

Foundations of HEFT

An Excerpt

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with the physical mass parameters

$$\begin{aligned}
 M_W^2 &= \frac{1}{4}g^2v^2 \left[1 + 2\frac{v^2}{\Lambda^2}\alpha_{\Phi W} \right], \\
 M_Z^2 &= \frac{1}{4}(g^2 + g'^2)v^2 \left[1 + \frac{v^2}{2\Lambda^2}(4\alpha_{ZZ} + \alpha_{\Phi D}) \right], \\
 M_H^2 &= \lambda v^2 \left[1 + \frac{v^2}{2\Lambda^2} \left(4\alpha_{\Phi\Box} - \frac{6}{\lambda}\alpha_{\Phi} - \alpha_{\Phi D} \right) \right], \\
 m_t &= \frac{1}{\sqrt{2}}U^{\dagger}\Gamma_t U^{t,\dagger}v \left[1 - \frac{1}{2}\frac{v^2}{\Lambda^2}\alpha_{t\phi} \right].
 \end{aligned} \tag{151}$$

In (150) we have used the usual 't Hooft–Feynman gauge-fixing term

$$\mathcal{L}_{\text{fix}} = -C_+C_- - \frac{1}{2}(C_Z)^2 - \frac{1}{2}(C_A)^2 - \frac{1}{2}C_G^A C_G^A \tag{152}$$

with

$$C_G^A = \partial_\mu G^{A\mu}, \quad C_A = \partial_\mu A^\mu, \quad C_Z = \partial_\mu Z^\mu + M_Z\phi^0, \quad C_\pm = \partial_\mu W^{\pm\mu} \pm iM_W\phi^\pm \tag{153}$$

in terms of the physical fields and parameters, which gives rise to the same propagators as in the SM.

In the following, the abbreviations c_w and s_w are defined via the physical masses

$$c_w = \frac{M_W}{M_Z}, \quad s_w = \sqrt{1 - c_w^2}. \tag{154}$$

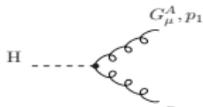
The parameters of the SM Lagrangian g , g' , λ , m^2 , and Γ_t keep their meaning in the presence of dimension-6 operators.

10.4.2 Higgs vertices

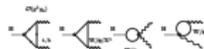
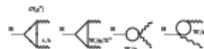
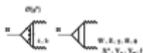
Here we list the most important Feynman rules for vertices involving exactly one physical Higgs boson. These are given in terms of the above-defined physical fields and parameters. In the coefficients of dimension-6 couplings we replaced v^2 by the Fermi constant via $v^2 = 1/(\sqrt{2}G_F)$.

The triple vertices involving one Higgs boson read:

Hgg coupling:



$$= i\frac{2g}{M_W}\frac{1}{\sqrt{2}G_F\Lambda^2} \left[\alpha_{GG}(p_{2\mu}p_{1\nu} - p_1p_2g_{\mu\nu}) + \alpha_{G\tilde{G}}\varepsilon_{\mu\nu\rho\sigma}p_1^\rho p_2^\sigma \right] \delta^{AB},$$

Figure 4: Diagram of a double Regge diagram with 21 channels contributing to the amplitude for $B \rightarrow \gamma$.Figure 5: Diagram of a double Regge diagram contributing to the amplitude for $B \rightarrow \gamma$.Figure 6: The same number of diagrams contributing to the amplitude for $B \rightarrow \gamma$. α_1, α_2 denote a W and a ρ meson, P a Pomeron.Figure 7: Diagram of a double Regge diagram contributing to the amplitude for $B \rightarrow \gamma$.

with the physical mass parameters

$$\alpha_1^2 = \frac{1}{2}(\alpha_1^2 + \alpha_2^2) - \frac{1}{2}(\alpha_1 - \alpha_2)^2$$

$$\alpha_2 = \frac{1}{2}(\alpha_1 + \alpha_2) + \frac{1}{2}(\alpha_1 - \alpha_2)$$

$$\alpha_1 = \frac{1}{2}(\alpha_1 + \alpha_2) - \frac{1}{2}(\alpha_1 - \alpha_2)$$

In (20) we have used the fact that the Pomeron spin is zero.

$$\alpha_1 = \alpha_2 = \alpha_P = \alpha_{(1,2)}$$

with

$$\alpha_P = \alpha_{(1,2)} = \alpha_1 = \alpha_2 = \alpha_P = \alpha_{(1,2)}$$

In terms of the physical fields and parameters, which gives rise to the unitarity problem in the (20).

In the following the differentiations are used as an alternative to the physical masses.

$$\frac{\partial}{\partial \alpha_1} = \frac{\partial}{\partial \alpha_P} + \frac{\partial}{\partial \alpha_{(1,2)}}$$

The expression of the (20) together with (19), (21) and (22) leads then directly to the problem of

the unitarity problem.

The (22) Regge vertex

There are two more important features which are common to all the physical Regge fields.

There are given in terms of the above defined physical fields and parameters. In the combination

of the physical fields and parameters (19) the third component $\alpha_{(1,2)}$ is the combination

of the physical fields and parameters (19) and (20).

Regge vertex

$$\alpha_1^2 = \frac{1}{2}(\alpha_1^2 + \alpha_2^2) - \frac{1}{2}(\alpha_1 - \alpha_2)^2$$

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Glancing at the headlines of the **complete** calculation for
 👁 $H \rightarrow \gamma\gamma$

- **SM** loops, dressed with admissible operators
- **New 33** loop-diagrams + **Counter-terms**
- at $\mathcal{O}(g^3, g^3 g_6)$, with $g_6 = 1/\sqrt{2} G_F \Lambda^2$ 

$$A_{\gamma\gamma}^{\text{HEFT}} = \kappa_W A_W^{\text{LO}} + \sum_F \kappa_f A_f^{\text{LO}} + A_{\text{NF}}$$

Note that for
 $\Lambda \approx 5 \text{ TeV}$
 we have

$$1/(\sqrt{2}G_F \Lambda^2) \approx g^2/(4\pi)$$

i.e. \Rightarrow the contributions of $d=6$ operators are \approx loop effects.
 $\Rightarrow \Rightarrow$ For higher scales, loop contributions tend to be more important (\gg)

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PTG - operators versus LG - operators, cf. Einhorn, Wudka, ...

It can be argued that (at LO) the basis operator

should be chosen from among the **PTG operators**

take $\mathcal{O}_{\text{LG}}^6$, contract two lines, is ren of some \mathcal{O}^4

a SM vertex with $\mathcal{O}_{\text{PTG}}^6$ required ... same order

$1/\Lambda$ expansion \rightarrow power-counting \checkmark

LG \rightarrow low-energy analytic structure \times

No hierarchy assumed \checkmark

$$A = \sum_{n=N}^{\infty} \sum_{l=0}^n \sum_{k=0}^{\infty} g_l^n g_{l+2k} A_{nlk}$$

$$g_{l+2k} = 1/(\sqrt{2}G_F \Lambda^2)^k$$

\curvearrowright N defines LO

PTG: T - generated in at least one extension of SM



PROPOSITION: There are two ways of formulating HEFT

- a) mass-dependent scheme(s) or **Wilsonian** HEFT
- b) mass-independent scheme(s) or **Continuum** HEFT (CHEFT)
 - only a) is conceptually consistent with the image of an EFT as a low-energy approximation to a high-energy theory
 - however, inclusion of NLO corrections is only meaningful in b) since we cannot regularize with a cutoff and NLO requires regularization
 - There is an additional problem, CHEFT requires evolving our theory to lower scales until we get below the "heavy-mass" scale where we use $\mathcal{L} = \mathcal{L}_{\text{SM}} + \mathbf{d}\mathcal{L}$, $\mathbf{d}\mathcal{L}$ encoding matching corrections at the boundary. Therefore, CHEFT does not integrate out heavy degrees of freedom but removes them compensating for by an appropriate matching calculation

Not quite the same as it is usually discussed (no theory approaching the boundary from above . . .) cf. low-energy SM, weak effects on $\mathbf{g}-2$ etc.



Footnotes
Annotations

$$\dim \phi = d/2 - 1$$

$$\dim \mathcal{O}^d = N_\phi \dim \phi + N_{\text{der}}$$

For $d \geq 3$ there is a finite number of relevant + marginal operators

For $d \geq 1$ there is a finite number of irrelevant operators

Sounds good for finite dependence on high-energy theory

This assumes that high-energy theory is weakly coupled

Dimensional arguments work for LO HEFT

In NLO HEFT scaling may break down, implying appeal to a particular renormalization scheme

Ren. group should only be applied to EFTs that are nearly massless

Decoupling theorem fails for CHEFT, but, arguably this does not prevent them from supporting a well defined scheme, but, decoupling must be inserted in the form of matching calculations (which we don't have ...)

Match Feynman diagrams \in HEFT with corresponding **1**(light)**PI** diagrams \in high-energy theory
(and discover that Taylor-expanding is not always a good idea)

Having said that ... no space left for annotations

Renormalisation

FP-sector: handle with care

✓ Make finite all Green's functions

Schemes: remember β_{QED} in large m_e -limit

$$g = g_{\text{ren}} \left[1 + \frac{g_{\text{ren}}^2}{16\pi^2} \left(dZ_g + g_6 dZ_g^{(6)} \right) \frac{1}{\epsilon} \right] \quad \checkmark \text{ Don't forget background}$$

$$M_W = M_W^{\text{ren}} \left[1 + \frac{1}{2} \frac{g_{\text{ren}}^2}{16\pi^2} \left(dZ_{M_W} + g_6 dZ_{M_W}^{(6)} \right) \frac{1}{\epsilon} \right]$$

etc.

Wilson coefficients $\rightarrow W_i$

$$W_i = \sum_j Z_{ij}^{\text{wc}} W_j^{\text{ren}}$$

$$Z_{ij}^{\text{wc}} = \delta_{ij} + \frac{g_{\text{ren}}^2}{16\pi^2} dZ_{ij}^{\text{wc}} \frac{1}{\epsilon}$$

$$\frac{1}{1/\omega} = \frac{2/\omega}{\omega} - \gamma - \ln \pi + \ln \mu R$$

Oops! ... $4f0$ needed for $H \rightarrow \bar{b}b$ $H \rightarrow \gamma\gamma$ not finite

Appendix C. Dimension-Six Basis Operators for the SM²².

X^3 (LG)		φ^6 and $\varphi^4 D^2$ (PTG)		$\psi^2 \varphi^3$ (PTG)	
Q_G	$f^{ABC} G_\mu^{Av} G_\nu^{B\rho} G_\rho^{C\mu}$	Q_φ	$(\varphi^\dagger \varphi)^3$	$Q_{e\varphi}$	$(\varphi^\dagger \varphi)(\bar{l}_p e_r \varphi)$
$Q_{\tilde{G}}$	$f^{ABC} \tilde{G}_\mu^{Av} G_\nu^{B\rho} G_\rho^{C\mu}$	$Q_{\varphi\Box}$	$(\varphi^\dagger \varphi)\Box(\varphi^\dagger \varphi)$	$Q_{u\varphi}$	$(\varphi^\dagger \varphi)(\bar{q}_p u_r \tilde{\varphi})$
Q_W	$\varepsilon^{IJK} W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$	$Q_{\varphi D}$	$(\varphi^\dagger D^\mu \varphi)^* (\varphi^\dagger D_\mu \varphi)$	$Q_{d\varphi}$	$(\varphi^\dagger \varphi)(\bar{q}_p d_r \varphi)$
$Q_{\tilde{W}}$	$\varepsilon^{IJK} \tilde{W}_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$				
$X^2 \varphi^2$ (LG)		$\psi^2 X \varphi$ (LG)		$\psi^2 \varphi^2 D$ (PTG)	
$Q_{\varphi G}$	$\varphi^\dagger \varphi G_{\mu\nu}^A G^{A\mu\nu}$	Q_{eW}	$(\bar{l}_p \sigma^{\mu\nu} e_r) \tau^I \varphi W_{\mu\nu}^I$	$Q_{\varphi l}^{(1)}$	$(\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi)(\bar{l}_p \gamma^\mu l_r)$
$Q_{\varphi \tilde{G}}$	$\varphi^\dagger \varphi \tilde{G}_{\mu\nu}^A G^{A\mu\nu}$	Q_{eB}	$(\bar{l}_p \sigma^{\mu\nu} e_r) \varphi B_{\mu\nu}$	$Q_{\varphi l}^{(3)}$	$(\varphi^\dagger i \overleftrightarrow{D}_\mu^I \varphi)(\bar{l}_p \tau^I \gamma^\mu l_r)$
$Q_{\varphi W}$	$\varphi^\dagger \varphi W_{\mu\nu}^I W^{I\mu\nu}$	Q_{uG}	$(\bar{q}_p \sigma^{\mu\nu} T^A u_r) \tilde{\varphi} G_{\mu\nu}^A$	$Q_{\varphi e}$	$(\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi)(\bar{e}_p \gamma^\mu e_r)$
$Q_{\varphi \tilde{W}}$	$\varphi^\dagger \varphi \tilde{W}_{\mu\nu}^I W^{I\mu\nu}$	Q_{uW}	$(\bar{q}_p \sigma^{\mu\nu} u_r) \tau^I \tilde{\varphi} W_{\mu\nu}^I$	$Q_{\varphi q}^{(1)}$	$(\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi)(\bar{q}_p \gamma^\mu q_r)$
$Q_{\varphi B}$	$\varphi^\dagger \varphi B_{\mu\nu} B^{\mu\nu}$	Q_{uB}	$(\bar{q}_p \sigma^{\mu\nu} u_r) \tilde{\varphi} B_{\mu\nu}$	$Q_{\varphi q}^{(3)}$	$(\varphi^\dagger i \overleftrightarrow{D}_\mu^I \varphi)(\bar{q}_p \tau^I \gamma^\mu q_r)$
$Q_{\varphi \tilde{B}}$	$\varphi^\dagger \varphi \tilde{B}_{\mu\nu} B^{\mu\nu}$	Q_{dG}	$(\bar{q}_p \sigma^{\mu\nu} T^A d_r) \varphi G_{\mu\nu}^A$	$Q_{\varphi u}$	$(\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi)(\bar{u}_p \gamma^\mu u_r)$
$Q_{\varphi WB}$	$\varphi^\dagger \tau^I \varphi W_{\mu\nu}^I B^{\mu\nu}$	Q_{dW}	$(\bar{q}_p \sigma^{\mu\nu} d_r) \tau^I \varphi W_{\mu\nu}^I$	$Q_{\varphi d}$	$(\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi)(\bar{d}_p \gamma^\mu d_r)$
$Q_{\varphi \tilde{WB}}$	$\varphi^\dagger \tau^I \varphi \tilde{W}_{\mu\nu}^I B^{\mu\nu}$	Q_{dB}	$(\bar{q}_p \sigma^{\mu\nu} d_r) \varphi B_{\mu\nu}$	$Q_{\varphi ud}$	$i(\tilde{\varphi}^\dagger D_\mu \varphi)(\bar{u}_p \gamma^\mu d_r)$

Table C.1: Dimension-six operators other than the four-fermion ones.

²²These tables are taken from [5], by permission of the authors.

Effective Lagrangians cannot be blithely used without acknowledging implications of their choice
 ex: non gauge-invariant, intended to be used in U-gauge
 ex: $\mathbf{H} \rightarrow \mathbf{W}\mathbf{W}^*$ is virtual \mathbf{W} + something else, depending on the operator basis

Murphy's law of Higgs Physics

- *Although skipping foundations is not specifically recommended*
- *Foundations without tools Is Worth Nothing*
- *(Tools without foundations have no scientific basis)*
- *Construction of H $\overline{\text{E}}$ F $\overline{\text{T}}$ follows Hofstadter's Law: It always takes longer than you expect, even when you take into account Hofstadter's Law (also check Hanlon's razor)*