Ye were not made to live like unto brutes, But for pursuit of QFT

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Mission impossible:

how to keep students away from the beach

This is a very complex problem and the solutions are not going to be easy and aren't going to be realized overnight, but we are putting mechanisms in place that we believe will have a major impact on easing the tensions within the beach community

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Which option for these lectures?

Feynman quoting Gibbons:

The power of instruction is seldom of much efficacy except in those happy dispositions where it is almost superfluous

Gell-Mann on Feynman

no, Dick's methods are not the same as the methods used here. Dick's method is this. You write down the problem. You think very hard. Then you write down the answer

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THE GREAT SUCCESS of modern particle physics is based on the possibility of describing the fundamental structure and behavior of matter within a theoretical framework called the standard model

Outlines

(1, 2,)

1. *From Lagrangians*
2. *to renormalization and Higgs Physics,*

what else, but the inevitable!

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Part I

Lecture I

(1, 2, 3,)

1. Our goal: New Physics
2. No SM background can give rise to a sharp peak (but for W, Z, t. all sources give rise to a continuum spectrum.
3. Caution is demanded in assuming that we know all that is needed in accurately predict the properties of LHC final states.

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The New Standard Model?

Fundamental theory of TeV scale?

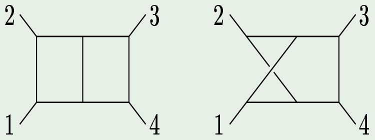
The first step in uncovering the NSM will be the rediscovery of the OSM at 14 TeV. *Many discrepancies between data and SM predictions will likely be uncovered most of which will not be signals of New Physics. Ultimately, we seek the effective* $\mathcal{L}_{\mathrm{TeV}}$ *but this goal will not be immediately attainable.*

Challenge

Problem

HO perturbative QFT is a rather challenging field requiring:
clever ideas and algorithms

Example



Solutions

- importing new ideas from twistor developments into QCD

- new ideas from QCD into EW physics to confront the *practical difficulties* there,

- especially as concerns *massive Feynman diagrams*.

Complexity of an NLO(NNLO) process

(1, 2, 3)

1. *A variety of important processes will benefit from NLO(NNLO) computations*
2. *some in conjuction with resummation of large logs*
3. *Ideally, one would like a NLO(NNLO) program that mimic the experimental situation*

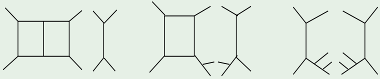
Complexity: $n\,!$ growth

Different amplitudes interfere

(a, b, c)

1. *virtual $\otimes$ tree*
2. *virtual $\otimes$ real*
3. *real $\otimes$ real*

Example



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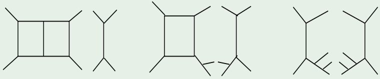

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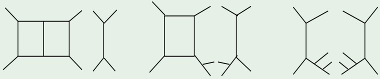

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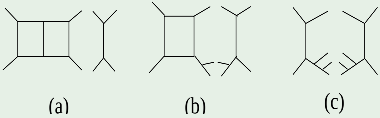

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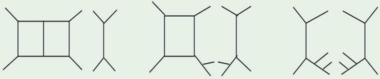

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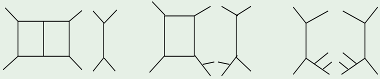

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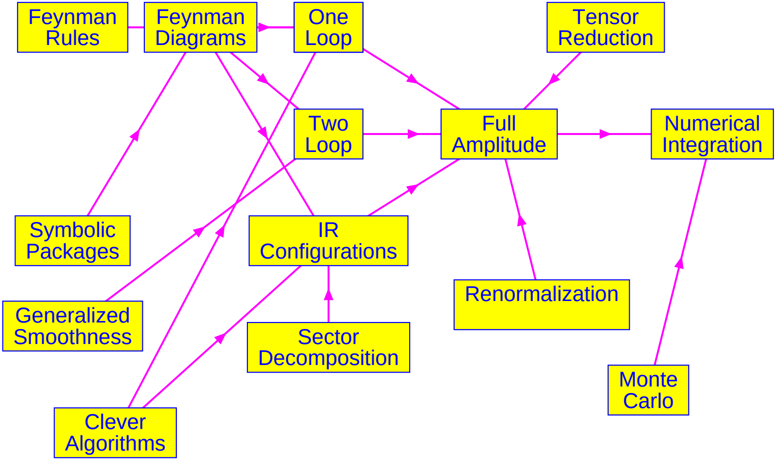
NNLO flowchart



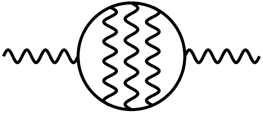
How to beat complexity?

Problems

- Memory
- Performance
- Software

Levels

- Analytic (approximations?): symbolic programs
- Numerical: stability & cancellations



3-loop 4-graviton $\approx 10^{21}$ terms / diag

Example

- parallelization
- automatization
- standardization

Concept of renormalization

Lagrangian

$\mathcal{L}(x)$, $x =$ parameters

Born

- no ambiguity
- one data $\rightarrow$ fix x
- compare with experiment(s)

Loops

- x(experiment) more complicated
- $x_{\text{loops}} - x_{\text{tree}} = \infty$

upshot

Use dimensional regularization

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Counter - term I

Lagrangian

Because *corrected* x and *tree* x are so different

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- one introduces the notion of *counter-term*

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- in the Lagrangian $x \to x\,(1 + \delta\,x)$

upshot

δx is chosen such that x remains in the neighborhood of the *tree* x. The only thing that ever emerges in the confrontation with the data is $x\,(1 + \delta x)$

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Counter - term II

δx

In order to have meaningful communication it is necessary, when talking about x, to specify what δx is used

Schemes

Stating one's conventions is termed **renormalization scheme**

Example

- prescribe what x is;
- prescribe what δx is.

Schemes

QED

- In the older days of QED method 1 was preferred
- The convention was to prescribe x and to use for that some very well defined experimental quantity; δx is then obtained from the data including radiative corrections

A case in point

Electron mass. $m\,(1+\delta m)$ is the *bare mass* and m itself the *experimental mass*

Example

Method 1 has the advantage of not being dependent on the choice of RS, but it offers a problem when there is no clear, precisely known experimental quantity that can play the role of defining x.

Renormalization I

The renormalization idea:

- Experimentally one observes never the lowest order alone, but the sum of all orders.
- Up to first order, the mass in a propagator is $m\,(1+\delta m)$ and that is what the experimenter observes.
- Therefore, m is the observed mass and the theory makes no predictions about the mass. It is a free parameter, and it must be fixed by comparing the results of the theory with the observed data.

The most important question is:

do all infinities of the theory appear in combination with a few parameters? If this is the case we call the theory renormalizable else, non-renormalizable.

Renormalization II

Example

We start by assuming the existence of some cutoff Λ, above which the theory eventually changes. The question now is what Λ-depemdent effects could we expect at low energy, characterized by some energy scale $E \ll \Lambda$.

UV

In working out perturbation theory (in some coupling constant g) we will encounter series in the variable $g\Lambda^2/E^2$. In a non-renormalizable theory any measurable quantity will correspond to a series that at sufficiently high order diverges as $\Lambda \to \infty$,

$$g^l \left[a_0 + a_2 g^2 + \cdots + a_k g^k \left(\frac{\Lambda}{E}\right)^2 + a_{k+2} g^{k+2} \left(\frac{\Lambda}{E}\right)^4 + \cdots\right],$$

Renormalization III

Example

consider the leptonic part of the Fermi theory of weak interactions,

$$\begin{aligned} \mathcal{L} &= G_F j_\mu j_\mu^\dagger, \quad G_F = \frac{g^2}{m_p^2}, \quad g^2 \sim 10^{-5}, \\ J_\alpha &= \sum_l \overline{\nu}_l \gamma_\alpha (1+\gamma_5) l. \end{aligned}$$

Consider $\nu_e e$ elastic scattering.

Example

- In lowest order the result is proportional to g^2,
- however in next order we have three diagrams proportional to $g^4 \Lambda^2 / E^2$.

Renormalization IV

The Λ effect

is not measurable at low energies simply because through renormalization of g this effect can be transformed away.

Example

Another amplitude suffering large corrections is that for μ-decay.

- The situation is precisely as before but the series has different coefficients and the renormalization of g on the basis of $\nu_e e$ scattering will not neutralize the series for μ-decay.

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Renormalization V

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Thus now the corrections become observable and we can rule out the values of Λ larger than E/g.

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In a renormalizable theory the cutoff dependence is not observable and can be absorbed in the parameters of the theory, e.g. coupling constants and masses.

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Gauge invariance

The quantum-mechanical counterpart

of the subsidiary condition that restricts the solutions in the classical theory, e.g. $\partial_\mu A^\mu = 0$, is that

$\partial_\mu A^\mu$

is a free field that decouples, i.e. does not interact with matter.

- To get rules for diagrams in a gauge theory, including the abelian one, difficulties manifest in the fact that the matrix that defines the propagator of the theory has no inverse.

$\mathcal{L}_{\mathrm{QED}}$

Example

Consider for instance the following Lagrangian:

$$\mathcal{L} \quad = \quad -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}, M^2 A_\mu A^\mu .$$

The propagator for the field A^μ is given by the inverse of $V_{\mu\nu} = -(p^2 + M^2)\delta_{\mu\nu} + p_\mu p_\nu$ which has a simple solution

$$V^{-1}_{\mu\nu} \quad = \quad \frac{1}{(2\pi)^4 i}\,\frac{1}{p^2 + M^2}\,\left(\delta_{\mu\nu} + \frac{p_\mu p_\nu}{M^2}\right) .$$

The gauge invariant theory corresponding to $M = 0$ is therefore singular since V is singular.

$\mathcal{L}_{\text{QED}}$

If, due to gauge invariance,

a Lagrangian is *singular*, then a good Lagrangian can be obtained by

- adding a term $-1/2\, C^2$ where C behaves non trivially with respect to the gauge transformation, $C \to C + t\,\Lambda$.
- Here t is an operator that may contain derivatives and be field-dependent.
- C will appear to be a free field and successively we must introduce the so-called Faddeev–Popov ghost fields to compensate for its introduction.

$\mathcal{L}_{\mathrm{QED}}$

A gauge-fixed

Lagrangian for QED is given by

$$\begin{aligned}\mathcal{L}_{\mathrm{QED}} &= -\frac{1}{4} F_{\mu\nu} F_{\mu\nu} - \frac{1}{2} (\mathcal{C}_A)^2 \\ &- \sum_f \overline{\psi}_f \left(\partial\!\!\!/ - ieQ_f A\!\!\!/ + m_f \right) \psi_f ,\end{aligned}$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu, \qquad \mathcal{C}_A = -\frac{1}{\xi} \partial_\mu A_\mu,$$

and where the sum runs over the fermion fields.

$\mathcal{L}_{\text{QED}}$

Each fermion has

- a charge, eQ_f, e, being the charge of the positron,
- and mass m_f.
- Within the SM we have leptons with charge $Q_l = -1$, up-quarks with $Q_f = \frac{2}{3}$, and down-quarks with charge $Q_f = -\frac{1}{3}$.

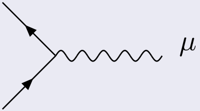
FR

The Feynman rules of QED

are particularly simple. They can be summarized as follows:

$$\frac{1}{(2\pi)^4 i} \frac{-i\not{p} + m_f}{p^2 + m_f^2 - i\epsilon},$$

$$\frac{1}{(2\pi)^4 i} \frac{1}{p^2 - i\epsilon} \left[\delta_{\mu\nu} + \left(\xi^2 - 1\right) \frac{p_\mu p_\nu}{p^2}\right],$$



$$(2\pi)^4 i \; ieQ_f \, \gamma_\mu .$$

Part II

Lecture II

Basic

EW

The electroweak theory is based on $SU(2) \otimes U(1)$ and we must discuss the field content of this theory in terms of representations of the group itself.

YM

There is a triplet of vector bosons B_μ^a, a singlet B_μ^0, a complex scalar field K, fermion families, and Faddeev–Popov ghost-fields (hereafter FP) $X^\pm$, Y^Z, Y^A. The physical fields Z and A are related to B_μ^3 and B_μ^0 by a rotation in terms of the so calledweak-mixing angle.

$\mathcal{L}_{\rm SM}$

The scalar field

in the minimal realization of the SM is

$$K = \frac{1}{\sqrt{2}}\begin{pmatrix} \chi \\ \sqrt{2} i \phi^- \end{pmatrix}, \qquad \chi = H + 2\frac{M}{g} + i\phi^0,$$

where by H we denote the physical Higgs boson and moreover M and g are Lagrangian parameters corresponding to the bare W mass and to the $SU(2)$ bare coupling constant.

Total $\mathcal{L}_{\rm SM}$

The total Lagrangian will be the sum of various pieces.

$\mathcal{L}_{\rm SM}$

$\mathcal{L}_{\rm YM} + \mathcal{L}_{\rm S}$

with the standard Yang–Mills terms given by

$$\mathcal{L}_{\rm YM} = -\frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu} - \frac{1}{4}F^0_{\mu\nu}F^0_{\mu\nu},$$

The *minimal* Higgs sector

$$\mathcal{L}_{\rm S} = -(D_\mu K)^+ D_\mu K - \mu^2 K^+ K - \frac{1}{2}\lambda\left(K^+K\right)^2,$$

where $\lambda > 0$ and **SB** requires $\mu^2 < 0$. **SB** is the mechanism of introducing masses for the vector bosons through the shift in the scalar field that allows for $< H >= 0$. The remaining degrees of freedom in K will be non-physical and connected with the longitudinal polarizations of the spin 1 particles.

$\mathcal{L}_{\rm SM}$

Moreover

$$
\begin{aligned}
F^a_{\mu\nu} &= \partial_\mu B^a_\nu - \partial_\nu B^a_\mu + g\varepsilon_{abc} B^b_\mu B^c_\nu , \\
F^0_{\mu\nu} &= \partial_\mu B^0_\nu - \partial_\nu B^0_\mu ,
\end{aligned}
$$

and

$$
D_\mu K = \left(\partial_\mu - \frac{i}{2} g B^a_\mu \tau^a - \frac{i}{2} g g_1 B^0_\mu \right) K ,
$$

with the standard Pauli matrices τ^a and $g_1 = -s_\theta / c_\theta$.

$\mathcal{L}_{\rm SM}$

$\mathcal{L}_{\rm YM} - (D_\mu K)^+ D_\mu K$ and $\mathcal{L}_{\rm S}^{\rm I}$

$$\mathcal{L}_{\rm YM} - (D_\mu K)^+ D_\mu K = \mathcal{L}_0 + M\Big(\frac{Z_\mu}{c_\theta}\partial_\mu\phi^0 + W_\mu^+\partial_\mu\phi^- + W_\mu^-\partial_\mu\phi^+\Big),$$

where the charged fields have been introduced as

$$W_\mu^\pm = \frac{1}{\sqrt{2}}\left(B_\mu^1 \mp iB_\mu^2\right), \quad \phi^\pm = \frac{1}{\sqrt{2}}\left(\phi^1 \mp i\phi^2\right), \; \phi^0 \equiv \phi^3.$$

This part of the Lagrangian contains $Z - \phi^0, W^\pm - \phi^\mp$ mixing terms; they are of $\mathcal{O}\left(g^0\right)$ and their contribution must be summed up. **There we discover the singularity of the Lagrangian**.

$\mathcal{L}_{\rm SM}$

$\mathcal{L}_{\rm SM}$ is invariant

under a set of transformations that are the generalization of the well known QED example $A_\mu \to A_\mu + \partial_\mu \Lambda$.

$$
\begin{aligned}
B^a_\mu &\to B^a_\mu + g\varepsilon_{abc}\Lambda^b B^c_\mu - \partial_\mu \Lambda^a, \qquad B^0_\mu \to B^0_\mu - \partial_\mu \Lambda^0, \\
K &\to \left(1 - \frac{i}{2} g \Lambda^a \tau^a - \frac{i}{2} g g_1 \Lambda^0\right) K, \qquad \text{with} \quad g_1 = -\frac{s_\theta}{c_\theta} \\
H &\to H - \frac{i}{2} g \left[\left(\Lambda^3 + g_1 \Lambda^0\right) \left(H + 2\frac{M}{g} + i\phi^0\right) + 2i\Lambda^+ \phi^- \right] \\
\phi^0 &\to \phi^0 - \frac{1}{2} g \left(\Lambda^3 + g_1 \Lambda^0\right) \left(H + 2\frac{M}{g}\right) + \frac{i}{2} g \left(\Lambda^- \phi^+ - \Lambda^+ \phi^-\right) \\
\phi^- &\to \phi^- - \frac{1}{2} g \Lambda^- \left(H + 2\frac{M}{g} + i\phi^0\right) - \frac{i}{2} g \left(-\Lambda^3 + g_1 \Lambda^0\right) \phi^-
\end{aligned}
$$

$\mathcal{L}_{\rm SM}$

add a *gauge-fixing* piece

to the Lagrangian ($\mathcal{L}_{\rm gf}$) that cancels these mixing terms.

However

it breaks the gauge invariance and successively we must introduce the so-called Faddeev–Popov ghost fields to compensate for this breaking. The gauge-fixing term transforms as

$$\mathcal{C}^i \quad \rightarrow \quad \mathcal{C}^i + \left(\mathrm{M}^{ij} + g\mathrm{L}^{ij}\right)\mathcal{N}^j .$$

M^{ij} must have an inverse and we thus have a permissible gauge. $g\mathrm{L}^{ij}$ defines the interaction with the gauge bosons.

$\mathcal{L}_{\text{SM}}$

Example

We now specify a set of gauges R_ξ depending on a single parameter ξ. We have

- a renormalizable gauge for finite ξ and
- the physical (unitary) gauge is obtained for $\xi \to \infty$.

That these two gauges belong to the same family and are connected through a continuous parameter is vital in proving renormalizability and unitarity of the theory.

$\mathcal{L}_{\rm SM}$

The gauge-fixing piece is

$$\mathcal{L}_{\rm gf} = -\frac{1}{2}\mathcal{C}^a\mathcal{C}^a - \frac{1}{2}\left(\mathcal{C}^0\right)^2 = -\mathcal{C}^+\mathcal{C}^- - \frac{1}{2}\left[\left(\mathcal{C}^3\right)^2 + \left(\mathcal{C}^0\right)^2\right],$$

where we can write

$$\mathcal{C}^a = -\frac{1}{\xi}\partial_\mu B^a_\mu + \xi M \phi^a.$$

$\mathcal{L}_{\rm SM}$

Example

The various components are given in the following equations: first

$$\mathcal{C}^{\pm} = -\frac{1}{\xi}\partial_\mu W^{\pm}_\mu + \xi M \phi^{\pm}, \qquad \mathcal{C}^0 = -\frac{1}{\xi}\partial_\mu B^0_\mu + \xi \frac{s_\theta}{c_\theta} M \phi^0.$$

Then, in the $Z - A$ basis, we obtain

$$\mathcal{C}_A = -\frac{1}{\xi}\partial_\mu A_\mu, \qquad \mathcal{C}_Z = -\frac{1}{\xi}\partial_\mu Z_\mu + \xi \frac{M}{c_\theta}\phi^0.$$

In the R_ξ gauge we have that

$$\mathcal{L}_{\rm YM} - \left(D_\mu K\right)^+ D_\mu K - \mathcal{C}^+\mathcal{C}^- - \frac{1}{2}\mathcal{C}_Z^2 - \frac{1}{2}\mathcal{C}_A^2 = \mathcal{L}_{\rm prop} + \mathcal{L}^{\rm bos,I}.$$

$\mathcal{L}_{\mathrm{SM}}$

$\mathcal{L}_{\mathrm{prop}}$, now reads

$$
\begin{aligned}
\mathcal{L}_{\mathrm{prop}} \quad = \quad & -\partial_\mu W^+_\nu \partial_\mu W^-_\nu + \left(1 - \frac{1}{\xi^2}\right) \partial_\mu W^+_\mu \partial_\nu W^-_\nu \\
& -\frac{1}{2}\partial_\mu Z_\nu \partial_\mu Z_\nu + \frac{1}{2}\left(1 - \frac{1}{\xi^2}\right)\left(\partial_\mu Z_\mu\right)^2 \\
& -\frac{1}{2}\partial_\mu A_\nu \partial_\mu A_\nu + \frac{1}{2}\left(1 - \frac{1}{\xi^2}\right)\left(\partial_\mu A_\mu\right)^2 \\
& -\frac{1}{2}\partial_\mu H \partial_\mu H - \partial_\mu \phi^+ \partial_\mu \phi^- - \frac{1}{2}\partial_\mu \phi^0 \partial_\mu \phi^0 \\
& -M^2 W^+_\mu W^-_\mu - \frac{1}{2}\frac{M^2}{c^2_\theta} Z_\mu Z_\mu \\
& -\xi^2 M^2 \phi^+ \phi^- - \frac{1}{2}\xi^2 \frac{M^2}{c^2_\theta}\phi^0 \phi^0 - \frac{1}{2} M_H H^2.
\end{aligned}
$$

$\mathcal{L}_{\mathrm{SM}}$

Those for the gauge fields are as follows:

$$
\begin{aligned}
\mathcal{L}_{\mathrm{prop}} \quad \rightarrow \quad W^{\pm} \qquad & \frac{1}{p^2+M^2}\left\{\delta_{\mu\nu}+\left(\xi^2-1\right)\frac{p_\mu p_\nu}{p^2+\xi^2 M^2}\right\} \\
&= \frac{1}{p^2+M^2}\left(\delta_{\mu\nu}+\frac{p_\mu p_\nu}{M^2}\right)-\frac{p_\mu p_\nu}{M^2\left(p^2+\xi^2 M^2\right)} \\
&= \frac{1}{p^2+M^2}\left(\delta_{\mu\nu}-\frac{p_\mu p_\nu}{p^2}\right)+\frac{\xi^2}{p^2+\xi^2 M^2}\frac{p_\mu p_\nu}{p^2}, \\
Z \qquad & \text{from } W^{\pm} \text{ by replacing } \quad M \rightarrow \frac{M}{c_\theta}, \\
A \qquad & \frac{1}{p^2}\left\{\delta_{\mu\nu}+\left(\xi^2-1\right)\frac{p_\mu p_\nu}{p^2}\right\}.
\end{aligned}
$$

$\mathcal{L}_{\mathrm{SM}}$

The scalar field propagators are

$\phi^{\pm}$

$$\frac{1}{p^2+\xi^2 M^2},$$

ϕ^{0}

$$\frac{1}{p^2+\xi^2 \frac{M^2}{c_\theta^2}}.$$

$\mathcal{L}_{\mathrm{SM}}$

Fermions will be arranged into isodoublets

.

$$\psi = \begin{pmatrix} u \\ d \end{pmatrix}, \qquad \psi_{L,R} = \frac{1}{2}(1 \pm \gamma_5)\,\psi,$$

- $u = \nu_l (l = e, \mu, \tau), u, c, t$-quark and
- $d = l(l = e, \mu, \tau), d, s, b$-quark.

Furthermore, we distinguish between left and right fields since a theory of weak interactions cannot be purely vectorial, in contrast with QED (and QCD).

$\mathcal{L}_{\text{SM}}$

The covariant derivative for the *L*-fields is

$$D_\mu \psi_L = \left(\partial_\mu + g B^i_\mu T^i\right)\psi_L, \qquad i = 0, ..., 3$$

$$T^a = -\frac{i}{2}\tau^a, \qquad T^0 = -\frac{i}{2} g_2 I.$$

$$D_\mu \psi_R = \left(\partial_\mu + g B^i_\mu t^i\right)\psi_R, \qquad i = 0, ..., 3,$$

$$t^a = 0, \qquad t^0 = -\frac{i}{2}\begin{pmatrix} g_3 & 0 \\ 0 & g_4 \end{pmatrix}.$$

$\mathcal{L}_{\rm SM}$

This part of the Lagrangian can be written as

$$\mathcal{L}_V^{\rm fer,I} = -\overline{\psi}_L \not{D} \psi_L - \overline{\psi}_R \not{D} \psi_R , \qquad g_i = -\frac{s_\theta}{c_\theta}\lambda_i .$$

- The parameters g_2, g_3 and g_4 are arbitrary constants.
- However, one can prove that $g_3 = g_1 + g_2$.
- In other words, these constants are not completely free if we want to generate fermion masses with the help of the Higgs system.

$\mathcal{L}_{\rm SM}$

Thus

- ψ_L transforms as a doublet under $SU(2)$ and the
- ψ_R as a singlet.

The parameters λ_i are then fixed by the requirement that the e.m. current has the conventional structure, $iQ_f e\bar{f}\gamma_\mu f$, without parity violating terms and with the right normalization. We put $e = gs_\theta$ and derive the solution as

$$\lambda_2 = 1 - 2Q_u = -1 - 2Q_d, \qquad \lambda_3 = -2Q_u, \qquad \lambda_4 = -2Q_d,$$

where the charge is $Q_f = 2I_f^{(3)}|Q_f|$.

$\mathcal{L}_{\text{SM}}$

Example

$$\begin{aligned}
\mathcal{L}_V^{\text{fer,I}} &= \sum_f \left[i g s_\theta Q_f A_\mu \bar{f} \gamma_\mu f \right. \\
&+ i \frac{g}{2c_\theta} Z_\mu \bar{f} \gamma_\mu \left(I_f^{(3)} - 2 Q_f s_\theta^2 + I_f^{(3)} \gamma_5 \right) f \left. \right] \\
&\quad + \sum_{\text{d}} \left[i \frac{g}{2\sqrt{2}} W_\mu^+ \bar{u} \gamma_\mu (1 + \gamma_5) d \right. \\
&+ i \frac{g}{2\sqrt{2}} W_\mu^- \bar{d} \gamma_\mu (1 + \gamma_5) u \left. \right],
\end{aligned}$$

$\mathcal{L}_{\rm SM}$

For the Higgs-fermion sector,

in the presence of quarks, we need not only the field K but its conjugate K^c too;

- that is, we need both K and K^c in order to give mass to the up- and down-partner of the fermionic isodoublet. The K^c is

$$K^c = -\frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{2}i\phi^+ \\ \chi^* \end{pmatrix},$$

$\mathcal{L}_{\text{SM}}$

with the corresponding part of the Lagrangian:

$$\mathcal{L}_S^{\text{fer}} = -\alpha_f \overline{\psi}_L K u_R - \beta_f \overline{\psi}_L K^c d_R + h.c.$$

The solution for the Yukawa couplings gives

$$\alpha_f = \frac{1}{\sqrt{2}} g \frac{m_u}{M}, \qquad \beta_f = -\frac{1}{\sqrt{2}} g \frac{m_d}{M}.$$

The last part of the Lagrangian is now

$$\mathcal{L}_S^{\text{fer}} = -\sum_f m_f \bar{f} f + \mathcal{L}_S^{\text{fer,I}},$$

$\mathcal{L}_{\rm SM}$

with an interaction Lagrangian given by

$$
\begin{aligned}
\mathcal{L}_S^{\rm fer,I} \;=\; & \sum_{\rm d} \left\{ i \frac{g}{2\sqrt{2}} \phi^+ \left[\frac{m_u}{M} \overline{u} \left(1+\gamma_5\right) d - \frac{m_d}{M} \overline{u} \left(1-\gamma_5\right) d \right] \right. \\
& \left. + i \frac{g}{2\sqrt{2}} \phi^- \left[\frac{m_d}{M} \overline{d} \left(1+\gamma_5\right) u - \frac{m_u}{M} \overline{d} \left(1-\gamma_5\right) u \right] \right\} \\
& + \sum_f \left(-\frac{1}{2} g H \frac{m_f}{M} \overline{f} f + i g I_f^{(3)} \phi^0 \frac{m_f}{M} \overline{f} \gamma_5 f \right),
\end{aligned}
$$

$\mathcal{L}_{\text{SM}}$

weak Lagrangian

Having fixed the propagators we can spell out the weak Lagrangian, describing the vector bosons and their interactions including interactions with the scalar system. The interested reader should consult any textbook for further details.

$\mathcal{L}_{\mathrm{SM}}$

This short section will be devoted

mainly to introducing some of the building blocks that are needed in order to discuss radiative corrections in any field theory. Beyond the Born-level loops will appear and they will depend on several variables, internal and external masses.

To cope with the complications of the SM,

we must derive a complete set of formulas valid for arbitrary internal and external masses. One has to deal with expressions for scalar diagrams with one, ... four external lines (or more).

Besides scalar functions

we also need tensor integrals with as many powers of momentum as allowed in a renormalizable theory. They can be reduced to linear combinations of scalar functions.

One-point

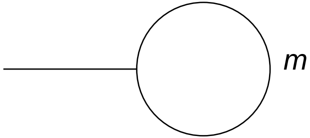


Figure: The one-point Green function.

Dimensional regularization

one-point scalar integrals

$$i\pi^2 A_0(m) \quad = \quad \mu^{4-n} \int d^n q \, \frac{1}{q^2 + m^2 - i\epsilon},$$

- μ is an arbitrary mass scale and
- we adopted DR defining an analytical continuation of the $\mathcal{S}$-matrix in the complex n-plane.
- Note the presence of a factor i as a consequence of a Wick rotation.

Within DR one obtains a consistent theory if it can be shown that the poles for $n = 4$ can be removed, oder by order in perturbation theory.

Integrals

One-point integral

can be easily evaluated in terms of the Euler Γ-function giving

$$A_0\left(m\right) \;=\; \pi^{n/2-2}\Gamma\left(1-\frac{n}{2}\right)m^2\left(\frac{m^2}{\mu^2}\right)^{n/2-2}.$$

If we introduce $\varepsilon = 4 - n$ and expand around $n = 4$, then the following expression is derived:

$$A_0\left(m\right) \;=\; m^2\left(-\frac{2}{\varepsilon}+\gamma+\ln\pi-1+\ln\frac{m^2}{\mu^2}\right)+\mathcal{O}\left(\varepsilon\right).$$

where $\gamma = 0.577216$ is the Euler constant.

Integrals

It is customary

to define a quantity $1/\bar{\varepsilon}$ by

$$\frac{1}{\bar{\varepsilon}} = \frac{2}{\varepsilon} - \gamma - \ln\pi,$$

and to write

$$A_0(m) = m^2\left(-\frac{1}{\bar{\varepsilon}} - 1 + \ln\frac{m^2}{\mu^2}\right) + \mathcal{O}(\varepsilon).$$

Explicit expressions for two and higher point scalar functions will not be discussed here. For the two-point function we have an expression that contains logarithms at most while for three- and four-point functions the final expression contains 12 and 108 (in the most general case) di-logarithms.

Two-point

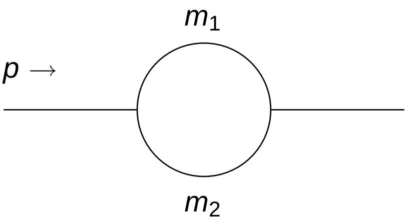


Figure: The two-point Green function.

Three-point

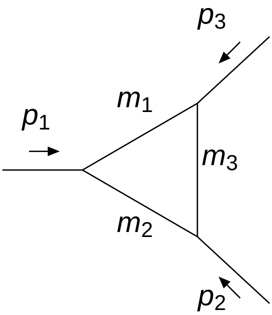


Figure: The three-point Green function.

Four-point

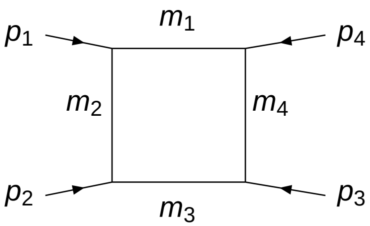


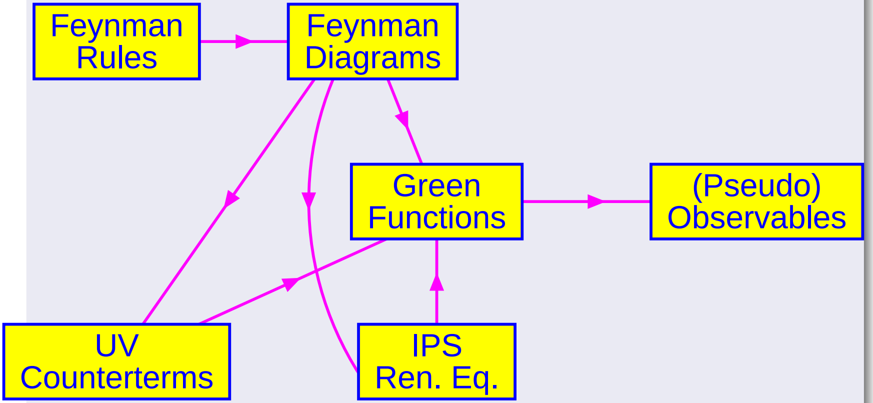
Figure: The four-point Green function.

What's new?

If one thinks for a while, everything is in the old papers of 't Hooft and Veltman; however, translating few formal properties into a working scheme is far from trivial; most of the times it is not a question of *how do I do it?*, rather it is a question of bookkeeping, namely

- *can I do it without exhausting the memory of my computer?*, or,
- *is there any practical way of presenting my results besides making my codes public?*.

Renormalization flowchart



From modern $1\,L$ to $2\,L$

$1\,L$ in a nutshell

$$
\begin{aligned}
S_{n\,;\,N}\,(f) &= \frac{\mu^{\epsilon}}{i\,\pi^2}\int d^n q\,\frac{f\,(q\,,\,\{p\})}{\prod_{i=0,N-1}(i)},\\
(i) &= (q+p_0+\cdots+p_i)^2+m_i^2\,.
\end{aligned}
$$

$$
\begin{aligned}
S_{n\,;\,N}\,(f) &= \sum_i b_i\,B_0(P_i^2)+\sum_{ij} c_{ij}\,C_0(P_i^2,P_j^2)\\
&+ \sum_{ijk} d_{ijk}\,D_0(P_i^2,P_j^2,P_k^2)+R,
\end{aligned}
$$

From modern $1\,L$ to $2\,L$

$1\,L$ in a nutshell

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$$(i) = (q+p_0+\cdots+p_i)^2+m_i^2.$$

$$S_{n\,;\,N}(f) = \sum_i b_i\,B_0(P_i^2)+\sum_{ij} c_{ij}\,C_0(P_i^2,P_j^2) + \sum_{ijk} d_{ijk}\,D_0(P_i^2,P_j^2,P_k^2)+R,$$

Technical problems

Although

HTF (usually) have nice properties,

- expansions are often available with good properties of convergence
- the expansion parameter has the same cut of the function

where is the limit?

- One - loop, Nielsen - Goncharov
- Two - loop, one scale ($s = 0, m^2$ cuts) harmonic polylogarithms
- Two - loop, two scales ($s = 4\,m^2$ cuts) generalized harmonic polylogarithms
- next? New higher transcendental functions?

Counterterms?

Then, there is the perennial question, with or without **counter-terms**? In a way, it is a fake question.

- The two approaches are fully equivalent and we will discuss the transition from **bare parameters** to **renormalized ones**.
- Finally we discuss the ultimate step in any renormalization procedure: the transition from renormalized parameters to a set of **physical** (pseudo-)observables.

One should try

to make a clear vocabulary of renormalization in QFT; a renormalization procedure is designed to bring you from a Lagrangian to theoretical predictions;

it includes,

- regularization (nowadays dimensional regularization is easy to understand),
- a renormalization scheme and
- an input parameter set.

Comments

- The scheme, being a transitory step, is almost irrelevant; it can be on-mass-shell or $\overline{MS}$ or complex poles, but unless you do something illegal (resummations that are not allowed or similar things) it really does not matter.
- One can define $\overline{MS}$ quantities as convenient landmarks but it is the last step that matters, at least as long as we have a convenient subtraction point (which we miss in QCD). Renormalized quantities should always be expressed in terms of a set of physical quantities.
- One may indulge to the introduction of an $\overline{MS}$ running e.m. coupling constant (importing from QCD to QED, which sounds strange anyway) but, finally, only cross sections matter.

Steps

- All the Green functions of the theory have to be made finite, up to two-loops, by introduction of counter-terms and all counter-terms are of non logarithmic nature, to respect unitarity.
- Renormalized Ward-Slavnov-Taylor identities must be satisfied.
- All ultraviolet finte parts must be classified and an algorithm has to be designed for their evaluation at any scale.

Of course, there are preliminar steps – not always the easy ones – but it is only the full control on the multi-scale level that pays off.

Multiplicative renormalization

c.t. not needed but useful

$$Z_i = 1 + \sum_{n=1}^{\infty} \left(\frac{g_R^2}{16\,\pi^2} \right)^n \delta Z_i^{(n)},$$

Example

masses, parameters

$$\begin{aligned} m &= Z_m^{1/2}\, m_R\,, \\ p &= Z_p\, p_R\,, \quad p = g\,, c_\theta\,, s_\theta \end{aligned}$$

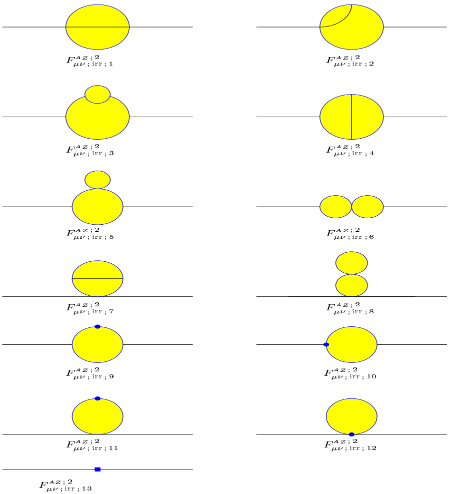
Example

Fields, gauge parameters

$$\begin{aligned} Z_{\xi_{AZ}} &= \sum_{n=1}^{\infty} \left(\frac{g_R^2}{16\,\pi^2} \right)^n \delta Z_{\xi_{AZ}}^{(n)} \\ \phi &= Z_\phi^{1/2}\, \phi_R \quad \psi^{L,R} = Z_{\psi_{L,R}}^{1/2}\, \psi_R^{L,R} \\ A^\mu &= Z_{AA}^{1/2}\, A_R^\mu + Z_{AZ}^{1/2}\, Z_R^\mu \\ Z_{AZ}^{1/2} &= \sum_{n=1}^{\infty} \left(\frac{g_R^2}{16\,\pi^2} \right)^n \delta Z_{AZ}^{(n)} \end{aligned}$$

FP ghost fields are not renormalized

$$G^{AZ\,;\,2}_{\mu\nu\,;\,\mathrm{irr}} =$$



The two facets of renormalization

Step 1

promote bare quantities p to renormalized ones p_R

Step 2

- fix the c.t. at $1\,L$ $\equiv$ to remove the UV poles from all $1\,L$ GF;
- check that $2\,L$ GF develop local UV residues;
- fix the $2\,L$ c.t. to remove $2\,L$ local UV poles.

Finite renomalization

the absorption of UV poles into local c.t. does not exhaust the procedure; we have to connect p_R to POs, thus making the theory predictive.

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Part III

Lecture III

Ren in QED

The QED Lagrangian

in the Feynman gauge ($\xi = 1$) is unambiguous at the tree level. Moving to higher orders, we assume that it is made of

- bare fields and parameters labelled with sup- or sub-indices 0 and
- specifies the renormalization constants for the two fields—A_μ and ψ—and the two QED parameters—the electron mass m and the charge e:

$$
\begin{aligned}
A^0_\mu &= Z_A^{1/2} A_\mu, \qquad \psi^0 = Z_\psi^{1/2}\,\psi, \\
e_0 &= Z_e\, e, \qquad m_0 = Z_m\, m = m + e^2 \delta m + \mathcal{O}\left(e^4\right), \\
Z_i &= 1 + e^2 \delta Z_i + \mathcal{O}\left(e^4\right).
\end{aligned}
$$

Ren in QED

The Lagrangian

can now be rewritten, up to terms $\mathcal{O}\left(e^2\right)$, as

$$\mathcal{L}^{\mathrm{R}}_{\mathrm{QED}} = \mathcal{L}_{\mathrm{QED}} + \mathcal{L}_{\mathrm{ct}},$$

with a counter-term Lagrangian:

$$\begin{aligned}
\mathcal{L}_{\mathrm{ct}} &= e^2\,\mathcal{L}^{(2)}_{\mathrm{ct}} + \mathcal{O}\left(e^4\right), \\
\mathcal{L}^{(2)}_{\mathrm{ct}} &= -\frac{1}{4}\,\delta Z_A\,F_{\mu\nu}F_{\mu\nu} - \frac{1}{2}\,\delta Z_A\,\left(\partial_\mu A_\mu\right)^2 - \delta Z_\psi \overline{\psi}\partial\!\!\!/\psi \\
&\quad - \left(\delta Z_\psi m + \delta m\right)\overline{\psi}\psi - i\left(\delta Z_e + \delta Z_\psi + \frac{1}{2}\delta Z_A\right) e\overline{\psi}A\!\!\!/\psi.
\end{aligned}$$

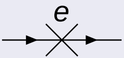
Ren in QED

the counter-term part

of the Lagrangian generates a new set of QED Feynman rules to be denoted by a cross. With their help we fix the counter-terms. First, the δZ_A counter-term:

$$\rightarrow \qquad -e^2 \delta Z_A .$$

Then the δZ_ψ and δm counter-terms:

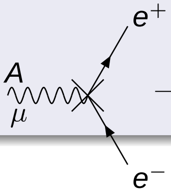


$$\rightarrow \qquad -e^2 \left(\delta Z_\psi i \not{p} + \delta Z_\psi m + \delta m \right) .$$

Ren in QED

And finally, the remaining combination

:



$$\rightarrow \qquad -ie\gamma_\mu e^3\left(\delta Z_e + \delta Z_\psi + \frac{1}{2}\delta Z_A\right).$$

Ren in QED

After

a relatively simple calculation one derives the following expressions:

$$\delta Z_A = \frac{1}{12\pi^2}\left(-\frac{1}{\bar{\varepsilon}} + \ln\frac{m^2}{\mu^2}\right).$$

$$\delta m = \frac{m}{16\pi^2}\left(-\frac{3}{\bar{\varepsilon}} + 3\ln\frac{m^2}{\mu^2} - 4\right),$$

$$\delta Z_\psi = \frac{1}{16\pi^2}\left(-\frac{1}{\bar{\varepsilon}} + \frac{2}{\hat{\varepsilon}} + 3\ln\frac{m^2}{\mu^2} - 4\right).$$

$$\delta Z_e \equiv -\frac{1}{2}\delta Z_A.$$

Ren in QED

At this point

the renormalization procedure can be carried through order by order. With the one-loop renormalized Lagrangian and with the one-loop counter Lagrangian we construct all two-loop diagrams and introduce $\mathcal{O}\left(e^2\right)$ new counter-terms.

- One obtains the correct result consistent with unitarity, provided that one has shown that overlapping diagrams contain new divergences behaving as local counter-terms.

The perturbative unitary bound

A very severe constraint on the Higgs boson mass comes from unitarity of the scattering amplitude.

$$\text{unitarity} \Longleftrightarrow \text{QM probability} < 1$$

Scattering probability bounded from above!

Considering the elastic scattering of longitudinally polarized Z bosons

$$Z_L Z_L \to Z_L Z_L$$

$$\mathcal{M} = -\frac{m_H^2}{v^2}\left[\frac{s}{s-m_H^2} + \frac{t}{t-m_H^2} + \frac{u}{u-m_H^2}\right] \qquad \text{in the } s \gg m_Z^2 \text{ limit}$$

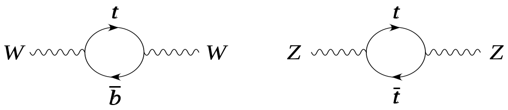
where s, t and u are the usual Mandelstam variables.

The perturbative unitary bound on the $J = 0$ partial amplitude takes the form

$$|\mathcal{M}_0|^2 = \left[\frac{3}{16\pi}\frac{m_H^2}{v^2}\right]^2 < 1 \qquad \Longrightarrow \qquad m_H < \sqrt{\frac{16\pi}{3}}v \approx 1 \text{ TeV}$$

Clues to the Higgs boson mass

From the sensitivity of electroweak observables to the mass of the top, we are able to measure its mass, even without directly producing it

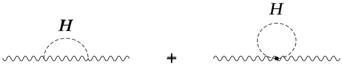


These quantum corrections alter the link between W and Z boson masses

$$m_W^2 = \frac{1}{\sin^2\theta_W\,(1-\Delta\rho)}\frac{\pi\alpha}{\sqrt{2}G_F} \qquad \Delta\rho_{(\text{top})} \approx -\frac{3G_F}{8\pi^2\sqrt{2}}\frac{1}{\tan^2\theta_W}m_t^2$$

The strong dependence on m_t^2 accounts for the precision of the top-quark mass estimates derived from electroweak observables.

The Higgs boson quantum corrections are typically smaller than the top-quark corrections, and exhibit a more subtle dependence on m_H than the m_t^2 dependence of the top-quark corrections.



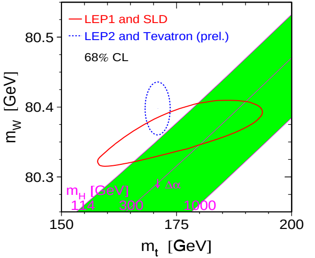
$$\Delta\rho_{(\text{Higgs})} = \frac{11 G_F m_Z^2 \cos^2\theta_W}{24\sqrt{2}\pi^2} \log\left(\frac{m_H^2}{m_W^2}\right)$$

Since m_Z has been determined at LEP to 23 ppm, it is interesting to examine the dependence of m_W upon m_t and m_H.

Indirect measurements of m_W and m_t (solid line)

Direct measurements of m_W and m_t (dotted line)
$m_t = 170.9 \pm 1.8$ GeV
$m_W = 80.398 \pm 0.025$ GeV

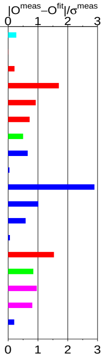
both shown as one-standard-deviation regions.



The indirect and direct determinations are in reasonable agreement and both favor a light Higgs boson, within the framework of the SM.

Summary of EW precision data

	Measurement	Fit
$\Delta\alpha_{had}^{(5)}(m_Z)$	0.02758 ± 0.00035	0.02768
m_Z [GeV]	91.1875 ± 0.0021	91.1875
Γ_Z [GeV]	2.4952 ± 0.0023	2.4957
σ_{had}^0 [nb]	41.540 ± 0.037	41.477
R_l	20.767 ± 0.025	20.744
$A_{fb}^{0,l}$	0.01714 ± 0.00095	0.01645
$A_l(P_\tau)$	0.1465 ± 0.0032	0.1481
R_b	0.21629 ± 0.00066	0.21586
R_c	0.1721 ± 0.0030	0.1722
$A_{fb}^{0,b}$	0.0992 ± 0.0016	0.1038
$A_{fb}^{0,c}$	0.0707 ± 0.0035	0.0742
A_b	0.923 ± 0.020	0.935
A_c	0.670 ± 0.027	0.668
A_l(SLD)	0.1513 ± 0.0021	0.1481
$\sin^2\theta_{eff}^{lept}(Q_{fb})$	0.2324 ± 0.0012	0.2314
m_W [GeV]	80.398 ± 0.025	80.374
Γ_W [GeV]	2.140 ± 0.060	2.091
m_t [GeV]	170.9 ± 1.8	171.3



Better estimates of the SM Higgs boson mass are obtained by combining all available data:

Summary of electroweak precision measurements (status winter 2007) as given on LEP-EWWG page:
http://lepewwg.web.cern.ch/LEPEWWG/

SM Higgs mass fit to EW precision data

$$m_H = 76^{+33}_{-24}\ \text{GeV}$$

Including theory uncertainty

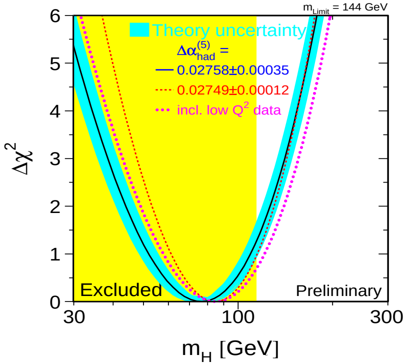
$$m_H < 144\ \text{GeV} \quad (95\%\ \text{CL})$$

Does not include
Direct search limit from LEP

$$m_H > 114\ \text{GeV} \quad (95\%\ \text{CL})$$

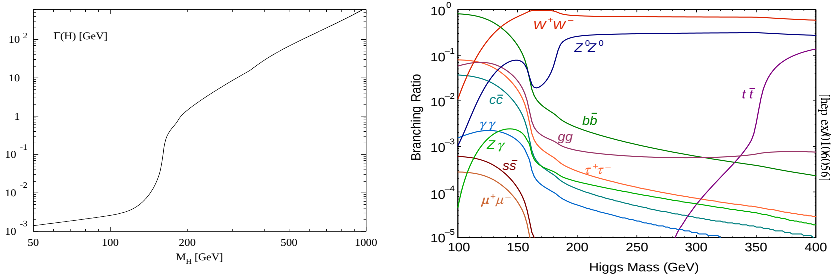
Renormalize probability for
$m_H > 114$ GeV to 100%:

$$m_H < 182\ \text{GeV} \quad (95\%\ \text{CL})$$

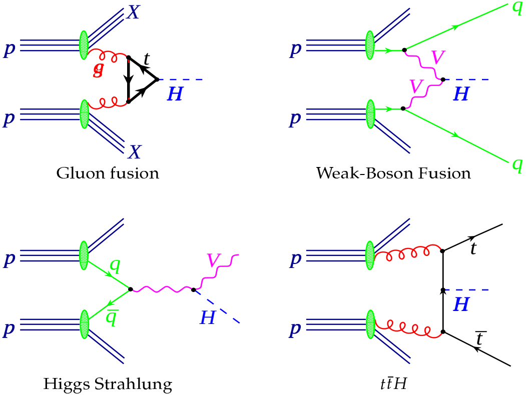


Decay of the SM Higgs

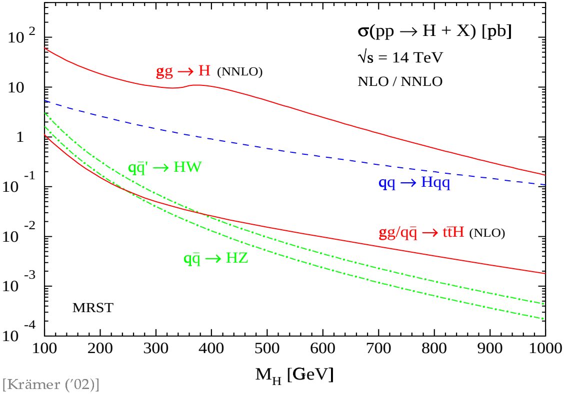
Higgs decay width and branching fractions within the SM



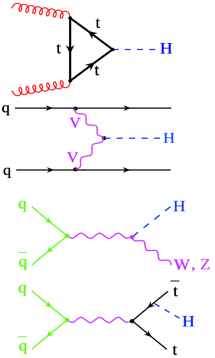
Higgs Production Modes at Hadron Colliders



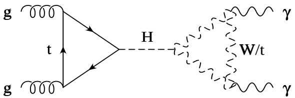
Total SM Higgs cross sections at the LHC

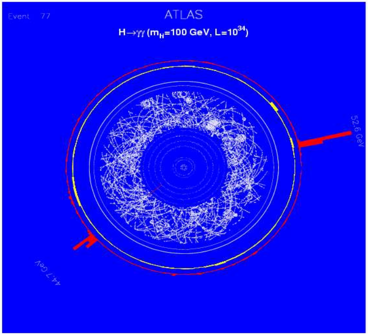


[Krämer ('02)]



$H \to \gamma\gamma$



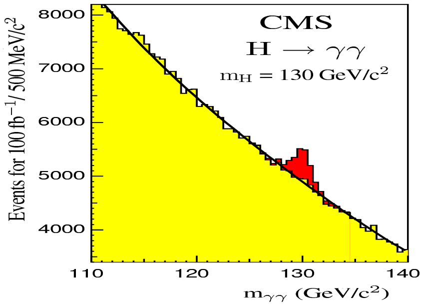


- ✗ BR$(H \to \gamma\gamma) \approx 10^{-3}$
- ✗ large backgrounds from $q\bar{q} \to \gamma\gamma$ and $gg \to \gamma\gamma$
- ✓ but CMS and ATLAS will have excellent photon-energy resolution (order of 1%)

Look for two isolated photons.

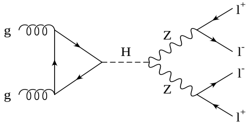
$H \longrightarrow \gamma\gamma$

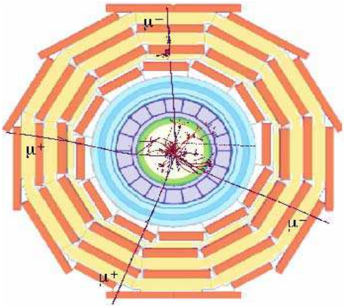
- ✓ Look for a narrow $\gamma\gamma$ invariant mass peak
- ✓ extrapolate background into the signal region from sidebands.



$H \longrightarrow ZZ \longrightarrow \ell^+\ell^-\ell^+\ell^-$

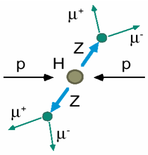
The gold-plated mode

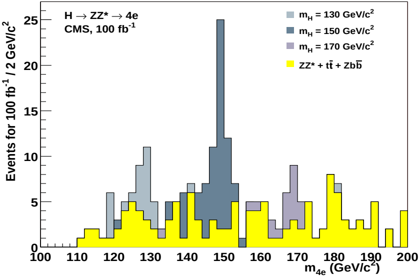




- ✓ This is the most important and clean search mode for $2m_Z < m_H < 600$ GeV.
- ✓ continuum, limited, irreducible background from $q\bar{q} \rightarrow ZZ$
- ✗ small $\mathrm{BR}(H \rightarrow \ell^+\ell^-\ell^+\ell^-) \approx 0.15\%$ (even smaller when $m_H < 2m_Z$)

$H \longrightarrow ZZ \longrightarrow \ell^+\ell^-\ell^+\ell^-$





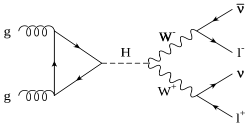
✓ invariant mass of the charged leptons fully reconstructed

For $m_H \approx 0.6$–1 TeV, use the “silver-plated” mode $H \to ZZ \to \nu\bar{\nu}\ell^+\ell^-$

✓ $\mathrm{BR}(H \to \nu\bar{\nu}\ell^+\ell^-) = 6\ \mathrm{BR}(H \to \ell^+\ell^-\ell^+\ell^-)$

✓ the large E_T missing allows a measurement of the transverse mass

$H \longrightarrow WW \longrightarrow \ell^+ \bar{\nu} \ell^- \nu$

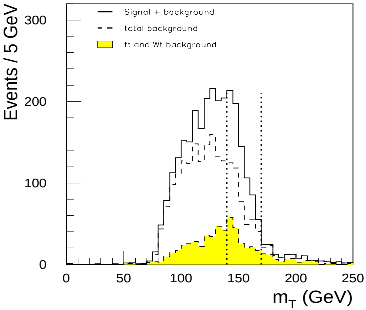


- ✓ Exploit $\ell^+\ell^-$ angular correlations
- ✓ measure the transverse mass with a Jacobian peak at m_H

$$m_T = \sqrt{2\, p_T^{\ell\ell}\, \not{E}_T\, (1 - \cos(\Delta\Phi))}$$

- ✗ background and signal have similar shape $\Longrightarrow$ must know the background normalization precisely

ATLAS TDR



$m_H = 170$ GeV

integrated luminosity $= 20\ \text{fb}^{-1}$

Higgs discovery potential