When Higgs Production & Decay Met EW

Giampiero PASSARINO

Dipartimento di Fisica Teorica, Università di Torino, Italy
INFN, Sezione di Torino, Italy
HEPTOOLS Network

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Based on work done in collaboration with
Stefano Actis, Christian Sturm and Sandro Uccirati

Outlines

(1, 2,)

1. *From the analytical structure of EW NNLOs*
2. *to their numerical evaluation*

what else, but the inevitable!

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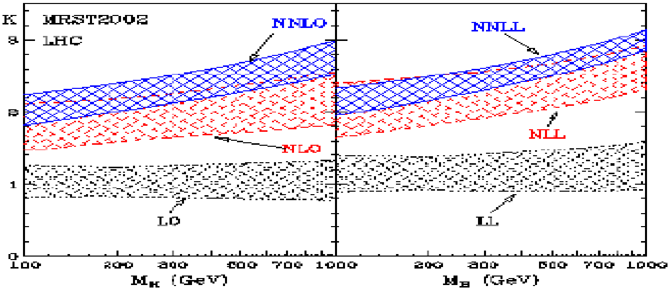
Part I

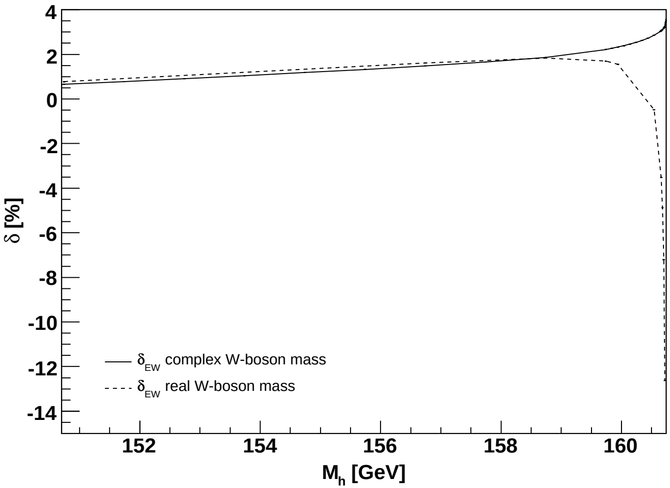
Preludio

QCD & K - factor(s)

QCD overview: from LO to NNLO and NNLL



What about EW? NLO for $\gamma\gamma$



$\downarrow -\infty$

Synopsis

From PO to RO

from $gg \to H$ to $pp \to gg(\to H) + X$

QCD, light Higgs

NLO K-fact.	$\approx$	$1.7 - 1.9$
NNLO K-fact.	$\approx$	$2.0 - 2.2$

EW $<$ 2008

- approximate
- incomplete
- divergent

Uncertainty

Remaining sources of large corrections?

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Part II

Intermezzo

GraphShot package

- A FORM code to generate and manipulate the amplitudes in the SM
- A link to FORTRAN libraries for numerical computation
- Authors: S. Actis, A. Ferroglia, G. Passarino, M. Passera, C. Sturm, S. Uccirati

The path to Feynman amplitudes · · ·

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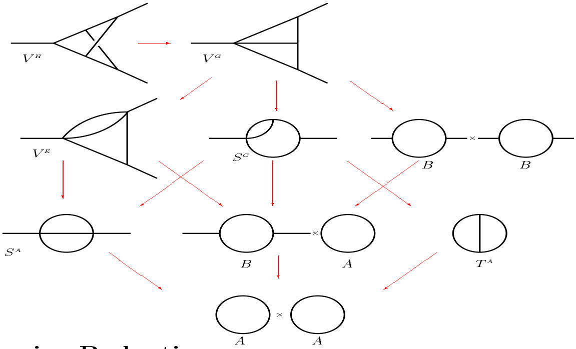
The path to Feynman amplitudes $\cdots$

Generating the Amplitude: reduction



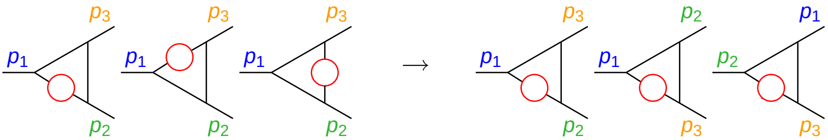
Recursive Reduction

Generic child topologies of the V^H parent topology. The five-line V^G diagram is obtained by removing one line of the V^H diagram; the second line contains the child topologies of V^G (V^E, S^C and $B \times B$). The third line contains the topologies S^A, $B \times A$ and T^A, obtained by removing one line from the diagrams above. The arrows indicate the correspondences between parent and child topologies.

Generating the Amptitude

Strategy

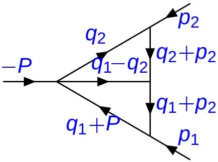
group diagrams into families, paying attention to permutation of external legs



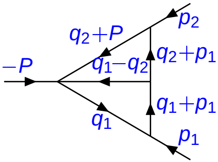
Rooting

Strategy

mapping onto a standard rooting for loop momenta



$\longrightarrow$

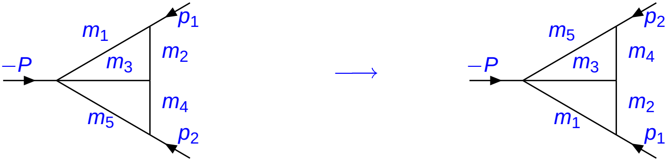


$$\begin{pmatrix} q_1 \to -q_1 - P \\ q_2 \to -q_2 - P \end{pmatrix}$$

Symmetry

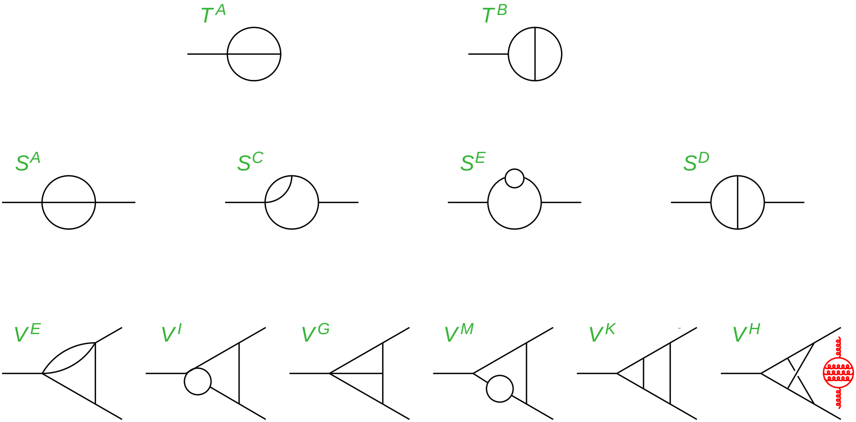
Strategy

apply symmetries to identify identical objects



$$\begin{pmatrix} q_1 \to -q_2 - P \\ q_2 \to -q_1 - P \end{pmatrix}$$

List-of-diagrams: all what is needed



All-you-can-do-analytic

rule-of-the-game

Adelante Numerics, cum judicio

UV

- UV poles, of course
- beware, *overlapping divergencies*

IR/Coll

- IR poles, of course
- Collinear logs, of course

upshot

Cancellations, if any, enforced analytically

All-you-can-do-analytic

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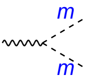
upshot

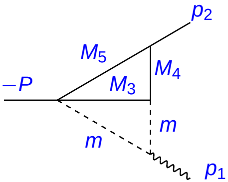
Cancellations, if any, enforced analytically

Extracting Collinear divergencies

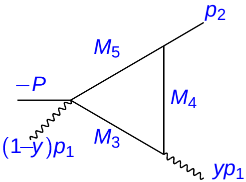
Theorem

Coefficients of collinear logarithms are integrals of one-loop functions





$$= \ln\frac{m^2}{s}\int_0^1 dy$$

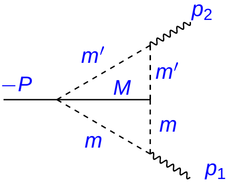


$$+ \quad \text{finite part}$$

Extracting Collinear divergencies

Example

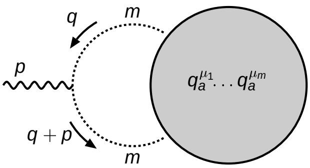
Sometimes the answer is explicit



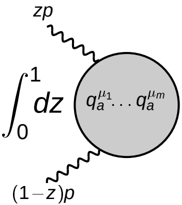
$$= \ln\frac{m^2}{s}\ln\frac{m'^2}{s}\mathrm{Li}_2\left(\frac{s}{M^2}\right) + \left(\ln\frac{m^2}{s} + \ln\frac{m'^2}{s}\right) \left[\mathrm{Li}_3\left(\frac{s}{M^2}\right) + 2\,S_{12}\left(\frac{s}{M^2}\right) - \ln\frac{M^2}{s}\mathrm{Li}_2\left(\frac{s}{M^2}\right)\right] + \text{finite part}$$

General results I

Coll. behavior of arbitrary two-loop q -scalar, UV-finite diagrams



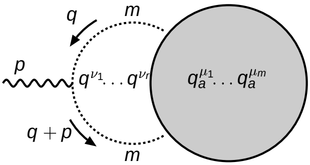
$$= \ln \frac{m^2}{s} \int_0^1 dz$$



$$+ \text{ coll. fin.}$$

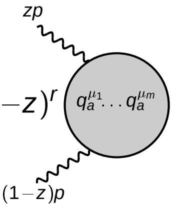
General results II

Generalization to tensor integrals



$$= \ln\frac{m^2}{s}\left[1-\frac{\epsilon}{2}\Delta_{UV}(s)-\frac{\epsilon}{4}\ln\frac{m^2}{s}\right]$$

$$\times \int_0^1 dz\,(-z)^r$$

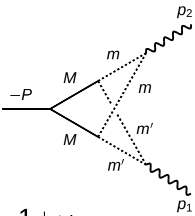


$$p^{\nu_1}\dots p^{\nu_r} + \text{c. f.}$$

General results III

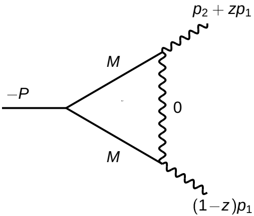
$$\omega = -P^2/M^2, \; l_\omega = \ln(1-\omega)$$

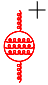
$$V^H_{\rm dc} = \left[P^2M^2 + 2P^2 q_1{\cdot}p_1 - 4(q_1{\cdot}p_1)^2\right]$$



$$= 2\left(1 - \frac{1+\omega}{\omega}\, l_\omega\right) L\, L' + 2\left[1 + \frac{1+\omega}{\omega}\, l_\omega\,(l_\omega - 1) + \mathrm{Li}_2(\omega)\right](L + L')$$

$$- 2\int_0^1 dz \left[(1-z)P^2\, L + (P^2 + 2\,q\cdot p_2)\, L'\right]$$

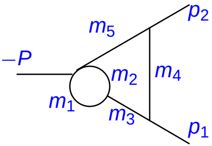




$+$

Extracting Ultraviolet divergencies

$V^I =$



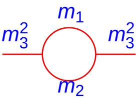
$$= \frac{1}{\pi^4} \int \frac{d^n q_1\, d^n q_2}{\underbrace{\underbrace{[1]\,[2]}_{x}\,[3]\,[4]\,[5]}_{y_1, y_2, y_3}},$$

$$\begin{aligned}
[1] &= q_1^2 + m_1^2 \\
[2] &= (q_1 - q_2)^2 + m_2^2 \\
[3] &= q_2^2 + m_3^2 \\
[4] &= (q_2 + p_1)^2 + m_4^2 \\
[5] &= (q_2 + P)^2 + m_5^2
\end{aligned}$$

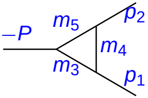
$$= C_\epsilon \int_0^1 dx \int dS_3(y_1, y_2, y_3)\, [x\,(1-x)]^{-\epsilon/2}\, (1 - y_1)^{\epsilon/2 - 1}\, V^{-1-\epsilon}$$

The single pole can always be expressed in terms of 1L.

$V^I =$



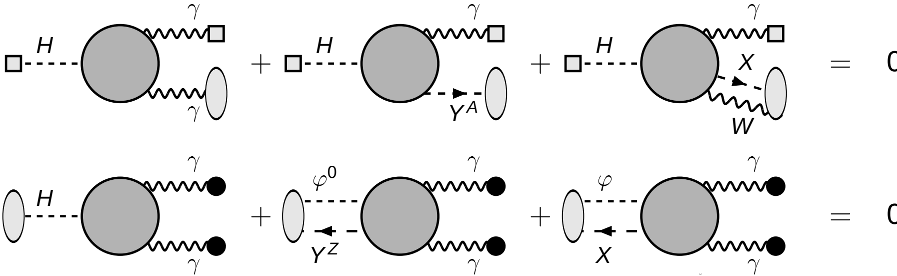
$\times$



$+$ finite part.

Checks

Off-shell WSTIs involving special sources
contracted sources $\longrightarrow$ black circles
physical ones $\longrightarrow$ gray boxes



Finite Renormalization

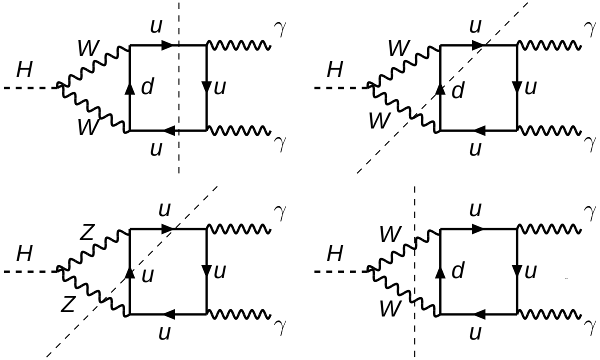
$$\mathcal{A}^{\mu\nu} = \mathcal{A}^{\mu\nu}_{(1)} \otimes (1 + \mathrm{FR}) + \mathcal{A}^{\mu\nu}_{(2)}$$

$$\begin{aligned}
m_B^2 &= M_B^2 \left[1 + \frac{G_F M_W^2}{2\sqrt{2}\,\pi^2}\,\mathrm{Re}\,\Sigma^{(1)}_{BB}(M_B^2)\right], \quad B = W, H, \\
m_t^2 &= M_t^2 \left[1 + \frac{G_F M_W^2}{\sqrt{2}\,\pi^2}\,\mathrm{Re}\,\Sigma^{(1)}_{t}(M_t^2)\right] \\
g^2\, s_\theta^2\, Z_A^{-1} &= 4\,\pi\,\alpha, \\
g\, Z_H^{-1/2} &= 2\,(\sqrt{2}\,G_F M_W^2)^{1/2} \left[1 - \frac{G_F M_W^2}{4\sqrt{2}\,\pi^2}\,\Pi_H(M_H^2)\right],
\end{aligned}$$

Part III

Andante

Around threshold



Singularities

- FD have a complicated analytical structure
- A frequently encountered singular behavior is associated with the so-called normal thresholds: the leading Landau singularities of self-energy-like diagrams
- which can appear, in more complicated diagrams, as sub-leading singularities.

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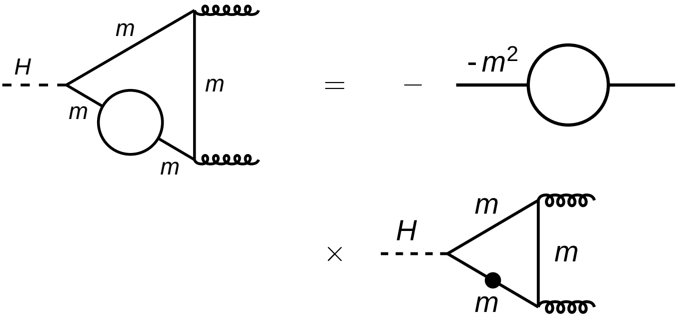
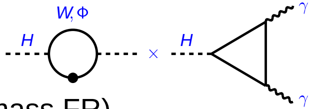
$1/\beta$ -behavior



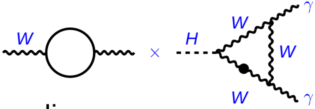
$$+ \left(\begin{array}{c} \text{reg. part} \\ \text{at } \beta = 0 \end{array} \right)$$

Origin of $1/\beta$

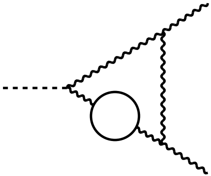
- (1-loop diagrams) $\otimes$ (H wave-function FR)



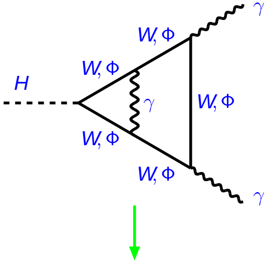
- (1-loop diagrams) $\otimes$ (W mass FR)



- Pure 2-loop diagrams

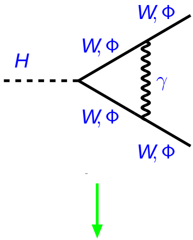


Logarithmic singularities



$\sim \ \ln \beta_W$

Remnant of Coulomb singularity



$\sim \ 1/\beta_W$

Part IV

Impetuoso

Solutions

RM scheme - none

where masses are the real on-shell ones; it gives the extension of the generalized minimal subtraction scheme up to two loop level.

MCM scheme - minimal

- start by removing the Re label in those terms that, coming from finite renormalization, violate WSTIs.
- split the amplitude

$$\mathcal{A}^{\mathrm{NLO}} = \sum_{i=W,Z} \frac{A_{\mathrm{SR},i}}{\beta_i} + A_{\mathrm{LOG}} \ln\left(-\beta_W^2 - i0\right) + A_{\mathrm{REM}},$$

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Solutions

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- After proving that all coefficients, gauge-parameter independent by construction, satisfy the WST identities, we minimally modify the amplitude introducing the complex-mass scheme of for the divergent terms.

$$m_i^2 = M_i^2 \left[1 + \frac{G_F M_W^2}{2\sqrt{2}\,\pi^2} \mathrm{Re}\Sigma_i^{(1)}(M_i^2) \right] \Rightarrow$$

$$m_i^2 = s_i \left[1 + \frac{G_F s_W}{2\sqrt{2}\,\pi^2} \Sigma_i^{(1)}(s_i) \right],$$

Solutions

pitfalls

A nice feature of the MCM scheme is its simplicity

MCM scheme - minimal

The MCM, however, does not deal with cusps associated with the crossing of normal thresholds.

MCM scheme - minimal

- The large and artificial effects arising around normal thresholds in the MCM scheme (or in RM scheme) are aesthetically unattractive.
- In addition, they represent a concrete problem in assessing the impact of two-loop EW corrections on processes relevant for the LHC.

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CM scheme - complete

- The procedure described for the divergent terms has been extended to the remainder A_{REM}. In particular, all two-loop diagrams have been computed with complex masses for the internal vector bosons.

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- In the full CM setup, the real parts of the W and Z self-energies induced by one-loop renormalization of the masses and the couplings have to be traded for the associated complex expressions.

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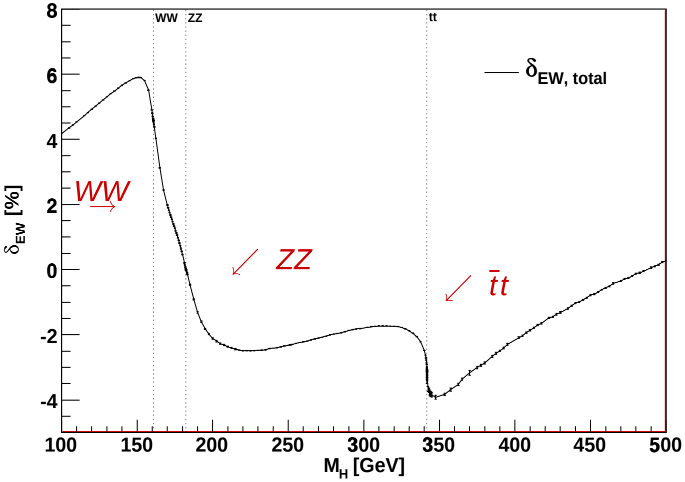
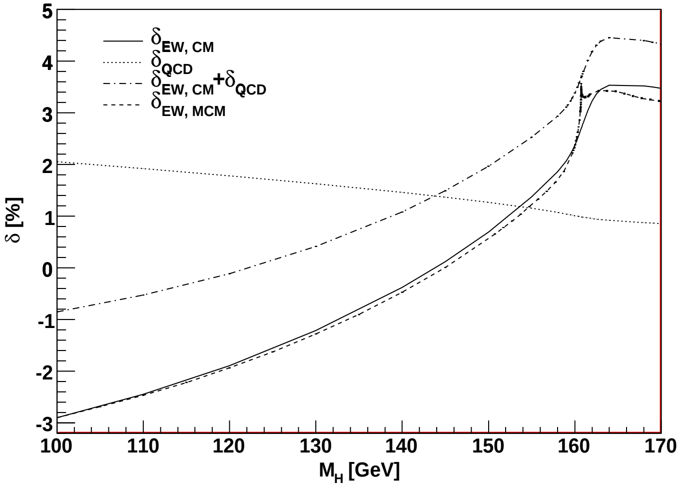
Part V

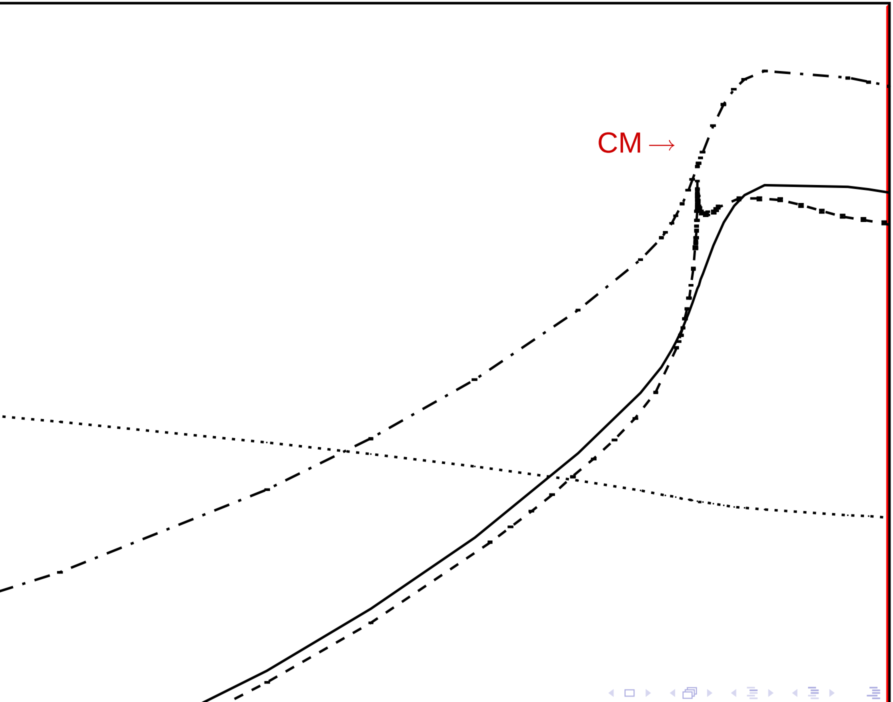
Allegro

EW on gluon-gluon fusion

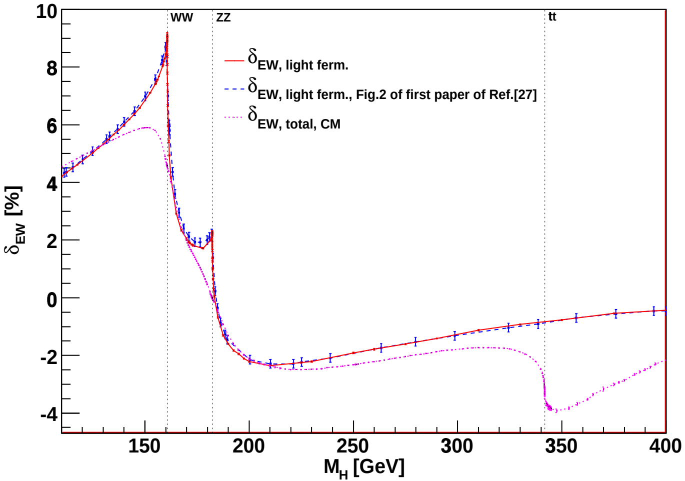


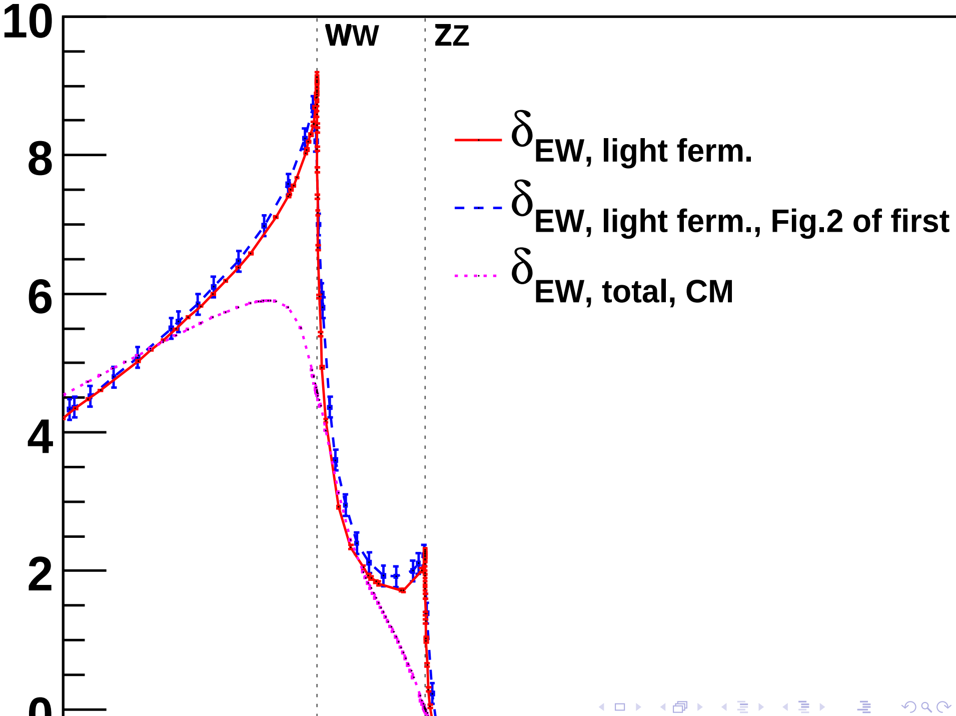
EW on decay ($\gamma\gamma$)




CM →

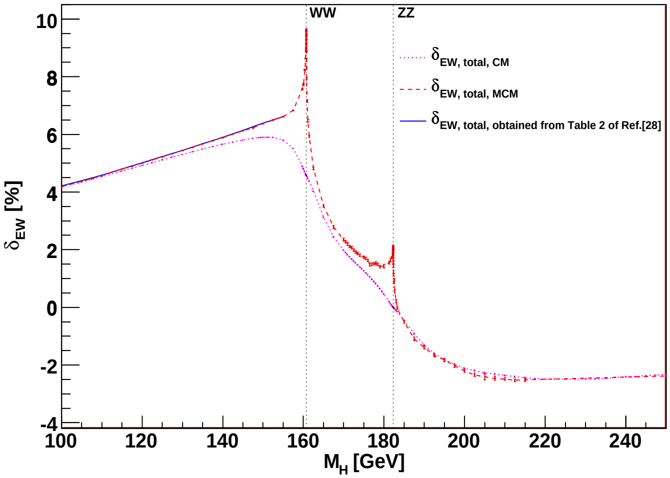
Comparing


10
8
6
4
2
0
-2
-4
WW
ZZ
tt
δEW, light ferm.
δEW, light ferm., Fig.2 of first paper of Ref.[27]
δEW, total, CM
δEW [%]
150
200
250
300
350
400
MH [GeV]



10
8
6
4
2
0
WW
ZZ
δ EW, light ferm.
δ EW, light ferm., Fig.2 of first
δ EW, total, CM

Comparing


WW
ZZ
$\delta_{EW,\ total,\ CM}$
$\delta_{EW,\ total,\ MCM}$
$\delta_{EW,\ total,\ obtained\ from\ Table\ 2\ of\ Ref.[28]}$
δ_{EW} [%]
M_H [GeV]
10
8
6
4
2
0
-2
-4
100
120
140
160
180
200
220
240

Part VI

Allegro Con Brio

EW on K-factors - uncertainty

We introduce two options for including NLO electroweak corrections

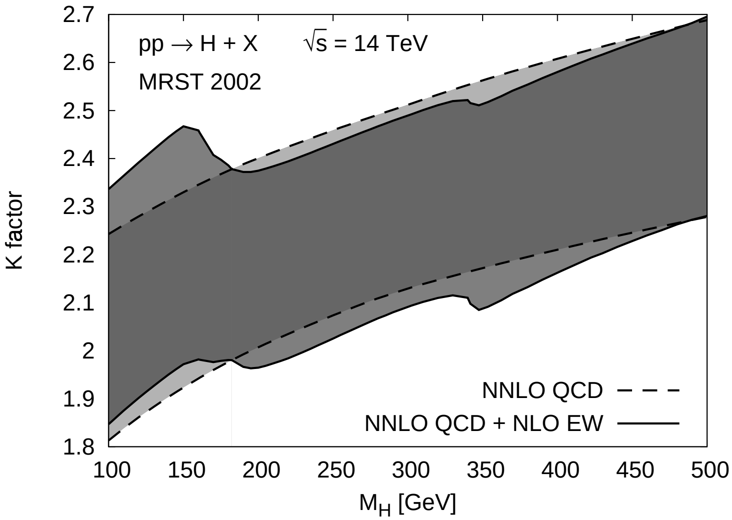
- CF (Complete Factorization):

$$\sigma^{(0)} G_{ij} \to \sigma^{(0)} \left(1 + \delta_{\rm EW}(M_H^2)\right) G_{ij};$$

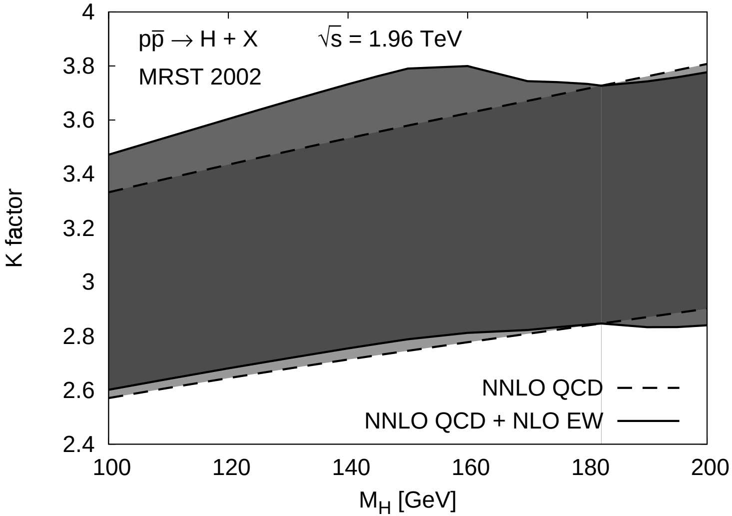
- PF (Partial Factorization):

$$\sigma^{(0)} G_{ij} \to \sigma^{(0)} \left[G_{ij} + \alpha_S^2(\mu_R^2)\delta_{\rm EW}(M_H^2)\, G_{ij}^{(0)}\right],$$

EW on K-factors - LHC



EW on K-factors - Tevatron



Conclusions

(1, 2, 3, 4, 5)

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   ($\approx$ 10 kilohour - project)
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   ($100\,\mathrm{GeV} < M_H < 500\,\mathrm{GeV}$),
3. *without incongruent large EW effects,*
4. *thereby showing that only a complete implementation of the computational scheme keeps two-loop corrections under control.*
5. *Se logra por repetición / meter el coso por dentro / en que está la cosa sagrata*

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