SMEFT, a theory for SM deviations
From Higgs discovery to Higgs properties

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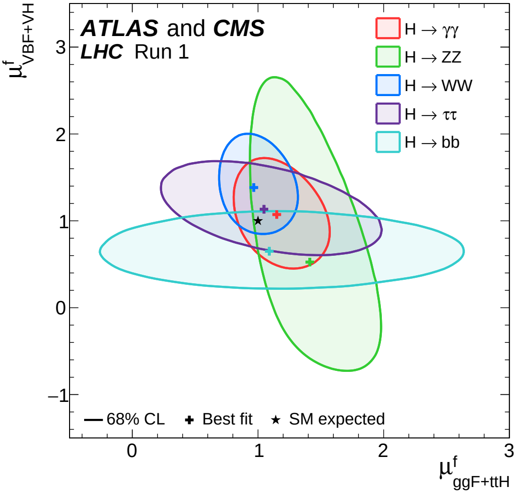
Synopsis

① Before the 2012 discovery the hypothesis was the SM and $m_{\mathbf{H}}$ the unknown, therefore bounds on $m_{\mathbf{H}}$ were derived through a comparison with high-precision data.

② At LHC, after the discovery, the unknowns are SM deviations, given that the SM is fully specified and deviations are constrainable. Of course, the definition of SM deviations requires a characterization of the underlying dynamics[1].

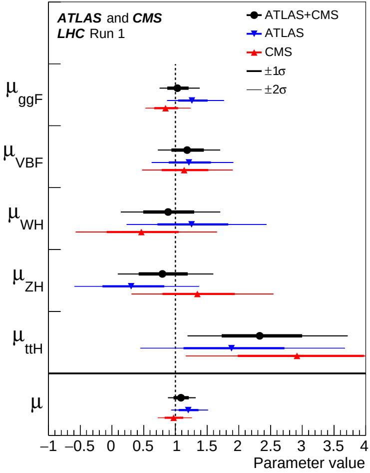
Remark *Notice that, so far, all the available studies on the couplings of the new resonance conclude it to be compatible with the Higgs boson of the SM within present precision, and, as of yet, there is no direct evidence for new physics phenomena beyond the SM.* Waiting for ③ *Ecco la fiera con la coda aguzza*

[1] every 20 bogus hypotheses you test, one of them will give you a p of < 0.05



SM predictions of signal yields scaled by a global signal strength $\boldsymbol{\mu}$

Negative log-likelihood contours at **68%** CL in the $[\mu_{\mathbf{ggF+tt}}, \mu_{\mathbf{VBF+VH}}]$ plane for the combination of ATLAS and CMS, for each of the final state analysed $PH \rightarrow \mathbf{ZZ}$, $\mathbf{H} \rightarrow \mathbf{WW}$, $\mathbf{H} \rightarrow \gamma\gamma$, $\mathbf{H} \rightarrow \tau\tau$, $\mathbf{H} \rightarrow \mathbf{bb}$, and their combination. The SM expectation is also shown as a black star.



Next step in SM deviations?

Best framework?

Best fit results for the production signal strengths for the combination of ATLAS and CMS data. Also shown are the results from each experiment. The error bars indicate the **1** $\boldsymbol{\sigma}$ (thick lines) and **2** $\boldsymbol{\sigma}$ (thin lines) intervals. The measurements of the global signal strength $\boldsymbol{\mu}$ are also shown.

Frameworks for SM deviations[2]

Definition (kappa framework)

A procedure used at LO, partially accommodating factorizable QCD [a] but not electroweak (EW) corrections, to parametrize SM deviations. It amounts to replace

$$\mathscr{L}_{\rm SM}\left(\{m\},\{g\}\right) \qquad \text{with} \qquad \mathscr{L}\left(\{m\},\{\kappa_g\, g\}\right)$$

where $\{m\}$ denotes the SM masses, $\{g\}$ the SM couplings and κ_g are the scaling parameters. This is the framework used during Run 1.

[a]  In general, there are contributions which induce sizeable corrections unrelated to the SM ones, [arXiv:1607.06354]

[2] A theory replaces a framework after testing confirms the hypothesis

Theory for SM deviations

Definition (EFT/SMEFT)

Exact non-perturbative solutions to quantum field theories are rarely known and approximate solutions that expand observables perturbatively in a small coupling constant and in a ratio of scales are generally developed. Such quantum field theories can be regarded as examples of effective field theory (EFT), e.g. SM effective field theory (SMEFT):

$$\mathscr{L} = \mathscr{L}_{\mathrm{SM}} + \sum_{n>4} \sum_{i=1}^{N_n} \frac{a_i^n}{\Lambda^{n-4}} \mathscr{O}_i^{(d=n)}$$

with arbitrary Wilson coefficients a_i^n which, however, give the leading amplitudes in an exactly unitary S-matrix at energies far below the scale of new physics, Λ. The theory is (strictly) non-renormalizable, which means that an infinite number of higher operators must be included. Nevertheless there is a consistent expansion of amplitudes in power of $\mathrm{v}/\Lambda, E/\Lambda$, where v is the Higgs VEV and E is the typical scale at which we measure the process.

Frameworks for SM deviations

Definition (Phenomenological Lagrangians[a])

[a] Theory deals with the well founded theoretical results obtained from first principles, while phenomenology deals with not so well founded effective models with a smaller domain of application, Hartmann (Studies in History and Philosophy of Modern Physics)

Any phenomenological approach, e.g. an extension of the SM Lagrangian with a limited number of interactions (like $\mathbf{HVV}$ and $\mathbf{H\bar{f}f}$), is a reasonable starting point to describe limits on SM deviations.

While this outcome is much less desirable than dealing with a consistent SMEFT it is important to recognize that the difference relates to possibility of including theory uncertainties and of having a MC tool.

Frameworks for SM deviations

Definition (PO)

Pseudo observables (POs) are a platform between realistic observables and theory parameters, allowing experimentalists and theorists to meet half way between, without theorists having to run full simulation and reconstruction and experimentalists fully unfolding to model-dependent parameter spaces.

Experimenters collapse some “primordial quantities” (say number of observed events in some pre-defined set-up) into some “secondary quantities” which we feel closer to the theoretical description of the phenomena [a]. In other words, POs answer the question
“**how to measure in order to preserve the data for a long time?**”

[a] [https://cds.cern.ch/record/2138023]

❶  SM augmented with the inclusion of higher dimensional operators ($\mathbf{T_1}$); not strictly renormalizable. Although workable to all orders, $\mathbf{T_1}$ fails above a certain scale, $\mathbf{\Lambda_1}$.

❷ Consider any BSM model that is strictly renormalizable and respects unitarity ($\mathbf{T_2}$); its parameters can be fixed by comparison with data, while masses of heavy states are presently unknown. $\mathbf{T_1} \neq \mathbf{T_2}$ in the UV but must have the same IR behavior.

❸ Consider now the whole set of data below $\mathbf{\Lambda_1}$.

> $\mathbf{T_1}$ should be able to explain them by fitting Wilson coefficients,
>
> $\mathbf{T_2}$ adjusting the masses of heavy states (as SM did with the Higgs mass at LEP) should be able to explain the data.

Goodness of both explanations are crucial in understanding how well they match and how reasonable is to use $\mathbf{T_1}$ instead of the full $\mathbf{T_2}$

❹ Does $\mathbf{T_2}$ explain everything? Certainly not, but it should be able to explain something more than $\mathbf{T_1}$.

❺ We could now define $\mathbf{T_3}$ as $\mathbf{T_2}$ augmented with (its own) higher dimensional operators; it is valid up to a scale $\mathbf{\Lambda_2}$.

❻ ...... 

INSIDE OUT

To explain SMEFT in a nutshell (for a complete description see [G. P., M. Trott, https://cds.cern.ch/record/2138031]) consider a process described by some SM amplitude

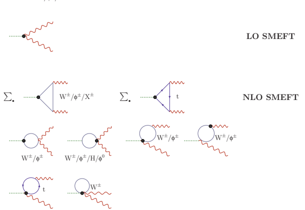
$$\mathscr{A}_{\mathrm{SM}} = \sum_{i=1,n} \mathscr{A}^{(i)}_{\mathrm{SM}}$$

where i labels gauge-invariant sub-amplitudes. In the extension the same process is given by a contact term or a collection of contact terms of $\mathrm{dim} = 6$; for instance, direct coupling of H to $\mathrm{VV}(\mathrm{V} = \gamma, \mathrm{Z}, \mathrm{W})$. In order to construct the theory one has to select a set of higher-dimensional operators and to start the complete procedure of renormalization.

$\cdots$ constructing SMEFT

- Experiments occur at finite energy and measure $\mathbf{S}^{\mathrm{eff}}(\boldsymbol{\Lambda})$
- Whatever QFT should give low energy $\mathbf{S}^{\mathrm{eff}}(\boldsymbol{\Lambda})$, $\forall \Lambda < \infty$
- There is no fundamental scale above which $\mathbf{S}^{\mathrm{eff}}(\boldsymbol{\Lambda})$ is not defined (K. Costello, Renormalization and EFT, AMS)
- $\mathbf{S}^{\mathrm{eff}}(\boldsymbol{\Lambda})$ loses its predictive power if a process at $E = \boldsymbol{\Lambda}$ requires ∞ renormalized parameters (J. Preskill, CALT-68-1493)

From SM to SMEFT



$$\mathscr{A}^{\rm LO}_{\rm SMEFT} = \sum_{i=1,n} \mathscr{A}^{(i)}_{\rm SM} + i\, g_6\, \kappa_c$$

$$\mathscr{A}^{\rm NLO}_{\rm SMEFT} = \sum_{i=1,n} \kappa_i\, \mathscr{A}^{(i)}_{\rm SM} + i\, g_6\, \kappa_c + g_6 \sum_{i=1,N} a_i\, \mathscr{A}^{(i)}_{\rm nf}$$

where $g_6^{-1} = \sqrt{2}\, G_F\, \Lambda^2$. The last term collects all loop contributions that do not factorize and the coefficients a_i are Wilson coefficients.

The κ_i are linear combinations of the a_i .

ATTENTION! We conclude that SMEFT gives the correct generalization of the original κ-framework at the price of introducing additional, non-factorizable, terms in the amplitude.

Resolved scaling factor in $gg \to H$

κ $\quad \kappa_t^2\, X_t + \kappa_b^2\, X_b + \kappa_t\, \kappa_b\, X_{t,b}$

LO SMEFT $\quad X_q + K^2\, a_{\phi G}^2 + K\, a_{\phi g}\, Y_q$

NLO SMEFT PTG $\quad \kappa_t^2\, X_t + \kappa_b^2\, X_b + \kappa_t\, \kappa_b\, X_{t,b} +$
$K^2\, a_{\phi G}^2 + K\, a_{\phi g}\, \kappa_t\, Y_t + K\, a_{\phi g}\, \kappa_b\, Y_b$

$$\boxed{A_{\kappa}^{\gamma\gamma}} = \kappa_t\, \mathscr{A}^{t}_{H\to\gamma\gamma} + \kappa_b\, \mathscr{A}^{b}_{H\to\gamma\gamma} + \kappa_V\, \mathscr{A}^{bos}_{H\to\gamma\gamma}$$

$$\boxed{A_{PO}^{\gamma\gamma}} = \varepsilon_\mu\, \varepsilon'_\nu\, \varepsilon_{\gamma\gamma}\, (g^{\mu\nu}\, q\cdot q' - q^\mu\, q'^\nu)$$

TOP-DOWN approach

When heavy and light fields mix

beware of subtleties

Matching Matters

[arXiv:1603.03660,arXiv:1607.08251]

$$\boxed{A_{SMEFT}^{gg}} = \frac{g\, g_S}{\pi^2}\, \sum_{q=t,b} \kappa_q^{gg}\, \mathscr{A}_q^{gg} + 2\, g_S\, g_6\, \frac{s}{M_W^2}\, a_{\phi g}$$
$$+ \frac{g\, g_S^2\, g_6}{\pi^2}\, \sum_{q=t,b} \mathscr{A}_q^{nf;gg}\, a_{qg}$$

all sets of gauge invariant, dimension d operators,
none of which is redundant, form a **basis** and
all **bases** are equivalent
A **basis** is closed under renormalization [G. P.]

Only a basis should be called a "basis"

The main message from Run 1: it is important to check the apparent minimality of the Higgs sector as it is important to anticipate deviations. Of course, not only is there LHC, there are EWPD. Measurements of the $\mathbf{W}$ mass provide an important consistency check of the SM and constrain BSM physics.

$$
\begin{aligned}
\frac{M_{\mathrm{W}}^2}{M_{\mathrm{Z}}^2} \quad = \quad & \hat{c}_\theta^2 + \frac{\alpha}{\pi}\,\mathrm{Re}\Big\{\Big(1-\frac{1}{2}\,g_6\,a_{\phi\mathrm{D}}\Big)\,\Delta_{\mathrm{B}}^{(4)}(M_{\mathrm{W}}) + \sum_{gen}\Big[\Big(1+4\,g_6\,a_{\phi\mathrm{l}}^{(3)}\Big)\,\Delta_{\mathrm{l}}^{(4)}(M_{\mathrm{W}}) \\
+ \quad & \Big(1+4\,g_6\,a_{\phi\mathrm{q}}^{(3)}\Big)\,\Delta_{\mathrm{q}}^{(4)}(M_{\mathrm{W}})\Big] + g_6\Big[\Delta_{\mathrm{B}}^{(6)}(M_{\mathrm{W}}) + \sum_{gen}\Big(\Delta_{\mathrm{l}}^{(6)}(M_{\mathrm{W}}) + \Delta_{\mathrm{q}}^{(6)}(M_{\mathrm{W}})\Big)\Big]\Big\}
\end{aligned}
$$

Global constraints of the SMEFT have been developed in [arXiv:1606.06693] with results that show how the SMEFT

theory error should not be neglected in future fit errors.


First
Results

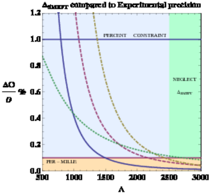
The work of [arXiv:1606.06502] has shown that the extra error introduced in these measurements due to SMEFT higher dimensional operators is subdominant to the current experimental systematic errors.

ATTENTION! This means that the leading challenge to interpreting these measurements in the SMEFT is the pure theoretical uncertainty in how these measurements are mapped to Lagrangian parameters[3].

[3] There is now an overall consensus on having a "truncation" error in SMEFT, and the recommendation is to quote it separately.

KEEP
CALM
AND
APPLY
PHYSICS

goglobal



Inclusion of EWPD in a global fit deserves additional comments. Usually bounds on the coefficients are obtained in two ways: individual coefficients are switched one one at the time, or marginalized in a simultaneous fit.

In [arXiv:1508.05060] the global constraint picture on SMEFT parameters has been updated with the conclusion that stronger constraints can be obtained by using some combinations of Wilson coefficients, when making assumptions on the UV completion of the SM.

Furthermore, strong bounds at the per-mille or sub-per-mille level on some combinations of Wilson coefficients in the Effective Lagrangian can be artificially enhanced in fits of this form in detail.

KEEP CALM AND APPLY PHYSICS

GO DIFFERENTIAL

As discussed in [arXiv:1510.03443,arXiv:1602.05202] a few select kinematic distributions can be used to collect information on modified Higgs couplings, for example in the gluon fusion production process.
In the top-gluon-Higgs sector one can compare three different analysis strategies:

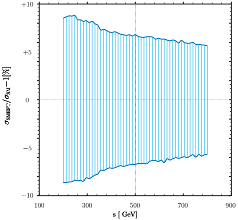
- a modified p_T spectrum of boosted Higgs production in gluon fusion [Banfi], [arXiv:1501.04103]
- off-shell Higgs production, and
- a measurement of the gluon fusion vs $\mathrm{\bar{t}tH}$ production rates.

Unfortunately, explicit threshold effects in boosted Higgs production are too small to be observable in the near future [arXiv:1604.06096].

Remark Unfortunately, global analyses including kinematic information in all Higgs channels cannot rely on the kappa framework, but they can be based on SMEFT. Such analyses provide potentialities and challenges at the same time.

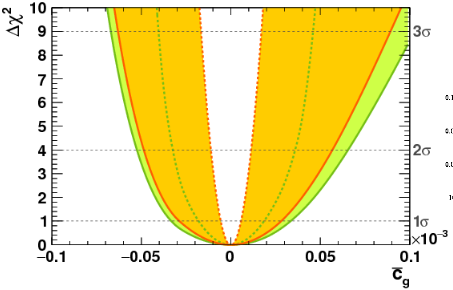
Several (theoretical) analyses have been performed with the available Run I data, as summarized in [arXiv:1511.05170,arXiv:1512.03429,arXiv:1410.7703].
These analyses always use a subset of the full Warsaw basis and show a good agreement, with differences due to different sets of assumptions.

Remark The results can be summarized by saying that current measurements show good agreement with the zero SM deviation hypothesis.

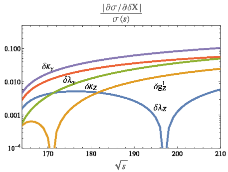


NLO off-shell SMEFT

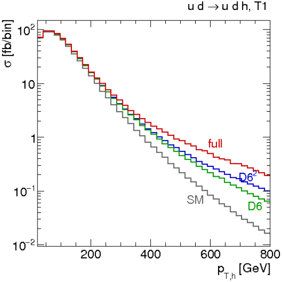
one-parameter fit



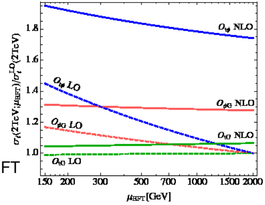
EWPD SMEFT THU



Matching



Fits, waiting for ATLAS, CMS



NLO QCD SMEFT

- ✓ Conventional vision: some very different physics occurs at Planck scale, SM is just an effective field theory. What about the next SM? A new weakly coupled renormalizable model? A tower of EFTs[4]?
- ✓ A different vision: is the SM close to a fundamental theory?

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[4] The SMEFT framework is useful because one can set limits on the effective coefficients in a model-independent way [arXiv:1508.05060]. This is why SMEFT in the bottom-up approach is so useful: we do not know what the tower of UV completions is (or if it exists at all) but we can formulate the SMEFT and perform calculations with it without needing to know what happens at arbitrarily high scales. On the other hand interpreting such limits as bounds on UV models does require some assumption of the UV dynamics [arXiv:1604.06444].

Thank you for your attention


Backup Slides

known

to be specified

still useful

Λ_1

T_2 (NSM)

T_1 (SMEFT)

to be constructed

e.g. [arXiv:1603.03660]

T_3 (NSMEFT)

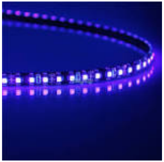
$$
\begin{aligned}
M^2_{\mathrm{H}^\pm} &= \Lambda^2 + \frac{1}{2}\mathrm{v}^2(\lambda_4+\lambda_5) \quad M^2_{\mathrm{A}^0} = \Lambda^2 + \mathrm{v}^2\lambda_5 \quad M^2_{\mathrm{H}} = \Lambda^2 - \frac{1}{4}\left[\mathrm{v}^2\left(\lambda_1 - 2\overline{\lambda}\right)\right. \\
\sin\beta &= 1 - \frac{1}{8}\frac{\mathrm{v}^4}{\Lambda^4} + \mathscr{O}\left(\frac{\mathrm{v}^6}{\Lambda^6}\right) \qquad \cos\beta = \frac{1}{2}\frac{\mathrm{v}^2}{\Lambda^2} + \mathscr{O}\left(\frac{\mathrm{v}^4}{\Lambda^4}\right) \\
\sin(\alpha-\beta) &= -1 + \mathscr{O}\left(\frac{\mathrm{v}^6}{\Lambda^6}\right) \qquad \cos(\alpha-\beta) = -\frac{1}{2}\left(M^2_{\mathrm{h}} + \mathrm{v}^2\overline{\lambda}\right)\frac{\mathrm{v}^2}{\Lambda^4} + \mathscr{O}\left(\frac{\mathrm{v}^6}{\Lambda^6}\right)
\end{aligned}
$$

methodological antireductionism It is possible that at some very large energy scale, all nonrenormalizable interactions disappear. This seems unlikely, given the difficulty with gravity. It is possible that the rules change drastically, it may even be possible that there is no end, simply more and more scales (Georgi).

This prompts the important question whether there is a last fundamental theory in this tower of EFTs which supersede each other with rising energies. Some people conjecture that this deeper theory could be a string theory, i.e. a theory which is not a field theory any more.

epistemological antifoundationalism Or should one ultimately expect from physics theories that they are only valid as approximations and in a limited domain? (Hartmann, Castellani)

Or ... one should not resort to arguments involving gravity: let us banish further thoughts about gravity and the damage it could do to the weak scale (J. D. Wells)



The UV connection

for off-peak

$$\mathscr{A} \;=\; \sum_{n=N}^{\infty} g^n \mathscr{A}_n^{(4)} + \sum_{n=N_6}^{\infty} \sum_{l=1}^{n} \sum_{k=1}^{\infty} g^n g^l_{4+2k} \mathscr{A}^{(4+2k)}_{n\,l\,k}$$

where g is the $SU(2)$ coupling constant and $g_{4+2k} = 1/(\sqrt{2}\,G_F\,\Lambda^2)^k = g_6^k$, where G_F is the Fermi coupling constant and Λ is the scale around which new physics (NP) must be resolved. For each process N defines the $\mathrm{dim} = 4$ LO (e.g. $N = 1$ for $\mathrm{H} \to \mathrm{VV}$ etc. but $N = 3$ for $\mathrm{H} \to \gamma\gamma$). $N_6 = N$ for tree initiated processes and $N - 2$ for loop initiated ones. Here we consider single insertions of $\mathrm{dim} = 6$ operators, which defines NLO SMEFT.

Ex: HAA (tree) vertex generated by $\mathscr{O}^{(6)}_{\phi\mathrm{W}} = \left(\Phi^\dagger\,\Phi\right) \mathrm{F}^{a\mu\nu}\,\mathrm{F}^a_{\mu\nu}$, by $\mathscr{O}^{(8)}_{\phi\mathrm{W}} = \Phi^\dagger\,\mathrm{F}^{a\mu\nu}\,\mathrm{F}^a_{\mu\rho}\,\mathrm{D}^\rho\,\mathrm{D}_\nu\,\Phi$ etc.

SMEFT ordertable for tree initiated $1 \to 2$ processes

$$\begin{array}{llll} g \,/\, \mathrm{dim} & \longrightarrow & & \\ \downarrow & g\,\mathscr{A}_1^{(4)} & + \; g\,g_6\,\mathscr{A}_{1,1,1}^{(6)} & + \; g\,g_8\,\mathscr{A}_{1,1,2}^{(8)} \\ & g^3\,\mathscr{A}_3^{(4)} & + \; g^3\,g_6\,\mathscr{A}_{3,1,1}^{(6)} & + \; g^3\,g_6^2\,\mathscr{A}_{3,2,1}^{(6)} \\ & \cdots\cdots & \cdots\cdots & \cdots\cdots \end{array}$$

- $g\,g_6\,\mathscr{A}_{1,1,1}^{(6)}$ LO SMEFT. There is also RG-improved LO ([arXiv:1308.2627]) and MHOU for LO SMEFT ([arXiv:1508.05060])
- $g^3\,g_6\,\mathscr{A}_{3,1,1}^{(6)}$ ([arXiv:1505.03706]) NLO SMEFT
- $g\,g_8\,\mathscr{A}_{1,1,2}^{(8)}$ ([arXiv:1510.00372]), $g^3\,g_6^2\,\mathscr{A}_{3,2,1}^{(6)}$ MHOU for NLO SMEFT

N.B. g_8 denotes a single $\mathscr{O}^{(8)}$ insertion, g_6^2 denotes two, distinct, $\mathscr{O}^{(6)}$ insertions

$\mathrm{CT}_{4,6} + \mathrm{Mix}$

$$\mathrm{A} = g^{N}\,\mathrm{A}^{(4)}_{\mathrm{LO}}(\{p\}) + g^{N}\,g_6\,\mathrm{A}^{(6)}_{\mathrm{LO}}(\{p\}\,,\,\{a\}) + \frac{1}{16\,\pi^2}\,g^{N+2}\,\mathrm{A}^{(4)}_{\mathrm{NLO}}(\{p\}) + \frac{1}{16\,\pi^2}\,g^{N+2}\,g_6\,\mathrm{A}^{(6)}_{\mathrm{NLO}}(\{p\}\,,\,\{a\})$$

CT_4

$\{p\} = \{g\,,\, g_s\,,\, \sin\theta_{\mathrm{W}}\,,\, M\,,\, M_{\mathrm{H}}\,,\, M_{\mathrm{t}}\} \;\in\;$ SM

$\{a\} =$ Wilson coeff. $\in$ Warsaw basis

$\{p\}\,,\,\{a\} \;\longrightarrow\; \{p_{\mathrm{ren}}\}\,,\,\{a_{\mathrm{ren}}\} \;\longrightarrow\;$ IPS , $\{a_{\mathrm{ren}}(\mu_R)\}$

$\underbrace{\mathrm{G}_F\,,\,\alpha_s\,,\,M_{\mathrm{W}}\,,\,M_{\mathrm{Z}}\,,\,M_{\mathrm{H}}}$

CT = counterterm

The role of $\mathrm{H} \to$ VEV

$$\mathscr{O} = \Lambda^{-n}\, M^{l}\, \overbrace{\partial^{c}\, \overbrace{\overline{\psi}^{a}\, \psi^{b}\, \left(\Phi^{\dagger}\right)^{d}\, \Phi^{e}\, \mathrm{A}^{f}}^{\mathrm{N_F}}}^{\mathrm{dim}}$$

$$\underbrace{\partial^{c}\, \overline{\psi}^{a}\, \psi^{b}\, \left(\Phi^{\dagger}\right)^{d}\, \Phi^{e}\, \mathrm{A}^{f}}_{\mathrm{codim}}$$

$$\frac{3}{2}(a+b)+c+d+e+f+l+n=4$$

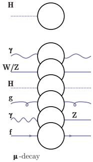
one loop renormalization is controlled by:

$$\boxed{\mathrm{dim}=6 \quad \mathrm{codim}=4 \quad \mathrm{N_F}>2 \quad \text{(Jargon: LO SMEFT)}}$$

The hearth of the problem: a large number of operators implodes into a small number of coefficients

$$\boxed{\text{92 SM vertices} \Longleftrightarrow \text{28 CP even operators (1 flavor, } \mathrm{N}_{\psi}=0,2)}$$

Self-energies



$$
\begin{aligned}
\mathrm{S}_{\mathrm{HH}} &= \frac{g^2}{16\,\pi^2}\,\Sigma_{\mathrm{HH}} = \frac{g^2}{16\,\pi^2}\,\left(\Sigma^{(4)}_{\mathrm{HH}} + g_6\,\Sigma^{(6)}_{\mathrm{HH}}\right) \\
S^{\mu\nu}_{\mathrm{AA}} &= \frac{g^2}{16\,\pi^2}\,\Sigma^{\mu\nu}_{\mathrm{AA}} \qquad \Sigma^{\mu\nu}_{\mathrm{AA}} = \Pi_{\mathrm{AA}}\,\mathrm{T}^{\mu\nu} \\
S^{\mu\nu}_{\mathrm{VV}} &= \frac{g^2}{16\,\pi^2}\,\Sigma^{\mu\nu}_{\mathrm{VV}} \qquad \Sigma^{\mu\nu}_{\mathrm{VV}} = \mathrm{D}_{\mathrm{VV}}\,\delta^{\mu\nu} + \mathrm{P}_{\mathrm{VV}}\,p^{\mu}\,p^{\nu} \\
\mathrm{D}_{\mathrm{VV}} &= \mathrm{D}^{(4)}_{\mathrm{VV}} + g_6\,\mathrm{D}^{(6)}_{\mathrm{VV}} \qquad \mathrm{P}_{\mathrm{VV}} = \mathrm{P}^{(4)}_{\mathrm{VV}} + g_6\,\mathrm{P}^{(6)}_{\mathrm{VV}} \\
S^{\mu\nu}_{\mathrm{ZA}} &= \frac{g^2}{16\,\pi^2}\,\Sigma^{\mu\nu}_{\mathrm{ZA}} + g_6\,\mathrm{T}^{\mu\nu}\,a_{\mathrm{AZ}} \qquad \Sigma^{\mu\nu}_{\mathrm{ZA}} = \Pi_{\mathrm{ZA}}\,\mathrm{T}^{\mu\nu} + \mathrm{P}_{\mathrm{ZA}}\,p^{\mu}\,p^{\nu} \\
\mathrm{S}_{\mathrm{f}} &= \frac{g^2}{16\,\pi^2}\,\left[\Delta_{\mathrm{f}} + \left(\mathrm{V}_{\mathrm{f}} - \mathrm{A}_{\mathrm{f}}\,\gamma^5\right)\, i\not{p}\right]
\end{aligned}
$$

Counterterms

$\Delta_{\rm UV} = \frac{2}{4-n} - \gamma - \ln \pi - \ln \frac{\mu_R^2}{\mu^2}$

n is space-time dimension

loop measure $\mu^{4-n}\, d^n q$

μ_R ren. scale

Warsaw basis

$$Z_i \;=\; 1 + \frac{g^2}{16\,\pi^2}\left(dZ_i^{(4)} + g_6\, dZ_i^{(6)}\right)\Delta_{\rm UV}$$

With field/parameter counterterms we can make

$\mathrm{S_{HH}}, \Pi_{\mathrm{AA}}, \mathrm{D_{VV}}, \Pi_{\mathrm{ZA}}, \mathrm{V_f}, \mathrm{A_f}$ and the corresponding Dyson resummed propagators UV finite at $\mathscr{O}(g^2 g_6)$ (Q.E.D.)

which is enough when working under the assumption that gauge bosons couple to conserved currents

A gauge-invariant description turns out to be mandatory

Mixing

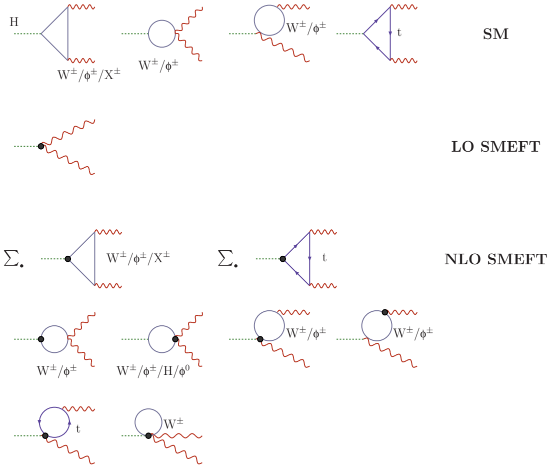
Field/parameter counterterms are not enough to make UV finite the Green's functions with more than two legs. A mixing matrix among Wilson coefficients is needed:

$$a_i = \sum_j Z^{\mathrm{w}}_{ij}\, a^{\mathrm{ren}}_j \qquad Z^{\mathrm{w}}_{ij} = \delta_{ij} + \frac{g^2}{16\,\pi^2}\, dZ^{\mathrm{w}}_{ij}\, \Delta_{\mathrm{UV}}$$

$$|\, g^{\mathrm{N}}\, \mathscr{A}^{(4)}_{\mathrm{N}} + g^{\mathrm{K}}\, g_6\, \mathscr{A}^{(6)}_{\mathrm{K},1,1}\,|^2 \quad \rightsquigarrow \quad |\, g^{\mathrm{N}}\, \mathscr{A}^{(4)}_{\mathrm{N}}\,|^2 + 2\, g^{\mathrm{N+K}}\, g_6\, \mathrm{Re}\left[\mathscr{A}^{(4)}_{\mathrm{N}}\right]^{\dagger} \mathscr{A}^{(6)}_{\mathrm{K},1,1}$$

Remark negative bin entries judge the validity of the $\dim = 6$ “linear” approach ([arXiv:1511.05170])

Nihil novi: for a similar problem in EWPD see [The standard model in the making]. Quadratize when/if needed.



Diagrams contributing to the amplitude for $\mathbf{H \to \gamma\gamma}$ in the $\mathbf{R_\xi}$ -gauge: SM (first row), LO SMEFT (second row), and NLO SMEFT. Black circles denote the insertion of one $\mathbf{dim = 6}$ operator. $\sum_\bullet$ implies summing over all insertions in the diagram (vertex by vertex). For triangles with internal charge flow ($\mathbf{t, W^\pm, \phi^\pm, X^\pm}$) only the clockwise orientation is shown. Non-equivalent diagrams obtained by the exchange of the two photon lines are not shown. Higgs and photon wave-function factors are not included. The Fadeev-Popov ghost fields are denoted by $\mathbf{X}$.

$\mathscr{G}$

❶ 

Define the following combinations of Wilson coefficients (where $s_\theta\,(c_\theta)$ denotes the sine(cosine) of the renormalized weak-mixing angle.

$$
\begin{aligned}
a_{ZZ} &= s_\theta^2\, a_{\phi B} + c_\theta^2\, a_{\phi W} - s_\theta\, c_\theta\, a_{\phi WB} \\
a_{AA} &= c_\theta^2\, a_{\phi B} + s_\theta^2\, a_{\phi W} + s_\theta\, c_\theta\, a_{\phi WB} \\
a_{AZ} &= 2\, c_\theta\, s_\theta\, \left(a_{\phi W} - a_{\phi B}\right) + \left(2\, c_\theta^2 - 1\right)\, a_{\phi WB}
\end{aligned}
$$

and compute the (on-shell) decay $\mathrm{H}(P) \to \mathrm{A}_\mu(p_1)\mathrm{A}_\nu(p_2)$ where the amplitude is

$$
\mathrm{A}^{\mu\nu}_{\mathrm{HAA}} = \mathscr{T}_{\mathrm{HAA}}\, T^{\mu\nu} \qquad M_{\mathrm{H}}^2\, T^{\mu\nu} = p_2^\mu\, p_1^\nu - p_1 \cdot p_2\, \delta^{\mu\nu}
$$

Remark The amplitude is made UV finite by mixing a_{AA} with a_{AA}, a_{AZ}, a_{ZZ} and a_{QW} Q.E.D.

❷ 

Compute the (on-shell) decay $\boxed{\mathrm{H}(P) \to \mathrm{A}_\mu(p_1)\mathrm{Z}_\nu(p_2)}$. After adding 1PI and 1PR components we obtain

$$\mathrm{A}^{\mu\nu}_{\mathrm{HAZ}} = \mathcal{T}_{\mathrm{HAZ}}\, T^{\mu\nu} \qquad M^2_{\mathrm{H}}\, T^{\mu\nu} = p_2^\mu\, p_1^\nu - p_1 \cdot p_2\, \delta^{\mu\nu}$$

Remark The amplitude is made UV finite by mixing a_{AZ} with $a_{\mathrm{AA}}, a_{\mathrm{AZ}}, a_{\mathrm{ZZ}}$ and a_{QW} Q.E.D.

❸

Compute the (on-shell) decay $\mathrm{H}(P) \to \mathrm{Z}_\mu(p_1)\mathrm{Z}_\nu(p_2)$. The amplitude contains

- a $\mathcal{D}_{\mathrm{HZZ}}$ part proportional to $\delta^{\mu\nu}$ and
- a $\mathcal{P}_{\mathrm{HZZ}}$ part proportional to $p_2^\mu\, p_1^\nu$.

Remark Mixing of a_{ZZ} with other Wilson coefficients makes $\mathcal{P}_{\mathrm{HZZ}}$ UV finite, while the mixing of $a_{\phi\Box}$ makes $\mathcal{D}_{\mathrm{HZZ}}$ UV finite Q.E.D.

❹ 

Compute the (on-shell) decay $\mathrm{H}(P) \to \mathrm{W}^{-}{}_{\mu}(p_1)\mathrm{W}^{+}{}_{\nu}(p_2)$. This process follows the same decomposition of $\mathrm{H} \to \mathrm{ZZ}$ and it is UV finite in the $\mathbf{dim} = \mathbf{4}$ part. However, for the $\mathbf{dim} = \mathbf{6}$ one, there are no Wilson coefficients left free in $\mathscr{P}_{\mathrm{HWW}}$ so that its UV finiteness follows from gauge cancellations

($\mathrm{H} \to \mathrm{AA}, \mathrm{AZ}, \mathrm{ZZ}, \mathrm{WW} = 6$ Lorentz structures controlled by 5 coefficients)

Proposition

This is the first part in proving closure of NLO SMEFT under renormalization Q.E.D.

Remark Mixing of $a_{\phi\mathrm{D}}$ makes $\mathscr{D}_{\mathrm{HWW}}$ UV finite Q.E.D.

❺ 

Compute the (on-shell) decay $\mathbf{H}(P) \to \mathbf{b}(p_1)\bar{\mathbf{b}}(p_2)$.

Remark

- It is $\mathbf{dim} = \mathbf{4}$ UV finite and
- mixing of $\boldsymbol{a}_{\mathbf{d}\phi}$ makes it UV finite also at $\mathbf{dim} = \mathbf{6}$ Q.E.D.

❻ 

Compute the (on-shell) decay $\mathrm{Z}(P) \to \mathrm{f}(p_1)\bar{\mathrm{f}}(p_2)$. It is $\mathbf{dim} = 4$ UV finite and we introduce

$$
\begin{aligned}
a_{\mathrm{l\,W}} &= s_\theta\, a_{\mathrm{l\,WB}} + c_\theta\, a_{\mathrm{l\,BW}} & a_{\mathrm{l\,B}} &= s_\theta\, a_{\mathrm{l\,BW}} - c_\theta\, a_{\mathrm{l\,WB}} \\
a_{\mathrm{d\,W}} &= s_\theta\, a_{\mathrm{d\,WB}} + c_\theta\, a_{\mathrm{d\,BW}} & a_{\mathrm{d\,B}} &= s_\theta\, a_{\mathrm{d\,BW}} - c_\theta\, a_{\mathrm{d\,WB}} \\
a_{\mathrm{u\,W}} &= s_\theta\, a_{\mathrm{u\,WB}} + c_\theta\, a_{\mathrm{u\,BW}} & a_{\mathrm{u\,B}} &= c_\theta\, a_{\mathrm{u\,WB}} - s_\theta\, a_{\mathrm{u\,BW}}
\end{aligned}
$$

$$
\begin{aligned}
a^{(3)}_{\phi\mathrm{l}} - a^{(1)}_{\phi\mathrm{l}} &= \frac{1}{2}\,(a_{\phi\mathrm{l\,V}} + a_{\phi\mathrm{l\,A}}) & a_{\phi\mathrm{l}} &= \frac{1}{2}\,(a_{\phi\mathrm{l\,A}} - a_{\phi\mathrm{l\,V}}) \\
a_{\phi\mathrm{u\,V}} &= a^{(3)}_{\phi\mathrm{q}} + a_{\phi\mathrm{u}} + a^{(1)}_{\phi\mathrm{q}} & a_{\phi\mathrm{u\,A}} &= a^{(3)}_{\phi\mathrm{q}} - a_{\phi\mathrm{u}} + a^{(1)}_{\phi\mathrm{q}} \\
a_{\phi\mathrm{d\,V}} &= a^{(3)}_{\phi\mathrm{q}} - a_{\phi\mathrm{d}} - a^{(1)}_{\phi\mathrm{q}} & a_{\phi\mathrm{d\,A}} &= a^{(3)}_{\phi\mathrm{q}} + a_{\phi\mathrm{d}} - a^{(1)}_{\phi\mathrm{q}}
\end{aligned}
$$

and obtain that (Q.E.D.)

$\mathbf{Z} \to \bar{\mathbf{l}}\mathbf{l}$ requires mixing of $\boldsymbol{a}_{\mathbf{l\,BW}}, \boldsymbol{a}_{\boldsymbol{\phi}\mathbf{l\,A}}$ and $\boldsymbol{a}_{\boldsymbol{\phi}\mathbf{l\,V}}$ with other coefficients,

$\mathbf{Z} \to \bar{\mathbf{u}}\mathbf{u}$ requires mixing of $\boldsymbol{a}_{\mathbf{u\,BW}}, \boldsymbol{a}_{\boldsymbol{\phi}\mathbf{u\,A}}$ and $\boldsymbol{a}_{\boldsymbol{\phi}\mathbf{u\,V}}$ with other coefficients,

$\mathbf{Z} \to \bar{\mathbf{d}}\mathbf{d}$ requires mixing of $\boldsymbol{a}_{\mathbf{d\,BW}}, \boldsymbol{a}_{\boldsymbol{\phi}\mathbf{d\,A}}$ and $\boldsymbol{a}_{\boldsymbol{\phi}\mathbf{d\,V}}$ with other coefficients,

$\mathbf{Z} \to \bar{\boldsymbol{\nu}}\boldsymbol{\nu}$ requires mixing of $\boldsymbol{a}_{\boldsymbol{\phi\nu}} = 2\,(\boldsymbol{a}^{(1)}_{\boldsymbol{\phi}\mathbf{l}} + \boldsymbol{a}^{(3)}_{\boldsymbol{\phi}\mathbf{l}})$ with other coefficients.

❼ 

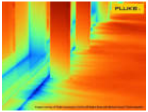
At this point we are left with the universality of the electric charge. In QED there is a Ward identity telling us that e is renormalized in terms of vacuum polarization and Ward-Slavnov-Taylor identities allow us to generalize the argument to the full SM.

We can give a quantitative meaning to the the previous statement by saying that the contribution from vertices (at zero momentum transfer) exactly cancel those from (fermion) wave function renormalization factors. Therefore,

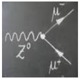
Compute the vertex $\mathbf{A\bar{f}f}$ (at $q^2 = \mathbf{0}$) and the $\mathbf{f}$ wave function factor in SMEFT, proving that the WST identity can be extended to $\mathbf{dim} = \mathbf{6}$; this is non trivial since there are no free Wilson coefficients in these terms (after the previous steps); (non-trivial) finiteness of $\mathbf{e^+e^-} \to \mathbf{\bar{f}f}$ follows.

Proposition

This is the second part in proving closure of NLO SMEFT under renormalization Q.E.D.



The IR connection (e.g. $\mathrm{Z} \to \bar{\mathrm{l}}\mathrm{l}$)



$$= \rho^{\mathrm{f}}_{\mathrm{Z}}\, \gamma^{\mu} \left[\left(I^{(3)}_{\mathrm{f}} + i\, a_{\mathrm{L}} \right) \gamma_{+} - 2\, Q_{\mathrm{f}}\, \kappa^{\mathrm{f}}_{\mathrm{Z}} \sin^2\theta + i\, a_{\mathrm{Q}} \right]$$

$$\mathscr{A}^{\mathrm{tree}}_{\mu} \quad = \quad g\, \mathscr{A}^{(4)}_{1\,\mu} + g\, g_6\, \mathscr{A}^{(6)}_{1\,\mu}$$

$$\mathscr{A}^{(4)}_{1\,\mu} = \frac{1}{4\,c_\theta}\, \gamma_\mu \left(v_{\mathrm{L}} + \gamma^5 \right) \qquad\qquad \mathscr{A}^{(6)}_{1\,\mu} = \frac{1}{4}\, \gamma_\mu \left(\mathrm{V}_{\mathrm{l}} + \mathrm{A}_{\mathrm{l}}\, \gamma^5 \right)$$

$$\begin{aligned}
\mathrm{V}_{\mathrm{l}} \quad &= \quad \frac{s^2_\theta}{c_\theta} \left(4\, s^2_\theta - 7 \right) a_{\mathrm{AA}} + c_\theta \left(1 + 4\, s^2_\theta \right) a_{\mathrm{ZZ}} + s_\theta \left(4\, s^2_\theta - 3 \right) a_{\mathrm{AZ}} \\
&+ \quad \frac{1}{4\,c_\theta} \left(7 - s^2_\theta \right) a_{\phi\,\mathrm{D}} + \frac{2}{c_\theta}\, a_{\phi\,1\,\mathrm{V}} \\
\mathrm{A}_{\mathrm{l}} \quad &= \quad \frac{s^2_\theta}{c_\theta}\, a_{\mathrm{AA}} + c_\theta\, a_{\mathrm{ZZ}} + s_\theta\, a_{\mathrm{AZ}} - \frac{1}{4\,c_\theta}\, a_{\phi\,\mathrm{D}} + \frac{2}{c_\theta}\, a_{\phi\,\mathrm{L\,A}}
\end{aligned}$$

After UV renormalization, i.e. after counterterms and mixing have been introduced, we perform analytic continuation in n (space-time dimension), $n = 4 + \varepsilon$ with ε positive.

Proposition

The infrared/collinear part of the one-loop virtual corrections shows double factorization.

$$\Gamma\left(Z \to \bar{l} + l\right)|_{\mathrm{div}} = -\frac{g^4}{384\,\pi^3} M_Z\, s_\theta^2\, \mathscr{F}^{\mathrm{virt}} \left[\Gamma_0^{(4)}\left(1 + g_6\,\Delta\Gamma\right) + g_6\,\Gamma_0^{(6)}\right]$$

Proposition

The infrared/collinear part of the real corrections shows double factorization.

$$\Gamma^{\mathrm{app}}\left(Z \to \bar{l} + l + (\gamma)\right) = \frac{g^4}{384\,\pi^3} M_Z\, s_\theta^2\, \mathscr{F}^{\mathrm{real}} \left[\Gamma_0^{(4)}\left(1 + g_6\,\Delta\Gamma\right) + g_6\,\Gamma_0^{(6)}\right]$$

Proposition

The total = virtual + real is IR /collinear finite at $\mathscr{O}(g^4\, g_6)$ (Q.E.D.).

Assembling everything gives

$$\Gamma^{\mathrm{l}}_{\mathrm{QED}} = \frac{3}{4}\,\Gamma^{\mathrm{l}}_{0}\,\frac{\alpha}{\pi}\left(1 + g_6\,\Delta^{(6)}_{\mathrm{QED}}\right) \qquad \Gamma^{\mathrm{l}}_{0} = \frac{G_{\mathrm{F}}\,M^3_{\mathrm{Z}}}{24\,\sqrt{2}\,\pi}\left(v^2_{\mathrm{l}} + 1\right)$$

$$\Delta^{(6)}_{\mathrm{QED}} = 2\left(2 - s^2_\theta\right) a_{\mathrm{AA}} + 2\,s^2_\theta\,a_{\mathrm{ZZ}} + 2\left(\frac{c^3_\theta}{s_\theta} + \frac{512}{26}\,\frac{v_{\mathrm{L}}}{v^2_{\mathrm{L}} + 1}\right) a_{\mathrm{AZ}}$$

$$- \frac{1}{2}\,\frac{c^2_\theta}{s^2_\theta}\,a_{\phi\,\mathrm{D}} + \frac{1}{v^2_{\mathrm{L}} + 1}\,\delta^{(6)}_{\mathrm{QED}}$$

$$\delta^{(6)}_{\mathrm{QED}} = \left(1 - 6\,v_{\mathrm{l}} - v^2_{\mathrm{l}}\right)\frac{1}{c^2_\theta}\left(s_\theta\,a_{\mathrm{AA}} - \frac{1}{4}\,a_{\phi\,\mathrm{D}}\right)$$

$$+ \left(1 + 2\,v_{\mathrm{l}} - v^2_{\mathrm{l}}\right)\left(a_{\mathrm{ZZ}} + \frac{s_\theta}{c_\theta}\,a_{\mathrm{AZ}}\right)$$

$$+ \frac{2}{c^2_\theta}\left(a_{\phi\,\mathrm{l\,A}} + v_{\mathrm{l}}\,a_{\phi\,\mathrm{l\,V}}\right)$$