\nu} + \frac{\mathrm{i}}{2}\Lambda\cdot\Lambda\, C_{\mu\nu}\right)^2} + \frac{1}{2} F_{\mu\nu}\widetilde{F}^{\mu\nu} - 2\bar{\Lambda}_{\dot{\alpha}} \bar{\not{D}}^{\dot{\alpha}\beta} \Lambda_\beta \right\} .$$

$\rightsquigarrow$ How is the instantonic sector affected?

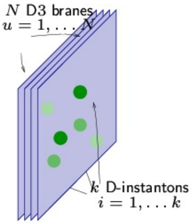
Instantonic effects in the deformed theory

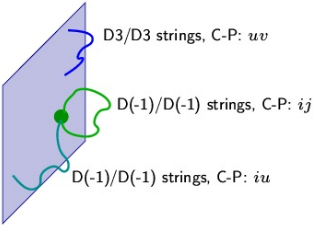




- As we saw in the previous talk, adding (fractional) D(-1) branes to the D3's $\rightsquigarrow$ instantonic sectors in the gauge theory.

Instantonic effects in the deformed theory

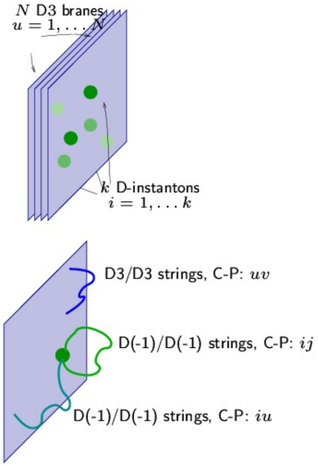




- As we saw in the previous talk, adding (fractional) D(-1) branes to the D3's $\rightsquigarrow$ instantonic sectors in the gauge theory.
- The open strings stretching
  - between a D(-1) and another D(-1);
  - between a D(-1) and a D3

  carry no momentum $\rightsquigarrow$ ADHM moduli in the gauge theory.
- Disks with D(-1) and mixed D(-1)/D3 boundary $\rightsquigarrow$ "measure" on moduli space

Instantonic effects in the deformed theory



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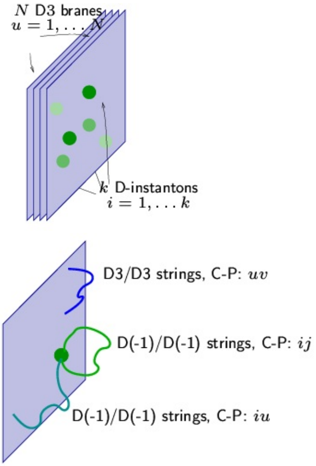
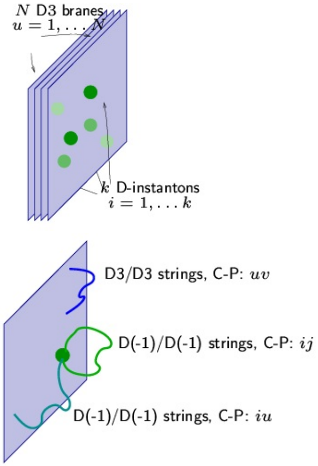


- As we saw in the previous talk, adding (fractional) D(-1) branes to the D3's $\rightsquigarrow$ instantonic sectors in the gauge theory.
- Mixed D(-1)/D3 disks can emit gauge theory fields $\rightsquigarrow$ produce the instantonic solutions of the gauge theory.

Instantonic effects in the deformed theory

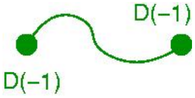


- As we saw in the previous talk, adding (fractional) D(-1) branes to the D3's $\rightsquigarrow$ instantonic sectors in the gauge theory.
- We shall now
  - Review this in the $\mathcal{N} = 1$ case;
  - Deform it with the graviphoton.

Moduli spectrum in the $\mathcal{N} = 1$ case

D(-1)/D(-1) strings

With k D(-1)'s, all vertices have Chan-Paton factors in the adjoint of $\mathrm{U}(k)$.



NS sector

The vertices surviving the orbifold projection are

$$V_a(y) = (2\pi\alpha')^{\frac{1}{2}}\, g_0\, a_\mu\ \psi^\mu(y)\, \mathrm{e}^{-\phi(y)}\ .$$

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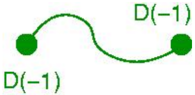

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- Here g_0 is the coupling on the D(-1) theory:

$$C_0 = \frac{1}{2\pi^2\alpha'^2}\,\frac{1}{g_0^2} = \frac{8\pi^2}{g_{\mathrm{YM}}^2} \ .$$

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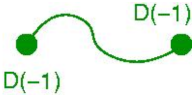

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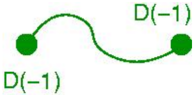
- C_0 = normaliz. of disks with (partly) D(-1) boundary. Since g_{YM} is fixed as $\alpha' \to 0$, g_0 blows up.

Moduli spectrum in the $\mathcal{N} = 1$ case



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- The moduli a_μ are *rescaled* with powers of g_0 so that their interactions survive when $\alpha' \to 0$ with g^2_{YM} fixed.

Moduli spectrum in the $\mathcal{N} = 1$ case

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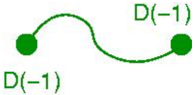

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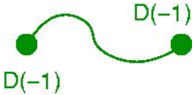
- The moduli a_μ have dimension (length) $\sim$ positions of the (multi)center of the instanton

Moduli spectrum in the $\mathcal{N}=1$ case

D(-1)/D(-1) strings



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NS sector

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$$V_a(y) = (2\pi\alpha')^{\frac{1}{2}}\, g_0\, a_\mu\, \psi^\mu(y)\, \mathrm{e}^{-\phi(y)} \ .$$

Moreover, we have the auxiliary vertex decoupling the quartic interactions

$$V_D(y) = (2\pi\alpha')\, \frac{D_c\, \bar{\eta}^c_{\mu\nu}}{2}\, \psi^\nu \psi^\mu(y) \ ,$$

Moduli spectrum in the $\mathcal{N} = 1$ case

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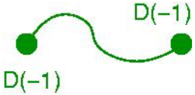


Ramond sector

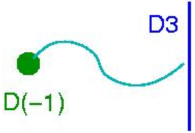
The vertices surviving the orbifold projection are

$$V_M(y) = (2\pi\alpha')^{\frac{3}{4}}\,\frac{g_0}{\sqrt{2}}\,M'^{\alpha}\,S_\alpha(y)S^{(-)}(y)\,\mathrm{e}^{-\frac{1}{2}\phi(y)} \quad ,$$

$$V_\lambda(y) = (2\pi\alpha')^{\frac{3}{4}}\,\lambda_{\dot\alpha}\,S^{\dot\alpha}(y)S^{(+)}(y)\,\mathrm{e}^{-\frac{1}{2}\phi(y)} \quad .$$

- M'^{α} has dimensions of $(\text{length})^{\frac{1}{2}}$;
  $\lambda_{\dot\alpha}$ has dimensions of $(\text{length})^{-\frac{3}{2}}$.

Moduli spectrum in the $\mathcal{N} = 1$ case



D(-1)/D3 strings

All vertices have Chan-Patons in the bifundamental of $\mathrm{U}(k) \times \mathrm{U}(N)$.

NS sector

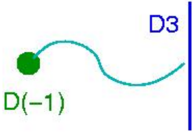
The vertices surviving the orbifold projection are

$$V_w(y) = (2\pi\alpha')^{\frac{1}{2}} \frac{g_0}{\sqrt{2}} w_{\dot\alpha}\, \Delta(y)\, S^{\dot\alpha}(y)\, \mathrm{e}^{-\phi(y)} \quad ,$$

$$V_{\bar w}(y) = (2\pi\alpha')^{\frac{1}{2}} \frac{g_0}{\sqrt{2}} \bar w_{\dot\alpha}\, \bar\Delta(y)\, S^{\dot\alpha}(y)\, \mathrm{e}^{-\phi(y)} \quad ,$$

- The (anti-)twist fields $\Delta, \bar\Delta$ switch the b.c.'s on the X^μ string fields.

Moduli spectrum in the $\mathcal{N}=1$ case



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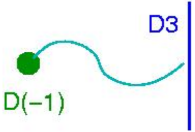
- w and w have dimensions of (length) and are related to the size of the instanton solution.

Moduli spectrum in the $\mathcal{N}=1$ case



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Ramond sector

The vertices surviving the orbifold projection are

$$V_{\mu}(y) = (2\pi\alpha')^{\frac{3}{4}} \, \frac{g_0}{\sqrt{2}} \, \mu \, \Delta(y) \, S^{(-)}(y) \, \mathrm{e}^{-\frac{1}{2}\phi(y)} \quad ,$$

$$V_{\bar{\mu}}(y) = (2\pi\alpha')^{\frac{3}{4}} \, \frac{g_0}{\sqrt{2}} \, \bar{\mu} \, \bar{\Delta}(y) \, S^{(-)}(y) \, \mathrm{e}^{-\frac{1}{2}\phi(y)} \quad .$$

- The fermionic moduli μ,$\bar{\mu}$ have dimensions of (length)$^{1/2}$.

The $\mathcal{N}=1$ moduli action

- (Mixed) disk diagrams with the above moduli, for $\alpha' \to 0$ yield

$$S_{\rm mod} = \mathrm{tr}\Big\{-\mathrm{i}D_c\Big(W^c + \mathrm{i}\bar\eta^c_{\mu\nu}\big[a'^\mu, a'^\nu\big]\Big) -\mathrm{i}\lambda^{\dot\alpha}\Big(w^u{}_{\dot\alpha}\,\bar\mu_u + \mu^u\bar w_{\dot\alpha u} + \big[a'_{\alpha\dot\alpha}, M'^\alpha\big]\Big)\Big\}$$

where

$$(W^c)_j{}^i = w^{iu}{}_{\dot\alpha}\,(\tau^c)^{\dot\alpha}{}_{\dot\beta}\,\bar w^{\dot\beta}{}_{uj}$$

- D_c and $\lambda^{\dot\alpha}$ $\sim$ Lagrange multipliers for the (super)ADHM constraints

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- D_c and $\lambda^{\dot\alpha}$ $\sim$ Lagrange multipliers for the (super)ADHM constraints.

The $\mathcal{N} = 1$ ADHM constraints

- The ADHM constraints are three $k \times k$ matrix eq.s

$$W^c + \mathrm{i}\bar{\eta}^c_{\mu\nu}\left[a'^{\mu}, a'^{\nu}\right] = \mathbf{0} \ .$$

- and their fermionic counterparts

$$w^u{}_{\dot{\alpha}}\,\bar{\mu}_u + \mu^u \bar{w}_{\dot{\alpha}u} + \left[a'_{\alpha\dot{\alpha}}, M'^{\alpha}\right] = \mathbf{0} \ .$$

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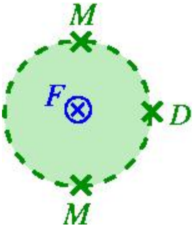
The graviphoton in D(-1) disks

- Inserting $V_{\mathcal{F}}$ in a disk with all boundary on D(-1)'s is perfectely analogous to the D3 case (but we have non momenta).



- The only possible diagram is

$$\langle\!\langle\, V_M V_M V_D V_{\mathcal{F}} \,\rangle\!\rangle$$

$$= \frac{\pi^2}{2} 2\pi\alpha')^{\frac{1}{2}} \mathrm{tr}\Big(M' \cdot M' D_c\Big) \bar{\eta}^c_{\mu\nu} \mathcal{F}_{\dot{\alpha}\dot{\beta}} \left(\bar{\sigma}_{\nu\mu}\right)^{\dot{\alpha}\dot{\beta}}$$

$$= -\frac{1}{2}\, \mathrm{tr}\Big(M' \cdot M' D_c\Big) C^c \ ,$$

where

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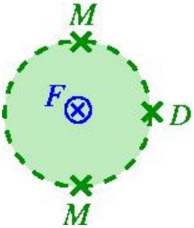

$$
\begin{aligned}
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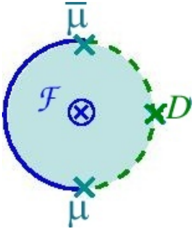
$$

The graviphoton in mixed disks

- We can also insert $V_{\mathcal{F}}$ in a disk with mixed b.c.'s.
  - There is a possible diagram

$$\langle\!\langle\, V_{\bar{\mu}} V_{\mu} V_D V_{\mathcal{F}} \,\rangle\!\rangle$$



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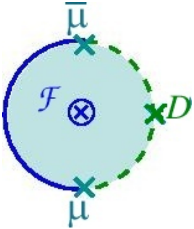

- We have different b.c.s on the two parts of the boundary, but the spin fields in the RR vertex $V_{\mathcal{F}}$ have the same identification on both:

$$S^{\dot\alpha} S^{(+)}(z) = \tilde{S}^{\dot\alpha} \tilde{S}^{(+)}(\bar{z}) \Big|_{z=\bar{z}} .$$

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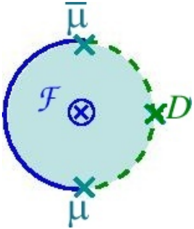

$$\langle\!\langle\, V_{\bar{\mu}} V_{\mu} V_D V_{\mathcal{F}} \,\rangle\!\rangle$$

  - This is because we chose D(-1)'s to represent instantons with self-dual f.s. and $\mathcal{F}_{\mu\nu}$ to be anti-self-dual.

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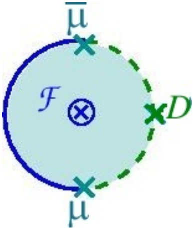

- The $\mu, \bar{\mu}$ vertices contain bosonic twist fields with correlator

$$\Delta(y_1)\,\bar{\Delta}(y_2) \sim (y_1 - y_2)^{-\frac{1}{2}} \;.$$

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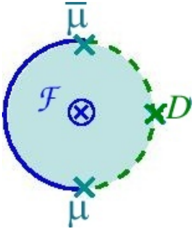

  - Taking into account all correlators, the $\mathrm{SL}(2,\mathbb{R})$ gauge fixing, the integrations and the normalizations, we find the result

$$-\frac{\pi^2}{2}(2\pi\alpha')^{\frac{1}{2}}\mathrm{tr}\Big(\bar{\mu}_u\mu^u D_c\Big)\bar{\eta}^c_{\mu\nu}\mathcal{F}_{\dot{\alpha}\dot{\beta}}\,(\bar{\sigma}^{\nu\mu})^{\dot{\alpha}\dot{\beta}}$$
$$=\frac{1}{2}\,\mathrm{tr}\Big(\bar{\mu}_u\mu^u D_c\Big)C^c\ .$$

Effects of the graviphoton on the moduli measure

- No other disk diagrams contribute in our $\alpha' \to 0$ limit.
- The two terms above are linear in the auxiliary field D_c $\rightsquigarrow$ deform the bosonic ADHM constraints to

$$W^c + \mathrm{i}\bar{\eta}^c_{\mu\nu}\left[a'^{\mu}, a'^{\nu}\right] + \boxed{\tfrac{\mathrm{i}}{2}\Big(M'\cdot M' + \mu^u\bar{\mu}_u\Big)C^c} = \mathbf{0}\ .$$

- This is the only effect of the chosen anti-self-dual graviphoton bckg.
- Had we chosen a self-dual graviphoton, we would have no effect.

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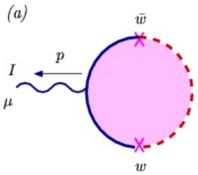
The emitted gauge field



- Mixed disks represent sources for the gauge theory fields. In particular, the amplitude for the emission of a gauge field A^I_μ results in

$$\langle\!\langle \, V_{\bar{w}} \, \mathcal{V}_{A^I_\mu}(-p) \, V_w \, \rangle\!\rangle$$
$$= \mathrm{i}\, (T^I)^v{}_u \, p^\nu \, \bar{\eta}^c_{\nu\mu} \Big(w^u{}_{\dot{\alpha}} \, (\tau^c)^{\dot{\alpha}}{}_{\dot{\beta}} \, \bar{w}^{\dot{\beta}}{}_v \Big) \, \mathrm{e}^{-\mathrm{i}p\cdot x_0} \; .$$

- The $\mathcal{V}_{A^I_\mu}(-p)$ has no polarization and outgoing momentum.

- N.B. From now on we set $k = 1$, i.e. we consider instanton number 1.

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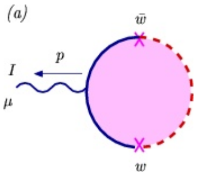

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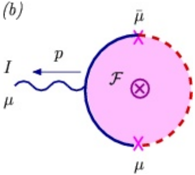
The emitted gauge field in presence of $C_{\mu\nu}$



- In the graviphoton background, we have the extra emission diagram

$$\langle\!\langle\, V_{\bar\mu}\, \mathcal{V}_{A^I_\mu}(-p)\, V_\mu V_{\mathcal{F}} \,\rangle\!\rangle$$
$$= 2\pi^2 (2\pi\alpha')^{\frac{1}{2}} (T^I)^v{}_u\, p^\nu (\bar\sigma_{\nu\mu})^{\dot\alpha\dot\beta}\, \mathcal{F}_{\dot\alpha\dot\beta}\, \mu^u \bar\mu_v\, \mathrm{e}^{-\mathrm{i}p\cdot x_0}$$
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- No other diagrams with only two moduli contribute to the emission of a gauge field.

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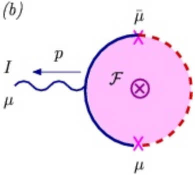

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- No other diagrams with only two moduli contribute to the emission of a gauge field.

The classical solution

- Altogether, the emission amplitude is

$$A^I_\mu(p) = \mathrm{i}\,(T^I)^v{}_u\, p^\nu\, \bar\eta^c_{\nu\mu}\Big[(T^c)^u{}_v + (S^c)^u{}_v\Big]\,\mathrm{e}^{-\mathrm{i}p\cdot x_0} \quad ,$$

where $(T^I)^v{}_u$ are the $\mathrm{U}(N)$ generators and

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- From this we obtain the profile of the classical solution

$$A^I_\mu(x) = \int\frac{d^4p}{(2\pi)^2}\,A^I_\mu(p)\,\frac{1}{p^2}\,\mathrm{e}^{\mathrm{i}p\cdot x}$$
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The classical solution

- The above solution will represents the leading term at long distance of the deformed instanton solution in the singular gauge.
- However, above appeared the unconstrained moduli $\mu, \bar{\mu}, w, \bar{w}$.
  - We need to enforce the deformed ADHM contraints, for $k = 1$:

$$W^c + \boxed{\tfrac{\mathrm{i}}{2}\Big(M'\cdot M' + \mu^u \bar{\mu}_u\Big)C^c} = \mathbf{0} \,,$$

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The classical solution in the true moduli space

- Using the ADHM constraints, the solution can be written as

$$\begin{aligned} A^I_\mu(x) &= 2\Big(\mathcal{M}^{cb}\,\mathrm{Tr}\big(T^I t^b\big) + W^c\mathrm{Tr}\big(T^I t^0\big) + \mathrm{Tr}\big(T^I S^c\big)\Big) \\ &\times\ \bar{\eta}^c_{\mu\nu}\,\frac{(x-x_0)^\nu}{(x-x_0)^4}\ . \end{aligned}$$

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- On the bosonic ADHM constraints,

$$W^c = -\frac{\mathrm{i}}{2}\Big(M'\cdot M' + \mu^u\bar\mu_u\Big)C^c \equiv \hat{W}^c.$$

Without the RR deformation, W^c would vanish.

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- The matrix $\mathcal{M}$ is $\mathcal{M}^{ab} = W^0\sqrt{W_0^2 - |\vec{W}|^2}\,\big(\mathcal{R}^{-\frac{1}{2}}\big)^{ab}$, with $\big(\mathcal{R}\big)^{ab} = W_0^2\,\delta^{ab} - W^a W^b$, where

$$W^0 = w^u{}_{\dot\alpha}\,\bar{w}^{\dot\alpha}{}_u\ .$$

At $C_c = 0$, $W^0 = 2\rho^2$, where ρ = size of the instanton.

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- The $N \times N$ matrices t^a and t^0, depending on the moduli $w, \bar w$, generate a $\mathrm{u}(2)$ subalgebra
  $\rightsquigarrow$ the instanton field contains an abelian factor, beside $\mathrm{su}(2)$.

- Moreover, the matrix $(S^c)^u{}_v = -\frac{\mathrm{i}}{2}\,\mu^u\bar\mu_v\,C^c$ commutes with this $\mathrm{u}(2)$ $\rightsquigarrow$ another abelian factor.

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An explicit case of the solution

- We can write the above general expression choosing a particular solution to the ADHM constraints, to make contact with the literature [Grassi et al, 2003, Britto et al, 2003].
- Decomposing $u = (\dot{\alpha}, i)$ with $\dot{\alpha} = 1, 2$ and $i = 3, \ldots, N$, the bosonic ADHM constraints are solved by

$$\begin{cases} w^{\dot{\beta}}{}_{\dot{\alpha}} = \rho\, \delta^{\dot{\beta}}{}_{\dot{\alpha}} + \frac{1}{4\rho}\, \hat{W}_c\, (\tau^c)^{\dot{\beta}}{}_{\dot{\alpha}}\ , \\ w^{i}{}_{\dot{\alpha}} = 0\ . \end{cases}$$

- Having fixed $w, \bar{w}$, the fermionic constraints are solved by

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An explicit case of the solution

- The instanton gauge field $(A_\mu)^u{}_v$ reduces then to

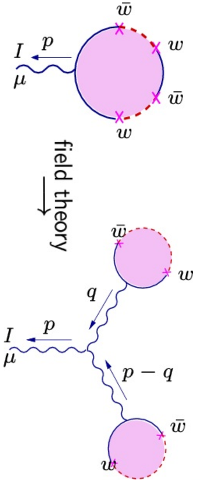
$$(A_\mu)^{\dot\alpha}{}_{\dot\beta} = \Big\{ \rho^2 (\tau_c)^{\dot\alpha}{}_{\dot\beta} - \frac{\mathrm{i}}{4}\Big(M'\cdot M' + \mu^3\bar\mu_3\Big) C_c\, \delta^{\dot\alpha}{}_{\dot\beta}$$
$$+ \frac{1}{32\rho^2}\Big(|\vec C|^2 (\tau_c)^{\dot\alpha}{}_{\dot\beta} - 2 C_c C^b (\tau_b)^{\dot\alpha}{}_{\dot\beta}\Big) M'\cdot M'\, \mu^3\bar\mu_3 \Big\}\, \bar\eta^c_{\mu\nu}\, \frac{(x-x_0)^\nu}{(x-x_0)^4}$$

and

$$(A_\mu)^3{}_3 = -\frac{\mathrm{i}}{2}\, \mu^3\bar\mu_3\, C_c\, \bar\eta^c_{\mu\nu}\, \frac{(x-x_0)^\nu}{(x-x_0)^4} \quad .$$

This agrees with [Britto et al, 2003].

Additional remarks



- The mixed disks emit also a gaugino $\Lambda^{\alpha, I}$ $\rightsquigarrow$ account for its leading profile in the super-instanton solution.
- Subleading terms in the long-distance expansion of the solution arise from emission diagrams with more moduli insertions.
- At the field theory level, they correspond to having more source terms.
- This, is exactly the field-theoretical procedure utilized in [Grassi et al, 2003, Britto et al, 2003] to determine the (deformed) super-instanton profile,

Conclusions

- The open string realization of gauge theories is a very powerful tool, also in discussing possible deformations (induced by closed string backgrounds).
- In particular, the deformation of $\mathcal{N} = 1$ gauge theory to $\mathcal{N} = 1/2$ gauge theory is exactly described in the open string set-up by the inclusion of a particular Ramond-Ramond background.
- The stringy description of gauge instantons and of their moduli space by means of D3/D(-1) systems extends to the deformed case, proving itself to be a valuable tool.

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Perspectives

- Deformations of $\mathcal{N} = 2$ theories:
  - deformations of $\mathcal{N} = 2$ superspace by RR backgrounds (work in progress);
  - stringy interpretation of the deformations leading to the localization á la Nekrasov of the integrals on instanton moduli space (under investigation, in collab. also with Tor Vergata).
- Derivation of the effects of constant Ramond-Ramond field strengths (gauge theory action, instantons, etc) using Berkovits' formalism instead of RNS (work in progress).
- Derivation of the instantonic sector of non-commutative gauge theory from the string realization with constant $B_{\mu\nu}$ background.

Very few references

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