NLO HEFT
landscape of κ layers

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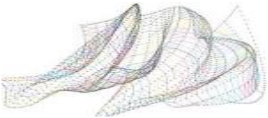
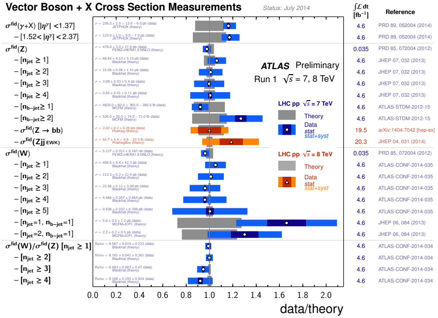


Higgs WG (N)NLO MC and Tools Workshop, 17 − 19 December 2014, CERN

André David, Michael Duehrssen, Sandro Uccirati and Margherita Ghezzi

chronicle of an idiosyncratic research

Appetizer



✗ Fiducial cross sections

Pro: maximal information
Pro: no extrapolation
Con: not universal, NAN

✗ Pseudo-observables

Pro: universal
Pro: simple
Con: almost model independent†

✗ SM deformations (this talk)

for nice introductions: David, Denner
Hamburg Workshop on Higgs Physics

for comprehensive EFT reading
Murayama et al. arXiv:1412.1837

definition **PO** any, uniquely defined, QFT-consistent, expression giving one number

PO is implementable in any SM deformation

† assuming $\Delta B \ll \Delta S$

drops of PO

Example

From Laurent expansion

$$\frac{f(z)}{z-z_0} = \frac{f(z_0)}{z-z_0} + \sum_{n=1}^{\infty} \frac{1}{n!} f^{(n)}(z_0)\,(z-z_0)^{n-1}$$

to *gauge-invariant* QFT splitting (only the POLE, the RESIDUE and the REMAINDER are gauge invariant)

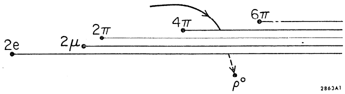
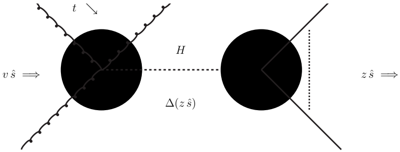
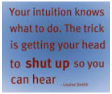


Fig. 1



$$\Longrightarrow\ \sigma_{gg\to H+X}(v\hat{s}\,,\,\hat{t}\,,\,s_H)\,\frac{(z\hat{s})^2}{\left|z\hat{s}-s_H\right|^2}\,\frac{\Gamma_{H\to f}(s_H)}{\left|s_H\right|^{1/2}}$$



$\mathbf{H} \to \mathbf{Z}^* \left(\to \bar{\mathbf{f}}\mathbf{f}\right) + \gamma$	unphysical
$\mathbf{H} \to \gamma^* \left(\to \bar{\mathbf{f}}\mathbf{f}\right) + \gamma$	unphysical
$\mathbf{H} \to \mathbf{Z}_\mathbf{c} \left(\to \bar{\mathbf{f}}\mathbf{f}\right) + \gamma$	PO

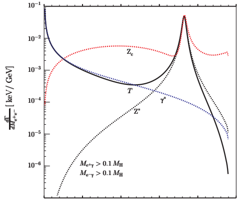
where $\mathbf{Z}^*$ is the off-shell $\mathbf{Z}$ boson and $\mathbf{Z}_\mathbf{c}$ is the $\mathbf{Z}$ boson at its complex pole. Understanding the problem of POs means understanding the difference between $\mathbf{H} \to \bar{\mathbf{f}}\mathbf{f}$ and $\mathbf{H} \to \bar{\mathbf{f}}\mathbf{f} + n\gamma$

As long as we have $\sigma^{\mathrm{fid}}\left(\mathrm{pp} \to \gamma\bar{\mathbf{f}}\mathbf{f}\right)$ and $\mathbf{H} \to \mathbf{Z}_\mathbf{c} + \gamma$ we know everything, we can communicate and always go back, should a new theory emerge 

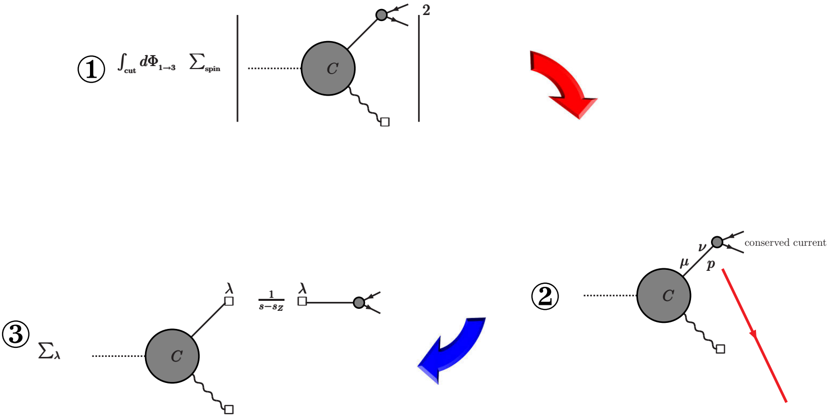
Table: The PO $\Gamma_c = \Gamma(H \to Z_c\gamma \to e^+e^-\gamma)$ and $\Gamma_{tot} = \Gamma(H \to e^+e^-\gamma)$. Here $M_{(e^+e^-)\gamma} > 0.1\,M_H$ and $M_Z - \xi\,\Gamma_Z < M_{e^+e^-} < M_Z + \xi\,\Gamma_Z$.

ξ	$\Gamma_{tot}[\ keV]$	$\Gamma_c[\ keV]$	$R_c = \Gamma_c/\Gamma_{tot}$
1	138.7	154.1	1.11
2	166.2	194.8	1.17
3	176.4	217.9	1.24
4	181.7	236.5	1.30
5	185.0	253.6	1.37

The ratio $R_c(\xi)$ gives the correction factor for extracting the PO



PO building manual



$\square =$ *polarization*

$\delta_{\mu\nu} \to \sum_\lambda \left[e^\lambda_\mu(p)\right]^* e^\lambda_\nu(p)$

$| \sum_\lambda f(\lambda) |^2 = \sum_\lambda | f(\lambda) |^2 + \text{rest}$

HZZ PO has been described

- in Sect. 7 of " NLO Inspired Effective Lagrangians for Higgs Physics" (e-Print: arXiv:1209.5538) and
- in Appendix C of "The Higgs Boson Lineshape" (e-Print: arXiv:1112.5517)

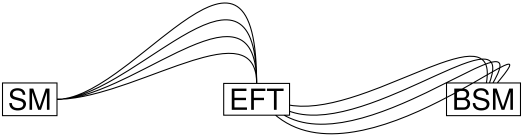
✓ How POs were used/defined from both the theory and the experimental side? Please, read **http://arxiv.org/abs/hep-ex**/0509008 (especially Sect. **1.5.4**)

NLO EFT for the Sorcerer’s Apprentice

… where it is proven that NLO EFT provides the general framework for consistent calculation of higher orders and allows for global fits, superseding any ad-hoc variation of the SM parameters

Ongoing and near future experiments can achieve an estimated per mille accuracy on precision Higgs and EW observables, thus providing a window to indirectly explore the theory space of BSM physics. That’s why you need NLO EFT

One more reason for NLO? Well, κ-framework is not fully consistent (violates gauge symmetry and unitarity); do you want to consistently differentiate loops in loop-induced processes? There is only one way …



In case you ask

Higgs couplings from LHC

- New: VH->bb included in ATLAS, updates for H->Zγ, VH/ttH->γγ (*)
- No BSM Higgs decay modes assumed

–Comparable numbers for κ_W,κ_Z, κ_t, and κ_γ between the experiments

–Couplings can be determined with 2-10% precision at 3000fb^{-1} for CMS Scenario 2

		κ_γ	κ_W	κ_Z	κ_g	κ_b	κ_t	κ_τ	$\kappa_{Z\gamma}$	κ_μ
300fb^{-1}	ATLAS	[9,9]	[9,9]	[8,8]	[11,14]	[22,23]	[20,22]	[13,14]	[24,24]	[21,21]
300fb^{-1}	CMS	[5,7]	[4,6]	[4,6]	[6,8]	[10,13]	[14,15]	[6,8]	[41,41]	[23,23]
3000fb^{-1}	ATLAS	[4,5]	[4,5]	[4,4]	[5,9]	[10,12]	[8,11]	[9,10]	[14,14]	[7,8]
3000fb^{-1}	CMS	[2,5]	[2,5]	[2,4]	[3,5]	[4,7]	[7,10]	[2,5]	[10,12]	[8,8]

– **Assumptions made: no particles other than those from SM in the loops, or contributing to the total width**

–ATLAS: [no theory uncert.; full theory uncert.]

–CMS: [uncertainties scale with $1/\sqrt{L}$ and theory reduced by ½; all uncertainties as today]

Higgs couplings in e+e- colliders

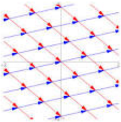
	Model-independent fit				Constrained fit		
Coupling	TLEP-240	TLEP		ILC	TLEP		ILC
g_{HZZ}	0.16%	**0.15%**	(0.18%)	0.9%	0.05%	(0.06%)	0.31%
g_{HWW}	0.85%	**0.19%**	(0.23%)	0.5%	0.09%	(0.11%)	0.25%
$g_{Hb\bar{b}}$	0.88%	**0.42%**	(0.52%)	2.4%	0.19%	(0.23%)	0.85%
$g_{Hc\bar{c}}$	1.0%	**0.71%**	(0.87%)	3.8%	0.68%	(0.84%)	3.5%
g_{Hgg}	1.1%	**0.80%**	(0.98%)	4.4%	0.79%	(0.97%)	4.4%
$g_{H\tau\tau}$	0.94%	**0.54%**	(0.66%)	2.9%	0.49%	(0.60%)	2.6%
$g_{H\mu\mu}$	6.4%	**6.2%**	(7.6%)	45%	6.2%	(7.6%)	45%
$g_{H\gamma\gamma}$	1.7%	**1.5%**	(1.8%)	14.5%	1.4%	(1.7%)	14.5%
BR_{exo}	0.48%	**0.45%**	(0.55%)	2.9%	0.16 %	(0.20%)	0.9%

Relative statistical uncertainty on the Higgs boson couplings, as expected from the physics programme at $\sqrt{s}$ = 240 and 350 GeV at FCC-ee (TLEP in the table).

The numbers between brackets indicates the uncertainties expected with two detectors instead of four.

my power counting

for **Λ < 5 *TeV*** NLO EFT effects greater or comparable to NLO EW in ggF the latter can reach **5%**



QFT is not flexible
We need matching of UV models onto EFT order-by-order in a loop expansion

Definition

$L = \{\mathcal{O}_1^{(d)}, \dots \mathcal{O}_n^{(d)}\}$ is a list of operators in $V^{(d)}$, then these operators form a basis iff every $\mathcal{O}^{(d)} \in V^{(d)}$ can be uniquely written as a linear combination of the elements in L

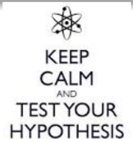
Remark Overcomplete L is useful for cross-checking

An L that is not a basis is useless, e.g. is not closed under renormalization and leads to violation of WST identities

Proposition

A basis is optimal insofar as it allows to write Feynman rules in arbitrary gauges

Awareness of assumptions in EFT



✗ one Higgs doublet (flexible)
✗ linear realization (flexible)
✗ no *light dof + decoupling (rigid)*
✗ UVC weakly-coupled and ren. (flexible)
✗ Neglecting $\dim = 8$ and NNLO EW $\mapsto$
$3\ TeV < \Lambda < 5\ TeV$

HEP phases

- **PREDICTIVE phase: in any (strictly) renormalizable theory with n parameters you need to match n data points, the $(n+1)$th calculation is a prediction, e.g. as doable in the SM**
- **FITTING (approximate predictive) phase: there are $(N_6 + N_8 + \cdots = \infty)$ renormalized Wilson coefficients that have to be fitted, e.g. measuring SM deformations due to a single $\mathcal{O}^{(6)}$ insertion (N_6 enough for per mille accuracy)**


PROPOSITION: There are two ways of formulating HEFT

a) mass-dependent scheme(s) or **Wilsonian** HEFT

b) mass-independent scheme(s) or **Continuum** HEFT (CHEFT)

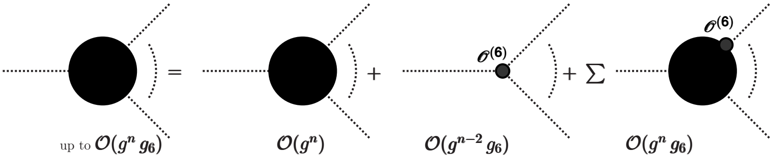
- only a) is conceptually consistent with the image of an EFT as a low-energy approximation to a high-energy theory
- however, inclusion of NLO corrections is only meaningful in b) since we cannot regularize with a cutoff and NLO requires regularization
  - There is an additional problem, CHEFT requires evolving our theory to lower scales until we get below the “heavy-mass” scale where we use $\mathscr{L} = \mathscr{L}_{\mathrm{SM}} + d\mathscr{L}$, $d\mathscr{L}$ encoding matching corrections at the boundary. Therefore, CHEFT **does not integrate out heavy degrees of freedom** but removes them compensating for by an appropriate matching calculation

Not quite the same as it is usually discussed (no theory approaching the boundary from above ...) cf. low-energy SM, weak effects on $g-2$ etc.

Having said that . . .

This is not a contribution to the EFT basis diatribe, it's NLO EFT building, capturing the convergency with future calculations from UV models

What can be said at all can be said clearly and whereof one cannot speak thereof one must be silent includes non-SM families

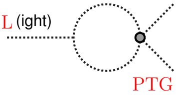


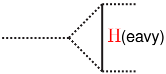
$$g_6 = 1/(\sqrt{2}\, G_F\, \Lambda^2)$$

PTG/LG

option 1: absent
option 2: PTG
option 3: PTG/LG

mixing under renormalization

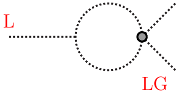


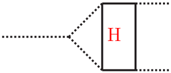


DIAGRAMMATICA of EFT

EFT

UVC





☛ Forget κs if you are using **1**

OPTIONS

1 only tree PTG&LG

2 tree PTG&LG, loops PTG

3 tree&loops PTG&LG ✓

3′ tree PTG&LG, loops “UV admissible”

Appendix C. Dimension-Six Basis Operators for the SM[22].

Einhorn, Wudka

$\mathcal{O}$ is PTG

$\mathcal{O}$ is LG

Grzadkowski, Iskrzynski, Misiak, Rosiek

X^3 **(LG)**		φ^6 and $\varphi^4 D^2$ **(PTG)**		$\psi^2\varphi^3$ **(PTG)**	
Q_G	$f^{ABC} G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	Q_φ	$(\varphi^\dagger\varphi)^3$	$Q_{e\varphi}$	$(\varphi^\dagger\varphi)(\bar l_p e_r \varphi)$
$Q_{\widetilde G}$	$f^{ABC} \widetilde G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	$Q_{\varphi\Box}$	$(\varphi^\dagger\varphi)\Box(\varphi^\dagger\varphi)$	$Q_{u\varphi}$	$(\varphi^\dagger\varphi)(\bar q_p u_r \widetilde\varphi)$
Q_W	$\varepsilon^{IJK} W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$	$Q_{\varphi D}$	$\left(\varphi^\dagger D^\mu\varphi\right)^\star \left(\varphi^\dagger D_\mu\varphi\right)$	$Q_{d\varphi}$	$(\varphi^\dagger\varphi)(\bar q_p d_r \varphi)$
$Q_{\widetilde W}$	$\varepsilon^{IJK} \widetilde W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$				
$X^2\varphi^2$ **(LG)**		$\psi^2 X\varphi$ **(LG)**		$\psi^2\varphi^2 D$ **(PTG)**	
$Q_{\varphi G}$	$\varphi^\dagger\varphi\, G^A_{\mu\nu} G^{A\mu\nu}$	Q_{eW}	$(\bar l_p \sigma^{\mu\nu} e_r)\tau^I \varphi W^I_{\mu\nu}$	$Q_{\varphi l}^{(1)}$	$(\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi)(\bar l_p \gamma^\mu l_r)$
$Q_{\varphi\widetilde G}$	$\varphi^\dagger\varphi\, \widetilde G^A_{\mu\nu} G^{A\mu\nu}$	Q_{eB}	$(\bar l_p \sigma^{\mu\nu} e_r)\varphi B_{\mu\nu}$	$Q_{\varphi l}^{(3)}$	$(\varphi^\dagger i \overleftrightarrow{D}^I_\mu \varphi)(\bar l_p \tau^I \gamma^\mu l_r)$
$Q_{\varphi W}$	$\varphi^\dagger\varphi\, W^I_{\mu\nu} W^{I\mu\nu}$	Q_{uG}	$(\bar q_p \sigma^{\mu\nu} T^A u_r)\widetilde\varphi\, G^A_{\mu\nu}$	$Q_{\varphi e}$	$(\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi)(\bar e_p \gamma^\mu e_r)$
$Q_{\varphi\widetilde W}$	$\varphi^\dagger\varphi\, \widetilde W^I_{\mu\nu} W^{I\mu\nu}$	Q_{uW}	$(\bar q_p \sigma^{\mu\nu} u_r)\tau^I \widetilde\varphi\, W^I_{\mu\nu}$	$Q_{\varphi q}^{(1)}$	$(\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi)(\bar q_p \gamma^\mu q_r)$
$Q_{\varphi B}$	$\varphi^\dagger\varphi\, B_{\mu\nu} B^{\mu\nu}$	Q_{uB}	$(\bar q_p \sigma^{\mu\nu} u_r)\widetilde\varphi\, B_{\mu\nu}$	$Q_{\varphi q}^{(3)}$	$(\varphi^\dagger i \overleftrightarrow{D}^I_\mu \varphi)(\bar q_p \tau^I \gamma^\mu q_r)$
$Q_{\varphi\widetilde B}$	$\varphi^\dagger\varphi\, \widetilde B_{\mu\nu} B^{\mu\nu}$	Q_{dG}	$(\bar q_p \sigma^{\mu\nu} T^A d_r)\varphi\, G^A_{\mu\nu}$	$Q_{\varphi u}$	$(\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi)(\bar u_p \gamma^\mu u_r)$
$Q_{\varphi WB}$	$\varphi^\dagger\tau^I\varphi\, W^I_{\mu\nu} B^{\mu\nu}$	Q_{dW}	$(\bar q_p \sigma^{\mu\nu} d_r)\tau^I \varphi\, W^I_{\mu\nu}$	$Q_{\varphi d}$	$(\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi)(\bar d_p \gamma^\mu d_r)$
$Q_{\varphi\widetilde WB}$	$\varphi^\dagger\tau^I\varphi\, \widetilde W^I_{\mu\nu} B^{\mu\nu}$	Q_{dB}	$(\bar q_p \sigma^{\mu\nu} d_r)\varphi\, B_{\mu\nu}$	$Q_{\varphi ud}$	$i(\widetilde\varphi^\dagger D_\mu\varphi)(\bar u_p \gamma^\mu d_r)$

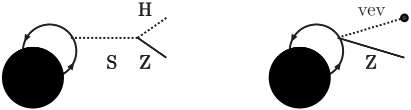
Table C.1: Dimension-six operators other than the four-fermion ones.

[22] These tables are taken from [5], by permission of the authors.

Effective Lagrangians cannot be blithely used without acknowledging implications of their choice

What in loops?

Example: PTG in loops



S heavy BSM particle

No LG in loops?

Scenario with $g^2/(16\pi^2)$ suppression, $g_6 = 0.068\left(\frac{TeV}{\Lambda}\right)^2$

$g_6 > \frac{g^2}{16\pi^2}$ requires $\Lambda < 5$ *TeV* $\quad \frac{g_6}{16\pi^2} > g_6^2$ requires $\Lambda > 3$ *TeV*

☛ There is no model independent EFT statement on some operators being big and other small arXiv:1305.0017

… no suppression in ggF series … multi-$\mathbf{H}$ interaction due to (multi) Manohar-Wise $(\mathbf{8},\mathbf{2})_{1/2}$ fields … ground-to-ground, once everything is computed, why bother?

De rerum rinormalizzazione

- Renormalization should make UV finite all off-shell Green functions (not done yet)

✓ all relevant, on-shell, S-matrix elements made finite, i.e.

① Introduce $\Phi = Z_{\Phi}\,\Phi_{\mathrm{ren}},\; p = Z_p\, p_{\mathrm{ren}}$ (fields & parameters)

$$Z_i = 1 + \frac{g^2}{16\,\pi^2}\left(\Delta Z_i^{(4)} + g_6\, Z_i^{(6)}\right)$$

② Construct self-energies, Dyson resum and make propagators UV finte

③ Construct $\mathbf{3}$-point (or higher) functions, check their $\mathscr{O}^{(4)}$-finiteness, remove remaining $\mathscr{O}^{(6)}$ divergencies by mixing Wilson coefficients 

Everything on my hard disk

```
id dZh=    + 1
           - 1/2*M^-2*ml^2*sumg
           - 3/2*M^-
           - 3/2*M^-
           + 1/2*ctl
           + 2*deg;
*
id dZmh= - (
           + 1
           + 2*M^-2*
           + 6*M^-2*
           + 6*M^-2*
           - 1/2*M^-
           - 3/2*M^-
           - 3/2*M^-
           - 3/2*M^-
           + 1/2*ctl
           - 3/2*M^2
           - 3*M^2*m
*
id dZa= 2*deg + stl
*
id dZw= 19/6 - 4/3*
*
id dM= - M^-2*(
           + 1/2*ml^2*sumg
           + 3/2*mb^2*sumg
           + 3/2*mt^2*sumg
           - M^2*cth^-2
           + 7/6*M^2
           - 4/3*M^2*NG);
*
id dZzz=   + 10/3
           - 1/6*cth^-2
           - 3/4*cth^-2*NG
           - 1/4*cth^-2*NG*vt^2
           - 1/4*cth^-2*NG*vb^2
           - 1/12*cth^-2*NG*vl^2
```

The CTs

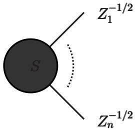
The mixing

} remove UV
induce running

The finite CTs

replace SM parameters with data

Why not anomalous dimension matrix only?
For moderate Λ the logs are modest
finite terms are important !



dim = 6 → dim = 4 normalization

H → vev in $\mathscr{O}^{(6)}$

$$\mathcal{L} = Z\,\phi\,\mathcal{O}\,\phi + \mathcal{L}_{\rm int} + J\,\phi$$

$\mathcal{O}$ = Klein-Gordon, Dirac, etc.

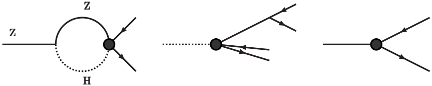
Z = **dim** = **6** remnant

$$\mathcal{L} = \overline{\phi}\,\mathcal{O}\,\overline{\phi} + \overline{\mathcal{L}_{\rm int}} + Z^{-1/2}\,J\,\overline{\phi}$$

S−matrix factors	
ϕ	$Z_\phi^{-1/2}$
H	$1 + g_6\,\left(a_{\phi\,\square} - \frac{1}{4}\,a_{\phi\,\mathrm{D}}\right)$
A	$1 + g_6\,a_{\mathrm{AA}}$
Z	$1 + g_6\,a_{\mathrm{ZZ}}$
W	$1 + g_6\,a_{\phi\,\mathrm{W}}$
...	...

Additional correlations  (the **vev tower game), e.g.**

$a^{(1,3)}_{\phi q}$, $a_{\phi b}$ correlate $\mathbf{HZ\bar{b}b}$ with $\mathbf{Z\bar{b}b}$



κ building manual

① Split the SM amplitude (e.g. $\mathbf{t},\mathbf{b}$ loops and bosonic loops in $\mathbf{H} \to \gamma\gamma$)

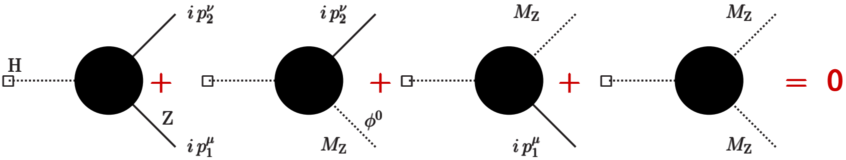
$$\mathscr{A}_{\mathrm{SM}} = \sum_{i=1,n} \mathscr{A}_i^{(4)}$$

② Recover these sub-amplitudes in the full answer

③ Classify the (non-factorizable) remainder and obtain

$$\mathscr{A}_{\mathrm{prc}} = \sum_{i=1,n} \kappa_i^{\mathrm{prc}} \mathscr{A}_i^{(4)} + \sum_{i=1,m} \kappa_i^{\mathrm{prc\,NF}} \mathscr{A}_i^{(6\,\mathrm{NF})}$$

BEWARE

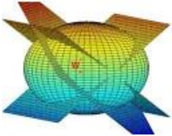


Lorentz structures are severely constrained

by WST identities

For *off-shell* **H** there are additional terms

κs form hyperplanes in the space of Wilson coefficients



- Each κ-plane describes (tangent) *flat*-directions
- Normal directions are blind
- κ-planes intersect correlations between different processes
- Only now you can start making approximations, e.g. only $\mathcal{PTG}$, only $\mathbf{H}$-induced SM deformations etc. (but why?)
- Finally, a global rescaling $\mathscr{A}/\mathscr{A}_{\mathbf{SM}}$ is a running parameter (i.e. phase-space dependent)

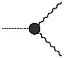
EFT

zooming in

$\mathbf{H} \to \gamma\gamma$ cut-away

starts at $\mathcal{O}(g^3)$ | 3κ | $6\kappa^{NF}$

is LG

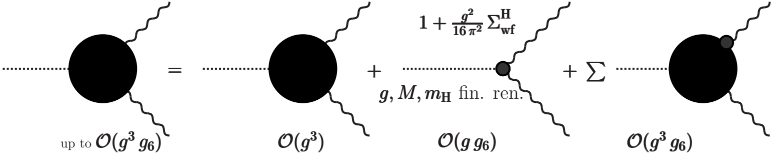


$\mathbf{dim} = 6$ tree $\propto$ a_{AA}

mixes a_{AA} a_{AZ} a_{ZZ} a_{bWB} a_{tWB} a_{tBW}

one row in the anomalous dimension matrix γ

κs are linear combinations of Wilson coeff.



Assembling the amplitude

Finite renormalization

$$s - M^2_{\mathrm{ren}} + \Sigma_{\mathrm{WW}}(s)\Big|_{s=M^2_{\mathrm{W}}} = 0 \qquad \text{etc.}$$

Fine points in renormalization

H WF renormalization à la LSZ

γ WF renormalization $e^2 \rightarrow 4\pi\alpha(0)$

(including IPS dependence)

Don't say *I only want to shift* **H** couplings

InputParameterSet $G_{\mathrm{F}}, M_{\mathrm{W}}, M_{\mathrm{Z}}, M_{\mathrm{H}}$ $\qquad p_{\mathrm{ren}} \neq p(IPS)$

$$\kappa_{\rm t}^{\gamma\gamma} = 1 + g_6 \Big\{ \left(6 - s_\theta^2\right) a_{\rm AA} + \frac{2 - s_\theta^2}{s_\theta}\, c_\theta\, a_{\rm AZ} - \frac{3}{2}\,\frac{M_{\rm t}^2}{M^2}\, c_\theta\, a_{\rm tBW}$$

$$+ \frac{3}{4}\,\frac{M_{\rm t}^2}{M^2}\,\frac{1 - 2\,s_\theta^2}{s_\theta}\, a_{\rm tWB} - \frac{1}{2\,s_\theta^2}\left[a_{\phi D} + 2\,s_\theta^2\left(c_\theta^2\, a_{\rm ZZ} - 2\,a_{\phi\Box} - a_{{\rm t}\phi}\right)\right]\Big\}$$

$$\kappa_{\rm b}^{\gamma\gamma} = 1 + g_6 \Big\{ \left(6 - s_\theta^2\right) a_{\rm AA} + \frac{2 - s_\theta^2}{s_\theta}\, c_\theta\, a_{\rm AZ} + \frac{3}{2}\,\frac{M_{\rm b}^2}{M^2}\, c_\theta\, a_{\rm bWB}$$

$$- \frac{1}{2\,s_\theta^2}\left[a_{\phi D} + 2\,s_\theta^2\left(c_\theta^2\, a_{\rm ZZ} - 2\,a_{\phi\Box} - a_{{\rm b}\phi}\right)\right]\Big\}$$

$$\kappa_{\rm W}^{\gamma\gamma} = 1 + \frac{g_6}{3} \Big\{ \left(14 + 5\,s_\theta^2 - 2\,\frac{M_{\rm H}^2}{M^2}\, s_\theta^2\right) a_{\rm AA} + \left(5 - 2\,\frac{M_{\rm H}^2}{M^2}\right) c_\theta^2\, a_{\rm ZZ}$$

$$+ \left(4 + 5\,s_\theta^2 - 2\,\frac{M_{\rm H}^2}{M^2}\, s_\theta^2\right) \frac{c_\theta}{s_\theta}\, a_{\rm AZ} - \frac{3}{2}\,\frac{1}{s_\theta^2}\left(a_{\phi D} - 4\,s_\theta^2\, a_{\phi\Box}\right)\Big\}$$

 **H → γγ** *Ad usum Delphini* (does not mean former member of Delphi)

 is PTG

$$\Delta\kappa^{\gamma\gamma} = -\frac{1}{2\,s_\theta^2}\left(a_{\phi D} - 4\,s_\theta^2\,a_{\phi\Box}\right)$$

$$\Delta\kappa_{\mathrm{W}}^{\gamma\gamma} = \Delta\kappa \quad \Delta\kappa_{\mathrm{t}}^{\gamma\gamma} = \Delta\kappa^{\gamma\gamma} + a_{\mathrm{t}\phi} \quad \Delta\kappa_{\mathrm{b}}^{\gamma\gamma} = \Delta\kappa^{\gamma\gamma} + a_{\mathrm{b}\phi}$$

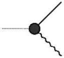
$$\mathscr{A}\,(\mathrm{H} \to \gamma\gamma) = \kappa^{\gamma\gamma}\,\mathscr{A}^{(4)} + \kappa_{\mathrm{t}}^{\gamma\gamma}\,\mathscr{A}_{\mathrm{t}}^{(4)} + \kappa_{\mathrm{b}}^{\gamma\gamma}\,\mathscr{A}_{\mathrm{b}}^{(4)} + 2\,i\,g g_6\,\frac{M_{\mathrm{H}}^2}{M_{\mathrm{W}}}\,a_{\mathrm{AA}}$$

probes $\left|\phi^\dagger D_\mu\,\phi\right|^2 \diamond \;|\,\phi\,|^2\,\Box\,|\,\phi\,|^2 \diamond \;|\,\phi\,|^2\,\overline{\mathrm{q}}\,\mathrm{t}\,\phi \diamond \;|\,\phi\,|^2\,\overline{\mathrm{q}}\,\mathrm{b}\,\phi^{\mathrm{c}}$

$\mathbf{H} \to \gamma Z$ cut-away

starts at $\mathscr{O}(g^3)$ | 3κ | $11\kappa^{NF}$

 is LG  is PTG

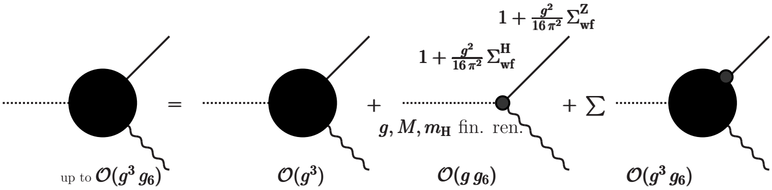


$\mathbf{dim} = \mathbf{6}$ tree $\propto$ a_{AZ}

mixes a_{AA} a_{AZ} a_{ZZ} a_{bWB} a_{tWB} a_{tBW} in γ

κ^{NF} contains

☛ a_{AA} a_{ZZ} a_{AZ} $a_{\phi D}$ a_{tWB} a_{tBW} a_{bWB}
a_{bBW} a_{tWB} $a_{\phi tV}$ $a_{\phi bV}$



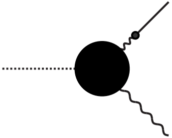
Assembling the amplitude

Finite renormalization

$$s - M^2_{ren} + \Sigma_{WW}(s)\Big|_{s=M^2_W} = 0 \qquad \text{etc.}$$

H, **Z** WF renormalization à la LSZ

γ WF renormalization $e^2 \to 4\,\pi\,\alpha(0)$



Crucial for WST identities

IPS G_F, M_W, M_Z, M_H

$$
\begin{aligned}
\kappa_{\mathrm{t}}^{Z\gamma} &= 1 + g_6 \left(6\, a_{AA} + 2\, a_{ZZ} - a_{\phi D} + 4\, a_{\phi \Box} + 2\, a_{t\phi}\right) \\
\kappa_{\mathrm{b}}^{Z\gamma} &= 1 + \frac{1}{2}\, g_6 \left(6\, a_{AA} + 2\, a_{ZZ} - a_{\phi D} + 4\, a_{\phi \Box} + 2\, a_{b\phi}\right) \\
\kappa_{\mathrm{W}}^{Z\gamma} &= 1 + g_6 \left[\left(3 + s_\theta^2\right) a_{AA} + \left(4 - s_\theta^2\right) a_{ZZ} + s_\theta\, c_\theta\, a_{AZ} + 2\, a_{\phi \Box}\right]
\end{aligned}
$$

Ad usum Delphini

$$
\begin{aligned}
\mathscr{A}\left(\mathrm{H} \to \gamma \mathrm{Z}\right) &= \kappa_{\mathrm{W}}^{\gamma Z}\, \mathscr{A}_{\mathrm{W}}^{(4)} + \kappa_{\mathrm{t}}^{\gamma Z}\, \mathscr{A}_{\mathrm{t}}^{(4)} + \kappa_{\mathrm{b}}^{\gamma Z}\, \mathscr{A}_{\mathrm{b}}^{(4)} + i\, g g_6 \frac{M_{\mathrm{H}}^2}{M_{\mathrm{W}}}\, a_{AZ} \\
&+ a_{\phi D}\, \mathscr{A}_{\mathrm{W}}^{\mathrm{NF}} + \sum_{\mathrm{f}=\mathrm{t,b}} \left(a_{\phi q}^{(3)} - a_{\phi q}^{(1)} - a_{\phi f}\right) \mathscr{A}_{\mathrm{f}}^{\mathrm{NF}}
\end{aligned}
$$

Intersecting $\mathbf{H} \to \gamma\gamma \bigcap \mathbf{H} \to \gamma Z \diamond \kappa_i^{\gamma Z} = 1 + g_6\, s_\theta^2\, \Delta\kappa_i^{\gamma\gamma} + g_6\, \Delta^{\rm rest}\kappa_i^{\gamma Z}$

$$\Delta^{\rm rest}\kappa_{\rm t}^{\gamma Z} = \left(\hat{s}_\theta^2 - 3\right) a_{\rm AA} + \frac{2 - s_\theta^2}{s_\theta}\left(s_\theta\, a_{\rm ZZ} - c_\theta\, a_{\rm AZ}\right) + \frac{1}{2}\frac{c_\theta^2}{s_\theta^2}\, a_{\phi D} - \frac{3}{4}\frac{1 - 2\, s_\theta^2}{s_\theta}\frac{M_{\rm t}^2}{M_{\rm W}^2}\, a_{\rm tWB} + \frac{3}{2}\frac{M_{\rm t}^2}{M_{\rm W}^2}\, c_\theta\, a_{\rm tBW}$$

$$\Delta^{\rm rest}\kappa_{\rm b}^{\gamma Z} = \left(s_\theta^2 - 3\right) a_{\rm AA} + \frac{2 - s_\theta^2}{s_\theta}\left(s_\theta\, a_{\rm ZZ} - c_\theta\, a_{\rm AZ}\right) + \frac{1}{2}\frac{c_\theta^2}{s_\theta^2}\, a_{\phi D} - \frac{3}{2}\frac{M_{\rm b}^2}{M_{\rm W}^2}\, a_{\rm bWB}$$

$$\Delta^{\rm rest}\kappa_{\rm W}^{\gamma Z} = -\frac{1}{3}\left\{\left[5 + 2\left(1 - \frac{M_{\rm H}^2}{M_{\rm W}^2}\right) s_\theta^2\right] a_{\rm AA} - \frac{3}{2}\frac{1}{s_\theta^2}\, a_{\phi D} - \left[9 - 2\left(1 - \frac{M_{\rm H}^2}{M_{\rm W}^2}\right) c_\theta^2\right] a_{\rm ZZ} + \left[2 + \left(1 - \frac{M_{\rm H}^2}{M_{\rm W}^2}\right) s_\theta^2\right] a_{\rm AZ}\right\}$$

$\mathbf{H \to ZZ, WW, bb}$

① Many more terms, start at $\mathscr{O}(g)$ requiring massive renormalization

② Need to account for real radiation in $\mathbf{H \to WW, bb}$

③ $\boldsymbol{\kappa}$ structure different in $\mathbf{H \to WW, bb}$, e.g. $\boldsymbol{\kappa}^{\mathbf{WW}}_{\mathbf{tb}}, \boldsymbol{\kappa}^{\mathbf{WW}}_{\mathbf{bt}}$ etc.
$\mathbf{H \to bb}$ includes $\mathbf{4f}$ operators

$\mathbf{H \to ZZ, WW}$ can only be defined as POs (at the peak) but can be used in building a DPA in conjunction with $\mathbf{Z \to \bar{f}f}$ etc. (beginning of Lep 2). All these elements are available now (on my hard disk), next full $\mathbf{4f}$ final state is required (work in progress)

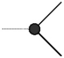
EXAMPLE: non factorizable $\mathbf{H} \rightarrow \gamma\gamma$

```
ANF =
    + 2*i_*g*r2^-1*g6*M*xh*aAA
    + i_*g^3*r2^-1*g6*pi^-2*M*aAA * (
       + 1/8*xh*(dmhf + degf - 1/2*dMf - 1/2*WFH)
       - 1/2*sth^4*xh*a0f(M)
       - 3/64*xh^2*B0f( - mh^2,[],mh,mh)
       - 1/32*xh^2*B0f( - mh^2,[],M0,M0)
       - 1/32*((1 - 2*sth^2)^2*xh + 8*cth^2*sth^2)*xh*B0f( - mh^2,[],M,M)
       + (2 - sth^2)*M^2*sth^2*C0( - mh^2,0,0,[],M,M,M)
       - 1/192*(16*sth^4*xh^2 + 48*(1 - 3*sth^2)*sth^2*xh - 48*(1 - 3
      *sth^2)*sth^2*omLR*xh - 32*(2 - 7*sth^2)*sth^2*xh + 96*(2 - sth^2
      )*sth^2 + 3*(7 - 8*sth^2 + 8*sth^4)*xh^2 - 3*(7 - 8*sth^2 + 8*
      sth^4)*omLR*xh^2)
       )
    + i_*g^3*r2^-1*g6*pi^-2*M*sth^2*aZZ * (
       - 1/2*cth^2*xh*a0f(M)
       + 1/8*(2 - xh)*cth^2*xh*B0f( - mh^2,[],M,M)
       - M^2*cth^2*C0( - mh^2,0,0,[],M,M,M)
       + 1/24*(12 - 10*xh - 5*xh^2 - 18*omLR*xh + 3*omLR*xh^2)*cth^2
       )
    + i_*g^3*r2^-1*g6*pi^-2*M*sth*aAZ * (
       - 1/2*cth*sth^2*xh*a0f(M)
       - 1/16*(1 - 2*sth^2)*(2 - xh)*cth*xh*B0f( - mh^2,[],M,M)
       + M^2*cth^3*C0( - mh^2,0,0,[],M,M,M)
       + 1/48*(3*(1 - 2*sth^2)*xh^2 - 3*(1 - 2*sth^2)*omLR*xh^2 - 4*sth^2*
      xh^2 - 24*cth^2 + 8*(1 - 7*sth^2)*xh - 6*(1 - 6*sth^2)*xh + 6*(
      1 - 6*sth^2)*omLR*xh)*cth
       )
    + i_*g^3*r2^-1*g6*pi^-2*M*sth^2*cth*xt*auBW * (
       + 1/4*(4*xt - omLR*xh)
       + 1/4*xh*B0f( - mh^2,[],mt,mt)
       - 2*M^2*xt^2*C0( - mh^2,0,0,[],mt,mt,mt)
       )
    + i_*g^3*r2^-1*g6*pi^-2*M*sth*xt*auWB * (
       - 1/4*(2*(1 - 2*sth^2)*xt + sth^2*omLR*xh)
       - 1/8*xh*a0f(mt)
       - 1/8*((1 - 2*sth^2))*xh*B0f( - mh^2,[],mt,mt)
       + ((1 - 2*sth^2))*M^2*xt^2*C0( - mh^2,0,0,[],mt,mt,mt)
       )
    + i_*g^3*r2^-1*g6*pi^-2*M*sth*xb*adWB * (
       - 1/8*(2*xb - omLR*xh)
       + 1/16*xh*a0f(mb)
       - 1/16*B0f( - mh^2,[],mb,mb)*xh
       + 1/2*C0( - mh^2,0,0,[],mb,mb,mb)*M^2*xb^2
       )
```

$\mathbf{H} \rightarrow \mathbf{ZZ}$ cut-away

starts at $\mathscr{O}(g)$

$$
\begin{aligned}
\mathscr{A}^{\mu\nu} &= \kappa^{\mathrm{ZZ}}_{\mathrm{LO}}\, \mathscr{A}^{\mathrm{LO}}\, \delta^{\mu\nu} + \mathscr{A}^{\mu\nu}_{\mathrm{NF}} \\
&+ \sum_{i=\mathrm{t,b,W}} \kappa^{\mathrm{ZZ}}_{\mathrm{NLO,i}} \left[\mathscr{A}^{\mathrm{NLO}}_{i,\mathrm{D}}\, \delta^{\mu\nu} + \mathscr{A}^{\mathrm{NLO}}_{i,\mathrm{P}}\, p_2^{\mu}\, p_1^{\nu} \right]
\end{aligned}
$$



dim $=$ **6** tree *contains* $a_{\mathrm{AA}}, a_{\mathrm{AZ}}, a_{\mathrm{ZZ}}$ *and* $a_{\phi\square}$

mixes a_{AA} a_{AZ} a_{ZZ} a_{bWB} a_{tWB} a_{tBW} $a_{\phi\square}$ in γ

$$\Delta\kappa^{ZZ}_{LO} = s^2_\theta\, a_{AA} + \left(4 + c^2_\theta - \frac{M^2_H}{M^2_Z}\right) a_{ZZ} + s^2_\theta c^2_\theta\, a_{AZ} + 2\, a_{\phi\Box}$$

$$\Delta\kappa^{ZZ}_{NLO,\,t} = 2\, a_{ZZ} + 2\, a_{\phi\Box} + a_{t\phi}$$

$$\Delta\kappa^{ZZ}_{NLO,\,b} = 2\, a_{ZZ} + 2\, a_{\phi\Box} - a_{b\phi}$$

$$\Delta\kappa^{ZZ}_{NLO,\,W} = 3\, a_{AA} + 2\, a_{ZZ} + 2\, a_{\phi\Box}$$

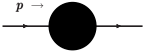
17 non-fact amplitudes with both PTG and LG coefficients

PTG only (in loops)

$\mathbf{H \to bb}$ Summary

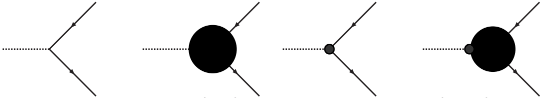
$m_b = Z_{m_b}\, m_b^{ren}$

$p\ \to$

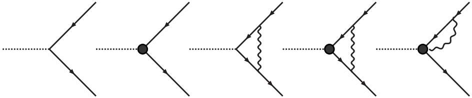
R_b  $\overline{R}_b \to$ UV pole $\propto\ g^2\, g_6\, p^2$

$R_b = Z_b^{+}\,\gamma^{+} + Z_b^{-}\,\gamma^{-}, \qquad \gamma^{\pm} = \frac{1}{2}\left(1 \pm \gamma^5\right)$

$Z_b^{\pm} = 1 - \frac{1}{2}\,\frac{g^2}{16\pi^2}\,\Delta Z_b^{\pm}$

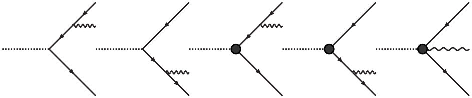


$\mathcal{O}(g)$ $\quad$ $\mathcal{O}(g^3,\ g\,g_s^2)$ $\quad$ $\mathcal{O}(g\,g_6)$ $\quad$ $\mathcal{O}(g^3\,g_6,\ g\,g_s^2\,g_6)$



$a_{\phi W},\ a_{\phi D},\ a_{\phi\Box},\ a_{b\phi}$

Infrared



$a_{bW},\ a_{bB}$

$\mathbf{H \to e^+e^-\mu^+\mu^-}$ at LO

Helicity	$\mathrm{dim} = 4$	$\mathrm{dim} = 6$ fact + non fact
$-+-+$	✓	$a^{(1)}_{\phi l}$, $a^{(3)}_{\phi l}$, $a_{\phi D}$, $a_{\phi \Box}$, $a_{\phi \mathrm{W}}$, $a_{\phi \mathrm{B}}$, $a_{\phi \mathrm{WB}}$
$-++-$	✓	$a_{\phi l}$, $a^{(1)}_{\phi l}$, $a^{(3)}_{\phi l}$, $a_{\phi D}$, $a_{\phi \Box}$, $a_{\phi \mathrm{W}}$, $a_{\phi \mathrm{B}}$, $a_{\phi \mathrm{WB}}$
$+--+$	✓	$a_{\phi l}$, $a^{(1)}_{\phi l}$, $a^{(3)}_{\phi l}$, $a_{\phi D}$, $a_{\phi \Box}$, $a_{\phi \mathrm{W}}$, $a_{\phi \mathrm{B}}$, $a_{\phi \mathrm{WB}}$
$+-+-$	✓	$a_{\phi l}$, $a_{\phi D}$, $a_{\phi \Box}$, $a_{\phi \mathrm{W}}$, $a_{\phi \mathrm{B}}$, $a_{\phi \mathrm{WB}}$

at NLO more amplitudes are populated

Provisional summary record, option tree PTG&LG, loops PTG

❶ Wilson coeff $\to \Delta\kappa$, ❷ $\Delta\kappa$ correlations and ❸ mappings 

Beware of mappings with LO-limited warranty

① $$a_{\phi\Box} = \frac{1}{2}\Delta\kappa_{\mathrm{W}}^{\gamma\mathrm{Z}} = \frac{1}{2}\Delta\kappa_{\mathrm{LO}}^{\mathrm{ZZ}} \qquad a_{\mathrm{t}\phi} = \Delta\kappa_{\mathrm{t}}^{\gamma\gamma} - \Delta\kappa_{\mathrm{W}}^{\gamma\gamma}$$

$$a_{\mathrm{b}\phi} = \Delta\kappa_{\mathrm{b}}^{\gamma\gamma} - \Delta\kappa_{\mathrm{W}}^{\gamma\gamma} \qquad a_{\phi D} = 2\, s_\theta^2\,(\Delta\kappa_{\mathrm{W}}^{\gamma\mathrm{Z}} - \Delta\kappa_{\mathrm{W}}^{\gamma\gamma})$$

② $$\Delta\kappa_{\mathrm{t}}^{\gamma\mathrm{Z}} = 2\,\Delta\kappa_{\mathrm{t}}^{\gamma\gamma} + 2\, c_\theta^2\,(\Delta\kappa_{\mathrm{W}}^{\gamma\mathrm{Z}} - \Delta\kappa_{\mathrm{W}}^{\gamma\gamma})$$

$$\Delta\kappa_{\mathrm{NLO,\,t}}^{\mathrm{ZZ}} = \Delta\kappa_{\mathrm{t}}^{\gamma\gamma} + \Delta\kappa_{\mathrm{W}}^{\gamma\mathrm{Z}} - \Delta\kappa_{\mathrm{W}}^{\gamma\gamma}$$

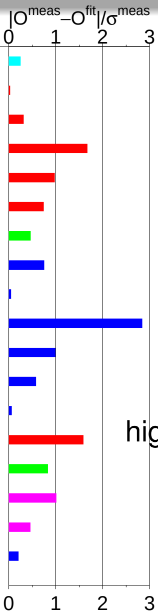
③ $$\frac{\mathrm{V_{H\bar{t}t}}}{\mathrm{V_{H\bar{t}t}^{SM}}} = 1 + g_6\left[\Delta\kappa_{\mathrm{t}}^{\gamma\gamma} + \frac{1}{2}\, c_\theta^2\,\Delta\kappa_{\mathrm{W}}^{\gamma\mathrm{Z}} - \frac{1}{2}\,(2 - s_\theta^2)\,\Delta\kappa_{\mathrm{W}}^{\gamma\gamma}\right]$$

etc. 

Oldies But Goodies

the EWPD

	Measurement	Fit
$\Delta\alpha^{(5)}_{had}(m_Z)$	0.02758 ± 0.00035	0.02767
m_Z [GeV]	91.1875 ± 0.0021	91.1874
Γ_Z [GeV]	2.4952 ± 0.0023	2.4959
σ^0_{had} [nb]	41.540 ± 0.037	41.478
R_l	20.767 ± 0.025	20.742
$A^{0,l}_{fb}$	0.01714 ± 0.00095	0.01643
$A_l(P_\tau)$	0.1465 ± 0.0032	0.1480
R_b	0.21629 ± 0.00066	0.21579
R_c	0.1721 ± 0.0030	0.1723
$A^{0,b}_{fb}$	0.0992 ± 0.0016	0.1038
$A^{0,c}_{fb}$	0.0707 ± 0.0035	0.0742
A_b	0.923 ± 0.020	0.935
A_c	0.670 ± 0.027	0.668
A_l(SLD)	0.1513 ± 0.0021	0.1480
$\sin^2\theta^{lept}_{eff}(Q_{fb})$	0.2324 ± 0.0012	0.2314
m_W [GeV]	80.410 ± 0.032	80.377
Γ_W [GeV]	2.123 ± 0.067	2.092
m_t [GeV]	172.7 ± 2.9	173.3



taking into account EWPD

S T U not enough

they live at $q^2 = 0$

high precision lives a bit higher

world TH accuracy record

EXAMPLE: M_W

☛ select the α, G_F, M_Z IPS where

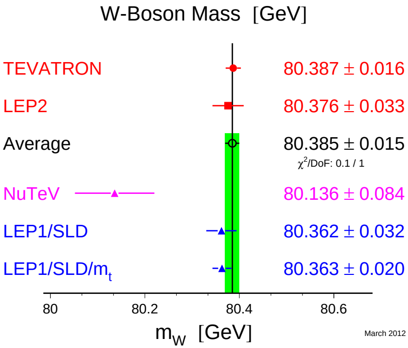
$$
\begin{aligned}
g^2\, s_\theta^2 &= 4\,\pi\alpha \\
\frac{g^2}{M^2} &= 4\,\sqrt{2}\,G_F \quad + \text{ Rad. Corr.} \\
\frac{M^2}{c_\theta^2} &= M_Z^2
\end{aligned}
$$

☞ Require $M_W^2 = M^2 - \frac{g^2}{16\pi^2}\,\mathrm{Re}\,S_{WW}(M_W^2)$

✌ Obtain the solution

$$
M_W^2 \;=\; M_W^2\Big|_{SM} + \frac{\alpha g_6}{\pi}\,\Delta_W^{(6)}
$$

$\Delta_W^{(6)}$ contains **9** PTG terms and **9** LG terms



NOT so easy to constrain

to be 

with the full list of POs

THE ★★★★★ Example

$$
\begin{aligned}
S_{\rm WW} &= \frac{g^2}{16\,\pi^2}\,\Sigma_{\rm WW} \\
S_{\rm ZZ} &= \frac{g^2}{16\,\pi^2\,c_\theta^2}\,\Big(\Sigma_{33} - 2\,s_\theta^2\,\Sigma_{3Q} - s_\theta^4\,\Pi_{\rm AA}\,s\Big) \\
\Sigma_{\rm F} &= \Sigma_{\rm WW}(0) - {\rm Re}\,\Sigma_{33}(M_{\rm Z}^2) + {\rm Re}\,\Sigma_{3Q}(M_{\rm Z}^2)
\end{aligned}
$$

Define $\boldsymbol{\rho}^{-1} = 1 + \frac{G_{\rm F}}{2\sqrt{2}\pi^2}\,\boldsymbol{\Sigma}_{\rm F} = 0.99490$, $\boldsymbol{\Delta\rho}$ contains ($\mathcal{PTG}$ only):
$\boldsymbol{a_{\phi D}},\ \boldsymbol{a_{\Phi\Box}},\ \boldsymbol{a_{\phi \rm f}},\ \boldsymbol{a_{\phi \rm f}^{(1,3)}}, \quad \mathbf{f = l,\ u,\ d}$

Leading term (don't use it for precision) is

$$
\begin{aligned}
\Delta\rho &= M_{\rm t}^2\left[\kappa_\rho\,\Delta\rho^{(4)} + g_6\sum_i F_i\,a_i\right] \qquad a_i = a_{\phi D},\ a_{\phi\rm t},\ a_{\phi\rm q}^{(1,3)} \\
\kappa_\rho &= 1 + \frac{g_6}{11}\left[\frac{7}{6}\,a_{\phi D} + 28\,(a_{\phi\rm q}^{(1)} + a_{\phi\rm q}^{(3)}) - 20\,a_{\phi\rm t}\right]
\end{aligned}
$$

Conclusions

NLO HEFT is ready and we have a consistent framework for testing compatibility of data with SM Higgs couplings at the projected level of accuracy

Results and Tools are ready/under way, they should provide useful for those wishing to use NLO EFT, but

… anyway, a Never-Ending Story

Good tests kill flawed theories; we remain alive to guess again.

Thank you for your attention