The Pillars of SM deviations
From Higgs discovery to Higgs properties

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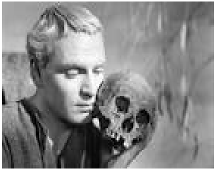


HEFT2016 workshop, 26-28 October 2016, Copenhagen

to André David and Mike Trott for keeping my dreams alive

Will  manage to kill those dreams? [1]

[1] "Non ti curar di lor, ma guarda e passa". Dante, Inferno, canto III, v.51

Synopsis

Is there a universal language for SM deviations?
. . . . . . wolf, goat, and cabbage

"Why don't you go and calculate for a change?"

"Why don't you prepare in advance?"

Deviations at LHC, parameterizations:

① external layer[a] (similar to LEP σ_f^{peak})

$$\left(\sum_f\right)\Gamma_{Vff} \quad A_{FB}^{ZZ} \quad N_{off}^{4l} \quad \text{etc.}$$

not NWA or truncated MPE

② intermediate layer (similar to LEP $g^e_{V\,A}$)

$\varepsilon_{\gamma\gamma}$ etc. See Marzocca talk

③ internal layer: the (generalized) kappas

$$\kappa_f^{\gamma\gamma} \quad \kappa_W^{\gamma\gamma} \quad \kappa_i^{\gamma\gamma\,NF} \quad \text{etc.}$$

④ innermost layer: Wilson coeff. in SMEFT or non-SM parameters in BSM (e.g. α, β, M_{sb} etc. in THDMs)

more

uni-

ver-

sal

[a] where kinematics cannot be manipulated

Topdown **Catch a glimpse of the top-down approach**

- The strategy here starts with a more fundamental theory which is valid on a given energy scale Λ and derive a systematic procedure for getting low-energy ($E \ll \Lambda$) results.

? spin partners

plenty of spin 1/2

plenty of spin 1

1 spin 0 ?

more ? Hierarchy of VEVs?

↙

serious fine-tuning

↘

small mixings

accidental?
systematic (i.e. symmetry)?

banishing scalars?
extra dimensions?
warped extra dimensions?

One model is falsifiable, but an endless stream of them is not

The integration of heavy scalar fields in BSM models, containing more that one representation for scalars and with mixing [arXiv:1602.00126,1603.03660,1604.01019]

- Interplay between integrating out heavy scalars and the Standard Model decoupling limit:
  - ✓ In general, the latter cannot be obtained in terms of only one large scale and can only be achieved by imposing further assumptions on the couplings
  - ✓ Systematic low-energy expansions in the more general, non-decoupling scenario, include mixed tree-loop and mixed heavy-light generated operators. The number of local operators is larger than the one usually reported in the literature

When the NLO-people come back ...... Beware of on-shell vs. $\overline{\mathrm{MS}}$ renormalization vs. gauge invariance:

- Tadpoles matter, i.e. they only cancel in on-shell renormalization [hep-ph/0612122,arXiv:1607.07352]

- When masses of heavy states and mixings are $\overline{\mathrm{MS}}$-renormalized there could be problems: i.e. the $\overline{\mathrm{MS}}$ renormalization of the mixing angles combined with popular on-shell renormalization schemes gives rise to gauge-dependent results already at the one-loop level [arXiv:1607.07352]

Examining the layers ......

Bottom-up or *methodological antireductionism* (not to be confused with *epistemological antifoundationalism*, i.e. there is no final theory): this position advocates the bottom-up EFT research strategy which is also favoured by pragmatically-minded physicists [2]

[2] S. Hartmann, Stud. Hist. Philos. Mod. Phys.

1. SM augmented with the inclusion of higher dimensional operators ($\mathbf{T_1}$); not strictly renormalizable. Although workable to all orders, $\mathbf{T_1}$ fails above a certain scale, $\mathbf{\Lambda_1}$.

2. Consider any BSM model that is strictly renormalizable and respects unitarity ($\mathbf{T_2}$); its parameters can be fixed by comparison with data, while masses of heavy states are presently unknown. $\mathbf{T_1} \neq \mathbf{T_2}$ in the UV but must have the same IR behavior.

3. Consider now the whole set of data below $\mathbf{\Lambda_1}$.
$\mathbf{T_1}$ should be able to explain them by fitting Wilson coefficients,
$\mathbf{T_2}$ adjusting the masses of heavy states (as SM did with the Higgs mass at LEP) should be able to explain the data.
Goodness of both explanations are crucial in understanding how well they match and how reasonable is to use $\mathbf{T_1}$ instead of the full $\mathbf{T_2}$

4. Does $\mathbf{T_2}$ explain everything? Certainly not, but it should be able to explain something more than $\mathbf{T_1}$.

5. We could now define $\mathbf{T_3}$ as $\mathbf{T_2}$ augmented with (its own) higher dimensional operators; it is valid up to a scale $\mathbf{\Lambda_2}$.

6. ...... 

What are the bases of SMEFT? [3]

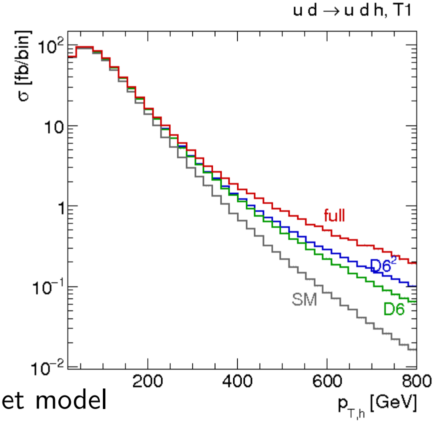
- Experiments occur at finite energy and "measure" an effective action $\mathbf{S}^{\text{eff}}(\boldsymbol{\Lambda})$
- whatever QFT should give low energy $\mathbf{S}^{\text{eff}}(\boldsymbol{\Lambda})\,,\ \forall \boldsymbol{\Lambda} < \infty$
- One also assumes that there is no fundamental scale above which $\mathbf{S}^{\text{eff}}(\boldsymbol{\Lambda})$ is not defined [Costello2011] and
- $\mathbf{S}^{\text{eff}}(\boldsymbol{\Lambda})$ loses its predictive power if a process at $E = \boldsymbol{\Lambda}$ requires ∞ renormalized parameters [Preskill:1990]

[3] It is remarkable that when constructive proofs are provided, their simplicity always seems to detract from their originality

Is SMEFT good?

D6*vs*.D6^2 in MHOU

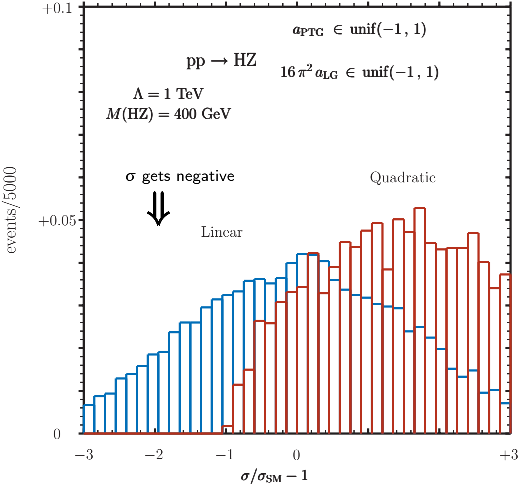
open for discussion



Full = vector triplet model

Quoting T. Plehn "Forcing the EFT approach into a spectacular breakdown was the original aim of this paper, but to our surprise this did not happen"

But ... don't get confused



MHOU is something, breakdown is something else

Is SMEFT consistent?

- All sets of gauge invariant, dimension d operators, none of which is redundant, form a basis and all bases are equivalent. For a formal definition of redundancy see [Einhorn:2013kja]. Avoid field reparameterization[4]

① What about closure w.r.t. renormalization?

② What about IR/collinear singularities?

[4] For different opinions 

① Technically speaking this would require proving removal of UV poles for ALL off-shell Green's functions, too much in the present moment. Closure is proven for all on-shell Green's functions relevant for Higgs physics and EWPD, [arXiv:1607.01236]

② Yes, they cancel

Anomalies?

Inclusion of triple/quadrupole gauge couplings: this brings us to gauge anomalies and anomaly cancellation; perhaps, a deeper understanding of SMEFT, a low-energy limit of an underlying anomaly-free theory?

Proposition

SMEFT anomalies are UV finite (it is good for renormalizability), restoring gauge invariance order-by-order by adding finite counterterms, i.e. it is possible to quantize an anomalous theory in a manner that respects WSTI [Preskill:1990fr] and local. The latter is good for unitarity, another tiny step forward.

Do we necessarily have to make UV assumptions?

- The pattern of suppressions for Wilson coefficients is not a SMEFT prediction but must be determined experimentally. Of course, it depends on the underlying UV completion but can be determined experimentally solely by using "low-energy" measurements that

  can be computed by using SMEFT

  as was done over the last 50 years, cfr. [arxiv:1601.07551]

$$\left\{ \begin{array}{ll} \text{Select low-energy measurements} \ll \Lambda & \longrightarrow \\ \text{Compute them using SMEFT} & \longrightarrow \\ \text{Determine the parameters of SMET} & \end{array} \right.$$

Low-energy theories, next resonances and all that

- Fermi theory, see section III.C of [arxiv:1601.07551]
- SM before LEP: how to use low-energy ($Q^2 \ll M_W^2$) data points

$$g^2 = 4\pi\alpha(0) \quad M_W^2 = \frac{g^2}{4\sqrt{2}\,G_F} \quad s_W^2 \text{ from } \frac{\sigma(\bar{\nu}_e\, e)}{\sigma(\nu_e\, e)}$$

plus radiative corrections. Back in the Eighties it was

$$\begin{aligned} \boldsymbol{M_W} &= \frac{37.281\ GeV}{s_W}(1 + \text{radiative corr.}) \\ s_W^2\,\big|_{exp} &= 0.224 \pm 0.014 \quad \text{SLAC e-deuteron DIS} \end{aligned}$$

not bad for a low-energy theory (SM expanded in Q^2) ......

- SM after LEP and before LHC: M_H? Look at the blue-band! ($M_H \to \infty$ breaks unitarity)

Why NLO SMEFT?


How do you allocate resolved coupling modifiers?

What is the point of arguing about the size of one loop calculations endlessly instead of just doing the calculations?

How to connect kappas with Wilson coefficients?

❶ Original kappa-framework: It amounts to replace $\mathscr{L}_{\rm SM}(\{m\},\{g\})$ with $\mathscr{L}(\{m\},\{\kappa_g\, g\})$, where $\{m\}$ denotes the SM masses, $\{g\}$ the SM couplings and κ_g are the scaling parameters. This is the framework used during Run 1.

❷ Going from SM to SMEFT we modify the amplitude as follows:

$$\mathscr{A}^{\rm LO}_{\rm SMEFT} = \sum_{i=1,n} \mathscr{A}^{(i)}_{\rm SM} + i\, g_6\, \kappa_{\rm c}$$

$$\mathscr{A}^{\rm NLO}_{\rm SMEFT} = \sum_{i=1,n} \kappa_i\, \mathscr{A}^{(i)}_{\rm SM} + i\, g_6\, \kappa_{\rm c} + g_6 \sum_{i=1,{\rm N}} a_i\, \mathscr{A}^{(i)}_{\rm nf}$$

where $g_6^{-1} = \sqrt{2}\, G_{\rm F}\, \Lambda^2$. The last term collects all loop contributions that do not factorize and the coefficients a_i are Wilson coefficients. The κ_i are linear combinations of the a_i.

How to connect intermediate POs with Wilson coefficients?

Example:

❶ The amplitude for the process $\mathbf{H}(P) \to \gamma_\mu(p_1)\gamma_\nu(p_2)$ can be written as

$$A^{\mu\nu}_{\mathrm{HAA}} = i\,\mathscr{T}_{\mathrm{HAA}}\,T^{\mu\nu} = -i\,\frac{2}{v_F\,M^2_{\mathrm{H}}}\,\varepsilon_{\gamma\gamma}\,T^{\mu\nu}$$

$$M^2_{\mathrm{H}}\,T^{\mu\nu} = p_2^\mu\,p_1^\nu - p_1 \cdot p_2\,g^{\mu\nu}$$

❷ A convenient way for writing the amplitude is the following: after renormalization we neglect all fermion masses but m_t, m_b and write

$$\mathscr{T}_{\mathrm{HAA}} = \frac{g_F^3\,s_W^2}{8\,\pi^2}\sum_{I=W,t,b}\rho_I^{\mathrm{HAA}}\,\mathscr{T}^I_{\mathrm{HAA;LO}} + g_F\,g_6\,\frac{M^2_{\mathrm{H}}}{M_W}\,a_{AA} + \frac{g_F^3\,g_6}{\pi^2}\,\mathscr{T}^{\mathrm{nf}}_{\mathrm{HAA}}$$

❸ Introduce $g_{\mathrm{F}}^2 = 4\sqrt{2}\, G_{\mathrm{F}}\, M_{\mathrm{W}}^2$ and $c_{w} = M_{\mathrm{W}}/M_{\mathrm{Z}}$ (note that, at this point we have selected the $\{G_{\mathrm{F}}, M_{\mathrm{Z}}, M_{\mathrm{W}}\}$ IPS, alternatively one could use the $\{\alpha, G_{\mathrm{F}}, M_{\mathrm{Z}}\}$) and derive

$$
\begin{aligned}
\kappa_{\mathrm{I}}^{\mathrm{HAA}} &= \frac{g_{\mathrm{F}}^3\, s_{\mathrm{W}}^2}{8\,\pi^2}\, \rho_{\mathrm{I}}^{\mathrm{HAA}} \\
\kappa_{\mathrm{c}}^{\mathrm{HAA}} &= g_{\mathrm{F}}\, \frac{M_{\mathrm{H}}^2}{M_{\mathrm{W}}}\, a_{\mathrm{AA}}
\end{aligned}
$$

$$
\begin{aligned}
a_{\mathrm{ZZ}} &= s_{\mathrm{W}}^2\, a_{\phi\mathrm{B}} + c_{\mathrm{W}}^2\, a_{\phi\mathrm{W}} - s_{\mathrm{W}}\, c_{\mathrm{W}}\, a_{\phi\mathrm{WB}} \\
a_{\mathrm{AA}} &= c_{\mathrm{W}}^2\, a_{\phi\mathrm{B}} + s_{\mathrm{W}}^2\, a_{\phi\mathrm{W}} + s_{\mathrm{W}}\, c_{\mathrm{W}}\, a_{\phi\mathrm{WB}} \\
a_{\mathrm{AZ}} &= 2\, c_{\mathrm{W}}\, s_{\mathrm{W}}\, \left(a_{\phi\mathrm{W}} - a_{\phi\mathrm{B}}\right) + \left(2\, c_{\mathrm{W}}^2 - 1\right)\, a_{\phi\mathrm{WB}}
\end{aligned}
$$

➍ The (process dependent) $\boldsymbol{\rho}$-factors are given by

$$\rho_{\mathrm{I}}^{\mathrm{proc}} \quad = \quad 1 + g_6\, \Delta\rho_{\mathrm{I}}^{\mathrm{proc}}$$

and there are additional, non-factorizable, contributions. For $\mathrm{H} \rightarrow \gamma\gamma$ the $\Delta\rho$ factors are as follows:

$$\begin{aligned}
\Delta\rho_{\mathrm{t}}^{\mathrm{HAA}} &= \frac{3}{16}\,\frac{M_{\mathrm{H}}^2}{\mathrm{s_W}\, M_{\mathrm{W}}^2}\, a_{\mathrm{t\,WB}} + (2 - \mathrm{s_W^2})\,\frac{\mathrm{c_W}}{\mathrm{s_W}}\, a_{\mathrm{AZ}} + (6 - \mathrm{s_W^2})\, a_{\mathrm{AA}} \\
&- \frac{1}{2}\Big[a_{\phi\,\mathrm{D}} + 2\,\mathrm{s_W^2}\,(\mathrm{c_W^2}\, a_{\mathrm{ZZ}} - a_{\mathrm{t}\,\phi} - 2\, a_{\phi\,\square})\Big]\,\frac{1}{\mathrm{s_W^2}} \\
\Delta\rho_{\mathrm{b}}^{\mathrm{HAA}} &= -\frac{3}{8}\,\frac{M_{\mathrm{H}}^2}{\mathrm{s_W}\, M_{\mathrm{W}}^2}\, a_{\mathrm{b\,WB}} + (2 - \mathrm{s_W^2})\,\frac{\mathrm{c_W}}{\mathrm{s_W}}\, a_{\mathrm{AZ}} + (6 - \mathrm{s_W^2})\, a_{\mathrm{AA}} \\
&- \frac{1}{2}\Big[a_{\phi\,\mathrm{D}} + 2\,\mathrm{s_W^2}\,(\mathrm{c_W^2}\, a_{\mathrm{ZZ}} + a_{\mathrm{b}\,\phi} - 2\, a_{\phi\,\square})\Big]\,\frac{1}{\mathrm{s_W^2}} \\
\Delta\rho_{\mathrm{W}}^{\mathrm{HAA}} &= (2 + \mathrm{s_W^2})\,\frac{\mathrm{c_W}}{\mathrm{s_W}}\, a_{\mathrm{AZ}} + (6 + \mathrm{s_W^2})\, a_{\mathrm{AA}} - \frac{1}{2}\Big[a_{\phi\,\mathrm{D}} - 2\,\mathrm{s_W^2}\,(2\, a_{\phi\,\square} + \mathrm{c_W^2}\, a_{\mathrm{ZZ}})\Big]\,\frac{1}{\mathrm{s_W^2}}
\end{aligned}$$

$\mathscr{G}$

❺ In the PTG [5] scenario we only keep $a_{t\phi}, a_{b\phi}, a_{\phi D}$ and $a_{\phi \Box}$. These results tell us that κ-factors can be introduced also at the loop level; they are combinations of Wilson coefficients but we have to extend the scheme with the inclusion of process dependent, non-factorizable, contributions.

We also derive the following result for the non-factorizable part of the amplitude (in the PTG scenario all non-factorizable amplitudes for $\mathrm{H} \to \gamma\gamma$ vanish):

$$\begin{aligned}
\mathscr{T}^{\mathrm{nf}}_{\mathrm{HAA}} &= M_{\mathrm{W}} \sum_{a \in \{A\}} \mathscr{T}^{\mathrm{nf}}_{\mathrm{HAA}}(a)\, a \\
\{A\} &= \{a_{\mathrm{t\,WB}}, a_{\mathrm{b\,WB}}, a_{\mathrm{AA}}, a_{\mathrm{AZ}}, a_{\mathrm{ZZ}}\}
\end{aligned}$$

[5] Potentially Tree Generated

❻ At LO we obtain the following relation between an intermediate PO (red), see Marzocca talk, and a Wilson coefficient (blue):

$$\varepsilon_{\gamma\gamma} = -\frac{1}{2}\,\frac{\mathrm{v_F}}{M_{\mathrm{H}}^2}\,\mathcal{T}_{\mathrm{HAA}}$$

$$\mathcal{T}_{\mathrm{HAA}}^{\mathrm{LO}} = \mathcal{T}_{\mathrm{HAA}}^{\mathrm{SM}} + g_{\mathrm{F}}\, g_6\, \frac{M_{\mathrm{H}}^2}{M_{\mathrm{W}}}\, a_{\mathrm{AA}}$$

NLO follows as well …… simply use $\rho_{\mathrm{I}}^{\mathrm{HAA}}$

Message for skeptics: the PO-language can be translated back into SMEFT-language

Do we need "physical" POs?

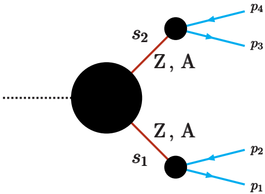
- It's about time that we stop reporting
  - Non existing objects, e.g. $\mathbf{H} \to \mathbf{ZZ}$
  - Non gauge invariant objects, e.g. $\mathbf{H} \to \mathbf{Z^*Z}$
- "physical"' POs allow for exp. cuts [6] and, most of the time, an inclusive setup is just nonsense (e.g. $\mathrm{Z}\gamma$, $\mathrm{tHW}\,/\,\bar{\mathrm{t}}\mathrm{tH}$)
  - "physical"' POs bypass ad hoc constructions like "diagram removal" (not g.i.) or "diagram subtraction" (ad hoc prefactor and BW profile)

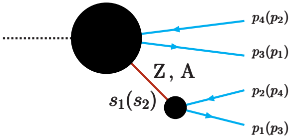
[6] However, care is needed if we want differential distributions with cuts, see App. C of [arXiv:1112.5517]

How to construct "physical" POs?
There are several steps

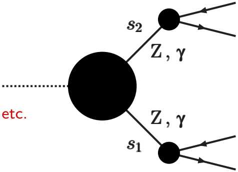
- Multi Pole Expansion
- PV - cuts
- Phase space factorization
- Helicity factorization

❶ Multi Pole Expansion (see Marzocca talk): poles and their residues are intimately related to the gauge invariant splitting of the amplitude (Nielsen identities); residues of poles (after squaring the amplitude and after integration over residual variables) can be interpreted as physical POs, which requires factorization into subprocesses.





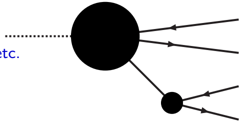
MPE: crab expansion



$\Gamma(\mathrm{H} \to \mathrm{Z}\bar{\mathrm{f}}\,\mathrm{f})$ etc.

$$\frac{\mathscr{A}_{\mathrm{DR}}(s_1, s_2; \dots)}{(s_1 - s_Z)(s_2 - s_Z)} = \frac{\mathscr{A}_{\mathrm{DR}}(s_Z, s_Z; \dots)}{(s_1 - s_Z)(s_2 - s_Z)} + \frac{\mathscr{A}^{(2)}_{\mathrm{DR}}(s_Z, s_2; \dots)}{s_1 - s_Z}$$

$$\dots + \mathscr{A}^{\mathrm{rest}}_{\mathrm{DR}}(s_1, s_2; \dots)$$

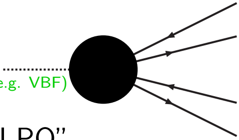


$\Gamma(\mathrm{H} \to \mathrm{Z}\gamma)$ etc.

$$\frac{\mathscr{A}_{\mathrm{SR}}(s_1; \dots)}{s_1 - s_Z} = \frac{\mathscr{A}_{\mathrm{SR}}(s_Z; \dots)}{s_1 - s_Z} + \mathscr{A}^{\mathrm{rest}}_{\mathrm{SR}}(s_1; \dots)$$

remember LEP

$$\sigma^{\mathrm{peak}}_{\mathrm{f}} = 12\pi \frac{\Gamma_{\mathrm{e}} \Gamma_{\mathrm{f}}}{M^2_{\mathrm{Z}} \Gamma^2_{\mathrm{Z}}}$$



the difficult part (e.g. VBF)

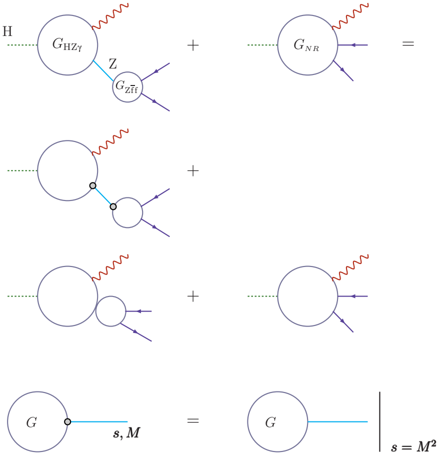
$$\mathscr{A}_{\mathrm{NR}}(\dots)$$

$$+ \; (\mathrm{Z} \to \gamma)$$

⇑ "physical PO"

●, ●, ● gauge invariant sets

Multi-pole-expansion for $\mathrm{H} \to \gamma \bar{\mathrm{f}} \mathrm{f}$. G stands for Green's function and G_{NR} denotes the non-resonant part of the amplitude. The sum of amplitudes in the second (third) row is gauge-parameter independent. In the last row, an amplitude with an external line of virtuality s and mass M is put on-shell



❷ Phase Space Factorization : gauge invariant splitting is not the same as "factorization" of the process into sub-processes, indeed phase space factorization requires the pole to be inside the physical region. It's $|$ amplitude $|^2$ that matters. Decompose the square of a propagator:

$$\Delta = \frac{1}{(s-M^2)^2+\Gamma^2 M^2} = \frac{\pi}{M\Gamma}\,\boldsymbol{\delta\left(s-M^2\right)} + \mathrm{PV}\left[\frac{1}{(s-M^2)^2}\right]$$

and use the n-body decay phase space

$$d\Phi_n(P,p_1\dots p_n) = \frac{1}{2\pi}\,dQ^2\,d\Phi_{n-j+1}\left(P,Q,p_{j+1}\dots p_n\right) \times d\Phi_j\left(Q,p_1\dots p_j\right)$$

✓ To "complete" the decay ($d\Phi_j$) we need the $\boldsymbol{\delta}$-function. We can say that the δ-part of the resonant (squared) propagator opens the corresponding line allowing us to define physical POs (t-channel propagators cannot be cut).

Consider the process $\mathbf{qq \to \bar{f}_1 f_1 \bar{f}_2 f_2} \boldsymbol{jj}$, according to the structure of the resonant poles we have different options in extracting physical POs, e.g.

$$\sigma(\mathrm{qq} \to \bar{\mathrm{f}}_1 \mathrm{f}_1 \bar{\mathrm{f}}_2 \mathrm{f}_2 jj) \quad \stackrel{PO}{\longmapsto} \quad \sigma(\mathrm{qq} \to \mathrm{H} jj)\,\mathrm{Br}(\mathrm{H} \to \mathrm{Z} \bar{\mathrm{f}}_1 \mathrm{f}_1)\,\mathrm{Br}(\mathrm{Z} \to \bar{\mathrm{f}}_2 \mathrm{f}_2)$$

$$\sigma(\mathrm{qq} \to \bar{\mathrm{f}}_1 \mathrm{f}_1 \bar{\mathrm{f}}_2 \mathrm{f}_2 jj) \quad \stackrel{PO}{\longmapsto} \quad \sigma(\mathrm{qq} \to \mathrm{ZZ} jj)\,\mathrm{Br}(\mathrm{Z} \to \bar{\mathrm{f}}_1 \mathrm{f}_1)\,\mathrm{Br}(\mathrm{Z} \to \bar{\mathrm{f}}_2 \mathrm{f}_2)$$

$\mathscr{G}$

❸ There are fine points when factorizing a process into "physical" sub-processes (POs): extracting the δ from the (squared) propagator does not finish the job. Consider and amplitude that can be factorized as follows:

$$\mathscr{A} \quad = \quad \mathscr{A}^{(1)}_{\mu}\, \Delta_{\mu\nu}(p)\, \mathscr{A}^{(2)}_{\nu}$$

where $\boldsymbol{\Delta_{\mu\nu}}$ enters the propagator for a resonant, spin-1 particle. We would like to replace (conserved currents)

$$\Delta_{\mu\nu} \quad \rightarrow \quad \frac{1}{s - s_{\rm c}} \sum_{\lambda} \varepsilon_{\mu}(p,\lambda)\, \varepsilon^{*}_{\nu}(p,\lambda)$$

where $s_{\rm c}$ is the complex pole and ε_{μ} are spin-1 polarization vectors. What we obtain is

$$|\,\mathscr{A}\,|^2 \quad = \quad \frac{1}{|\, s - s_{\rm c}\,|^2} \Big| \sum_{\lambda} \Big[\mathscr{A}^{(1)} \cdot \varepsilon(p,\lambda) \Big] \Big[\mathscr{A}^{(2)} \cdot \varepsilon^{*}(p,\lambda) \Big] \Big|^2$$

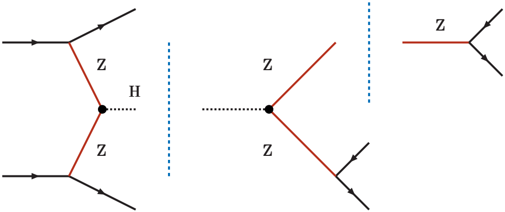
Which means that we do not have what we need,

$$\sum_{\lambda}\left|\mathscr{A}^{(1)}\cdot\varepsilon(p,\lambda)\right|^{2}\sum_{\sigma}\left|\mathscr{A}^{(2)}\cdot\varepsilon(p,\sigma)\right|^{2}$$

Is there a solution?

Iff cuts are not introduced, the interference terms among different helicities oscillate over the phase space and drop out, i.e. we achieve factorization, see [Uhlemann:2008pm]. Effects of cuts can be computed.

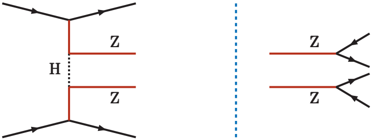
ADDITIONAL READING Furthermore, MPE should be understood as "asymptotic expansion", [Nekrasov:2007ta,Tkachov:1999qb], not as Narrow-Width-Approximation (NWA). The phase space decomposition obtains by using the two parts in the propagator expansion: the δ-term is what we need to reconstruct POs, the PV-term (understood as a distribution) gives the remainder and POs are extracted without making any approximation. It is worth noting that, in extracting POs, analytic continuation (on-shell masses into complex poles) is performed only after integrating over residual variables [Goria:2011wa]



$\mathbf{A_{PO}(qq \rightarrow H\mathit{jj})}$ $\mathbf{A_{PO}(H \rightarrow Z\bar{f}_1 f_1)}$ $\mathbf{A_{PO}(Z \rightarrow \bar{f}_2 f_2)}$

beware: not a Feynman diagram

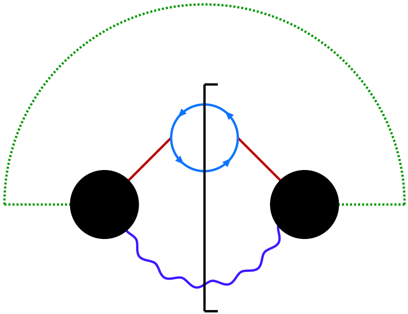
but a convenient way to visualize $\mathrm{A_{PO}}$



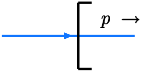
beware: not a Feynman diagram

but a convenient way to visualize A_{PO}

A brief introduction to cut - 

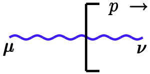


These are Cutkosky's cutting rules



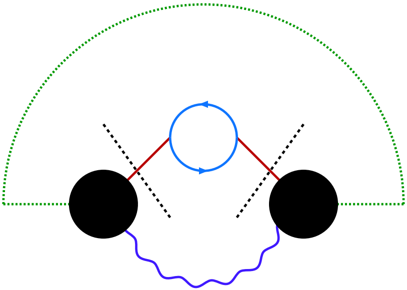
$$= \frac{1}{(2\,\pi)^3}\,(-i\,\not{p} + m)\,\theta(p_0)\,\delta(p^2 + m^2)$$

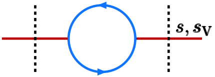
Never cut an unstable line, learn from [Veltman:1963th]



$$= \quad \frac{1}{(2\,\pi)^3}\,\delta_{\mu\nu}\,\theta(p_0)\,\delta(p^2)$$

PV - cuts





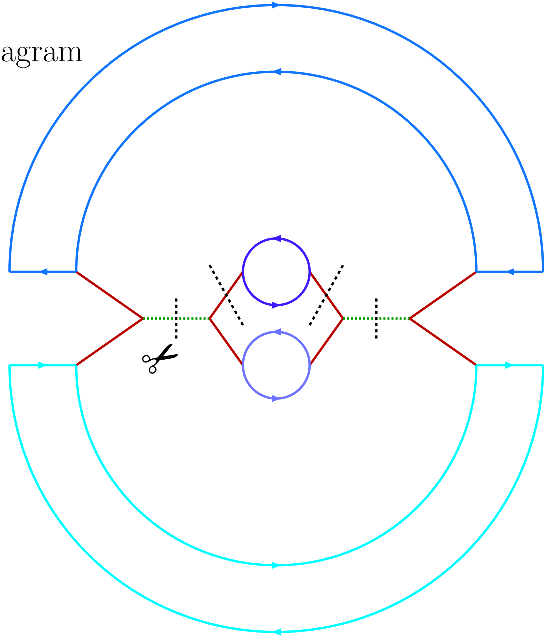
$$\frac{1}{\left(s-\mu_V^2\right)^2+\mu_V^2\,\gamma_V^2} \to \frac{\pi}{\mu_V\,\gamma_V}\,\delta\left(s-\mu_V^2\right)$$

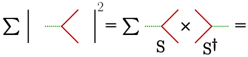
Proceed with Cutkosky's cutting rules

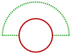
Pernickety:

$$\mathrm{PV}\,\frac{1}{x^n} = \frac{(-1)^{n-1}}{\Gamma(n)}\,\frac{d^n}{dx^n}\ln\mid x\mid \text{ in distribution sense}$$

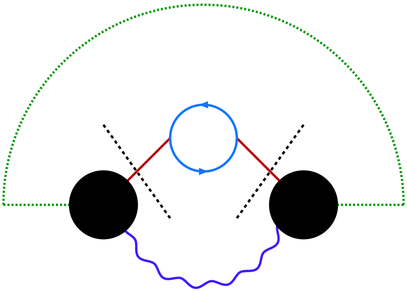
K-diagram



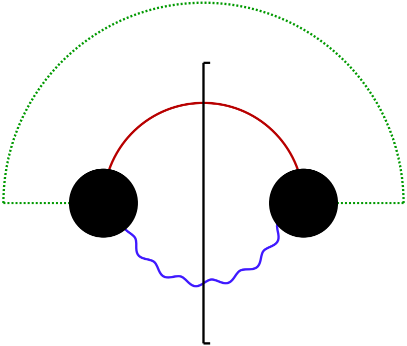


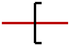


K -diagram



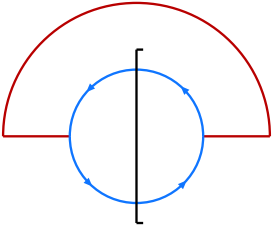
K -diagram for SR

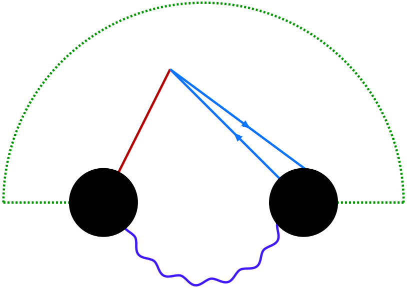




$$\frac{1}{(2\,\pi)^3}\,(\delta_{\mu\nu} + \frac{p_\mu p_\nu}{M_Z^2})\,\theta(p_0)\,\delta(p^2 + M_Z^2)$$

$$\delta_{\mu\nu} + \frac{p_\mu p_\nu}{M_Z^2} = \sum_\lambda e^\lambda_\mu\,[e^\lambda_\nu]^\dagger$$





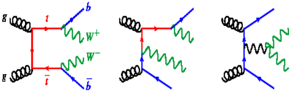
K-diagram for Interference

A complex example:

$$\left.\begin{array}{ll} \bar{t}\,t & \text{DR part of} \\ t(\bar{t})\,W^-(W^+)\,\bar{b}(b) & \text{SR part of} \end{array}\right\{ W\,b\,W\,b \quad \text{production}$$

- do not "kill" diagrams
- write the K-diagrams [7] for $W\,b\,W\,b$ production and find the appropriate factorization (δ-parts)

---

[7] i.e. square 

A simple but non-trivial example: Dalitz decay of the Higgs boson. Consider the process

$$\mathbf{H}(P) \quad\rightarrow\quad \bar{\mathrm{f}}(p_1) + \mathrm{f}(p_2) + \gamma(p_3)$$

The physical POs are:

$$\boldsymbol{\Gamma_{\mathrm{PO}}\left(\mathrm{H} \rightarrow \mathrm{Z}\gamma\right)} \quad = \quad \frac{1}{16\,\pi}\,\frac{1}{M_{\mathrm{H}}}\left(1 - \frac{\mu_{\mathrm{Z}}^2}{M_{\mathrm{H}}^2}\right)\mathrm{F}_{\mathrm{H}\rightarrow\mathrm{Z}\gamma}\left(s_{\mathrm{Z}}\,,\,\mu_{\mathrm{Z}}^2\right)$$

$$\boldsymbol{\Gamma_{\mathrm{PO}}\left(\mathrm{Z} \rightarrow \bar{\mathrm{f}}\mathrm{f}\right)} \quad = \quad \frac{1}{48\,\pi}\,\frac{1}{\mu_{\mathrm{Z}}}\,\mathrm{F}_{\mathrm{Z}\rightarrow\bar{\mathrm{f}}\mathrm{f}}\left(s_{\mathrm{Z}}\,,\,\mu_{\mathrm{Z}}^2\right)$$

$$\boldsymbol{\Gamma_{\mathrm{SR}}\left(\mathrm{H} \rightarrow \bar{\mathrm{f}}\mathrm{f}\gamma\right)} \quad = \quad \frac{1}{2}\,\Gamma_{\mathrm{PO}}\left(\mathrm{H} \rightarrow \mathrm{Z}\gamma\right)\,\frac{1}{\gamma_{\mathrm{Z}}}\,\Gamma_{\mathrm{PO}}\left(\mathrm{Z} \rightarrow \bar{\mathrm{f}}\mathrm{f}\right) + \text{ remainder}$$

$\mathrm{F}_{\mathrm{H}\rightarrow\mathrm{Z}\gamma}\,,\,\mathrm{F}_{\mathrm{Z}\rightarrow\bar{\mathrm{f}}\mathrm{f}}$ not shown here

$\mathscr{G}$

The interpretation in terms of SMEFT is based on

$$\mathscr{T}_{\mathrm{HAZ}} = \frac{g_{\mathrm{F}}^3}{\pi^2 M_{\mathrm{Z}}} \sum_{\mathrm{I=W,t,b}} \rho_{\mathrm{I}}^{\mathrm{HAZ}} \mathscr{T}^{\mathrm{I}}_{\mathrm{HAZ;LO}} + g_{\mathrm{F}}\, g_6\, \frac{M_{\mathrm{H}}^2}{M}\, a_{\mathrm{AZ}} + \frac{g_{\mathrm{F}}^3\, g_6}{\pi^2}\, \mathscr{T}^{\mathrm{nf}}_{\mathrm{HAZ}}$$

The factorizable part is defined in terms of ρ-factors

$$\Delta\rho_{\mathrm{q}}^{\mathrm{HAZ}} = \left(2\,\mathrm{I}_{\mathrm{q}}^{(3)}\, a_{\mathrm{q}\,\phi} + 2\, a_{\phi\,\square} - \frac{1}{2}\, a_{\phi\,\mathrm{D}} + 3\, a_{\mathrm{AA}} + 2\, a_{\mathrm{ZZ}}\right)$$

$$\Delta\rho_{\mathrm{W}}^{\mathrm{HAZ}} = \frac{1+6\,\mathrm{c}_{\mathrm{W}}^2}{\mathrm{c}_{\mathrm{W}}^2}\, a_{\phi\,\square} - \frac{1}{4}\,\frac{1+4\,\mathrm{c}_{\mathrm{W}}^2}{\mathrm{c}_{\mathrm{W}}^2}\, a_{\phi\,\mathrm{D}} - \frac{1}{2}\,\frac{1+\mathrm{c}_{\mathrm{W}}^2 - 24\,\mathrm{c}_{\mathrm{W}}^4}{\mathrm{c}_{\mathrm{W}}^2}\, a_{\mathrm{AA}}$$

$$+ \frac{1}{4}\left(1+12\,\mathrm{c}_{\mathrm{W}}^2 - 48\,\mathrm{c}_{\mathrm{W}}^4\right)\frac{\mathrm{s}_{\mathrm{W}}}{\mathrm{c}_{\mathrm{W}}^3}\, a_{\mathrm{AZ}} + \frac{1}{2}\,\frac{1+15\,\mathrm{c}_{\mathrm{W}}^2 - 24\,\mathrm{c}_{\mathrm{W}}^4}{\mathrm{c}_{\mathrm{W}}^2}\, a_{\mathrm{ZZ}}$$

In the PTG scenario we only keep $\boldsymbol{a_{\mathrm{t}\,\phi}, a_{\mathrm{b}\,\phi}, a_{\phi\,\mathrm{D}}}$ and $\boldsymbol{a_{\phi\,\square}}$. We also derive the following result for the non-factorizable part of the amplitude:

$$\mathscr{T}^{\mathrm{nf}}_{\mathrm{HAZ}} = \sum_{a\in\{A\}} \mathscr{T}^{\mathrm{nf}}_{\mathrm{HAZ}}(a)\, a$$

where $\{A\} = \{a_{\phi\,\mathrm{t\,V}}, a_{\mathrm{t\,BW}}, a_{\mathrm{t\,WB}}, a_{\phi\,\mathrm{b\,V}}, a_{\mathrm{b\,WB}}, a_{\mathrm{b\,BW}}, a_{\phi\,\mathrm{D}}, a_{\mathrm{AZ}}, a_{\mathrm{AA}}, a_{\mathrm{ZZ}}\}$. In the PTG scenario there are only 3 non-factorizable amplitudes for $\mathbf{H} \to \boldsymbol{\gamma}\mathbf{Z}$, those proportional to $a_{\phi\,\mathrm{t\,V}}, a_{\phi\,\mathrm{b\,V}}$ and $a_{\phi\,\mathrm{D}}$.

To summarize: $\mathrm{H} \rightarrow \bar{\mathrm{f}} + \mathrm{f} + \gamma$

intermediate POs[8]	$\boldsymbol{\varepsilon_{Z\gamma}}$, $\boldsymbol{\varepsilon_{\gamma\gamma}}$
Wilson coeff.[9]	$\boldsymbol{a_{AZ}}$ etc.
physical PO[10]	$\boldsymbol{\Gamma_{PO}(H \rightarrow Z\gamma)}$

everything well defined and interconnected. Remember, this should cover SMEFT, not only HEFT. Different measurements should be combined with as few assumptions as possible. Poles and tails [11] are complementary.

[8] residues of the poles in (one-particle-reducible) Green's functions (in well-defined kinematic limits)

[9] SMEFT

[10] MPE, resonant propagator expansion, phase space factorization

[11] But beware of EFT interpretation is a series expanded in $E/\Lambda > 1$

We have shown that there are different layers of LHC POs:

1. An external layer (where kinematics is kept exact), e.g. $\Gamma_{\mathsf{SR}}\left(\mathrm{H}\to \bar{\mathrm{f}}\mathrm{f}\gamma\right)$, which is similar to LEP $\sigma_{\mathrm{f}}^{\mathsf{peak}}$. Note that it is not trivial NWA.
2. An intermediate layer, similar to LEP $g^{\mathrm{e}}_{\mathrm{V\,A}}$
3. An internal layer, the kappas, and finally,
4. the innermost layer: Wilson coefficients or non-SM parameters in BSM, e.g. $\alpha, \beta, \mathrm{M}_{\mathrm{sb}}$ etc. in THDMs.

When moving to the innermost layer we still have the option of performing the tree-level SMEFT translation, which is well defined and should be integrated with the corresponding estimate of MHOU, or we can go to SMEFT at the loop level, again with its own MHOU.

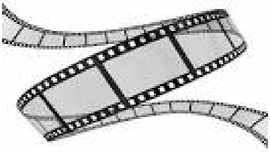
We often say that SMEFT and POs are a bridge allowing experimentalists and theorists to meet half way between. However, at present, it looks more like this

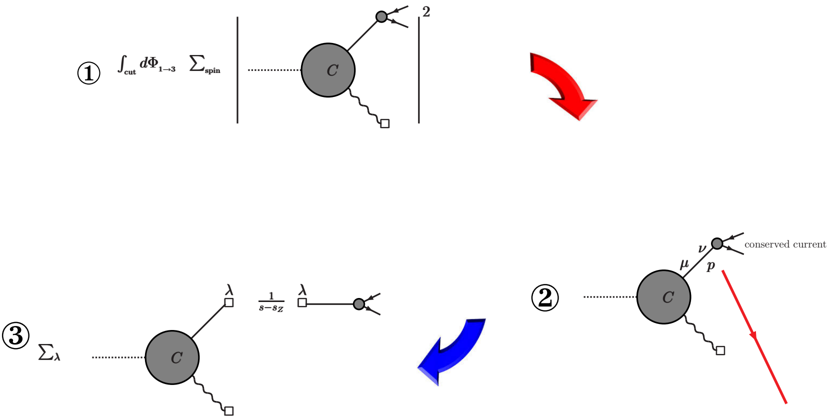
The Battle of Remagen, Ludendorff Bridge over the Rhine

- The task is not to see what no else has seen but to think what no else has thought about that which everyone else has seen
- The problem is not how to imagine wild scenarios, the problem is how to arrive to the correct scenario by making only small steps, without having to make unreasonable assumptions.
- We have the Standard Model of particle physics with coupling strengths that we do not know how to derive, but which can be measured accurately.

Thank you for your attention


Backup Slides

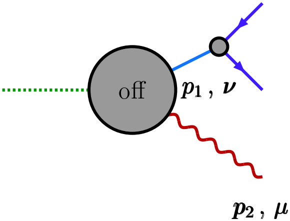
 The PO in the making slide collection

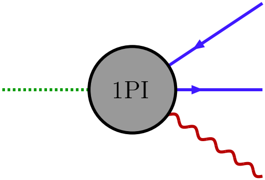


$\delta_{\mu\nu} \to \sum_\lambda \left[e^\lambda_\mu(p)\right]^* e^\lambda_\nu(p)$

□ = *polarization*

$|\sum_\lambda f(\lambda)|^2 = \sum_\lambda |f(\lambda)|^2 + \text{rest}$

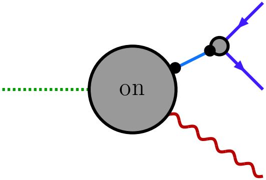


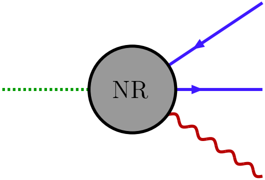


intermediate POs are everywhere, e.g.

$$\left.\begin{array}{c}\kappa^{\mathrm{v}}_{\mathrm{HAZ}}\,(p_{2\nu}\,p_{1\mu} - \delta_{\mu\nu}\,p_1\cdot p_2)\,\gamma^{\nu} \\ \kappa^{\mathrm{a}}_{\mathrm{HAZ}}\,(p_{2\nu}\,p_{1\mu} - \delta_{\mu\nu}\,p_1\cdot p_2)\,\gamma^{\nu}\,\gamma^{5}\end{array}\right\}\quad \text{Res.}$$

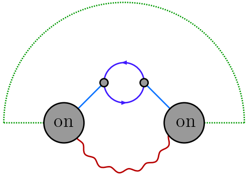
$$\kappa_{\mathrm{HA\bar{l}l}}\,\sigma^{\mu\alpha}\,(1\pm\gamma^5)\,p_{2\alpha}$$





$$\frac{R(s_1)}{s_1 - s_Z} = \frac{R(s_Z)}{s_1 - s_Z} + \Delta R(s_1)$$

$$1\,\mathrm{PI} \quad + \Delta\,R = \quad \mathrm{NR}$$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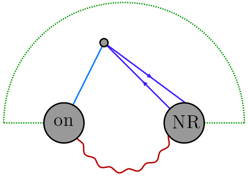
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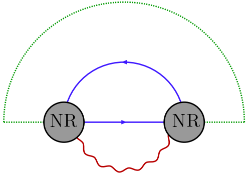
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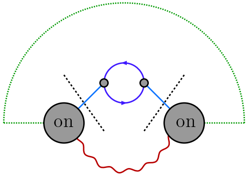
NR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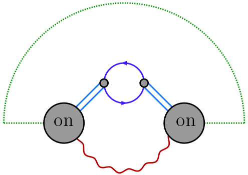
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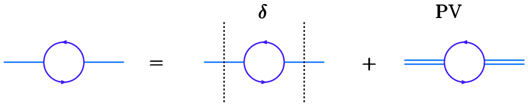

=
δ
+
PV

$$\mathcal{A}^{(1)}_{\mu}\,\Delta_{\mu\nu}(p)\,\mathcal{A}^{(2)}_{\nu} \quad\rightarrow\quad \mathcal{A}^{(1)}_{\mu}\,\sum_{\lambda}\,\epsilon_{\mu}(p,\lambda)\,\epsilon^{*}_{\nu}(p,\lambda)\mathcal{A}^{(2)}_{\nu}$$

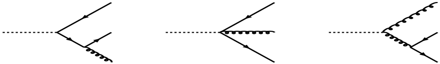
$$\left|\sum_{\lambda}\left[\mathcal{A}^{(1)}\cdot\epsilon(p,\lambda)\right]\left[\mathcal{A}^{(2)}\cdot\epsilon^{*}(p,\lambda)\right]\right|^{2} \quad\rightarrow\quad \sum_{\lambda}\left|\mathcal{A}^{(1)}\cdot\epsilon(p,\lambda)\right|^{2}\sum_{\sigma}\left|\mathcal{A}^{(2)}\cdot\epsilon(p,\sigma)\right|^{2} + \ \mathbf{Rest(cut)}$$

The effect of exp. cuts can be computed and included


FAQ

FAQ and misunderstandings

- There is a correspondence SMEFT $\to$ POS. Therefore, in building POs we can use SMEFT as a guide, as a matter of fact NLO SMEFT. Consider $\mathrm{H} \to \overline{\mathrm{b}}\,\mathrm{b}\,\mathrm{g}$.



- It has been claimed that PO parameterization is only valid at LO and that only the first diagram can be obtained through PO parameterization.
- On the contrary, one includes all structures, e.g.

$$a_{\mathrm{b\,g}}\,\lambda^{a}_{ij}\,\sigma^{\mu\nu}\,\gamma_{\pm}\,(p_1 + p_2)_{\nu}$$

from contact $\mathrm{H}\overline{\mathrm{b}}\mathrm{bg}$ interaction, hides Wilson coefficients into kappas and obtains the (intermediate) PO parameterization. In general

$$\mathscr{O}(\hat{s}) \quad = \quad \int dz\,\mathrm{K}_{\mathrm{QCD}}(z\,,\,\hat{s})\,\hat{\mathscr{O}}(z\,,\,\hat{s}) + \mathscr{O}_{\mathrm{NF}}$$

- The "to square or not to square problem" has been analyzed (for specific model) only at LO
  - At NLO we have the "not to square",

$$| g^{\mathrm{N}} \mathscr{A}^{(4)}_{\mathrm{N}} + g^{\mathrm{K}} g_6 \mathscr{A}^{(6)}_{\mathrm{K},1,1} |^2 \quad \rightsquigarrow \quad | g^{\mathrm{N}} \mathscr{A}^{(4)}_{\mathrm{N}} |^2 + 2\, g^{\mathrm{N+K}} g_6 \,\mathrm{Re}\left[\mathscr{A}^{(4)}_{\mathrm{N}} \right]^{\dagger} \mathscr{A}^{(6)}_{\mathrm{K},1,1}$$

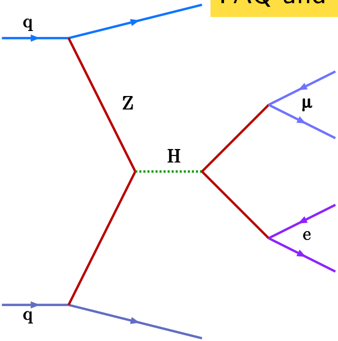
  and the "square", i.e. the addition of

$$| g^{\mathrm{K}} g_6 \mathscr{A}^{(6)}_{\mathrm{K},1,1} |^2$$

  - In both cases we are missing something: e.g. at NLO the interference of $\mathscr{A}^{(4)}_{\mathrm{N}}$ with $g_6^2 \mathscr{A}^{(6)}_{\mathrm{K},2,1}$, i.e. double insertion of $\dim = 6$ operators (not to mention $\dim = 8$ operators)

$$\mathscr{A} \quad = \quad \sum_{n=\mathrm{N}}^{\infty} g^{n} \mathscr{A}^{(4)}_{n} + \sum_{n=\mathrm{N}_6}^{\infty} \sum_{l=1}^{n} \sum_{k=1}^{\infty} g^{n} g^{l}{}_{4+2k} \mathscr{A}^{(4+2k)}_{n\,l\,k}$$

FAQ and misunderstandings



$$\mathcal{A}_{\rm LO} = \alpha^3\,\mathcal{A}^{(\rm dim=4)} + \alpha^3 \sum_{n=1,6} g_6^n\,\mathcal{A}_n^{(\rm dim=6)}$$

where n denotes the number of $\mathbf{dim} = 6$ insertions

“not to square” $+\,\alpha^6\,g_6 \left[\mathcal{A}^{(\rm dim=4)}\right]^\dagger \mathcal{A}_1^{(\rm dim=6)}$

“to square” $+\,\alpha^6\,g_6^2\,\mid \mathcal{A}_1^{(\rm dim=6)} \mid^2$

“not to forget” $+\,\alpha^6\,g_6^2 \left[\mathcal{A}^{(\rm dim=4)}\right]^\dagger \mathcal{A}_2^{(\rm dim=6)}$

FAQ and misunderstandings

- The "to square" argument: "There are phase space region with a suppressed $\dim = 4$ prediction where the SMEFT expansion holds"
  - "expansion holds" is a questionable statement and, most likely, means "positive"
  - if $(\dim = 4) \times (\dim = 6)$ (LO) is suppressed what about $(\dim = 4) \times (\dim = 6)^2$ (LO)?
  - if $(\dim = 4) \times (\dim = 6)$ (LO) is suppressed what about $(\dim = 4) \times (\dim = 6)$ (NLO)?
- "to square" vs. "not to square" is certainly part of MHOU, the problem (process dependent) is on the central value

$\mathbf{pp} \to \mathbf{HZ}$ helicity amplitudes, large $M(\mathrm{HZ})$ behavior

helicity	SM	one insertion	two insertions
$-+-$	$M_\mathrm{Z}/M(\mathrm{HZ})$	$g_6\, M(\mathrm{HZ})/M_\mathrm{Z}$	$g_6^2\, M(\mathrm{HZ})/M_\mathrm{Z}$
Wilson	$-$	$a_\mathrm{ZZ}\; a^{(3)}_{\phi\mathrm{q}}\; a^{(1)}_{\phi\mathrm{q}}$	$a_\mathrm{AA}\; a_\mathrm{AZ}\; a_\mathrm{ZZ}\; a_{\phi\mathrm{D}}\; a_{\phi\Box}\; a^{(3)}_{\phi\mathrm{q}}\; a^{(1)}_{\phi\mathrm{q}}$
$-+0$	const	$g_6\, M^2(\mathrm{HZ})/M_\mathrm{Z}^2$	g_6^2
Wilson	$-$	$a^{(3)}_{\phi\mathrm{q}}\; a^{(1)}_{\phi\mathrm{q}}$	$a_\mathrm{AA}\; a_\mathrm{AZ}\; a_\mathrm{ZZ}\; a_{\phi\mathrm{D}}\; a_{\phi\Box}\; a^{(3)}_{\phi\mathrm{q}}\; a^{(1)}_{\phi\mathrm{q}}$
$-++$	$M_\mathrm{Z}/M(\mathrm{HZ})$	$g_6\, M(\mathrm{HZ})/M_\mathrm{Z}$	$g_6^2\, M(\mathrm{HZ})/M_\mathrm{Z}$
Wilson	$-$	$a_\mathrm{ZZ}\; a^{(3)}_{\phi\mathrm{q}}\; a^{(1)}_{\phi\mathrm{q}}$	$a_\mathrm{AA}\; a_\mathrm{AZ}\; a_\mathrm{ZZ}\; a_{\phi\mathrm{D}}\; a_{\phi\Box}\; a^{(3)}_{\phi\mathrm{q}}\; a^{(1)}_{\phi\mathrm{q}}$

FAQ and misunderstandings

- The frequently used statement that "processes can be consistently analyzed in terms of a Lagrangian and not by parameterizing scattering amplitudes" is wrong
  - Observables are S-matrix elements, not "interaction terms", a decomposition into residues of complex poles and non-resonant parts is meaningful. Formal manipulations at the Lagrangian level lead to wrong results unless re-interpreted in terms of the S-matrix.

Parameterizing scattering amplitudes infeasible. Uhm?

The colour-summed result is given by a combination of previously computed colour–Born interference terms (2.36). For each phase-space point, this requires *a single evaluation* of the non-trivial colour-stripped amplitude $\mathcal{A}^{(\Gamma)}$ of each (sub)diagram.

2.2.3 Algebraic reduction of helicity structures and helicity sums

The helicity structures encountered in the explicit evaluation of all Feynman diagrams are algebraically reduced to a common basis of standard matrix elements (SMEs). The general form of SMEs for the $a(k_1)b(k_2) \to \mathrm{W}^+(k_3)\mathrm{W}^-(k_4)\mathrm{b}(k_5)\bar{\mathrm{b}}(k_6)$ channel for the initial states $ab = q\bar{q}/\mathrm{gg}$ is

$$\begin{aligned}\hat{\mathcal{M}}^{q\bar{q}}_{m,\sigma\tau} &= Q^{\nu_1\ldots\nu_l}_{m;\mu_3\mu_4}\Big[\bar{v}_{\bar{q}}(k_1)\gamma_{\nu_1}\ldots\gamma_{\nu_k}\omega_\sigma u_q(k_2)\Big]\varepsilon^{\mu_3 *}_{\mathrm{W}^+}(k_3)\varepsilon^{\mu_4 *}_{\mathrm{W}^-}(k_4)\\ &\quad\times\Big[\bar{v}_{\mathrm{b}}(k_5)\gamma_{\nu_{k+1}}\ldots\gamma_{\nu_l}\omega_\tau u_{\bar{\mathrm{b}}}(k_6)\Big],\\ \hat{\mathcal{M}}^{\mathrm{gg}}_{m,\tau} &= Q^{\nu_1\ldots\nu_l}_{m;\mu_1\ldots\mu_4}\varepsilon^{\mu_1}_{\mathrm{g}}(k_1)\varepsilon^{\mu_2}_{\mathrm{g}}(k_2)\varepsilon^{\mu_3 *}_{\mathrm{W}^+}(k_3)\varepsilon^{\mu_4 *}_{\mathrm{W}^-}(k_4)\left[\bar{v}_{\mathrm{b}}(k_5)\gamma_{\nu_1}\ldots\gamma_{\nu_l}\omega_\tau u_{\bar{\mathrm{b}}}(k_6)\right],\end{aligned}\tag{2.39}$$

where $Q^{\nu_1\ldots\nu_l}_{m;\mu_3\mu_4}$ and $Q^{\nu_1\ldots\nu_l}_{m;\mu_1\ldots\mu_4}$ consist of combinations of metric tensors and external momenta, and $\sigma,\tau=\pm$ refer to the chirality projectors $\omega_\pm = (1\pm\gamma_5)/2$. In the double-pole approximation (see Section 2.2.7), W-boson decays are described via effective polarisation vectors

$$\begin{aligned}\varepsilon^{\mu *}_{\mathrm{W}^+}(k_3) &= \frac{e\,\bar{u}(k_{\nu_\mathrm{e}})\gamma^\mu\omega_- v(k_{\mathrm{e}^+})}{\sqrt{2}s_\mathrm{w}\Big((k_{\nu_\mathrm{e}}+k_{\mathrm{e}^+})^2 - M_\mathrm{W}^2 + \mathrm{i}M_\mathrm{W}\Gamma_\mathrm{W}\Big)},\\ \varepsilon^{\mu *}_{\mathrm{W}^-}(k_4) &= \frac{e\,\bar{u}(k_{\mu^-})\gamma^\mu\omega_- v(k_{\bar{\nu}_\mu})}{\sqrt{2}s_\mathrm{w}\Big((k_{\mu^-}+k_{\bar{\nu}_\mu})^2 - M_\mathrm{W}^2 + \mathrm{i}M_\mathrm{W}\Gamma_\mathrm{W}\Big)},\end{aligned}\tag{2.40}$$

which include the left-handed lepton currents and the W-boson propagators. In our calculation we encounter about 800 and 2000 SMEs for the $q\bar{q}$ and gg channels, respectively. These compact spinor chains permit to decouple helicity information from the remnant


Supporting
Slides
AND
Additional
Information

$\mathscr{G}$

How to derive δ-part and PV part

1. Let $s_{\rm V}$ be the complex pole for a particle ${\rm V}$; it is parametrized as $s_{\rm V} = \mu_{\rm V}^2 - i\,\mu_{\rm V}\,\gamma_{\rm V}$. Consider the following integral

$$\mathrm{I}_n\,(a\,,\,b\,,\,s_{\rm V}) \;=\; \int_a^b ds\,\frac{s^n}{\mid s-s_{\rm V}\mid^2} = \int_a^b ds\,\frac{s^n}{(s-\mu_{\rm V}^2)^2+\mu_{\rm V}^2\,\gamma_{\rm V}^2}$$

which appears in the calculation of ${\rm V}$-resonant amplitudes, s being the virtuality.

2. For $n=0$ we obtain

$$\mathrm{I}_0\,(a\,,\,b\,,\,s_{\rm V}) \;=\; \frac{\pi}{(\mathrm{A}\,\lambda)^{1/2}}\,\theta(X)\,\theta(1-X) + \mathrm{I}_0^{\rm PV}\,(a\,,\,b\,,\,s_{\rm V})$$

$$\mathrm{I}_0^{\rm PV}\,(a\,,\,b\,,\,s_{\rm V}) \;=\; -\frac{1}{\mathrm{A}}\sum_{l=1,2}{}_2\mathrm{F}_1\left(1\,,\,\frac{1}{2}\,;\,\frac{3}{2}\,;\,-\frac{\lambda}{\mathrm{A}X_l^2}\right)$$

where ${}_2F_1$ is the hypergeometric function and

$$\mathrm{A} \;=\; b-a \quad X = \frac{\mu_{\rm V}^2-a}{b-a} \quad \lambda = \frac{\mu_{\rm V}^2\gamma_{\rm V}^2}{b-a}$$

$$X_1 \;=\; -X \qquad X_2 = 1-X$$

The issue of gauge invariance

- Consider a process with two components: a resonant one, with the exchange of a particle of mass M and virtuality s, a the continuum (N). The corresponding amplitude is

$$\mathscr{A} = \frac{V_i(\xi, s, M, \ldots)\, V_f(\xi, s, M, \ldots)}{s - M^2} + \mathrm{N}(\xi, s, \ldots)$$

where $V_i(V_f)$ are the initial (final) sub-amplitudes in the resonant part, ξ is a gauge parameter and the dependence on additional invariants is denoted by $\ldots$.

- It can ve shown, in full generality, that

$$V_{i,f}(\xi, s, M \ldots) = V^{\mathrm{inv}}_{i,f}\left(M^2 = s, \ldots\right) + (s - M^2)\,\Delta V_{i,f}(\xi, s, M, \ldots)$$

i.e., only the on-shell production $\times$ decay is gauge-parameter independent. Therefore, we need to expand the resonant part,

$$\mathscr{A} = \frac{V^{\mathrm{inv}}_i(M^2, M^2, \ldots)\, V^{\mathrm{inv}}_f(M^2, M^2, \ldots)}{s - M^2} + \mathrm{B}(s, \ldots)$$

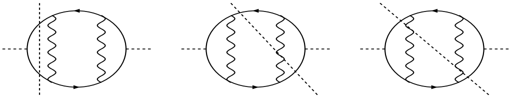
with an impact for the number of off-shell events. Technically speaking, the mass M should be replaced by the corresponding complex pole.

$\mathscr{G}$

- Typical example: off-shell Higgs boson, e.g. $\mathrm{H} \to \gamma\gamma$

$$
\begin{aligned}
\mathscr{M}^{\mu\nu}(\mathrm{H} \to \gamma\gamma) &= i\,\frac{g^3 \mathrm{s}_\mathrm{W}^2}{8\,\pi^2}\, M_\mathrm{W} \Big\{ \mathscr{M}_\mathrm{OS}\, T^{\mu\nu} \\
&+ \Big(s - M_\mathrm{H}^2\Big) \Big[\mathscr{M}^T_\mathrm{off} + \Big(\xi_\mathrm{W}^2 - 1\Big)\, \mathscr{M}^T_{\mathrm{off},\,\xi} \Big]\, T^{\mu\nu} \\
&+ \mathscr{M}^d_\mathrm{off} + \Big(\xi_\mathrm{W}^2 - 1\Big)\, \mathscr{M}^d_{\mathrm{off},\,\xi} \Big]\, \delta^{\mu\nu} \Big\} \\
T^{\mu\nu} &= \frac{p_1^\nu p_2^\nu}{s} + \frac{1}{2}\,\delta^{\mu\nu}
\end{aligned}
$$

Do we need experimental cuts?



Moral: Unless you isolate photons you don't know which process you are talking about

$\mathbf{H} \to \bar{\mathbf{f}}\mathbf{f}$ at NNLO

$\mathbf{H} \to \bar{\mathbf{f}}\mathbf{f}\boldsymbol{\gamma}$ at NLO

The complete $\mathbf{S}$-matrix element will read as follows:

$$
\begin{aligned}
\mathrm{S} \;=\;& \left|\mathrm{A}^{(0)}\left(\mathrm{H}\to\bar{\mathrm{f}}\,\mathrm{f}\right)\right|^{2} \\
+\;& 2\,\mathrm{Re}\left[\mathrm{A}^{(0)}\left(\mathrm{H}\to\bar{\mathrm{f}}\,\mathrm{f}\right)\right]^{\dagger}\mathrm{A}^{(1)}\left(\mathrm{H}\to\bar{\mathrm{f}}\,\mathrm{f}\right) \\
+\;& \left|\mathrm{A}^{(0)}\left(\mathrm{H}\to\bar{\mathrm{f}}\,\mathrm{f}\gamma\right)\right|^{2} \text{ ✗} \\
+\;& 2\,\mathrm{Re}\left[\mathrm{A}^{(0)}\left(\mathrm{H}\to\bar{\mathrm{f}}\,\mathrm{f}\right)\right]^{\dagger}\mathrm{A}^{(2)}\left(\mathrm{H}\to\bar{\mathrm{f}}\,\mathrm{f}\right) \\
+\;& 2\,\mathrm{Re}\left[\mathrm{A}^{(0)}\left(\mathrm{H}\to\bar{\mathrm{f}}\,\mathrm{f}\gamma\right)\right]^{\dagger}\mathrm{A}^{(1)}\left(\mathrm{H}\to\bar{\mathrm{f}}\,\mathrm{f}\gamma\right) \text{ ✗} \\
+\;& \left|\mathrm{A}^{(0)}\left(\mathrm{H}\to\bar{\mathrm{f}}\,\mathrm{f}\gamma\gamma\right)\right|^{2}.
\end{aligned}
$$

SMEFT, not only decay

$$\mathrm{u}(p_1)+\mathrm{u}(p_2)\to\mathrm{u}(p_3)+\mathrm{e}^-(p_4)+\mathrm{e}^+(p_5)+\mu^-(p_6)+\mu^+(p_7)+\mathrm{u}(p_8)\quad \text{LO SMEFT}$$

$$\begin{aligned}
J^{\mu}_{\pm}(p_i\,,p_j) &= \overline{u}(p_i)\,\gamma^{\mu}\,\gamma_{\pm}\,u(p_j)\\
\mathscr{A}^{\mathrm{TR}}_{\mathrm{LO}} &= \Big[J^{\mu}_{-}(p_4\,,p_5)\,(1-v_{\mathrm{l}})+J^{\mu}_{+}(p_4\,,p_5)\,(1+v_{\mathrm{l}})\Big]\\
&\times \Big[J^{-}_{\mu}(p_6\,,p_7)\,(1-v_{\mathrm{l}})+J^{+}_{\mu}(p_6\,,p_7)\,(1+v_{\mathrm{l}})\Big]\\
&\times \Big[J^{\nu}_{-}(p_3\,,p_2)\,(1-v_{\mathrm{u}})+J^{\nu}_{+}(p_3\,,p_2)\,(1+v_{\mathrm{u}})\Big]\\
&\times \Big[J^{-}_{\nu}(p_8\,,p_1)\,(1-v_{\mathrm{u}})+J^{+}_{\nu}(p_8\,,p_1)\,(1+v_{\mathrm{u}})\Big]\\
\Delta^{-1}_{\Phi}(p) &= p^2+M^2_{\Phi}
\end{aligned}$$

$$\mathscr{A}^{\mathrm{TR}}_{\mathrm{SMEFT}} = \frac{g^6}{4096}\,\Delta_{\mathrm{H}}(q_1+q_2)\,\prod_{i=1,4}\Delta_{\mathrm{Z}}(q_i)\,\frac{M^2_{\mathrm{W}}}{c^8_{\theta}}\,\kappa_{\mathrm{LO}}\,\mathscr{A}^{\mathrm{TR}}_{\mathrm{LO}}+g^6\,g_6\,\mathscr{A}^{\mathrm{TR;nf}}_{\mathrm{SMEFT}}$$

$$\Delta\kappa_{\mathrm{LO}} = 2\,a_{\phi\,\square}+\frac{2\,M^2_{\mathrm{Z}}-2\,M^2_{\mathrm{H}}+q_1\cdot q_2+q_2\cdot q_2}{M^2_{\mathrm{W}}}\,c^2_{\theta}\,a_{\mathrm{ZZ}}$$

$$q_1=p_8-p_1,\;q_2=p_3-p_2,\;q_3=p_4+p_5,\;q_4=p_6+p_7$$

SMEFT and "background"

- Consider $\overline{\mathrm{u}}\mathrm{u} \to \mathrm{ZZ}$: the following Wilson coefficients appear:

$$
\begin{aligned}
W_1 &= a_{\gamma\gamma} = s_\theta c_\theta\, a_{\Phi\mathrm{WB}} + c_\theta^2\, a_{\phi\mathrm{B}} + s_\theta^2\, a_{\phi\mathrm{W}} \\
W_2 &= a_{\mathrm{ZZ}} = -s_\theta c_\theta\, a_{\Phi\mathrm{WB}} + s_\theta^2\, a_{\phi\mathrm{B}} + c_\theta^2\, a_{\phi\mathrm{W}} \\
W_3 &= a_{\gamma\mathrm{Z}} = 2\, s_\theta\, c_\theta \left(a_{\phi\mathrm{W}} - a_{\phi\mathrm{B}} \right) + \left(c_\theta^2 - s_\theta^2\right) a_{\Phi\mathrm{WB}} \\
W_4 &= a_{\phi\mathrm{D}} \\
W_5 &= a_{\phi\mathrm{q}}^{(3)} + a_{\phi\mathrm{q}}^{(1)} - a_{\phi\mathrm{u}} \\
W_6 &= a_{\phi\mathrm{q}}^{(3)} + a_{\phi\mathrm{q}}^{(1)} + a_{\phi\mathrm{u}}
\end{aligned}
$$

- Define

$$
\mathrm{A}^{\mathrm{LO}} = \frac{M_{\mathrm{Z}}^4}{t^2} + \frac{M_{\mathrm{Z}}^4}{u^2} - \frac{t}{u} - \frac{u}{t} - 4\,\frac{M_{\mathrm{Z}}^2 s}{tu}
$$

- Obtain the result ($\bar{\mathbf{u}}\mathbf{u} \to \mathbf{ZZ}$)

$$\sum_{\text{spin}} \left| \mathrm{A}^{(4+6)} \right|^2 = g^4\, \mathrm{A}^{\mathrm{LO}} \left[\mathrm{F}^{\mathrm{LO}}(s_\theta) + \frac{g_6}{\sqrt{2}} \sum_{i=1}^{6} \mathrm{F}^i(s_\theta)\, W_i \right]$$

- Background changes! Note that

$$\begin{aligned} \mathrm{F}^{\mathrm{LO}} &\approx -0.57 \quad \mathrm{F}^1 \approx +2.18 \quad \mathrm{F}^2 \approx -3.31 \\ \mathrm{F}^3 &\approx +4.07 \quad \mathrm{F}^4 \approx -2.46 \quad \mathrm{F}^4 \approx -2.46 \quad \mathrm{F}^6 \approx -5.81 \end{aligned}$$