# Exotic instantons in d=8 and the heterotic/type I’ duality

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*Problemi attuali di Fisica teorica*
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# Foreword

Mostly based on

- M. Billo, M. Frau, L. Gallot, A. Lerda and I. Pesando, "Classical solutions for exotic instantons?,", JHEP **03** (2009) 056, arXiv:0901.1666 [hep-th].
- Same authors + L. Ferro, in preparation.

It builds over a vast literature

- I apologize for missing references...

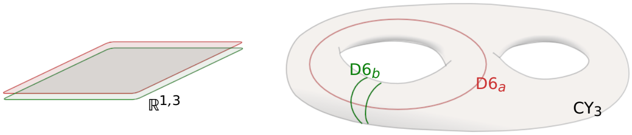
# Plan of the talk

1. Motivations
2. “Exotic” instantons in type I’
3. Interpretation as 8d instanton solutions
4. Rôle in type I’/Heterotic duality
5. Conclusions and perspectives

# Motivations

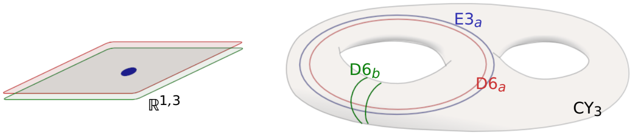
# Non-perturbative sectors

## in brane-worlds




- (Susy) gauge and matter sectors on the uncompactified part of (partially wrapped) D-branes
  - chiral matter, families from multiple intersections, tuning different coupling constants...

# Non-perturbative sectors
## in brane-worlds



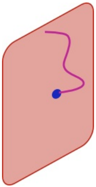

- (Susy) gauge and matter sectors on the uncompactified part of (partially wrapped) D-branes
  - chiral matter, families from multiple intersections, tuning different coupling constants...
- Non-perturbative sectors from partially wrapped E(uclidean)-branes
  - Pointlike in the $\mathbb{R}^{1,3}$ space-time: “instanton configurations”

# Ordinary vs. exotic

- E-branes identical to D-branes in the internal directions: gauge instantons
  - ADHM from strings attached to the instantonic branes Witten, 1995; Douglas, 1995-1996; ...
  - non-trivial instanton profile of the gauge field Billo et al, 2001
- E-branes different from D-branes in internal directions do not represent gauge instantons
  - They are called exotic or stringy instantons
  - in certain cases can give important contributions to the effective action, .e.g. Majorana masses for neutrinos, moduli stabilizing terms, ...

Blumenhagen et al 0609191; Ibanez and Uranga, 0609213; (long list)... ; Petersson 0711.1837

# World-sheet properties

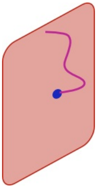


- Consider the strings stretching between the gauge D-branes and the E-branes
  - NS sector (lowest KK level) physicity condition

$$L_0 - \frac{1}{2} = N_X + N_\psi + \sum_{i=1}^{3} \frac{\theta_i}{2} = 0 \,,$$

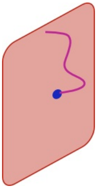
    - Ordinary case: internal twists $\theta_i = 0$. There are bosonic moduli $w_{\dot\alpha}$ typical of ADHM construction, related to the size
    - Exotic case: $\theta_i > 0$, i.e., there are "more than 4 ND directions" . The moduli $w_{\dot\alpha}$ are absent. Hints at zero-size limit of some gauge field configuration.

# World-sheet properties



- Consider the strings stretching between the gauge D-branes and the E-branes
  - In the R sector, the ground states always: fermionic anti-chiral moduli $\lambda_{\dot\alpha}$
    - Ordinary case: Lagrange multipl. of fermionic ADHM constraints
    - Exotic case: the the abelian component of the $\lambda$'s is a true fermionic zero-mode since the abelian part of ADHM constraint vanishes (would cointain the $w_{\dot\alpha}$)

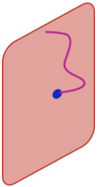
# World-sheet properties



- Consider the strings stretching between the gauge D-branes and the E-branes
  - Exotic case: need to remove the fermionic zero-mode to get non-zero correlators
    - orientifold projections Argurio et al, 2007; ...,
    - lift with closed string fluxes
      Blumenhagen et al, 2007; Billo et al, 2008; ...
    - other mechanisms Petersson, 2007; ....

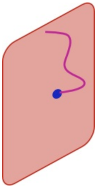
# World-sheet properties

- Consider the strings stretching between the gauge D-branes and the E-branes



- These w.s. properies of the exotic systems must be taken into account when trying to answer the question:
  - Do exotic instantonic branes correspond to some classical configuration of gauge/matter fields?

# World-sheet properties



- Consider the strings stretching between the gauge D-branes and the E-branes

- We will consider this problem in a simplified setting: D(-1)/D7 in type I' on $T^2$
  - It has more than 4 ND directions (eight, in fact): exotic
  - Built-in orientifod projection that eliminates the $\lambda_{\dot{\alpha}}$ ferm. 0-modes
  - However, the gauge theory part lives in 8 dimensions

# Instanton counting and dualities

- Beside providing (new) non-perturbative couplings in the effective actions see Alberto's talk instantons (also exotic ones?) play a key rôle in duality statements, both at the field theory and the string theory level

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- In theories with higher susy, sum over all instanton # needed to check dualities and exact results
  - D-instanton partition function from matrix integrals Moore et al, 1998 checks against expression derived by self-duality of type IIB Green-Gutperle, 1997
  - Seiberg-Witten sol. by instanton counting Nekrasov, 2002

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  - Seiberg-Witten sol. by instanton counting Nekrasov, 2002
- Effective action at $O(F^4)$ summing over D(-1)'s in type I' must match by string duality the Het $SO(4)^4$ one: → 2nd part of this talk

# “Exotic” instantons in type I’

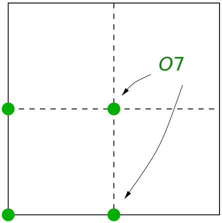
# A D(-1)/D7 system in type I'

- Type I' is type IIB on $T_2$ modded out by

$$\Omega = \omega\,(-1)^{F_L}\,\mathcal{I}_2$$

  where $\omega$ = w.s. parity, $F_L$ = left-moving w.s. fermion #, $I_2$ = inversion on $T_2$

- $\Omega$ has four fixed-points on $T_2$ where four O7-planes are placed



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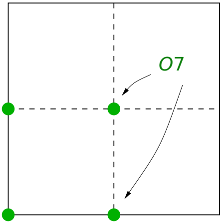

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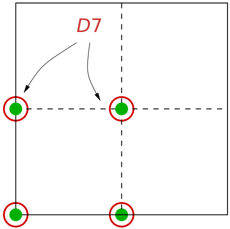
- Local RR tadpole cancellation requires 8 D7-branes at each fix point



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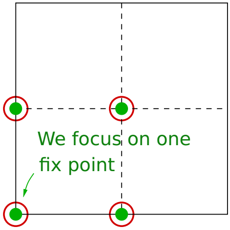

# The gauge theory on the D7's

- From the D7/D7 strings we get $\mathcal{N} = 1$ vector multiplet in $d = 8$ in the adjoint of SO(8):

$$\left\{A_\mu, \Lambda^\alpha, \phi_m\right\}, \quad \mu = 1, \ldots 8, \quad m = 8, 9$$

- Can be assembled into a "chiral" superfield Back

$$\Phi(x,\theta) = \phi(x) + \sqrt{2}\,\theta\Lambda(x) + \frac{1}{2}\theta\gamma^{\mu\nu}\theta\,F_{\mu\nu}(x) + \ldots$$

  where $\phi = (\phi_9 + i\phi_{10})/\sqrt{2}$.
  - Formally very similar to $\mathcal{N} = 2$ in $d = 4$

# Effective action on the D7's

Effective action in $F_{\mu\nu}$ an its derivatives: NABI ▸ Back

$$
\begin{aligned}
S_{D7} &= \frac{1}{8\pi g_s}\int d^8x\, Tr\left[\frac{F_{\mu\nu}F^{\mu\nu}}{(2\pi\sqrt{\alpha'})^4} - \frac{1}{3\,(2\pi)^2}\,t_8\,F^4\right] \\
&\quad + \frac{\alpha'}{g_s}\int d^8x\,\mathcal{L}_{(5)}(F, DF) + \cdots \\
&= S_{\rm YM} + S_{(4)} + S_{(5)} + \cdots\,,
\end{aligned}
$$

- The Yang-Mills action $S_{\rm YM}$ has a dimensionful coupling $g^2_{\rm YM} \equiv 4\pi g_s(2\pi\sqrt{\alpha'})^4$

# Effective action on the D7's

Effective action in $F_{\mu\nu}$ an its derivatives: NABI ▸ Back

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- The quartic action ▸ Detail has a dimensionless coupling $\lambda^4 \equiv 4\pi^3 g_s$:

$$S_{(4)} = -\frac{1}{4!\lambda^4}\int d^8x\, \mathrm{Tr}\left(t_8 F^4\right)$$

# Effective action on the D7's

Effective action in $F_{\mu\nu}$ an its derivatives: NABI Back

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&= S_{\rm YM} + S_{(4)} + S_{(5)} + \cdots ,
\end{aligned}
$$

- Adding the WZ term, we can write Back

$$
S_{(4)} = -\frac{1}{4!\,4\pi^3 g_s}\int d^8x\,\mathrm{Tr}\left(t_8 F^4\right) - 2\pi i\, C_0\, c_{(4)}
$$

where $c_{(4)}$ is the fourth Chern number

# Effective action on the D7's

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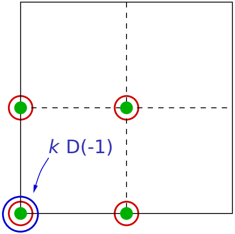
$$+\frac{\alpha'}{g_s}\int d^8x\,\mathcal{L}_{(5)}(F, DF) + \cdots$$
$$= S_{\text{YM}} + S_{(4)} + S_{(5)} + \cdots\,,$$

- Contributions of higher order in $(\alpha')$: rôle to be discussed later

# Adding D-instantons

- Add $k$ D-instantons.
- D7/D(-1) form a 1/2 BPS system with 8 ND directions



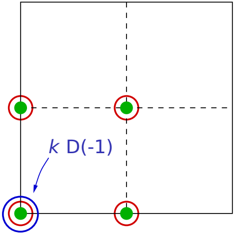

  - D(-1) classical action ▸ Back ▸ Back'

$$\mathcal{S}_{cl} = k\left(\frac{2\pi}{g_s} - 2\pi i C_0\right) \equiv -2\pi i k\tau \,,$$

  - Coincides with the quartic action ▸ Recall on the D7 for gauge fields $F$ with $c_{(4)} = k$ and

$$\int d^8x \, Tr\left(t_8 F^4\right) = -\frac{1}{2}\int d^8x \, Tr\left(\epsilon_8 F^4\right) = -\frac{4!}{2}(2\pi)^4 \, c_{(4)}$$

# Adding D-instantons




- Add $k$ D-instantons.
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  - D(-1) classical action Back Back'

$$\mathcal{S}_{cl} = k\left(\frac{2\pi}{g_s} - 2\pi i C_0\right) \equiv -2\pi i k \tau \,,$$

  - Analogous to relation with self-dual YM config.s in D3/D(-1)
  - Suggests relation to some 8d instanton of the quartic action

# The moduli spectrum

- Besides the classical action, we must consider the spectrum and interactions of strings ending on a D(-1).

# The moduli spectrum

Spectrum: Back

| Sector | | Name | Meaning | Chan-Paton | Dimension |
|---|---|---|---|---|---|
| $-1/-1$ | NS | $a_\mu$ | centers | symm SO$(k)$ | (length) |
| | | $\chi, \bar{\chi}$ | | adj SO$(k)$ | $(\text{length})^{-1}$ |
| | | $D_m$ | Lagr. mult. | $\vdots$ | $(\text{length})^{-2}$ |
| | R | $M^\alpha$ | partners | symm SO$(k)$ | $(\text{length})^{\frac{1}{2}}$ |
| | | $\lambda_{\dot\alpha}$ | Lagr. mult. | adj SO$(k)$ | $(\text{length})^{-\frac{3}{2}}$ |
| $-1/7$ | R | $\mu$ | | $\mathbf{8} \times \mathbf{k}$ | (length) |

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- The $SO(k)$ rep. is determined by the orientifold projection $\Omega$

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- Abelian part of $a_\mu$, $M_\alpha$ $\sim$ Goldstone modes of the (super)translations on the D7 broken by D(-1)'s
- Identified with coordinates $x_\mu$,$\theta_\alpha$

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- Abelian part of $\lambda_{\dot{\alpha}}$ would be a dangerous 0-mode
- Removed by the orientifold projection

# The moduli spectrum

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- One needs auxiliary fields $D_m$, $m = 1, \ldots 7$ to disentangle the quartic interaction $\left[a_\mu, a_\nu\right][a^\mu, a^\nu]$.
- "Octonionic" analogue of the fact that for D(-1)/D3 systems one needs $D_c$, $c = 1, 2, 3$

# The moduli spectrum

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- For "mixed" strings, no bosonic moduli from the NS sector
- This is a characteristic of "exotic" instantons

# The moduli action

Beside the classical action ▸ Recall we have ▸ Back

$$\mathcal{S} = \mathcal{S}_{quartic} + \mathcal{S}_{cubic} + \mathcal{S}_{mixed}$$

# The moduli action

Beside the classical action Recall we have Back

$$\mathcal{S} = \mathcal{S}_{quartic} + \mathcal{S}_{cubic} + \mathcal{S}_{mixed}$$

- The quartic part can be (partly) disentangled with auxiliary fields $D_m$:

$$\mathcal{S}_{quartic} = \frac{1}{g_0^2} tr \left\{ \frac{1}{2} D_m D^m + \frac{1}{2} D_m (\tau^m)_{\mu\nu} [a^\mu, a^\nu] - \left[a_\mu, \bar{\chi}\right] [a^\mu, \chi] + \frac{1}{2} [\bar{\chi}, \chi]^2 \right\}$$

# The moduli action

Beside the classical action Recall we have Back

$$\mathcal{S} = \mathcal{S}_{quartic} + \mathcal{S}_{cubic} + \mathcal{S}_{mixed}$$

- The cubic part reads:

$$\mathcal{S}_{cubic} = \frac{1}{g_0^2} \operatorname{tr}\Big\{ i\lambda_{\dot\alpha} (\gamma^\mu)^{\dot\alpha\beta} \big[ a_\mu, M_\beta \big] - i\lambda_{\dot\alpha} [\chi, \lambda^{\dot\alpha}] - iM_\alpha \, [\bar\chi, M^\alpha] \Big\} \, .$$

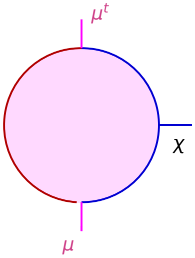
# The moduli action

Beside the classical action ▸ Recall we have ▸ Back

$$\mathcal{S} = \mathcal{S}_{quartic} + \mathcal{S}_{cubic} + \mathcal{S}_{mixed}$$

- From diagrams with mixed b.c.'s

$$\mathcal{S}_{mixed} = \frac{1}{g_0^2}\mathrm{tr}\Big\{-i\,\mu^T\chi\mu\Big\}\ ;$$



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$$\mathcal{S} = \mathcal{S}_{quartic} + \mathcal{S}_{cubic} + \mathcal{S}_{mixed}$$

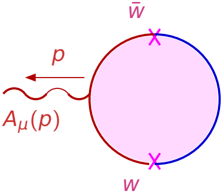
- In the case $k = 1$ all of these contributes vanish (no adjoint moduli!). We are left with the classical part only
$$\mathcal{S}_{cl} = -2\pi i\tau$$
- Coincides with the quartic action on the D7 for gauge fields $F$ s.t. ▸ Back
  - $c_{(4)} = 1$
  - $Tr(t_8 F^4) = -1/2 Tr(\epsilon_8 F^4)$
- Suggests interpretation as “instantons” but ...

# No gauge field emission

- Ordinary" instantonic brane systems (such as D(-1)/D3): classical instanton profile due to the emission of gauge field from mixed disks Billo et al, 2001

$$A_\mu^i = 2\rho^2 \bar{\eta}^i_{\mu\nu} \frac{x^\nu}{|x|^4} + \dots$$

(SU(2), sing. gauge, large-$|x|$, $2\rho^2 = tr\, \bar{w}_{\dot\alpha} w^{\dot\alpha}$ )



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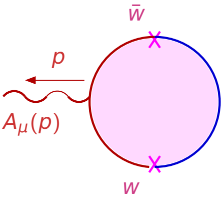


$$A^i_\mu = 2\rho^2 \bar{\eta}^i_{\mu\nu} \frac{x^\nu}{|x|^4} + \ldots$$

(SU(2), sing. gauge, large-$|x|$, $2\rho^2 = tr\, \bar{w}_{\dot{\alpha}} w^{\dot{\alpha}}$ )

- In the "exotic" D(-1)/D7 system there is no emission diagram for $A_\mu$ because there are no bosonic mixed moduli $w$

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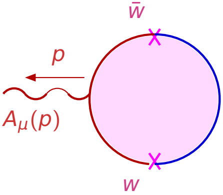
- Ordinary" instantonic brane systems (such as D(-1)/D3): classical instanton profile due to the emission of gauge field from mixed disks Billo et al, 2001




$$A^i_\mu = 2\rho^2 \bar{\eta}^i_{\mu\nu} \frac{x^\nu}{|x|^4} + \dots$$

(SU(2), sing. gauge, large-$|x|$, $2\rho^2 = tr\, \bar{w}_{\dot\alpha} w^{\dot\alpha}$ )

- In the "exotic" D(-1)/D7 system there is no emission diagram for $A_\mu$ because there are no bosonic mixed moduli $w$
- The classical profile vanishes outside the location of the D(-1)

# Interpretation as 8d instanton solutions

# Expected features

- A D(-1) inside the D7's should correspond to the zero-size limit of some "instantonic" configuration of the SO(8) gauge field such that
  - has 4-th Chern number $c_{(4)} = 1$
  - the quartic action reduces to the D(-1) action ▸ Recall
  - preserves SO(8) "Lorentz" invariance
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- Many generalizations of 4d instantons to 8d In particular "linear" instantons [Corrigan, 1982; Fubini-Nicolai, 1985;...]

$$F_{\mu\nu} + \frac{1}{2} T_{[\mu\nu\rho\sigma]} F^{\rho\sigma} = 0,$$

  - Bianchis imply YM e.o.m $D^\mu F_{\mu\nu} = 0$
  - do not fully preserve the SO(8) Lorentz group, and are less than 1/2 BPS

# The SO(8) instanton

- All our requirements met by the SO(8) instanton

[Grossmann e al, 1985]

$$\left[A_\mu(x)\right]^{\alpha\beta} = \frac{(\gamma_{\mu\nu})^{\alpha\beta}\, x^\nu}{r^2 + \rho^2}$$

with $\rho$ = instanton size and $r^2 = x_\mu x^\mu$, while $\alpha\beta \in$ adjoint of the SO(8) gauge group.

  - is "self-dual" in the sense that $F \wedge F = (F \wedge F)^*$
  - satisfies $t_8 F^4 = -1/2 \epsilon_8 F^4$ from Clifford Algebra
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- However, it is not a solution of Y.M. e.o.m. in $d = 8$:

$$D^\mu F_{\mu\nu}(x) = \frac{4(d-4)\rho^2}{(r^2+\rho^2)^3}\, \gamma_{\mu\nu} x^\nu \,.$$

# Consistency conditions

- Eff. action on the D7 is the NABI action ▸ Recall
- To keep the quartic action and the instanton effects the field-theory limit must be

$$\alpha' \to 0\,, \qquad g_s \text{ fixed}$$

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- This limit is dangerous on the YM action $S_{YM}$ since $g^2_{YM} \propto g_s \alpha'^2$. On the SO(8) instanton, however, we have ($R$ regulates the volume):

$$S_{YM} \to \frac{\rho^4}{\alpha'^2 g_s} \log\left(\rho/R\right)\ ,$$

which vanishes in the zero-size limit $\rho \to 0$ if $\rho^2/\alpha'^2 \to 0$ (done before removing $R$)

# Consistency conditions

- Eff. action on the D7 is the NABI action ▸ Recall
- To keep the quartic action and the instanton effects the field-theory limit must be

$$\alpha' \to 0 \,, \qquad g_s \text{ fixed}$$

- Consider the higher order $\alpha'$ corrections to the NABI action. On the SO(8) instanton, by dimensional reasons, must be

$$\rho^{d-8} \sum_{n=1}^{\infty} a_n \left( \frac{\alpha'}{\rho^2} \right)^n \,,$$

- The coefficients $a_n$ should vanish for consistency!

# $O(F^5)$ terms in the NABI

- The first coefficient $a_1$ arises from the integral of $\mathcal{L}^{(5)}(F, DF)$, i.e.the term of order $\alpha'^3$ w.r.t to the YM action.
- We would like to check that it vanishes. Crucial point: which is the form of $\mathcal{L}^{(5)}(F, DF)$?

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  - differing by terms which vanish "on-shell", i.e. upon use of the YM e.o.m.

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  - obtained by different methods
  - differing by terms which vanish "on-shell", i.e. upon use of the YM e.o.m.
- One proposal is singled out by admitting a susy extension Collinucci et al, 2002

# Check at $O(F^5)$ in the NABI

- The bosonic part of the supersimmetrizable $O(\alpha'^3)$ lagrangian is

$$
\begin{aligned}
\mathcal{L}^{(5)} &= \frac{\zeta(3)}{2}\mathrm{Tr}\Big\{4\left[F_{\mu_1\mu_2},F_{\mu_3\mu_4}\right]\left[\left[F_{\mu_1\mu_3},F_{\mu_2\mu_5}\right],F_{\mu_4\mu_5}\right] \\
&+ 2\left[F_{\mu_1\mu_2},F_{\mu_3\mu_4}\right]\left[\left[F_{\mu_1\mu_2},F_{\mu_3\mu_5}\right],F_{\mu_4\mu_5}\right] \\
&+ 2\left[F_{\mu_1\mu_2},D_{\mu_5}F_{\mu_1\mu_4}\right]\left[D_{\mu_5}F_{\mu_2\mu_3},F_{\mu_3\mu_4}\right] \\
&- 2\left[F_{\mu_1\mu_2},D_{\mu_4}F_{\mu_3\mu_5}\right]\left[D_{\mu_4}F_{\mu_2\mu_5},F_{\mu_1\mu_3}\right] \\
&+ \left[F_{\mu_1\mu_2},D_{\mu_5}F_{\mu_3\mu_4}\right]\left[D_{\mu_5}F_{\mu_1\mu_2},F_{\mu_3\mu_4}\right]\Big\}
\end{aligned}
$$

# Check at $O(F^5)$ in the NABI

- Plugging the instanton profile into $\frac{\alpha'}{g_s}\int d^8x\,\mathcal{L}^{(5)}$ we get [Using the CADABRA program by Kasper Peeters]

$$\frac{\alpha'\,\zeta(3)}{g_s}\,2^{d/2+9}\,\frac{\pi^{d/2}\,\Gamma(9-d/2)}{9!\,\rho^{10-d}}$$
$$\times\,(d-1)(d-2)(d-4)\left(-d\left(9-\frac{d}{2}\right)+\left(d+2\right)\frac{d}{2}\right)$$

namely a result proportional to

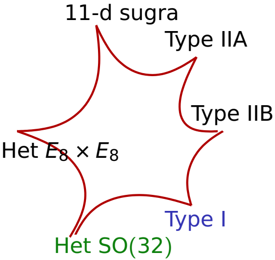
$$d(d-1)(d-2)(d-4)(d-8)$$

- The quintic action vanishes on the SO(8) instanton! The check is successful

# D(-1)’s and type I’/Heterotic duality

# Type I'/Heterotic duality

- In the web of string dualities, the S-duality between Het. SO(32) and Type I plays a fundamental rôle
- Upon compactification (e.g. on $T_2$), other dualities follow from it



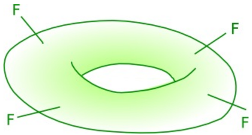

  - The Type I' theory is S-dual to Het $SO(8)^4$ (obtained with Wilson lines on $T_2$)
  - The mapping of parameters involve the relation

$$\tau \longleftrightarrow T$$

    - $\tau$: (complexified) string coupling in type I'
    - $T$: the Kähler param. of the torus in the Het.

# The heterotic $F^4$ effective action

- Het $SO(8)^4$ is a theory of closed strings. Focus on the effective action for one of the SO(8) gauge factors
- The BPS-saturated $t_8 F^4$ terms arise just at 1-loop. Threshold corrections organize as a series in $q \equiv e^{2\pi i T}$



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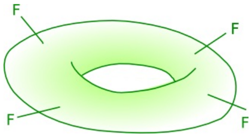


- At even powers in $q$ one gets contributions to the following structures: Lerche-Stieberger, Gutperle,... Back

$$t_8 Tr F^4 \Big[\frac{1}{2}\sum_k \big(\sum_{l|k}\frac{1}{l}\big) q^{2k} - \frac{1}{2}\big(\sum_{l|k}\frac{1}{l}\big) q^{4k}\Big]$$

$$-t_8 (Tr F)^2 \Big[\frac{1}{4}\sum_k \big(\sum_{l|k}\frac{1}{l}\big) q^{2k} - \frac{1}{8}\big(\sum_{l|k}\frac{1}{l}\big) q^{4k}\Big]$$

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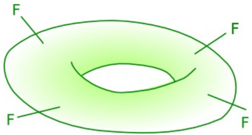


- For SO(8) there is another quartic gauge invariant, $Pf(F) = \epsilon_{a_1 \ldots a_8} F^{a_1 a_2} \ldots F^{a_7 a_8}$ (Lorentz indices omitted). Gets contributions at odd powers in $q$

Gava et al

$$8\, t_8\, Pf(F) \sum_{k odd} \Big(\sum_{l|k} \frac{1}{l}\Big) q^k$$

# The type I' $F^4$ effective action

- Under the Het/type I' duality,

$$q = e^{2\pi i T} \longleftrightarrow q = e^{2\pi i \tau}$$

- The series of threshold corrections maps to a series of non-perturbative corrections
- These corrections can be provided by D-instantons: indeed $q^k$ is the weight $e^{-\mathcal{S}_{cl}}$ for $k$ D(-1) ▸ Recall
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- Much discussed in this setting Gutperle, 1999 or in the T-dual one (D1/D9 systems in Type I)

Bachas et al, 1997; Kiritsis-Obers, 1997; . . .

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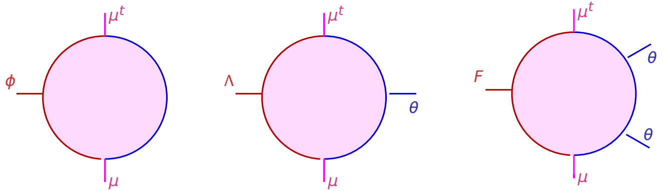
- The explicit derivation of the coefficients in the type I' side has never been performed (to our knowledge)

# Interaction with the multiplet $\Phi$

How can D(-1) contribute to the $F^4$ effective action?

- There's no emission diagram leading to a classical profile, but there are mixed disks (related by SUSY) involving D7/D7 fields




- Net effect: moduli action Recall dependence on the superfield $\Phi(x, \theta)$ Recall

$$\mathcal{S}_{(k)}[\Phi] = -2\pi i \tau k + \mathcal{S}_{quartic} + \mathcal{S}_{cubic} + \mathcal{S}_{mixed} + Tr\, \mu^t \Phi \mu$$

# The moduli integration

- The effective action for the gauge fields is obtained integrating, for each $k$, over the D-instanton moduli $\mathcal{M}_{(k)} = (x, \theta, \widehat{\mathcal{M}}_{(k)})$: ▸ Recall

$$Z[\phi, \Lambda, F] = \int d^8x d^8\theta \sum_k \int d\widehat{\mathcal{M}}_{(k)} e^{-\mathcal{S}_{(k)}[\Phi(x,\theta)]}$$

$$= \int d^8x d^8\theta \, \text{quartic inv.} (\Phi(x, \theta))$$

Second step from dimensionality: $[d\widehat{\mathcal{M}}_{(k)}] = l^{-4}$

- The integration over $\theta$ yields then a $t_8 F^4$ term:

$$\int d^8\theta \, (\theta\gamma^{\mu_1\nu_1}\theta)F_{\mu_1\nu_1} \ldots (\theta\gamma^{\mu_4\nu_4}\theta)F_{\mu_4\nu_4} = t_8 F^4$$

# Integration at $k = 1$

- At $k = 1$ spectrum of moduli is extremely reduced

$$Z_1 = e^{-2\pi i\tau} \int d^8x \, d^8\theta \, d^8\mu \, e^{-Tr\,\mu^t \Phi \mu} = e^{-2\pi i\tau} \int d^8x \, d^8\theta \, Pf(\Phi)$$

- To go to higher $k$, exploit the susy of the moduli action leading to
  - an (equivariant) cohomological BRS structure
  - localization of the integrals (upon suitable deformations from closed string backgrounds)
- Similar kind of techniques to those used for
  - D(-1) partition function in type IIB Moore et al, ...; ...
  - resummation of instantons in $\mathcal{N} = 2$ SYM leading to the SW solution Nekrasov, 2002 re-obtained by D3/D(-1) on orbifold Billo et al, 2006

# BRS reformulation

- Single out one of the supercharges $Q_{\dot{\alpha}}$, say $Q = Q_8$.
- After relabeling some of the moduli:

$$M_\alpha \to M_\mu \equiv (M_m, -M_8) \;, \quad \lambda_{\dot{\alpha}} \to (\lambda_m, \eta) \equiv (\lambda_m, \lambda_8)$$

one has

$$Qa^\mu = M^\mu \;, \;\; Q\lambda_m = -D_m \;, \;\; Q\bar{\chi} = -i\sqrt{2}\eta \;, \;\; Q\chi = 0 \;, \;\; Q\mu = W$$

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  - $T(\chi)$ = SO($k$) rotation in the appropriate rep
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  - $T(\chi)$ = SO($k$) rotation in the appropriate rep
  - $R(\phi)$ a gauge SO(8) rot.
- The complete moduli action is Q-exact

$$\mathcal{S} = Q\Xi$$

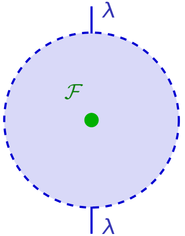
# Deformations from RR background

- To perform the computation, it is convenient to simply cosider the part. function with $\Phi(x, \theta) \to \phi = \langle \Phi \rangle$.
- Integration over the moduli $x, \theta$ would then diverge
- Introduce suitable deformations that
  - regulate the divergence
  - help to fully localize the integral

- Arise from RR field-strengths 3-form with one index on $T_2$

$$\mathcal{F}_{\mu\nu} \equiv F_{\mu\nu z}\ , \quad \bar{\mathcal{F}}_{\mu\nu} \equiv F_{\mu\nu z}$$

- Disk diagrams RR insertions modify the moduli action



# BRS deformation

- Let us parametrize the RR backround as follows:

$$\mathcal{F}_{\mu\nu} = \frac{1}{2} f_{mn} (\tau^{mn})_{\mu\nu} + h_m (\tau^m)_{\mu\nu} \quad , \quad \bar{\mathcal{F}}_{\mu\nu} = \frac{1}{2} \bar{f}_{mn} (\tau^{mn})_{\mu\nu} \; ,$$

- The moduli action is modified to

$$\mathcal{S}' = Q \Xi'$$

- $\bar{f}_{mn}$ only appear in the “gauge fermion” $\Xi'$: the final result does not depende on them
- $f_{mn}$, $h_m$ parametrize SO(7)⊂ SO(8) (Lorentz) with spinorial embedding and modify the action of $Q$

$$Q'^{\,2} \bullet = T(\chi) \bullet + R(\phi) \bullet + G(\mathcal{F}) \bullet$$

where $G$ is the appropriate SO(7) action

# Symmetries of the moduli

- The Action of the BRS charge $Q$ is determined by the symmetry properties of the moduli

| | SO($k$) | SO(7) | SO(8) |
|---|---|---|---|
| $a^\mu$ | symm | $\mathbf{8}_s$ | **1** |
| $M^\mu$ | symm | $\mathbf{8}_s$ | **1** |
| $D_m$ | adj | **7** | **1** |
| $\lambda_m$ | adj | **7** | **1** |
| $\bar{\chi}$ | adj | **1** | **1** |
| $\eta$ | adj | **1** | **1** |
| $\chi$ | adj | **1** | **1** |
| $\mu$ | **k** | **1** | $\mathbf{8}_v$ |

# Scaling to localization

- The BRS structure allows to suitably rescale the bosonic and fermionic moduli in such a way that
  - the (super)Jacobian of the rescaling is one (the measure is unaffected)
  - one can take a limit in which the exponent reduces to a quadratic expression
- The integration over some moduli is trivially done, and one is left with (at inst. # $k$)

$$
\begin{aligned}
Z_k &= \mathcal{N}_k \, \mathrm{e}^{2\pi i \tau k} \int \{d\chi da^\mu dM^\mu dD_{\hat{m}} d\lambda_{\hat{m}} d\mu\} \\
&\quad \times \mathrm{e}^{-\mathrm{tr}\left(\frac{g}{2} D_{\hat{m}} D^{\hat{m}} - \frac{g}{2} \lambda_{\hat{m}} Q'^2 \lambda^{\hat{m}} + \frac{t}{4} a_\mu \bar{\mathcal{F}}^{\mu\nu} Q'^2 a'_\nu + \frac{t}{4} M_\mu \bar{\mathcal{F}}^{\mu\nu} M_\nu - tr \mu^t Q'^2 \mu\right)} \\
&= \mathcal{N}_k \, \mathrm{e}^{2\pi i \tau k} \int \{d\chi\} \, \frac{Pf_{(adj,\mathbf{6}\subset\mathbf{7},\mathbf{1})}(Q'^2) Pf_{(\mathbf{k},\mathbf{1},\mathbf{8}_v)}(Q'^2)}{\det^{1/2}_{(symm,\mathbf{8}_s,\mathbf{1})}(Q'^2)}
\end{aligned}
$$

# General expressions

- Considering $\phi$ in the Cartan of SO(8), $f$ in the Cartan of SO(7) and bringing $\chi$ to the Cartan of SO(k) one gets (here for $k = 2n+1$) Back

$$Z_{2n+1} = \tilde{\mathcal{N}}_{2n+1} e^{2\pi i \tau(2n+1)} \frac{(f_1 f_2 f_3)^n}{\mathcal{E}^{(n+1)}} \int \prod_{l=1}^{n} \frac{d\chi^l}{2\pi i}$$

$$\times \frac{\prod_k \chi_k^2 R_\phi(\chi_k) R_f(\chi_k) \prod_{i<j} (\chi_{ij}^-)^2 (\chi_{ij}^+)^2 R_f(\chi_{ij}^-) R_f(\chi_{ij}^+)}{\prod_m R_E(2\chi_m) R_E(\chi_m) \prod_{p<q} R_E(\chi_{pq}^-) R_E(\chi_{pq}^+)}$$

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Here $\chi_{ij}^\pm = \chi_i \pm \chi_j$, $E_1 = 1/2(-f_1 + f_2 + f_3), \ldots, \mathcal{E} = E_1 E_2 E_3 E_4$ and

$$R_\phi(x) \equiv \prod_{u=1}^{4} \left(x^2 - \phi_u^2\right) ,$$

$$R_f(x) \equiv \prod_{a=1}^{3} \left(x^2 - f_a^2\right) , \quad R_E(x) \equiv \prod_{A=1}^{4} \left(x^2 - E_A^2\right)$$

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Analogous expression for $k = 2n$

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The $\chi$ integration are actually contour integrals to be done with certain prescriptions on the Im parts of the poles Moore et al, ... (follows from BRS structure)

# The prepotential

- Cosider the complete part. function

$$Z(\phi, f) = \sum_k \int d\mathcal{M}_{(k)} \, e^{-\mathcal{S}_{(k)}(\phi, f)} = \sum_k \hat{Z}_k \, e^{2\pi i \tau k} = \sum_k \hat{Z}_k \, q^k$$

- To a given order in $q$, contribute also "disconnected" configurations (instantons of lower numbers $k_i$, with $\sum k_i = k$).
  - To isolate the connected components, take the logarithm

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- In $d\mathcal{M}_{(k)}$ we have included the "center of mass" $dx^\mu$ and $d\theta$ integrals.
  - Without deformations these would diverge (with $\phi$ constant), now they give $\frac{1}{\mathcal{E}}$

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- The effective action for $\Phi(x, \theta)$ is written as

$$\int d^8x\, d^8\theta\, F(\Phi(x, \theta), f = 0)$$

where

$$F(\phi, f) = \mathcal{E} \log\left(1 + Z(\phi, f)\right)$$
$$F(\phi, f) = \sum_k F_k(\phi, f) q^k$$

# Explicit results at low k

By direct integration of the expression of $Z_k$ Recall and taking the log we get

$$F = Tr\,\Phi^4\left(\frac{1}{2}q^2+\frac{1}{4}q^4+\dots\right) - \left(\mathrm{Tr}\,\Phi^2\right)^2\left(\frac{1}{4}q^2+\frac{1}{4}q^4+\dots\right)$$
$$+8\,Pf\,\Phi\left(q+\frac{4}{3}q^3+\frac{6}{5}q^5+\dots\right)$$

in perfect agreement with the Heterotic results! Recall

- The fact that for $F$ is finite in the $f \to 0$ limit is highly non-trivial, requires very delicate cancellations

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- The fact that for $F$ is finite in the $f \to 0$ limit is highly non-trivial, requires very delicate cancellations
- If we keep the RR background turned on to the prepotential, we compute also gravitational corrections of the form $t_8\, tr\, R^4$ and $t_8\, (tr\, R^2)^2$

# Conclusions and perspectives

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- Integrating over the D(-1) moduli reproduces the $F^4$ effective action of the dual Het $SO(8)^4$ theory
  - Checked up to $k = 5$ (next step: all $k$ proof)
  - Gravitational corrections to be checked against Heterotic

# The tensor $t_8$

- We have ▸ Back

$$\mathrm{Tr}\left(t_8 F^4\right) \equiv \frac{1}{16} t_8^{\mu_1\mu_2\cdots\mu_7\mu_8} \mathrm{Tr}\left(F_{\mu_1\mu_2}\cdots F_{\mu_7\mu_8}\right)$$

$$= \mathrm{Tr}\Big(F_{\mu\nu}F^{\nu\rho}F^{\lambda\mu}F_{\rho\lambda} + \frac{1}{2}F_{\mu\nu}F^{\rho\nu}F_{\rho\lambda}F^{\mu\lambda}$$

$$-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}F_{\rho\lambda}F^{\rho\lambda} - \frac{1}{8}F_{\mu\nu}F_{\rho\lambda}F^{\mu\nu}F^{\rho\lambda}\Big)$$