Pseudo-Observables from LEP to LHC
jam sessions

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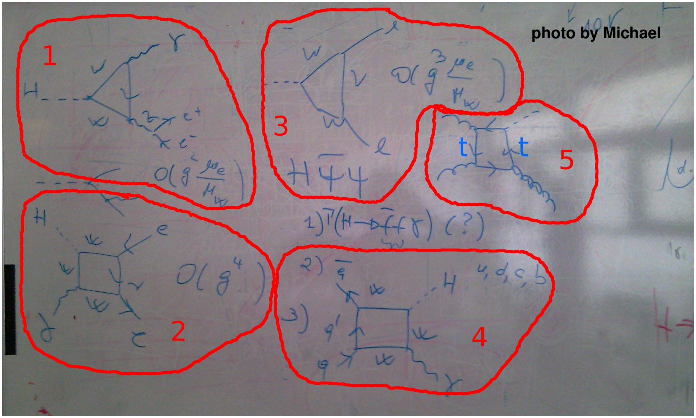
CERN



Supporting material for in depth sessions using the blackboard

Not a real lecture. Scattered notes on Higgs Physics - from Lep to LHC - originally left unfinished

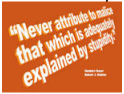
Murphy's law of Higgs Physics

Although skipping foundations is not specifically recommended

- *Foundations without tools is worth nothing*
- *Tools without foundations have no scientific basis*

The study of SM deviations follows Hofstadter's Law: It always takes longer than you expect, even when you take into account Hofstadter's Law

(also check Hanlon's razor)



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Part I

Mostly Lep

❶ POs at Lep, the role of the Z-pole

From $$\frac{V^{\mu}_{e^+e^-\gamma}\, V^{\mu}_{\bar{f}f\gamma}}{s} + \frac{V^{\mu}_{e^+e^-Z}\, V^{\mu}_{\bar{f}fZ}}{s - M_Z^2} + \text{Boxes}$$

To $$\sigma_f^{\text{peak}} = 12\pi\, \frac{\Gamma_e \Gamma_f}{M_Z^2 \Gamma_Z^2}$$

From on-shell mass M_Z $\rightarrowtail$ To complex pole s_Z

For a field Φ let $\Sigma_{\Phi\Phi}(s)$ be the self energy

① Define the Dyson resummed propagator

$$\overline{\Delta}_{\Phi} = \left[\mathrm{Z}_{\Phi}\left(s - \mathrm{Z}_{\mathrm{M}} M^2\right) + \Sigma_{\Phi\Phi}(s)\right]^{-1} = \left[s - M^2_{\text{ren}} + \Sigma^{\text{fin}}_{\Phi\Phi}(s)\right]^{-1}$$

where M is the bare mass and Z_i are renormalization constants

② Define the on-shell mass or the comples pole as

$$\begin{aligned} M^2_{\mathrm{OS}} - M^2_{\text{ren}} + \mathrm{Re}\,\Sigma^{\text{fin}}_{\Phi\Phi}\left(M^2_{\mathrm{OS}}\right) &= 0 \\ s_{\Phi} - M^2_{\text{ren}} + \Sigma^{\text{fin}}_{\Phi\Phi}(s_{\Phi}) &= 0 \end{aligned}$$

only s_{Φ} is gauge parameter independent to all orders (Nielsen identities)

Consequences for $\mathbf{W},\mathbf{Z}$

- Write $s_{\rm V} = \mu_{\rm V}^2 - i\,\gamma_{\rm V}\,\mu_{\rm V}$ and obtain

$$
\begin{aligned}
\mu_{\rm V}^2 &= M_{\rm V\,,OS}^2 - \Gamma_{\rm V\,,OS}^2 + \text{ h.o.} \\
\gamma_{\rm V} &= \Gamma_{\rm V\,,OS}\left(1 - \frac{1}{2}\,\frac{\Gamma_{\rm V\,,OS}^2}{M_{\rm V\,,OS}^2}\right) + \text{ h.o.}
\end{aligned}
$$

☛ Numerically irrelevant for a light SM $\mathbf{H}$

Off-shell is different (more later)

- Indeed, in the R_ξ gauge, at lowest order, one has the following expression for the bosonic part of the Higgs self-energy:

$$\mathrm{Im}\, S_{\mathrm{HH}\,,\mathrm{bos}}(s) = \frac{g^2}{4\,M_{\mathrm{W}}^2}\, s^2 \Big[\left(\frac{M_{\mathrm{H}}^4}{s^2} - 1 \right) \left(1 - 4\,\xi_{\mathrm{W}}\, \frac{M_{\mathrm{W}}^2}{s} \right)^{1/2} \theta\left(s - 4\,\xi_{\mathrm{W}}\, M_{\mathrm{W}}^2 \right) + \frac{1}{2}\,(\mathrm{W} \to \mathrm{Z}) \Big],$$

where ξ_{V} ($\mathrm{V} = \mathrm{W}, \mathrm{Z}$) are gauge parameters. Note that “expansions” involve derivatives.

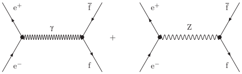


Figure 1: The process $e^+e^- \rightarrow (Z, \gamma) \rightarrow f\overline{f}$ in the Born approximation.

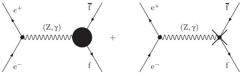


Figure 2: The process $e^+e^- \rightarrow (Z, \gamma) \rightarrow f\overline{f}$; final fermion vertex and its counter-terms.

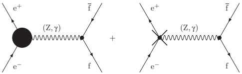


Figure 3: Process $e^+e^- \rightarrow (Z, \gamma) \rightarrow f\overline{f}$; electron vertex and its counter-terms

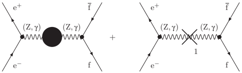


Figure 4: Process $e^+e^- \rightarrow (Z, \gamma) \rightarrow f\overline{f}$; self-energies and kinetic counter-terms

DIAGRAMMATICA
at Lep1

role of theory:
delivering boxes and crosses
with maniacal care for gauge invariance

The complete amplitude for the four-fermion process should be presented in all schemes and all gauges with a general structure,

$$
\begin{aligned}
\mathscr{A} \quad \sim \quad & \frac{1}{s}\Big\{\alpha^{\text{fer}}(s)\gamma^{\mu} \otimes \gamma_{\mu} + \chi(s) \\
& \Big[\mathscr{F}^{\text{ef}}_{\text{QQ}}(s,t)\,\gamma^{\mu} \otimes \gamma_{\mu} + \mathscr{F}^{\text{ef}}_{\text{LL}}(s,t)\,\gamma^{\mu}\gamma_{+} \otimes \gamma_{\mu}\,\gamma_{+} \\
& + \mathscr{F}^{\text{ef}}_{\text{QL}}(s,t)\,\gamma^{\mu} \otimes \gamma_{\mu}\,\gamma_{+} + \mathscr{F}^{\text{ef}}_{\text{LQ}}(s,t)\,\gamma^{\mu}\,\gamma_{+} \otimes \gamma_{\mu}\Big]\Big\}
\end{aligned}
$$

$$
\chi(s) = s\,\chi_{\text{Z}}(s)
$$

Again the *raison d'être* of any renormalization scheme is deeply connected to the possibility of defining the form factors in a gauge-invariant manner.

Where are the PO's?

$$\frac{d\sigma_f}{d\Omega} = \frac{\alpha^2}{4s} N_f^c \beta_f \Big[\Big(1+c^2\Big) \mathscr{F}_1(s) + 4\mu_f^2 (1-c^2) \mathscr{F}_2(s) + 2\beta_f c \mathscr{F}_3(s) \Big]$$

where $c = \cos\theta$ is the cosine of the scattering angle and $\beta_f^2 = 1 - 4\mu_f^2$ with $\mu_f^2 = m_f^2/s$.

The energy dependence is confined in the $\mathscr{F}$ -functions

$$\mathscr{F}_1(s) = Q_e^2 Q_f^2 + 2\, Q_e Q_f g_V^e g_V^f \ \mathrm{Re}\ \chi(s)$$
$$+ \left[(g_V^e)^2 + (g_A^e)^2 \right] \left[\left(g_V^f\right)^2 + \left(g_A^f\right)^2 - 4\,\mu_f^2 \right] \left|\chi(s)\right|^2,$$
$$\mathscr{F}_2(s) = Q_e^2 Q_f^2 + 2\, Q_e Q_f g_V^e g_V^f \ \mathrm{Re}\ \chi(s)$$
$$+ \left[(g_V^e)^2 + (g_A^e)^2 \right] \left(g_V^f\right)^2 \left|\chi(s)\right|^2,$$
$$\mathscr{F}_3(s) = 2\, Q_e Q_f g_A^e g_A^f \,\mathrm{Re}\ \chi(s) + 4\, g_V^e g_V^f g_A^e g_A^f \left|\chi(s)\right|^2$$

χ is the reduced $\gamma/\mathbf{Z}$ propagator ratio. The form factors $\mathscr{F}$ include weak loop corrections but, in their construction, we have completely ignored a few ingredients:

- ♣ QED radiation,
- ♢ weak boxes and
- ♠ all the imaginary parts

❷ Running of the parameters and gauge invariance, Lep guidance: the case of $\alpha_{\rm QED}$

- ☛ Any SM-deviation environment must be reducible to the "best" SM prediction
- ☛ Any manipulation you do must respect gauge invariance

The simplest example: let $\Pi_{\rm ren}(s)$ be the renormalized vacuum polarization: the running is given by

$$\alpha(s) = \frac{\alpha}{1 - \frac{\alpha}{4\pi}\Pi^{\rm fer}_{\rm ren}(s)} \qquad \text{not by} \qquad \alpha(s) = \frac{\alpha}{1 - \frac{\alpha}{4\pi}\Pi_{\rm ren}(s)}$$

❸ TH "options" and their role, e.g. the blue band. Limitations of the Model Independent (MI) approach, the role of the SM remnant.

The renormalization of any theory based on a local (renormalizable) Lagrangian is a procedure that starts from a set of bare—unrenormalized—amplitudes and after making use of the knowledge of very precisely measured quantities gives a finite answer for all remaining predictions.

Remark A rather intuitive notion of naturalness in radiative corrections:
independently of any specific detail, all realizations of radiative corrections single out two main components in each observable

$$\mathbf{O} = \mathbf{O_B} + \Delta\mathbf{O}$$

☞ The term $\mathbf{O_B}$ is supposed to give the bulk of the answer, or the leading contribution to $\mathbf{O}$

☛ The term $\mathbf{\Delta O}$ is supposed to represent small perturbation

The real difference in different renormalization procedures has little to do with the mechanism for absorbing infinities and a lot to do with the splitting between $\mathbf{O_B}$ and $\mathbf{\Delta O}$.

While everybody agrees at $\mathscr{O}(\boldsymbol{\alpha})$, there are differences which start at $\mathscr{O}(\boldsymbol{\alpha^2})$. Usually, the splitting between $\mathbf{O_B}$ and $\mathbf{\Delta O}$ is not uniquely defined, even within one renormalization procedure.

☛ The splitting is usually motivated by the re-summation of irreducible one-loop terms in a situation where nothing is known about irreducible higher-order terms.

The Lep unwritten rule: *never trust a lonely calculation*

We always compared the predictions for physical observables. For that two answers are equivalent if they lie—in the default setup—within the respective bands obtained by varying in all possible ways the theoretical options associated with the procedure.

Remark The theoretical options are obtained from the chosen setup by allowing all the alternatives consistent with the original scheme. Again, two options at $\mathcal{O}(\alpha)$ differ by terms of $\mathcal{O}(\alpha^2)$ and the discrepancy of this order can be eliminated once the complete $\mathcal{O}(\alpha^2)$ calculation—or at least a part of the sub-leading terms—is performed.

The main ingredients that enter the pure weak corrections are

1. the re-summation of the one-particle irreducible vector boson self-energies
2. the scale in vertex corrections and
3. the linearization of the corresponding S-matrix elements

☛ Suppose that a given quantity $\mathrm{O}(a)$ is given in perturbation theory by the following expansion:

$$\begin{aligned}\mathrm{O} &= a+g\left[a^2+f_1(a)\right]+g^2\left[a^3+f_2(a)\right]+\mathcal{O}(g^3)\\ &= \bar{a}+g\,f_1(a)+\mathcal{O}(g^2),\end{aligned}$$

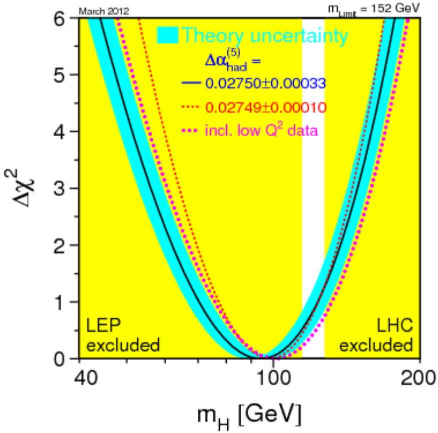
where $\bar{a}=a/(1-ga)$.

☛ Suppose that only the f_1 term is actually known. It could be decided that $\bar{a}$ is the effective expansion parameter (or that in the full expression we change variable $a\to\bar{a}$)

☛ This is equivalent, in the truncated expansion, to introduce the option

$$\mathrm{O}=\bar{a}+g\,f_1(a)=\bar{a}+g\,f_1(\bar{a}), \qquad \text{giving} \qquad \Delta\mathrm{O}=g^2\,f_1'(a)$$

as our estimate of the associated theoretical uncertainty.



Here we go, the blue band

Dima, Wolfgang and I should have patented the idea

Part II

κ frameworks

❹ The κ-framework: origin and problems.

The original framework is defined in **e-Print: arXiv:1209.0040** and has the following limitations:

- ☛ no κ touches kinematics. Therefore it works at the level of total cross-sections, not for differential distributions
- ☛ it is LO, partially accomodating factorizable QCD but not EW corrections
- ☛ ☛ it is not QFT-compatible (ad-hoc variation of the SM parameters, violates gauge symmetry and unitarity)

❺ The role of EFT in resetting the $\boldsymbol{\kappa}$-framework.

The role of $\mathcal{EFT}$ in paving the (as) Model Independent (as possible) road cannot be undermined.

Crumple the Warsaw basis basis) to capture your favorite scenario (LO $\boldsymbol{\kappa}$-vectors) is not the solution, bringing $\mathcal{EFT}$ to NLO is the correct way for focusing in consistency of the $\boldsymbol{\kappa}$-framework. The latter is crucial in describing SM deviations.

No NLO EFT 

see "HEFT beyond LO approximation" https://indico.cern.ch/event/345455/

Proposition

NLO EFT provides the general framework★ for consistent calculation of higher orders and allows for global fits, superseding any ad-hoc variation of the SM parameters. Furthermore, it allows for consistently branching out loops in loop-induced processes, in the spirit of the original framework.

★) within a (well defined) set of assumptions

In the following we discuss these assumptions and the (often misunderstood) properties of couplings in models with more than one scalar field

① one Higgs doublet and linear representation (flexible)

The scalar field $\mathbf{\Phi}$ (with hypercharge **1/2**) is defined by

$$\Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} \mathrm{H} + 2\,\frac{M}{g} + i\,\phi^0 \\ \sqrt{2}\,i\,\phi^- \end{pmatrix}$$

$\mathbf{H}$ is the custodial singlet in $(\mathbf{2_L} \otimes \mathbf{2_R}) = \mathbf{1} \oplus \mathbf{3}$.

□ Building blocks for the Lagrangian are matter fields (including $\mathbf{\Phi}$), field strength tensors and covariant derivatives of those objexts. Extensions are doable but “difficult”, e.g. THDM

$$\Phi \quad \rightarrow \quad \Phi_i \qquad\qquad \Phi_i = R_{ij}\,(\beta)\,\Psi^j$$

with additional diagonalization of the mass matrix for the CP-even scalars, return to □

② no *light dof (where are they anyway?) + decoupling of heavy dof are rigid assumptions*

Decoupling and $SU(2)_C$

- Heavy degrees of freedom $\hookrightarrow H \to \gamma\gamma$: to be fully general one has to consider effects due to heavy fermions $\in R_f$ and heavy scalars $\in R_s$ of $SU(3)$. Colored scalars disappear from the low energy physics as their mass increases.
☛ However, the same is not true for fermions.

¶ Renormalization: whenever $\rho_{LO} \neq 1$, quadratic power-like contribution to $\Delta\rho$ are absorbed by renormalization of the new parameters of the model $\rightsquigarrow \rho$ is not a measure of the custodial symmetry breaking.
☛ Alternatively one could examine models containing $SU(2)_L \otimes SU(2)_R$ multiplets.

Fine points. To be precise we define the following terminology: for a given amplitude, in the limit $m \to \infty$ we will distinguish

- *decoupling* $\mathscr{A} \sim 1/m^2$ (or more). The corresponding higher order operators are called “irrelevant”
- *screening* $\mathscr{A} \to$ const (or $\ln m^2$). The operators are called “marginal”
- *enhancement* $\mathscr{A} \sim m^2$ (or more). The operators are called “relevant”

③ Mixing. Absence of mass mixing of the new heavy scalars with the SM Higgs doublet is required. Mixings change the scenario

① consider a model with two doublets and $Y = 1/2$ (THDM). These doublets are first rotated (with an angle β) to the Georgi-Higgs basis and successively a mixing-angle α diagonalizes the mass matrix for the CP-even states, $\mathbf{h}$ and $\mathbf{H}$. The couplings of $\mathbf{h}$ to SM particles are almost the same of a SM Higgs boson with the same mass (at LO) only if we assume $\mathbf{\sin(\beta - \alpha) = 1}$

- ☛ Therefore, interpreting large deviations in the couplings within a THDM should be done only after relaxing this assumption

③ Mixing

② The case of triplet-like scalars is evem more complex; in the simplest case of a triplet with $Y = 1$ there are four mixing angles. Only in a very special case, requiring also zero VEV for the triplet, the couplings assume the simple form

$$c_{\mathrm{hH^{+}H^{-}}} = 2\,\frac{M_{+}^{2}}{v} \qquad c_{\mathrm{hH^{++}H^{--}}} = 2\,\frac{M_{++}^{2}}{v},$$

☛ where v is the SM Higgs VEV. Furthermore, decoupling of the charged Higgs partners depends on the mixing angles and it is the exception not the rule.

Remark It is the set of scalar fields that break EW symmetry by developing a VEV. The problem with more VEVs, or one VEV different from $(T, Y) = (\frac{1}{2}, 1)$ (T is isospin and Y is hypercharge), is partially related to the rho-parameter which at tree-level is given by

$$\rho_{\mathrm{LO}} = \frac{1}{2} \frac{\sum_i \left[c_i \mid v_i \mid^2 + r_i u_i^2 \right]}{\sum_i Y_i^2 \mid v_i \mid^2} \quad c_i = T_i (T_i + 1) - Y_i^2 \quad r_i = T_i (T_i + 1)$$

where the sum is over all Higgs fields,

- $v_i(u_i)$ gives the VEV of a complex(real) Higgs field with hypercharge Y_i and weak-isospin T_i.

☛ The experimental limit on $\rho - 1$ are rather stringent

More on custodial symmetry

① The SM Higgs potential is invariant under $SO(4)$; furthermore, $SO(4) \sim SU(2)_{\mathrm{L}} \otimes SU(2)_{\mathrm{R}}$ and the Higgs VEV breaks it down to the diagonal subgroup $SU(2)_{\mathrm{V}}$. It is an approximate symmetry since the $U(1)_{\mathrm{Y}}$ is a subgroup of $SU(2)_{\mathrm{R}}$ and only that subgroup is gauged.

② Furthermore, the Yukawa interactions are only invariant under $SU(2)_{\mathrm{L}} \otimes U(1)_{\mathrm{Y}}$ and not under $SU(2)_{\mathrm{L}} \otimes SU(2)_{\mathrm{R}}$ and therefore not under the custodial subgroup.

☛ Therefore, if we require a new CP-even scalar, which is also in a custodial representation of the group, the W/Z-bosons can only couple to a singlet or a 5-plet

If $(N_{\rm L}, N_{\rm R})$ denotes a representation of $SU(2)_{\rm L} \otimes SU(2)_{\rm R}$

- the usual Higgs doublet scalar is a $(2, \bar{2})$, while
- the $(3, \bar{3}) = 1 \oplus 3 \oplus 5$ contains the Higgs-Kibble ghosts (the 3), a real triplet (with $Y = 2$) and a complex triplet (with $Y = 0$)
- The Georgi - Machaceck model has EWSB from both a $(2, \bar{2})$ and a $(3, \bar{3})$

☛ Custodial symmetry is a statement on the ρ parameter, translation to $\rm SVV$ couplings requires care:

① a single source of EWSB. custodial symmetry $\rightrightarrows$ $\frac{g_{\rm S^0WW}}{g_{\rm S^0ZZ}} = \frac{M^2_{\rm W}}{M^2_{\rm Z}}$

② In general $\frac{g_{\rm SWW}}{g_{\rm SZZ}} = \lambda \frac{M^2_{\rm W}}{M^2_{\rm Z}}$, e.g. $\lambda = -1/2$ for a 5-plet (already excluded)

Fine points

① The Higgs doublet ϕ and its conjugate $\tilde{\phi} = i\,\tau_2\,\phi^*$ compose the columns of the matrix

$$\Phi = (\tilde{\phi}\,,\,\phi)$$

② In absence of the hypercharge coupling (g')

$$D_\mu\,\Phi = \partial_\mu\,\Phi + g\,\mathrm{W}_\mu\,\Phi - \frac{1}{2}\,i\,g'\,\mathrm{B}_\mu\,\Phi\,\tau_3 \quad \mathrm{W}_\mu = -\frac{1}{2}\,i\,\mathrm{W}^a_\mu\,\tau_a$$

The Lagrangian possess a global $SU(2) \otimes SU(2)$ invariance

$$\Phi \to \mathrm{G}\,\Phi\,\mathrm{H}^\dagger \quad \mathrm{W}_\mu \to \mathrm{G}\,\mathrm{W}_\mu\,\mathrm{G}^\dagger \quad \mathrm{B}_\mu \to \mathrm{B}_\mu$$

where $\mathrm{G}, \mathrm{H} \in SU(2)$

③ Because of SSB $\mathbf{\Phi}$ develops a vev that breaks $SU(2) \otimes SU(2)$

④ There remains a "diagonal" unbroken $SU(2)$, the "isospin"

$$\Phi \to G \Phi G^\dagger \qquad W_\mu \to G W_\mu G^\dagger$$

Another source of isospin breaking comes when fermions are included with Yukawa interactions

❍ One-loop contributions to the $\boldsymbol{\rho}$ parameter: isospin transformation properties of the mass matrix of heavy degrees of freedom are those determining the sign of the deviation of ρ from one.

EFT perturbative expansion

$$\mathscr{A} = \sum_{n=\mathrm{N}}^{\infty} g^n \mathscr{A}_n^{(4)} + \sum_{n=\mathrm{N}_6}^{\infty} \sum_{l=0}^{n} \sum_{k=1}^{\infty} g^n g_{4+2k}^{l} \mathscr{A}_{nlk}^{(4+2k)}$$

☞ g is the $SU(2)$ coupling constant, $g_{4+2k} = 1/(\sqrt{2}\, G_{\mathrm{F}} \Lambda^2)^k$. For each process N defines the $\mathrm{dim} = 4$ LO (e.g. $N = 1$ for $\mathrm{H} \to \mathrm{VV}$ etc. But $N = 3$ for $\mathrm{H} \to \gamma\gamma$). $N_6 = N$ for tree initiated processes and $N - 2$ for loop initiated ones.

What to do with $\left|\mathscr{A}\right|^2$ in the truncated version? Is $\mathrm{dim}_6 \otimes \mathrm{dim}_4$ interference enough? Do we need dim_6^2 and $\mathrm{dim}_8 \otimes \mathrm{dim}_4$? Examine the $\mathrm{dim}_6 \otimes \mathrm{dim}_4$ scenario

① Λ cannot be too small, otherwise one cannot neglect $\mathbf{dim} = \mathbf{8}$ (breaking of the E/Λ expansion)

② Λ cannot be too large, otherwise

☛ $\mathbf{1/(\sqrt{2}G_F\Lambda^2) \approx g^2/(4\pi)}$ $\Lleftarrow$ one more loop

i.e. $\mathbf{dim_4}$ higher loops are more important than $\mathbf{dim_6}$ interference.

Remark It does not mean that EFT becomes inconsistent! It only means that higher dimensional operators must be included as well ...

Remark Push Λ, neglect higher EW orders and you will end up discovering NP …

Remark The scale at which EFT can be tested is a completely different issue

$Q^2 \ll \Lambda^2$

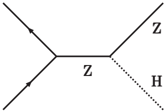
Remark Introducing form factors, with another (completely different) cutoff, … do we want to go back to the sixties (unitarization, $N/D, \ldots$)?

What is the meaning of $\mathbf{dim = N}$?

The role of gauge invariance

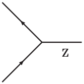
The role of $\mathbf{H} \to \mathrm{VEV}$

Consequences when “expanding” form factors



$$\mathscr{A} \propto g^2\, \overline{v}\, \not{\epsilon} \left(v_{\mathrm{q}} + a_{\mathrm{q}}\, \gamma^5\right) u\, \frac{M}{s - M_{\mathrm{Z}}^2}$$

Why $\mathbf{dim} = 4$?

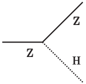


$$\frac{i g}{2 c_\theta}\, \gamma^\mu \left(v_{\mathrm{q}} + a_{\mathrm{q}}\, \gamma^5\right) \Leftarrow -\sum_{\mathrm{I=L,R}} \overline{\psi}_{\mathrm{I}}\, \not{D}\, \psi_{\mathrm{I}} \qquad (\not{D} \to \not{Z})$$

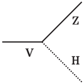
$$-\frac{g M}{c_\theta^2}\, \delta_{\mu\nu} \Leftarrow -\left(D_\mu \Phi\right)^\dagger D^\mu \Phi$$

$$\downharpoonleft$$

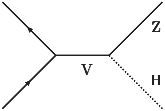
$$\mathrm{Z\ H} \quad \mathrm{Z} \quad \mathrm{vev}$$



It's gauge invariance of $\mathbf{dim} = 4$ operators



$g\,\mu_{\rm V}\,\delta_{\mu\nu}$



$$\mathscr{A} \propto g^2\,\overline{v}\,\not\epsilon\,\left(v_{\rm q}^6 + a_{\rm q}^6\,\gamma^5\right)\,u\,\frac{\mu_{\rm V}}{s - M_{\rm V}^2}$$

expand

$$\frac{1}{s - M_{\rm V}^2} = -\frac{1}{M_{\rm V}^2}\,\left(1 + \frac{s}{M_{\rm V}^2} + \cdots\right)$$

Identify $M_{\rm V} = \Lambda$. Where is this $\mathscr{A}$ coming from?
From gauge invariant ($\mathrm{dim} = 6$) operators, $\mathscr{O}_{\phi\rm q}^{(1,3)}$ e.g.

$$\mathscr{O}_{\phi\rm q}^{(1)} = \Phi^\dagger\,\left(\overrightarrow{D}_\mu - \overleftarrow{D}_\mu\right)\Phi\,(\overline{\rm q}\gamma^\mu{\rm q}) \;\rightrightarrows\; {\rm vev}\,{\rm Z}_\mu\,{\rm H}\,(\overline{\rm q}\gamma^\mu{\rm q})$$

Before you see the slope (s), you need $\mathrm{dim} = 8$ operators

A Layman's guide to renormalization

$$\begin{aligned}\mathscr{A}_{\rm EFT} &= \kappa_{\rm LO}(\{a\})\,\mathscr{A}_{\rm LO}(\{p_0\}) + \kappa_{\rm NLO}(\{a\})\,\mathscr{A}_{\rm NLO}(\{p_0\}) \\ &+ \mathscr{A}_{\rm nf}(\{a\,,p_0\})\end{aligned}$$

- where $\{p_0\}$ is the set of bare parameters (masses and couplings), $\{a\}$ a set of Wilson coefficients; furthermore $\mathscr{A}_{\rm LO}(\mathscr{A}_{\rm NLO})$ is the LO(NLO) SM amplitude. Since $\mathscr{A}_{\rm NLO}$ contains UV divergences we introduce counterterms

$$p_0 = p_{\rm ren} + \delta Z_p,$$

where $p_{\rm ren}$ is the renormalized parameter and δZ_p contains counterterms

- If $\mathscr{A}'$ denotes the derivative of the amplitude w.r.t. parameters we obtain

$$\begin{aligned}\mathscr{A}_{\rm EFT} &= \kappa_{\rm LO}(\{a\})\,\mathscr{A}_{\rm LO}(\{p_{\rm ren}\}) + \kappa_{\rm LO}(\{a\})\,\mathscr{A}'_{\rm LO}(\{p_{\rm ren}\}) \otimes \{Z_p\} \\ &+ \kappa_{\rm NLO}(\{a\})\,\mathscr{A}_{\rm NLO}(\{p_{\rm ren}\}) + \mathscr{A}_{\rm nf}(\{a, p_{\rm ren}\})\end{aligned}$$

The combination

$$\mathscr{A}'_{\rm LO}(\{p_{\rm ren}\}) \otimes \{Z_p\} + \mathscr{A}_{\rm NLO}(\{p_{\rm ren}\})$$

is now UV finite; $\mathscr{A}_{\rm EFT}$ is still UV divergent (in general)

- If we know the UV completion ren. must be discussed at the level of its parameters
- EFT ren. continues with a (renormalized) mixing of the Wilson coefficients
- There is a final step in the procedure, finite ren., where we relate $p_{\rm ren}$ to physical quantities (e.g. $e^2 = g^2 s_\theta^2 = \alpha/(4\,\pi)$)

$$p_{\rm ren} = p_{\rm exp} + F\left(\{p_{\rm exp}\}\right)$$

This substitution induces another shift in the amplitude

$$\mathscr{A}_{\rm LO}\left(\{p_{\rm ren}\}\right) \to \mathscr{A}_{\rm LO}\left(\{p_{\rm exp}\}\right) + \mathscr{A}'_{\rm LO}\left(\{p_{\rm exp}\}\right)\,F\left(\{p_{\rm exp}\}\right)$$

with $p_{\text{ren}} = p_{\text{exp}}$ in both $\mathscr{A}_{\text{NLO}}$ and $\mathscr{A}_{\text{nf}}$.

☛ This set of replacements completely defines our renormalization procedure.

However, there is no such a thing as a_{exp}

☞ A dependence on the renormalization scale will remain. This could be removed only by introducing matching conditions ……

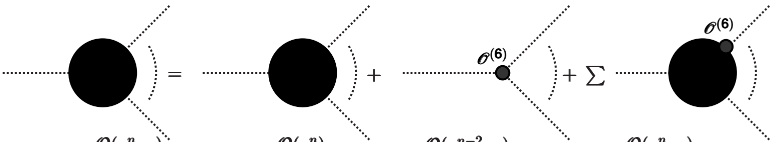
$$\mathscr{L}_{\text{EFT}} = \sum_{n=0}^{4} b_i \Lambda^{4-n} \mathscr{O}_n + \sum_{n>4} \sum_{i=1}^{N_n} \frac{a_i^n}{\Lambda^{n-4}} \mathscr{O}_i^{(d=n)}$$

① first sum is SM (not embedded): means $\boldsymbol{b_{1,2} = 0}$, *it's renormalization*!

② SM (embedded, Wilsonian scenario), $\boldsymbol{b_2}$ not suppressed by any symmetry

- $\boldsymbol{M_{\mathrm{H}}}$ should be $\mathscr{O}(\Lambda)$ and it is light, thus $\boldsymbol{\delta M_{\mathrm{H}}^2 \sim \Lambda^2}$
- $\boldsymbol{M_{\mathrm{H}} \approx 125}$ ***GeV*** which means $\boldsymbol{\Lambda \approx 1}$ ***TeV*** (which doesn't seem to be the case) or FINE TUNING (**not a theorem!)**

includes non-SM families



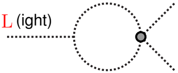
up to $\mathcal{O}(g^n\, g_6)$ = $\mathcal{O}(g^n)$ + $\mathcal{O}(g^{n-2}\, g_6)$ + $\sum$ $\mathcal{O}(g^n\, g_6)$

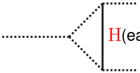
$$g_6 = 1/(\sqrt{2}\, G_F\, \Lambda^2)$$

PTG/LG

option 1: absent
option 2: PTG
option 3: PTG/LG

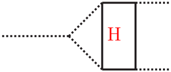
mixing under renormalization





DIAGRAMMATICA of EFT

EFT UVC



Forget κs if you are using **1**

OPTIONS

- **1** only tree PTG&LG
- **2** tree PTG&LG, loops PTG
- **3** tree&loops PTG&LG ✓
- **3′** tree PTG&LG, loops “UV admissible”

$\mathcal{K}$ building manual for $\mathbf{dim = 6}$

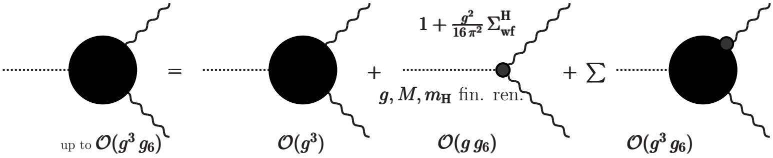
① Split the SM amplitude (e.g. $\mathbf{t}, \mathbf{b}$ loops and bosonic loops in $\mathbf{H} \to \gamma\gamma$)

$$\mathscr{A}_{\mathrm{SM}} = \sum_{i=1,n} \mathscr{A}_i^{(4)}$$

② Recover these sub-amplitudes in the full answer

③ Classify the (non-factorizable) remainder and obtain

$$\mathscr{A}_{\mathrm{prc}} = \sum_{i=1,n} \kappa_i^{\mathrm{prc}} \mathscr{A}_i^{(4)} + \sum_{i=1,m} \kappa_i^{\mathrm{prc\,NF}} \mathscr{A}_i^{(6\,\mathrm{NF})}$$



Assembling the amplitude

Finite renormalization

$$s - M^2_{\mathrm{ren}} + \Sigma_{\mathrm{WW}}(s)\Big|_{s=M^2_{\mathrm{W}}} = 0 \qquad \text{etc.}$$

H WF renormalization à la LSZ

γ WF renormalization $e^2 \to 4\,\pi\,\alpha(0)$

Fine points in renormalization

(including IPS dependence)

Don't say *I only want to shift* **H** couplings

InputParameterSet $G_{\mathrm{F}}, M_{\mathrm{W}}, M_{\mathrm{Z}}, M_{\mathrm{H}}$

$p_{\mathrm{ren}} \neq p(IPS)$

❻ How to write observables in the κ-EFT approach.

Remark $H \to \gamma\gamma$ and $H \to Z\gamma$ are "simple" (loop induced)

$$H \to ZZ,\ WW,\ bb$$

① Many more terms, start at $\mathscr{O}(g)$ requiring massive renormalization

② Need to account for real radiation in $H \to WW,\ bb$

③ κ structure different in $H \to WW,\ bb$, e.g. $\kappa_{tb}^{WW}, \kappa_{bt}^{WW}$ etc.
$H \to bb$ includes $4f$ operators

Appendix C. Dimension-Six Basis Operators for the SM[22].

Einhorn, Wudka

$\mathcal{O}$ is PTG

$\mathcal{O}$ is LG

Grzadkowski, Iskrzynski, Misiak, Rosiek

X^3 (LG)		φ^6 and $\varphi^4 D^2$ (PTG)		$\psi^2\varphi^3$ (PTG)	
Q_G	$f^{ABC} G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	Q_φ	$(\varphi^\dagger\varphi)^3$	$Q_{e\varphi}$	$(\varphi^\dagger\varphi)(\bar l_p e_r \varphi)$
$Q_{\widetilde G}$	$f^{ABC} \widetilde G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	$Q_{\varphi\Box}$	$(\varphi^\dagger\varphi)\Box(\varphi^\dagger\varphi)$	$Q_{u\varphi}$	$(\varphi^\dagger\varphi)(\bar q_p u_r \widetilde\varphi)$
Q_W	$\varepsilon^{IJK} W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$	$Q_{\varphi D}$	$\left(\varphi^\dagger D^\mu\varphi\right)^\star \left(\varphi^\dagger D_\mu\varphi\right)$	$Q_{d\varphi}$	$(\varphi^\dagger\varphi)(\bar q_p d_r \varphi)$
$Q_{\widetilde W}$	$\varepsilon^{IJK} \widetilde W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$				

$X^2\varphi^2$ (LG)		$\psi^2 X\varphi$ (LG)		$\psi^2\varphi^2 D$ (PTG)	
$Q_{\varphi G}$	$\varphi^\dagger \varphi\, G^A_{\mu\nu} G^{A\mu\nu}$	Q_{eW}	$(\bar l_p \sigma^{\mu\nu} e_r)\tau^I \varphi W^I_{\mu\nu}$	$Q_{\varphi l}^{(1)}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar l_p \gamma^\mu l_r)$
$Q_{\varphi\widetilde G}$	$\varphi^\dagger \varphi\, \widetilde G^A_{\mu\nu} G^{A\mu\nu}$	Q_{eB}	$(\bar l_p \sigma^{\mu\nu} e_r)\varphi B_{\mu\nu}$	$Q_{\varphi l}^{(3)}$	$(\varphi^\dagger i\overleftrightarrow{D}^I_\mu \varphi)(\bar l_p \tau^I \gamma^\mu l_r)$
$Q_{\varphi W}$	$\varphi^\dagger \varphi\, W^I_{\mu\nu} W^{I\mu\nu}$	Q_{uG}	$(\bar q_p \sigma^{\mu\nu} T^A u_r)\widetilde\varphi\, G^A_{\mu\nu}$	$Q_{\varphi e}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar e_p \gamma^\mu e_r)$
$Q_{\varphi\widetilde W}$	$\varphi^\dagger \varphi\, \widetilde W^I_{\mu\nu} W^{I\mu\nu}$	Q_{uW}	$(\bar q_p \sigma^{\mu\nu} u_r)\tau^I \widetilde\varphi\, W^I_{\mu\nu}$	$Q_{\varphi q}^{(1)}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar q_p \gamma^\mu q_r)$
$Q_{\varphi B}$	$\varphi^\dagger \varphi\, B_{\mu\nu} B^{\mu\nu}$	Q_{uB}	$(\bar q_p \sigma^{\mu\nu} u_r)\widetilde\varphi\, B_{\mu\nu}$	$Q_{\varphi q}^{(3)}$	$(\varphi^\dagger i\overleftrightarrow{D}^I_\mu \varphi)(\bar q_p \tau^I \gamma^\mu q_r)$
$Q_{\varphi\widetilde B}$	$\varphi^\dagger \varphi\, \widetilde B_{\mu\nu} B^{\mu\nu}$	Q_{dG}	$(\bar q_p \sigma^{\mu\nu} T^A d_r)\varphi\, G^A_{\mu\nu}$	$Q_{\varphi u}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar u_p \gamma^\mu u_r)$
$Q_{\varphi WB}$	$\varphi^\dagger \tau^I \varphi\, W^I_{\mu\nu} B^{\mu\nu}$	Q_{dW}	$(\bar q_p \sigma^{\mu\nu} d_r)\tau^I \varphi\, W^I_{\mu\nu}$	$Q_{\varphi d}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar d_p \gamma^\mu d_r)$
$Q_{\varphi\widetilde WB}$	$\varphi^\dagger \tau^I \varphi\, \widetilde W^I_{\mu\nu} B^{\mu\nu}$	Q_{dB}	$(\bar q_p \sigma^{\mu\nu} d_r)\varphi\, B_{\mu\nu}$	$Q_{\varphi ud}$	$i(\widetilde\varphi^\dagger D_\mu\varphi)(\bar u_p \gamma^\mu d_r)$

Table C.1: Dimension-six operators other than the four-fermion ones.

[22] These tables are taken from [5], by permission of the authors.

✍ Warsaw basis

In the next few slides I will show you beauty in a handful of κs

- Start with EFT at a given order (here NLO)
- write any amplitude as a sum of κ-deformed SM sub-amplitudes
- add another sum of κ-deformed non-SM amplitudes
- show that κs are linear combinations of Wilson coefficients
- discover correlations among the κs

Rationale for this course of action

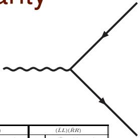
- ❍ Physics is symmetry plus dynamics
- ❍ Symmetry is quintessential (gauge invariance etc.)
- ❍ Symmetry without dynamics don't bring you this far
- ① At Lep dynamics was SM, unknowns were $M_{\mathrm{H}}\,(\alpha_{\mathrm{s}}(M_{\mathrm{Z}}),\dots)$
- ② At LHC (post SM) unknowns are SM-deviations, dynamics?
  - ☛ BSM is a choice. Something more model independent?
    - ❶ An unknown form factor?
    - ❷ A decomposition where dynamics is controlled by $\dim = 4$ amplitudes (with known analytical properties) and deviations (with a direct link to UV completions) are Wilson coefficients?
- ❍ It is for posterity to judge (for me deviations need a SM basis)

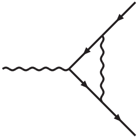
On-shell studies will tell us a lot, off-shell ones will tell us (hopefully) everything

- If we run away from the H peak with a SM-deformed theory, up to some reasonable value $s \ll \Lambda^2$, we need to reproduce (deformed) SM low-energy effects, e.g. VV and tt thresholds. The BSM loops will remain unresolved (as SM loops are unresolved in the Fermi theory).

- ☛ That is why you need to expand SM-deformed into a SM basis with the correct (low energy) behavior. If you stay in the neighbouhood of the peak any function will work, if you run you have to know more of the analytical properties

A similarity





$(\bar{L}L)(\bar{L}L)$		$(\bar{R}R)(\bar{R}R)$		$(\bar{L}L)(\bar{R}R)$	
Q_{ll}	$(\bar{l}_p\gamma_\mu l_r)(\bar{l}_s\gamma^\mu l_t)$	Q_{ee}	$(\bar{e}_p\gamma_\mu e_r)(\bar{e}_s\gamma^\mu e_t)$	Q_{le}	$(\bar{l}_p\gamma_\mu l_r)(\bar{e}_s\gamma^\mu e_t)$
$Q_{qq}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{q}_s\gamma^\mu q_t)$	Q_{uu}	$(\bar{u}_p\gamma_\mu u_r)(\bar{u}_s\gamma^\mu u_t)$	Q_{lu}	$(\bar{l}_p\gamma_\mu l_r)(\bar{u}_s\gamma^\mu u_t)$
$Q_{qq}^{(3)}$	$(\bar{q}_p\gamma_\mu\tau^I q_r)(\bar{q}_s\gamma^\mu\tau^I q_t)$	Q_{dd}	$(\bar{d}_p\gamma_\mu d_r)(\bar{d}_s\gamma^\mu d_t)$	Q_{ld}	$(\bar{l}_p\gamma_\mu l_r)(\bar{d}_s\gamma^\mu d_t)$
$Q_{lq}^{(1)}$	$(\bar{l}_p\gamma_\mu l_r)(\bar{q}_s\gamma^\mu q_t)$	Q_{eu}	$(\bar{e}_p\gamma_\mu e_r)(\bar{u}_s\gamma^\mu u_t)$	Q_{qe}	$(\bar{q}_p\gamma_\mu q_r)(\bar{e}_s\gamma^\mu e_t)$
$Q_{lq}^{(3)}$	$(\bar{l}_p\gamma_\mu\tau^I l_r)(\bar{q}_s\gamma^\mu\tau^I q_t)$	Q_{ed}	$(\bar{e}_p\gamma_\mu e_r)(\bar{d}_s\gamma^\mu d_t)$	$Q_{qu}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{u}_s\gamma^\mu u_t)$
		$Q_{ud}^{(1)}$	$(\bar{u}_p\gamma_\mu u_r)(\bar{d}_s\gamma^\mu d_t)$	$Q_{qu}^{(8)}$	$(\bar{q}_p\gamma_\mu T^A q_r)(\bar{u}_s\gamma^\mu T^A u_t)$
		$Q_{ud}^{(8)}$	$(\bar{u}_p\gamma_\mu T^A u_r)(\bar{d}_s\gamma^\mu T^A d_t)$	$Q_{qd}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{d}_s\gamma^\mu d_t)$
				$Q_{qd}^{(8)}$	$(\bar{q}_p\gamma_\mu T^A q_r)(\bar{d}_s\gamma^\mu T^A d_t)$

$(\bar{L}R)(\bar{R}L)$ and $(\bar{L}R)(\bar{L}R)$		B-violating	
Q_{ledq}	$(\bar{l}_p^j e_r)(\bar{d}_s q_t^j)$	Q_{duq}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\left[(d_p^\alpha)^T C u_r^\beta\right]\left[(q_s^{\gamma j})^T C l_t^k\right]$
$Q_{quqd}^{(1)}$	$(\bar{q}_p^j u_r)\varepsilon_{jk}(\bar{q}_s^k d_t)$	Q_{qqu}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(u_s^\gamma)^T C e_t\right]$
$Q_{quqd}^{(8)}$	$(\bar{q}_p^j T^A u_r)\varepsilon_{jk}(\bar{q}_s^k T^A d_t)$	$Q_{qqq}^{(1)}$	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\varepsilon_{mn}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(q_s^{\gamma m})^T C l_t^n\right]$
$Q_{lequ}^{(1)}$	$(\bar{l}_p^j e_r)\varepsilon_{jk}(\bar{q}_s^k u_t)$	$Q_{qqq}^{(3)}$	$\varepsilon^{\alpha\beta\gamma}(\tau^I\varepsilon)_{jk}(\tau^I\varepsilon)_{mn}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(q_s^{\gamma m})^T C l_t^n\right]$
$Q_{lequ}^{(3)}$	$(\bar{l}_p^j \sigma_{\mu\nu} e_r)\varepsilon_{jk}(\bar{q}_s^k \sigma^{\mu\nu} u_t)$	Q_{duu}	$\varepsilon^{\alpha\beta\gamma}\left[(d_p^\alpha)^T C u_r^\beta\right]\left[(u_s^\gamma)^T C e_t\right]$

Table 3: Four-fermion operators.

isospin and colour indices in the upper part of Tab. 3. In the lower-left block of that table, colour indices are still contracted within the brackets, while the isospin ones are made explicit. Colour indices are displayed only for operators that violate the baryon number B (lower-right block of Tab. 3). All the other operators in Tabs. 2 and 3 conserve both B and L.

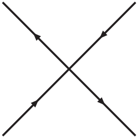
The bosonic operators (classes X^3, $X^2\varphi^2$, φ^6 and $\varphi^4 D^2$) are all Hermitian. Those containing $\widetilde{X}_{\mu\nu}$ are CP-odd, while the remaining ones are CP-even. For the operators containing fermions, Hermitian conjugation is equivalent to transposition of generation indices in each of the fermionic currents in classes $(\bar{L}L)(\bar{L}L)$, $(\bar{R}R)(\bar{R}R)$, $(\bar{L}L)(\bar{R}R)$, and $\psi^2\varphi^2 D^2$ (except for $Q_{\varphi ud}$). For the remaining operators with fermions, Hermitian conjugates are not listed explicitly.

If CP is defined in the weak eigenstate basis then $Q \underset{(+)}{-} Q^\dagger$ are CP-odd (-even) for all the fermionic operators. It follows that CP-violation by any of those operators requires a non-vanishing imaginary part of the corresponding Wilson coefficient. However, one should remember that such a CP is not equivalent to the usual ("experimental") one defined in the mass eigenstate basis, just because the two bases are related by a *complex* unitary transformation.

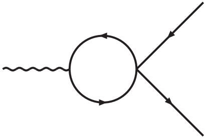
Counting the entries in Tabs. 2 and 3, we find 15 bosonic operators, 19 single-fermionic-current ones, and 25 B-conserving four-fermion ones. In total, there are 15+19+25=59 independent dimension-six operators, so long as B-conservation is imposed.

4

$+\mathrm{dim} = 6$ Fermi operators $=$ EFT



$\bar{\mathrm{e}}_{\mathrm{L}}\,\gamma^\mu\,\mathrm{e}_{\mathrm{L}}\,\bar{\mathrm{e}}_{\mathrm{L}}\,\gamma_\mu\,\mathrm{e}_{\mathrm{L}}$ etc.



extended at NLO

$\kappa \times \mathrm{QED}\; +$ non-fact $\mathrm{dim} = 6$

make sure to recover the low-energy QED (Bhabha ...)
By allowing for the most general set of Fermi couplings
use this EFT to study muon decay
predict $\nu_e\,\mathrm{e}$ scattering
realize the possibility of having neutral current
realize that YM theory could match our theory at very low scales
wait for 't Hooft and Veltman

First $\mathbf{H} \to \boldsymbol{\gamma\gamma}$

$$
\begin{aligned}
\mathscr{A}\left(\mathrm{H} \to \gamma\gamma\right) &= \kappa^{\gamma\gamma}_{\mathrm{W}}\, \mathscr{A}^{(4)}_{\mathrm{W}} + \kappa^{\gamma\gamma}_{\mathrm{t}}\, \mathscr{A}^{(4)}_{\mathrm{t}} + \kappa^{\gamma\gamma}_{\mathrm{b}}\, \mathscr{A}^{(4)}_{\mathrm{b}} \\
&+ \; 2\, i\, g g_6 \, \frac{M^2_{\mathrm{H}}}{M_{\mathrm{W}}}\, a_{\mathrm{AA}} + g_6 \sum_i \kappa^{\gamma\gamma}_{\mathrm{NF}\, i}\, \mathscr{A}^{(6\, i)}_{\mathrm{NF}}
\end{aligned}
$$

where $\boldsymbol{a}_{\mathrm{X}}$ is a Wilson coefficient, $\boldsymbol{\kappa_i}$ are linear combinations of of the $\boldsymbol{a}_{\mathrm{X}}$, $\mathscr{A}^{\mathbf{(4)}}_{\boldsymbol{i}}$ are SM i-loops and $\mathscr{A}^{\mathbf{(6\,\mathit{i})}}_{\mathbf{NF}}$ are non factorizable terms. Thus, the $\mathscr{A}$s, of $\mathscr{O}(\boldsymbol{g}^{\mathbf{3}})$, form a basis. Furthermore

$$
\kappa^{\gamma\gamma}_i = 1 + g_6\, \Delta\kappa^{\gamma\gamma}_i \qquad i = \mathrm{W}, \mathrm{t}, \mathrm{b}
$$

and (in the following) red means PTG

factorizable κ-coeff. for $H \to \gamma\gamma$

$$\kappa_t^{\gamma\gamma} = 1 + g_6 \Big\{ \big(6 - s_\theta^2\big)\, a_{AA} + \frac{2 - s_\theta^2}{s_\theta}\, c_\theta\, a_{AZ} - \frac{3}{2}\,\frac{M_t^2}{M^2}\, c_\theta\, a_{tBW} + \frac{3}{4}\,\frac{M_t^2}{M^2}\,\frac{1 - 2\,s_\theta^2}{s_\theta}\, a_{tWB} - \frac{1}{2\,s_\theta^2}\Big[a_{\phi D} + 2\,s_\theta^2\,\big(c_\theta^2\, a_{ZZ} - 2\,a_{\phi\Box} - a_{t\phi}\big)\Big]\Big\}$$

$$\kappa_b^{\gamma\gamma} = 1 + g_6 \Big\{ \big(6 - s_\theta^2\big)\, a_{AA} + \frac{2 - s_\theta^2}{s_\theta}\, c_\theta\, a_{AZ} + \frac{3}{2}\,\frac{M_b^2}{M^2}\, c_\theta\, a_{bWB} - \frac{1}{2\,s_\theta^2}\Big[a_{\phi D} + 2\,s_\theta^2\,\big(c_\theta^2\, a_{ZZ} - 2\,a_{\phi\Box} - a_{b\phi}\big)\Big]\Big\}$$

$$\kappa_W^{\gamma\gamma} = 1 + \frac{g_6}{3} \Big\{ \Big(14 + 5\,s_\theta^2 - 2\,\frac{M_H^2}{M^2}\, s_\theta^2\Big)\, a_{AA} + \Big(5 - 2\,\frac{M_H^2}{M^2}\Big)\, c_\theta^2\, a_{ZZ} + \Big(4 + 5\,s_\theta^2 - 2\,\frac{M_H^2}{M^2}\, s_\theta^2\Big)\,\frac{c_\theta}{s_\theta}\, a_{AZ} - \frac{3}{2}\,\frac{1}{s_\theta^2}\,\big(a_{\phi D} - 4\,s_\theta^2\, a_{\phi\Box}\big)\Big\}$$

H $\to \gamma\gamma$ *Ad usum Delphini* (does not mean former member of Delphi)

is PTG

$$\Delta\kappa^{\gamma\gamma} = -\frac{1}{2\, s_\theta^2}\left(a_{\phi\, D} - 4\, s_\theta^2\, a_{\phi\,\square}\right)$$

$$\Delta\kappa^{\gamma\gamma}_{\rm W} = \Delta\kappa^{\gamma\gamma} \quad \Delta\kappa^{\gamma\gamma}_{\rm t} = \Delta\kappa^{\gamma\gamma} + a_{{\rm t}\phi} \quad \Delta\kappa^{\gamma\gamma}_{\rm b} = \Delta\kappa^{\gamma\gamma} + a_{{\rm b}\phi}$$

$$\mathscr{A}\left({\rm H}\to\gamma\gamma\right) = \kappa^{\gamma\gamma}\,\mathscr{A}^{(4)} + \kappa^{\gamma\gamma}_{\rm t}\,\mathscr{A}^{(4)}_{\rm t} + \kappa^{\gamma\gamma}_{\rm b}\,\mathscr{A}^{(4)}_{\rm b} + 2\, i\, g g_6\,\frac{M_{\rm H}^2}{M_{\rm W}}\, a_{\rm AA}$$

Next $\mathbf{H \to Z\gamma}$

$$\kappa_{t}^{Z\gamma} = 1 + g_6 \left(6\, a_{AA} + 2\, a_{ZZ} - a_{\phi D} + 4\, a_{\phi \Box} + 2\, a_{t\phi}\right)$$

$$\kappa_{b}^{Z\gamma} = 1 + \frac{1}{2} g_6 \left(6\, a_{AA} + 2\, a_{ZZ} - a_{\phi D} + 4\, a_{\phi \Box} + 2\, a_{b\phi}\right)$$

$$\kappa_{W}^{Z\gamma} = 1 + g_6 \left[\left(3 + s_\theta^2\right) a_{AA} + \left(4 - s_\theta^2\right) a_{ZZ} + s_\theta\, c_\theta\, a_{AZ} + 2\, a_{\phi \Box}\right]$$

Ad usum Delphini

$$\mathscr{A}\left(H \to \gamma Z\right) = \kappa_{W}^{\gamma Z}\, \mathscr{A}_{W}^{(4)} + \kappa_{t}^{\gamma Z}\, \mathscr{A}_{t}^{(4)} + \kappa_{b}^{\gamma Z}\, \mathscr{A}_{b}^{(4)} + i\, g g_6 \frac{M_H^2}{M_W}\, a_{AZ}$$

$$+ \quad a_{\phi D}\, \mathscr{A}_{W}^{NF} + \sum_{f=t,b} \left(a_{\phi q}^{(3)} - a_{\phi q}^{(1)} - a_{\phi f}\right) \mathscr{A}_{f}^{NF}$$

$\mathbf{H \to \gamma\gamma \bigcap H \to \gamma Z}$, i.e. $\kappa_i^{\gamma Z} = 1 + g_6\, s_\theta^2\, \Delta\kappa_i^{\gamma\gamma} + g_6\, \Delta^{\rm rest}\kappa_i^{\gamma Z}$

$$
\begin{aligned}
\Delta^{\rm rest}\kappa_{\rm t}^{\gamma Z} &= \left(\hat{s}_\theta^2 - 3\right) a_{\rm AA} + \frac{2 - s_\theta^2}{s_\theta} \left(s_\theta\, a_{\rm ZZ} - c_\theta\, a_{\rm AZ}\right) \\
&+ \frac{1}{2}\,\frac{c_\theta^2}{s_\theta^2}\, a_{\phi D} - \frac{3}{4}\,\frac{1 - 2\, s_\theta^2}{s_\theta}\,\frac{M_{\rm t}^2}{M_{\rm W}^2}\, a_{\rm tWB} + \frac{3}{2}\,\frac{M_{\rm t}^2}{M_{\rm W}^2}\, c_\theta\, a_{\rm tBW}
\end{aligned}
$$

$$
\begin{aligned}
\Delta^{\rm rest}\kappa_{\rm b}^{\gamma Z} &= \left(s_\theta^2 - 3\right) a_{\rm AA} + \frac{2 - s_\theta^2}{s_\theta} \left(s_\theta\, a_{\rm ZZ} - c_\theta\, a_{\rm AZ}\right) \\
&+ \frac{1}{2}\,\frac{c_\theta^2}{s_\theta^2}\, a_{\phi D} - \frac{3}{2}\,\frac{M_{\rm b}^2}{M_{\rm W}^2}\, a_{\rm bWB}
\end{aligned}
$$

$$
\begin{aligned}
\Delta^{\rm rest}\kappa_{\rm W}^{\gamma Z} &= -\frac{1}{3}\left\{\left[5 + 2\left(1 - \frac{M_{\rm H}^2}{M_{\rm W}^2}\right) s_\theta^2\right] a_{\rm AA} - \frac{3}{2}\,\frac{1}{s_\theta^2}\, a_{\phi D}\right. \\
&- \left.\left[9 - 2\left(1 - \frac{M_{\rm H}^2}{M_{\rm W}^2}\right) c_\theta^2\right] a_{\rm ZZ} + \left[2 + \left(1 - \frac{M_{\rm H}^2}{M_{\rm W}^2}\right) s_\theta^2\right] a_{\rm AZ}\right\}
\end{aligned}
$$

$\mathscr{G}$

$$\mathbf{H \to ZZ} \qquad \boxed{\text{starts at } \mathscr{O}(g)}$$

$$\mathrm{H}(P) \quad \to \quad \mathrm{Z}^{\mu}(p_1) + \mathrm{Z}^{\nu}(p_2)$$

$$\begin{aligned}
\mathscr{A}^{\mu\nu} &= \kappa^{\mathrm{ZZ}}_{\mathrm{LO}}\, \mathscr{A}^{\mathrm{LO}}\, g^{\mu\nu} + \mathscr{A}^{\mu\nu}_{\mathrm{NF}} \\
&+ \sum_{i=\mathrm{t,b,W}} \kappa^{\mathrm{ZZ}}_{\mathrm{NLO,\,i}} \left[\mathscr{A}^{\mathrm{NLO}}_{\mathrm{D},i}\, g^{\mu\nu} + \mathscr{A}^{\mathrm{NLO}}_{\mathrm{P},i}\, p_2^{\mu}\, p_1^{\nu} \right] \\
\kappa^{\mathrm{ZZ}}_{i} &= 1 + g_6\, \Delta\kappa^{\mathrm{ZZ}}_{i}
\end{aligned}$$

$$\begin{aligned}
\Delta\kappa^{ZZ}_{LO} &= s_\theta^2\, a_{AA} + \left(4 + c_\theta^2 - \frac{M_H^2}{M_Z^2}\right) a_{ZZ} + s_\theta^2 c_\theta^2\, a_{AZ} + 2\, a_{\phi\Box} \\
\Delta\kappa^{ZZ}_{NLO,\,t} &= 2\, a_{ZZ} + 2\, a_{\phi\Box} + a_{t\phi} \\
\Delta\kappa^{ZZ}_{NLO,\,b} &= 2\, a_{ZZ} + 2\, a_{\phi\Box} - a_{b\phi} \\
\Delta\kappa^{ZZ}_{NLO,\,W} &= 3\, a_{AA} + 2\, a_{ZZ} + 2\, a_{\phi\Box}
\end{aligned}$$

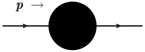
17 non-fact amplitudes with both PTG and LG coefficients

PTG only (in loops)

$\mathbf{H \to bb(\tau\tau)}$ Summary

$m_b = Z_{m_b}\, m_b^{\rm ren}$

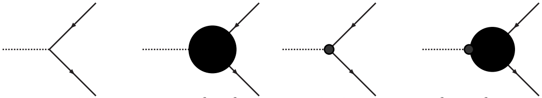
$p \to$



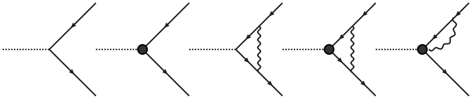
R_b → $\overline{R}_b \to$ UV pole $\propto\ g^2\, g_6\, p^2$

$$R_b = Z_b^+\,\gamma^+ + Z_b^-\,\gamma^-, \qquad \gamma^\pm = \frac{1}{2}\left(1 \pm \gamma^5\right)$$

$$Z_b^\pm = 1 - \frac{1}{2}\,\frac{g^2}{16\,\pi^2}\,\Delta Z_b^\pm$$

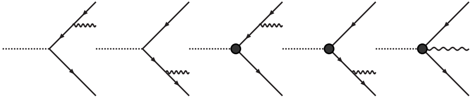


$\mathcal{O}(g)$ $\quad$ $\mathcal{O}(g^3,\ g\, g_s^2)$ $\quad$ $\mathcal{O}(g\, g_6)$ $\quad$ $\mathcal{O}(g^3\, g_6,\ g\, g_s^2\, g_6)$



$a_{\phi W},\ a_{\phi D},\ a_{\phi \Box},\ a_{b\phi}$

Infrared



$a_{bW},\ a_{bB}$

Lep heritage

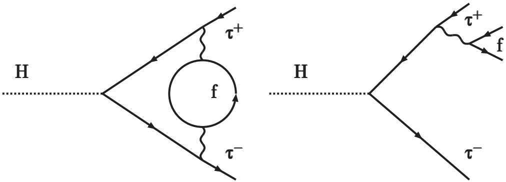
$$\mathbf{H} \rightarrow \boldsymbol{\tau}^{+} + \boldsymbol{\tau}^{-} + \bar{\mathbf{f}} + \mathbf{f}$$

① Is it the four-body decay of the Higgs or

② $\bar{\mathbf{f}}\mathbf{f}$ pair production corrections to the two-body decays $\mathbf{H} \rightarrow \boldsymbol{\tau}^{+}\boldsymbol{\tau}^{-}$ (with a primary $\boldsymbol{\tau}$ pair and a secondary $\mathbf{f}$ pair)?

③ Differentiate according to “invariant mass” of the pairs?

Virtual pairs   Real pairs



Needed when $\boldsymbol{M^2\left(\bar{\mathrm{f}}\mathrm{f}\right) \to 4\, m_{\mathrm{f}}^2}$
At Lep1 it was included through a radiator

process dependent kernel

$$\frac{\Gamma\left(\bar{\mathrm{f}}_1\mathrm{f}_1\bar{\mathrm{f}}_2\mathrm{f}_2\right)}{\Gamma\left(\bar{\mathrm{f}}_1\mathrm{f}_1\right)} = \left(\frac{\alpha}{\pi}\right)^2 \int_{4\,\mu_1^2}^{(1-2\,\mu_2)^2} dx \int_{4\,\mu_2^2}^{1-\sqrt{x})^2} dy\, \mathrm{K}(x,y)$$

❼ The κ-framework for BSM models (Singlet, THDMs, etc).

THDM (here type I)

$$
\begin{aligned}
\mathbf{H} \to \gamma\gamma \quad \mapsto \quad & i\,\frac{g^2 s_\theta^2}{8\,\pi^2}\,\left(p_1 \cdot p_2\, g^{\mu\nu} - p_2^{\mu} p_1^{\nu}\right) \\
\times \quad & \left\{ \frac{\cos\alpha}{\sin\beta} \sum_{\mathrm{f}} \mathscr{A}_{\mathrm{f}}^{\mathrm{SM}} - \sin(\alpha - \beta)\, \mathscr{A}_{\mathrm{bos}}^{\mathrm{SM}} \right. \\
+ \quad & \left[\left(M_{\mathrm{sb}}^2 + M_h^2\right) \cos(\alpha-\beta)\cos 2\beta \right. \\
- \quad & \left. \left. \left(2\,M_{\mathrm{sb}}^2 + M_h^2 + 2\,M_{\mathrm{H}^+}^2\right) \sin(\alpha-\beta)\sin 2\beta \right] \mathscr{A}_{\mathrm{H}^+}^{\mathrm{SM}} \right\}
\end{aligned}
$$

where M_{sb} is the Z_2 soft-breaking scale, $h(\mathbf{H})$ are the light(heavy) scalar Higg bosons.

aren't coeff κs?

Perturbative unitarity

Before LHC (no informations on the Higgs boson mass) there were two interesting scenarios in $\mathrm{V_L V_L \to V_L V_L}$ scattering:

① $M_{\mathrm{W}}^2, M_{\mathrm{Z}}^2 \ll M_{\mathrm{H}}^2 \ll s$

② $M_{\mathrm{W}}^2, M_{\mathrm{Z}}^2 \ll s \ll M_{\mathrm{H}}^2$

Assuming a light Higgs boson we analyze a new option

③ $M_{\mathrm{W}}^2, M_{\mathrm{Z}}^2, M_{\mathrm{H}}^2 \ll s$. The SM result is well-known

$$\frac{d}{dt}\sigma_{\mathrm{V_L V_L \to V_L V_L}} = \frac{\left|T(s,t)\right|^2}{16,\pi s^2}, \qquad T^0_{\mathrm{LO}} = \frac{1}{16\pi s}\int_{-s}^{0} dt\, T_{\mathrm{LO}}$$

$$T^0_{\mathrm{LO}}\left(\mathrm{W_L^+ W_L^- \to W_L^+ W_L^-}\right) \sim -\frac{G_{\mathrm{F}} M_{\mathrm{H}}^2}{4\sqrt{2}\pi}, \qquad s \to \infty$$

Anomalous couplings violates perturbative unitarity. However, one has to be careful in formulating the problem:

- ❍ the region of interest is $M^2_{\mathrm{W}}, M^2_{\mathrm{Z}}, M^2_{\mathrm{H}} \ll s \ll \Lambda^2$
- ☛ When s approaches Λ^2 the EFT must be replaced by its UV completion and it makes no sense to study the limit $s \to \infty$ in the EFT.
- ☞ However, it is well known that heavy degrees of freedom may induce effects of *delayed* unitatity cancellation in the intermediate region and these effects could be detectable

$$T^0_{\mathrm{SM+EFT}} \sim \sum_{n=0}^{2} T_n \left(G_{\mathrm{F}}\, s\right)^n$$

As expected the SM part contributes to the constant part while $\mathrm{dim} = 6$ operators have positive powers of s (up to power two). The leading behavior is controlled by the $\mathscr{O}_{\phi \mathrm{WB}}$ operator.

Part III

The role of gauge invariance, MHOU

❽ On-shell and off-shell for LHC physics.
The role of gauge invariance, definition of Signal. What is the problem with unstable particles?
☛ Why off-shell is problematic and why one should not take derivatives.

Certainly, LHC is not Lep, mostly due to the peculiar character of the Higgs boson: even for a light SM Higgs boson the 4f decays are 40$\%$ of the 2f decays.

☛ As a consequence we always face the problem of off-shell, unstable, particles, even at the H peak.

Remark Therefore, how to interpret $\boldsymbol{\Gamma(\mathrm{H} \to \mathrm{WW} \to \nu l \nu' l')}$ vs. $\boldsymbol{\Gamma(\mathrm{H} \to \mathrm{WW})}$? Stated differently, how to define $\boldsymbol{\Gamma(\mathrm{H} \to \mathrm{W}(\mathrm{W}^*)\mathrm{W})}$?

The short answer

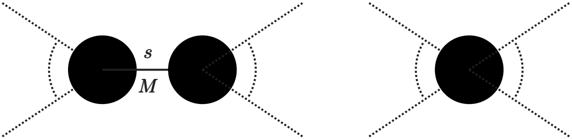
① Never introduce quantities that are not well-defined

② the Higgs couplings can be extracted from Green's functions in well-defined kinematical limits

☛ e.g. residue of the poles after extracting the parts which are 1P reducible

These are well-defined QFT objects, that we can probe both in production and in decays. From this perspective, VH or VBF are on equal footing with ggF and Higgs decays

Now, the long answer …



Once again we describe an arbitrary process with two components:

① a resonant one, with the exchange of a particle of mass ***M*** and virtuality ***s***

② a the continuum (N)

The corresponding amplitude is

$$\mathscr{A} \quad = \quad \frac{V_i\,(\xi, s, M, \dots)\; V_f\,(\xi, s, M, \dots)}{s - M^2} + \mathrm{N}\,(\xi, s, \dots)$$

where $V_i(V_f)$ are the inital(final) sub-amplitudes in the resonant part, ξ is a gauge parameter and the dependence on additional invariants is denoted by $\dots$. It can be shown, in full generality, that

$$V_{i,f}\,(\xi, s, M \dots) \quad = \quad V_{i,f}^{\mathrm{inv}}\Big(M^2 = s, \dots\Big) + (s - M^2)\,\Delta V_{i,f}\,(\xi, s, M, \dots)$$

☛ ☛ only the on-shell production $\times$ decay is gauge-parameter independent.

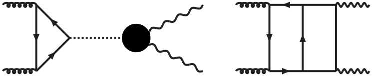
Therefore, we need to expand the resonant part,

$$\mathscr{A} \quad = \quad \frac{V_i^{\text{inv}}\left(M^2 = s, \ldots\right) V_f^{\text{inv}}\left(M^2 = s, \ldots\right)}{s - M^2} + \mathrm{B}\left(s, \ldots\right)$$

with an impact for the number of off-shell events. Note that $B \neq N$ is the remainder of the Laurent expansion around the pole. Technically speaking, the mass M should be replaced by the corresponding complex pole.

The q^2-derivative of a Form Factor is gauge dependent.

Facts of life (frequently forgotten)



prod decay(ξ) n/a two-loop bckg(ξ)

① Put all gluons you want in production (still gauge invariant)

② NLO decay: shift off-shell (ξ-dependent) part to non-resonant

③ this would require the two-loop non-resonant

❾ How to define “simple” quantities without destroying internal consistency:

- production cross sections (ggH, VH VBF)
- partial decay widths (with/without QED/QCD?)
- asymmetries
- off-shell events
- etc.

From $\boldsymbol{\kappa}$ to POs, a tentative list of POs.

The LHC problem

Generally speaking, at LHC the EW core is always embedded into a QCD environment, subject to large perturbative corrections and we expect considerable progress in the "evolution" of these corrections. Even worse is the situation when the t-quark is involved (multi-scale, two classes of logarithms to be resummed). The same considerations apply to PDFs when studying high-mass (large x) final states.

☛ Does it make sense to 'fit' the EW core? Note that this is not confined to introducing POs.

☞ If your answer is "stay fiducial", please use next exit.

From Lep to LHC

❶ What POs do is just collapsing (and/or transforming) some "primordial quantities" (say number of observed events in some pre-defined set-up) into some "secondary quantities" which we fill closer to the theoretical description of the phenomena.

❷ if the number of quantities is reduced, this implies that

☛ some assumptions have been made on the behaviour of the primordial quantities.

The validity of these assumptions is judged on statistical grounds. Within these assumptions (for Lep: QED deconvolution, resonance approach, etc.) the secondary quantities are as "observable" as the first ones.

Therefore, the LHC problem is a) list the assumptions, b) judge them on statistical grounds

To repeat the argument: we oscillate between

① you will fit only my “optimized” (reduced) Wilson coeff.

② the huge QCD background and the associated uncertainty are such that, yes, fit whatever you want but for each new QCD calculation your result will change substantially and not multiplicatively

It is obvious that ② is not limited to PO’s but refers to fitting the EW core, no matter how it is parametrized. The suggested procedure is:

❶ write the answer in terms of SM deviations, i.e. the dynamical parts are SM/dim_4 and

❷ certain combinations of the deviation parameters will define the POs and will be fitted. Optimally, part of the factorizing QCD corrections could enter the PO definition

The suggested procedure is based on

★ The parametrization must be as general as possible, no a priori dropping of terms

☞ this will allow us to “reweight” when new (differential) K-factors become available. New input will touch only the dim_4 components

☛ From this point of view we will differ from Lep where the number of quantities was reduced

☛ PDFs changing is the most serious problem. At Lep the $\mathrm{e}^+\mathrm{e}^-$ structure functions were know to very high accuracy (we tested the effect by using different QED radiators, differing by higher orders treatment). A change of PDFs at LHC will change the convolution ...... Sic transit gloria mundi

More on PDFs

① use codes (e.g.*POWHEG*) that provide weights such that one can use any PDF set and encode PDF variations in the likelihood function (changes $\Lleftarrow$ reevaluate the likelihood).

② Before or after showering? After parton showering, the PDFs enter also in the parton shower and a simple reweighting is no longer possible.

When people say “QCD factorization”, they usually mean

$$\mathrm{g}(p_1)+\mathrm{g}(p_2)\to \mathrm{A}(p_a)+\mathrm{B}(p_b)+\mathrm{X} \quad (p_1 = zx_1 P_1 \; p_2 = x_2 P_2)$$

where $(p_a+p_b)^2 = Q^2$ and $\tau s = Q^2$ and $z\to 1$ is the soft limit

$$\begin{aligned} d\sigma\left(\tau, Q^2, \ldots\right) &= \int dx_1\, dx_2\, dz\, f_{\mathrm{g}}(x_1, \mu_{\mathrm{F}})\, f_{\mathrm{g}}(x_2, \mu_{\mathrm{F}}) \\ &\times \delta(\tau - x_1 x_2 z)\, d\hat{\sigma}\left(z, \alpha_{\mathrm{s}}, \frac{Q^2}{\mu_{\mathrm{R}}^2}, \frac{Q^2}{\mu_{\mathrm{F}}^2} \ldots\right) \end{aligned}$$

$$\begin{aligned} d\hat{\sigma} &= d\hat{\sigma}^0 z\, G \\ G^{\mathrm{NLO}}(z, \alpha_{\mathrm{s}})\Big|_{\mathrm{soft}} &= \delta(1-z) + \frac{\alpha_{\mathrm{s}}}{2\pi}\Big[d_1 D_1(z) + (c_0 + c_1)\,\delta(1-z)\Big] \end{aligned}$$

Comments

- ☛ non universal NLO corrections (process dependent) only enter through the coefficient $\boldsymbol{c_1}$
- ☞ $\boldsymbol{D_n(z) = \left[\ln^n(1-z)/(1-z)\right]_+}$ plus subleading terms, implies convolution

$$\int_0^1 dz\, D_n(z)\, f(z) = \int_0^1 dz \frac{\ln^n(1-z)}{1-z}\left[f(z) - f(0)\right]$$

and dominates the cross-section in the soft limit. For reevaluation it is important to have $\boldsymbol{f(z) = \kappa f_{\mathrm{SM}}(z)}$.

Example

① define LO $\mathscr{A} = \sum_i \kappa_i \mathscr{A}_i^{(4)} \Leftarrow \hat{\sigma}^0 = \sum_{ij} \kappa_i \kappa_j \hat{\sigma}_{ij}^0$

② Introduce $\Delta\hat{\sigma}_{ij} = \int_0^1 dz\, z\, D_1(z)\, \hat{\sigma}_{ij}^0(z)$

③ define NLO

$$\begin{aligned}\hat{\sigma}^1 &= \sum_{ij} \kappa_i \kappa_j \left\{ \left[1 + \frac{\alpha_s}{2\pi}(c_0 + c_1)\right] \hat{\sigma}_{ij}^0(1) + \frac{\alpha_s}{2\pi} d_1 \Delta\hat{\sigma}_{ij} \right\} \\ &= \sum_{ij} \overline{\kappa}_i \overline{\kappa}_j \hat{\sigma}_{ij}^0(1)\end{aligned}$$

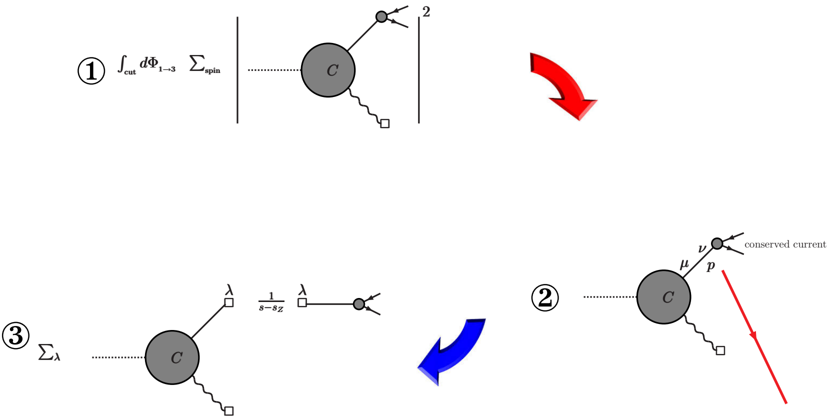
④ put $\overline{\kappa}_i = \kappa_i + \alpha_s/(2\pi) \sum_l X_{il} \kappa_l$ and derive

$$2 \sum_{il} \hat{\sigma}_{ij}^0(1)\, X_{il}\, \kappa_l = \sum_i \left[(c_0 + c_1)\, \hat{\sigma}_{ij}^0(1) + d_1\, \delta\hat{\sigma}_{ij}\right] \kappa_i$$

Part IV

POs at work

PO building manual



□ = *polarization*

$$\delta_{\mu\nu} \to \sum_\lambda \left[e^\lambda_\mu(p)\right]^* e^\lambda_\nu(p)$$

$$|\sum_\lambda f(\lambda)|^2 = \sum_\lambda |f(\lambda)|^2 + \text{rest}$$

Primordial POs: the κ-framework

➲ Of course, any amplitude admits a decomposition

Form factors(invariants) $\times$ Lorentz Structures

- Avoid using Form Factors, whose parametrization is arbitrary and does not reproduce the correct analytic structure (normal thresholds)

- The κ-framework, as seen from the point of view of EFT, allows you to deform both S and B in a consistent way. All "dynamical" parts are SM induced and they are deformed by constant κ-parameters, e.g.

$$\rho_{\mathrm{H}}^{\gamma \mathrm{Z}} = \mathscr{A}(\mathrm{H} \to \gamma \mathrm{Z}) = \kappa_{\mathrm{W}}^{\gamma \mathrm{Z}} \mathscr{A}_{\mathrm{W}}^{(4)} + \kappa_{\mathrm{t}}^{\gamma \mathrm{Z}} \mathscr{A}_{\mathrm{t}}^{(4)} + \kappa_{\mathrm{b}}^{\gamma \mathrm{Z}} \mathscr{A}_{\mathrm{b}}^{(4)} + i\, g g_6 \frac{M_{\mathrm{H}}^2}{M_{\mathrm{W}}} a_{\mathrm{AZ}}$$
$$+ a_{\phi D} \mathscr{A}_{\mathrm{W}}^{\mathrm{NF}} + \sum_{\mathrm{f}=\mathrm{t,b}} \left(a_{\phi \mathrm{q}}^{(3)} - a_{\phi \mathrm{q}}^{(1)} - a_{\phi \mathrm{f}} \right) \mathscr{A}_{\mathrm{f}}^{\mathrm{NF}}$$

$\mathscr{G}$

Next step: Introduce *effective* NLO $\mathbf{H}$ couplings, e.g.

$$\mathbf{HVV} \quad \mapsto \quad \rho_{\mathbf{H}}^{\mathbf{V}} \left(M g^{\mu\nu} + \frac{\mathscr{G}_{\mathbf{L}}^{\mathbf{V}}}{M} p_2^{\mu} p_1^{\nu} \right)$$

etc. After that start computing $\mathbf{\Gamma}$s and $\mathbf{A}$s

- ✗ e.g. F-asymmetry ($\pi/4$) WRT $|\cos\phi|$, ϕ being the angle between the decay planes of the reconstructed Z bosons, e.g. in the decay $\mathrm{H} \to \mathrm{eeqq}$
- ✗ e.g. FB-asymmetry in the angle between e and W reconstructed from qq pair in $\mathrm{H} \to \mathrm{e\nu qq}$

N.B. $\{\rho, \mathscr{G}\}_{\mathrm{NLO}} \neq \kappa$

The same coupling can be expressed in terms of Wilson coefficients within EFT.

$$\textbf{At LO} \quad \mathbf{HZZ} \quad \mapsto \quad g\,\frac{M}{c_\theta^2}\,g^{\mu\nu}\left[1 + g_6\left(a_{\phi\mathrm{W}} + a_{\phi\Box} + \frac{1}{4}\,a_{\phi D}\right)\right] \qquad (\Longleftarrow \;\kappa)$$

$$- \quad 2\,\frac{g g_6}{M}\,a_{\mathrm{ZZ}}\left(p_1 \cdot p_2\, g^{\mu\nu} - p_2^{\mu} p_1^{\nu}\right)$$

Secondary POs:

$$
\begin{aligned}
H \to \gamma\gamma\,(\gamma Z) &\quad\mapsto\quad \rho_H^{\gamma\gamma(Z)}\,\frac{p_1\cdot p_2\, g^{\mu\nu} - p_2^{\mu}\, p_1^{\nu}}{M} \\
HVV &\quad\mapsto\quad \rho_H^{V}\left(M\, g^{\mu\nu} + \frac{\mathscr{G}_L^{V}}{M}\, p_2^{\mu}\, p_1^{\nu}\right) \\
\Gamma(H \to bb) &\qquad\qquad \text{etc.}
\end{aligned}
$$

- ☛ None of these parametrizations represent an approximation (IBA-like)
- ☞ The full FOs are complete (to the best of our technology) and will be written as FO(PO,rest).

Off-shell POs 

- ✗ Going off-shell explains that there is no free lunch in search and optimization

Furthermore, POs should be as inclusive as possible, without requiring extrapolation of FOs; we can nevertheless define off-shell POs, e.g.

$$R^{4l}_{\mathrm{off}} = \frac{N^{4l}_{\mathrm{off}}}{N^{4l}_{\mathrm{tot}}} \qquad N^{4l}_{\mathrm{off}} = N^{4l}\,(M_{4l} > M_0)$$

where N^{4l} is the number of 4-leptons events.

Since the $\mathbf{K}$-factor has a relatively small range of variation with virtuality, the ratio is much less sensitive also to higher order terms.

function of the invariant mass μ^2 and of the momentum transfer $t = -(K_1 - k_1)^2$. Therefore going to the $K_1 K_2$ CM system we get

$$\int \mathrm{d}k_1\,\mathrm{d}k_2\,\delta^4(K_1 + K_2 - k_1 - k_2)\,\delta(\mu^2 + (k_1 + k_2)^2)\,\delta(t + (K_1 + k_1))^2 V = \tfrac{1}{2}(\pi/\mu^2)\,\delta(Q^2 + \mu^2)\,V(\mu^2, t)\,,$$

where once more we have neglected masses. Finally we fix the outgoing polarizations to be longitudinal. Even if there are not measurable we are expecting a strong signal only from $\mathrm{V_L V_L}$ scattering. Collecting the results we obtain

$$\frac{\partial^2}{\partial\mu^2\,\partial t}\,\sigma(\mathrm{e^+e^-} \to \bar{\ell}\ell \mathrm{V_L V_L}) = \frac{\alpha^2 A_+ A_-}{16\pi^5 \sin^4\theta_\mathrm{w}}\,\frac{M^4}{s}\int \mathrm{d}^4Q\,\mathrm{d}q_+\,\mathrm{d}q_-\,\delta(Q^2 + \mu^2)\,\delta^4\Big(\sum p - \sum q - Q\Big)\,\mathcal{L}\,\frac{V(\mu^2, t)}{\mu^2}\,,$$

where $A_\pm = (a_\pm^2 + b_\pm^2)$ and $\mathcal{L}$ denotes collectively the leptonic contribution. The vector Q is timelike with positive time component and we can replace $\mathrm{d}^4Q\,\delta(Q^2 + \mu^2)$ with $\mathrm{d}Q = \mathrm{d}^4Q\,\theta(Q)\,\delta(Q^2 + \mu^2)$. The phase space is reduced to a three-body problem which we evaluate in the p_+, p_- CM system with p_- along the positive z-axis, q_- in the x–z plane and θ_-, $\pi - \theta_+$ being the q_-, q_+ polar angles. The Q-integration is performed by using the last δ-function and $Q^2 = -\mu^2$ evaluated at $\theta_+ = \theta_- = 0$ gives the relation

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$$E_+ = \tfrac{1}{2}(s - \mu^2 - 2\sqrt{s}E_-)/(\sqrt{s} - 2E_-)\,, \quad 0 \leqslant E_- \leqslant (1/2\sqrt{s})(s - \mu^2)\,,$$

where $E_\pm$ are the $q_\pm$ time components. Proceeding in the approximation scheme we set $\theta_+ = \theta_- = 0$ everywhere but in the two propagator factors. In this way the integrations over θ_+, θ_- and E_- can be done exactly.

$$\frac{\partial^2}{\partial\mu^2\,\partial t}\,\sigma(\mathrm{e^+e^-} \to \bar{\ell}\ell \mathrm{V_L V_L}) = \frac{\alpha^2 A_+ A_-}{16\pi^2 \sin^4\theta_\mathrm{w}}\,\frac{\mu^2}{s^2}\,\frac{V(\mu^2, t)}{16\pi\mu^4}\int_0^{E_\mathrm{max}} \frac{F(E)}{\sqrt{s} - 2E}\,\mathrm{d}E\,,$$

$$(E) = [(4/\mu^4)(\tfrac{1}{2}\mu^2 - s + 2\sqrt{s}E)^2 - 1]\,\{[(\sqrt{s} + 2E)/(\sqrt{s} - 2E)]^2 - 1\}\,.$$

he last step is to recognize in $V(\mu^2, t)/16\pi\mu^4$ the expression for $\mathrm{d}\sigma(\mathrm{V_L V_L} \to \mathrm{V_L V_L})/\mathrm{d}t$ computed at the center mass energy μ. Integrating over t we obtain

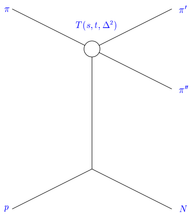
$$\mu^2\,\mathrm{d}\sigma(\mathrm{e^+e^-} \to \bar{\ell}\ell \mathrm{V_L V_L})/\mathrm{d}\mu^2 = (\alpha^2 A_+ A_-/2\pi^2 \sin^4\theta_\mathrm{w})\,f(\mu^2/s)\,\sigma(\mathrm{V_L V_L} \to \mathrm{V_L V_L})\,,$$

where $f(x) = (1 + x)\ln(1/x) - 2(1 - x)$ is essentially the luminosity factor of ref. [6]. The scattered leptons will be in the forward region and therefore it will not be possible to distinguish between electrons and neutrinos. By measuring the $\mathrm{W^+W^-}$ and $\mathrm{Z^0Z^0}$ (or even the exotic channels WZ) invariant mass distributions we can study combinations of cross sections as a function of their center of mass energy μ. For instance

$$\mu^2\,\mathrm{d}\sigma(\mathrm{e^+e^-} \to \mathrm{X W_L^+ W_L^-})/\mathrm{d}\mu^2 = (\alpha^2/32\pi^2 \sin^4\theta_\mathrm{w})\,f(\mu^2/s)\,[h(\theta_\mathrm{w})\,\sigma(\mathrm{Z_L^0 Z_L^0} \to \mathrm{W_L^+ W_L^-}) + \sigma(\mathrm{W_L^+ W_L^-} \to \mathrm{W_L^+ W_L^-})]\,,$$

$$h(\theta_\mathrm{w}) = (1/16\cos^4\theta_\mathrm{w})\,[(4\sin^2\theta_\mathrm{w} - 1)^2 + 1]^2\,.$$

In conclusion, following the idea introduced by the Berkeley group we have verified the existence of a relation between certain combinations of cross sections for longitudinal vector boson scattering and distributions which hopefully will be measurable in a near future. In general we are expecting a quite low statistics but at the same time we are waiting for some spectacular effect coming from the short-range strong part of the Yang–Mills force



4

▷ $\pi\pi$-scattering from $\pi + p \to N + \pi + \pi$

$$
\begin{aligned}
< \pi\pi | T | \pi\pi > &= T(s,t), \\
< \pi\pi N | T | \pi p > &\propto T(s,t,\Delta^2),
\end{aligned}
$$

$$
\begin{aligned}
(q_N - q_p)^2 &= \Delta^2 m^2, \\
s = -(k' + k'')^2, \quad &\text{and} \;\; t = -(k - k'')^2
\end{aligned}
$$

▷ Procedure

1. Extract

$$
|\; T(s,t,\Delta^2 \;|^2 \quad \text{from} \quad \frac{\partial^3 \sigma}{\partial s \partial t \partial \Delta^2} \qquad (1)
$$

2. Compute

$$
\frac{\partial \sigma_{\pi\pi}}{\partial t} = \frac{1}{16\,\pi} \frac{|T(s,t,-1)|^2}{\lambda(s,m^2,m^2)}
$$

- Observe that $\Delta^2 = -1$ while it can only be positive in Eq.(1).

▷ Conclusion: We can investigate $\sigma_{\pi\pi}$ from a

measurement of

▷ the differential σ in $\pi + p \to N + \pi + \pi$

▷ if the **extrapolation procedure** can be reliably performed

▷ see D.D. Carmony and R.T. Van de Walle, Phys. Rev. 127(1962)959.

How to inlude EWPD? The case of the **W** mass

Working in the α-scheme we can predict M_{W}. The solution is

$$
\begin{aligned}
\frac{M_{\mathrm{W}}^2}{M_{\mathrm{Z}}^2} &= \hat{c}_\theta^2 + \frac{\alpha}{\pi}\,\mathrm{Re}\Big\{ \Big(1 - \frac{1}{2}\,g_6\,a_{\phi\,\mathrm{D}}\Big)\,\Delta_{\mathrm{B}}^{(4)}(M_{\mathrm{W}}) \\
&+ \sum_{\text{gen}} \Big[\Big(1 + 4\,g_6\,a_{\phi\mathrm{l}}^{(3)}\Big)\,\Delta_{\mathrm{l}}^{(4)}(M_{\mathrm{W}}) + \Big(1 + 4\,g_6 a_{\phi\,\mathrm{q}}^{(3)}\Big)\,\Delta_{\mathrm{q}}^{(4)}(M_{\mathrm{W}})\Big] \\
&+ g_6\,\Big[\Delta_{\mathrm{B}}^{(6)}(M_{\mathrm{W}}) + \sum_{\text{gen}}\,\Big(\Delta_{\mathrm{l}}^{(6)}(M_{\mathrm{W}}) + \Delta_{\mathrm{q}}^{(6)}(M_{\mathrm{W}})\Big)\Big]\Big\}
\end{aligned}
$$

The expansion can be improved when working within the SM ($\mathrm{dim} = 4$). Any equation that gives $\mathrm{dim} = 6$ corrections to the SM result will always be understood as

$$
\mathscr{O} = \mathscr{O}^{\mathrm{SM}}\Big|_{\mathrm{imp}} + \frac{\alpha}{\pi}\,g_6\,\mathscr{O}^{(6)}
$$

in order to match the *TOPAZ0/Zfitter* SM results whe $g_6 \to 0$.

How to inlude EWPD?

① By reducing (a priori) the number of $\mathbf{dim} = \mathbf{6}$ operators

② By imposing penalty functions ω on the global fit, that is functions defining an ω-penalized LS estimator for a set of global penalty parameters (perhaps using *merit functions* and the *homotopy method*)

③ Using a Bayesian approach, with a flat prior for the parameters. One κ at the time? Fit first to the EWPD and then to H observables? Combination of both?

Of course, all EWPO must be rewritten in the κ-EFT approach

…

➓ How to treat the Background (e.g. in the κ-framework).

It is done similar to the previously examined signal. The amplitude is decomposed into Lorentz structures compatible with symmetries (e.g. Bose symmetry in $gg \to VV$) and with Ward identities. An EFT calculation is performed and κ factors (w or w/o factorization) are extracted.

☛ The whole process changes …

Example: $\mathrm{g}(p_1)\mathrm{g}(p_2) \to \mathrm{Z}(p_3)\mathrm{Z}(p_4)$ polarization tensor

$$\mathrm{Z}_\mu \, \overline{\mathrm{q}} \, \gamma^\mu \left(v_\mathrm{q} + a_\mathrm{q} \, \gamma^5 \right) \mathrm{q}$$
$$P^{\mu\nu\alpha\beta} \propto v_\mathrm{q}^2 \, P_\mathrm{V}^{\mu\nu\alpha\beta} + a_\mathrm{q}^2 \, P_\mathrm{A}^{\mu\nu\alpha\beta}$$

① charge conjugation invariance $\mapsto$ no $v_\mathrm{q} \, a_\mathrm{q}$

② P transversal to gluon momenta, P_V transversal to Z momenta, P_A also transversal for light quarks ($m_\mathrm{q} = 0$)

$$P^{\mu\nu\alpha\beta} = \mathrm{A}_1^{(4)} \left(g^{\mu\nu} + \frac{p_1^\nu p_2^\mu}{p_1 \cdot p_2} \right) g^{\alpha\beta} + \cdots \;\; \rightarrow \;\; \kappa_1^{\mathrm{ggZZ}} \, \mathrm{A}_1^{(4)} + \cdots$$

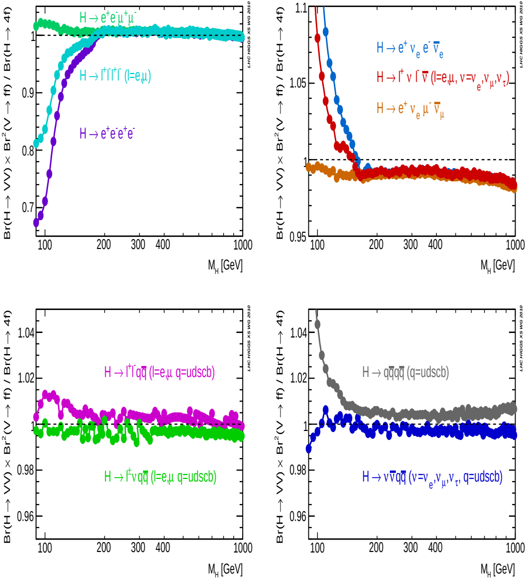
involving $a_{\phi\,\mathrm{g}}, a_{\mathrm{u}\,\mathrm{g}}$ etc.

Of course, we always have TH remnants. This means that (understating the problem) we face a decomposition

$$\mathbf{FO} = \mathbf{PO} \oplus \mathbf{T}_{\textbf{remnant}}$$

and the choice of PO must be such that $\mathbf{T}_{\textbf{remnant}}$ is not a source of large errors due to bias (as using a phonebook to select participants in a survey). For example, as more terms are added to $\mathbf{T}_{\textbf{remnant}}$, the greater the resulting model's complexity will be.

★ κ-EFT needed for the full process



⓫ How to "insert" POs into Fiducial Observables (FOs).

A schetchy example

$$\mathscr{A} = \frac{V_i(s, s_{\mathrm{H}}, \xi, \dots)\, V_f(s, s_{\mathrm{H}}, \xi, \dots)}{s - s_{\mathrm{H}}} + B(s, \xi, \dots)$$

$$V_{i,f}(s, s_{\mathrm{H}}, \xi, \dots) = V_i^{\mathrm{inv}}(s, s, \dots) + (s - s_{\mathrm{H}})\, \Delta V_{i,f}(s, s_{\mathrm{H}}, \xi, \dots)$$

where s_{H} is the H complex pole, s the H virtuality, ξ the gauge parameter(s) and where $\dots$ represent other invariants

$$\mathscr{A} = \mathscr{A}_{\mathrm{S}} + \mathscr{A}_{\mathrm{B}} \qquad \mathscr{A}_{\mathrm{S}} = \frac{V_i^{\mathrm{inv}}\, V_f^{\mathrm{inv}}}{s - s_{\mathrm{H}}}$$

$$\begin{aligned}
\mathrm{FO} &= \int_{\mathrm{cut}} d\Phi \sum_{\mathrm{spin}} \left| \mathrm{A_S} + \mathrm{A_B} \right|^2 = \int_{\mathrm{cut}} d\Phi \sum_{\mathrm{spin}} \left| \mathrm{A_S} \right|^2 + \mathrm{FO}_{\mathrm{rest}} \\
&= \int d\Phi \sum_{\mathrm{spin}} \left| \mathrm{A_S} \right|^2 + \left(\int_{\mathrm{cut}} - \int \right) d\Phi \sum_{\mathrm{spin}} \left| \mathrm{A_S} \right|^2 + \mathrm{FO}_{\mathrm{rest}} \\
&= \mathrm{PO} + \mathrm{rest}
\end{aligned}$$

A schetchy example (cont'd)

As far as Signal (for a given F final state) is concerned we can also write as follows:

$$\sigma\,(ij \to \mathrm{H} \to \mathrm{F}) = \frac{1}{\pi}\,\sigma_{ij\to\mathrm{H}}(s)\,\frac{s^2}{\left|s - s_{\mathrm{H}}\right|^2}\,\frac{\Gamma_{\mathrm{H}\to\mathrm{F}}(s)}{\sqrt{s}}$$

and write $\Gamma_{\mathrm{H}\to\mathrm{F}}$ in terms of POs, e.g. $\Gamma_{\mathrm{H}\to\mathrm{ZZ}}$ and $\Gamma_{\mathrm{Z}\to\mathrm{ll}}$, where all unstable particles are computed at their complex pole.

☛ Compare $\mathrm{PO}_{\mathrm{ATLAS}}$, $\mathrm{PO}_{\mathrm{CMS}}$

Constructing POs in $\mathbf{H} \to \mathbf{4\,f}$

$$\mathscr{M} \quad = \quad \mathscr{M}_{\rm fc}^{\nu\nu}\,(p_1\,,p_2)\,\Delta_{\mu\alpha}\,(p_1)\,\Delta_{\nu\beta}\,(p_2)\,J^{\alpha}\,(q_1\,,k_1)\,J^{\beta}\,(q_2,k_2) + \mathscr{M}_{\rm nf}\,(p_1\,,p_2)$$

fermion currents

non 2PR

$$J^{\mu}\,(q,k) \quad = \quad g\,\bar{u}(q)\,\gamma^{\mu}\,\left(v_{\rm f} + a_{\rm f}\,\gamma^5\right)\,v(k), \qquad p = q + k$$

$\Delta^{\mu\nu}(p)$ is the Z propagator and $\mathscr{M}_{\rm nf}$ collects all diagrams that are not doubly (Z) resonant

$$\mathscr{M}_{\rm fc}^{\mu\nu} \quad = \quad F_{\rm D}\,\delta^{\mu\nu} + F_{\rm T}\,T^{\mu\nu} \qquad T^{\mu\nu} = \frac{p_1^{\nu}\,p_2^{\mu}}{p_1 \cdot p_2} - \delta^{\mu\nu}$$

$$\Delta^{\mu\nu}(p) \to \sum_{\lambda} e_{\mu}\,(p,\lambda)\,e^{*}_{\nu}\,(p,\lambda)\,\Delta(p^2) \qquad \Delta(p^2) = \frac{1}{s - M_{\rm Z}^2}$$

mapping virtual $\mapsto$ real

𝒢

Constructing POs in $\mathbf{H} \to \mathbf{4f}$ (cont'd)

$$P_{ij} \quad = \quad \Big[\mathscr{M}_{\rm D}\,\delta^{\mu\nu} + \mathscr{M}_{\rm T}\,T^{\mu\nu}\Big]\,e_{\mu}(p_1,i)\,e_{\nu}(p_2,j)$$

$$D_{ij}(p) = \sum_{\rm spin} E_i(p)\,E^{\dagger}_j(p) \qquad\qquad E_i(p) = J^{\mu}\,(q,k)\;e^{*}_{\mu}(p,i)$$

where $i,j = -1,0,+1$ and $p = q+k$. We obtain

$$\sum_{\rm spin}\Big|\mathscr{M}_{\rm fc}\Big|^2 \;=\; \sum_{ijkl} P_{ij}\,P^{\dagger}_{kl}\,D_{ik}(p_1)\,D_{jl}(p_2)\,\Big|\Delta(s_1)\,\Delta(s_2)\Big|^2 = \sum_{ijkl} A_{ijkl}\,\Big|\Delta(s_1)\,\Delta(s_2)\Big|^2$$

$$= \;\Big[\sum_{i} A_{iiii} + \sum_{ij} A_{ijij} + \sum_{\substack{k,j\neq i\\ l\neq j}} A_{ijkl}\Big]\,\Big|\Delta(s_1)\,\Delta(s_2)\Big|^2$$

where $\mathscr{M}$ is the matrix element comprising all factorizable contributions, not only the SM ones. A_{iiii} gives informations on H decaying into two Z of the same helicity $(0,0$ etc.), A_{ijij} on mixed helicities $(0,1$ etc.) while the third term gives the interference

Constructing POs in $\mathrm{H} \to 4\,\mathrm{f}$ (cont'd)

$$
\begin{aligned}
\mathscr{M}_{\text{fc}} &= \sum_{ij} a_{ij}\,(s, s_1, s_2, \dots)\,\Delta(s_1)\,\Delta(s_2) \\
&= \sum_{ij} a_{ij}\,\left(s_{\mathrm{H}}, s_{\mathrm{Z}}, s_{\mathrm{Z}}\,\dots\right)\,\Delta(s_1)\,\Delta(s_2) + N\,(s, s_1, s_2, \dots)
\end{aligned}
$$

where N denotes the remainder of the double expansion around $s_{1,2} = s_{\mathrm{Z}}$, $s = -(p_1 + p_2)^2$ and

$$
\Delta(s) = \frac{1}{s - s_{\mathrm{Z}}},
$$

$s_{\mathrm{H}}, s_{\mathrm{Z}}$ being the H, Z complex poles. Therefore, we define pseudo-observables PO-number!

$$
\mathbf{\Gamma}_{\boldsymbol{i}} = \int d\Phi_{1\to 4}\,\sum_{\text{spin}} \left|\,a_{ii}\,\left(s_{\mathrm{H}}, s_{\mathrm{Z}}, s_{\mathrm{Z}}\,\dots\right)\,\Delta(s_1)\,\Delta(s_2)\,\right|^2
$$

with similar definitions for Γ_{ij}

$\mathscr{G}$

POs (container) at LHC: summary table

① external layer (similar to σ_f^{peak})

$$\Gamma_{VV} \quad A_{FB}^{ZZ} \quad N_{off}^{4l} \quad \text{etc}$$

② intermediate layer (similar to g_{VA}^{e})

$$\rho_H^V \quad \mathscr{G}_L^V \quad \rho_H^{\gamma\gamma}, \rho_H^{\gamma Z} \quad \rho_H^f$$

③ internal layer

$$\kappa_f^{\gamma\gamma} \quad \kappa_W^{\gamma\gamma} \quad \kappa_i^{\gamma\gamma\,NF} \quad \text{etc}$$

④ internal layer (contained): Wilson coeff. or non-SM parameters in BSM (e.g. α, β, M_{sb} etc. in THDMs)

Lep heritage: fine points to remember when building POs (but not only)

- $\mathbf{H} \to \bar{\mathbf{f}}\mathbf{f}\boldsymbol{\gamma}$ defines Dalitz decay for isolated photons but is part of the real corrections to $\mathbf{H} \to \bar{\mathbf{f}}\mathbf{f}$ for IR/collinear photons.
- $\mathbf{H} \to \mathbf{4f}$ defines
  1. the four-body decay of the Higgs or
  2. pair production corrections to the two-body decays (with a primary and a secondary pair), depending on the invariant masses of the fermion pairs.

☛ Strategies? The whole $\mathbf{4f}$ is included in $\mathbf{H} \to \mathbf{2f}$ or part of it defines the $\mathbf{2f}$ signal and part the $\mathbf{4F}$ signal

❶❷ Who should provide POs?
Who should provide interpretation of POs, e.g. using LO EFT, NLO EFT, BSMs?

Well, Well, Well, its certainly a compelling provocative exciting to think about idea

In general, there should be a mapping between code parameters and whatever POs we define. Ideally, nothing in the calculation would change apart from the data card format that provides the input parameters.

The LHC M-code:

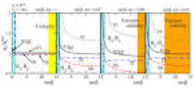
- ✗ For each process write down some (QFT-compatible) amplitude allowing for SM-deviations, both for signal and background (NLO EFT is a good example). Compute FOs.
- ✗ Insert Signal expressed through POs without altering the total. Please, do not subtract SM background (B changes too)
- ✗ Fit POs, Γ_{ZZ} (conventionally defined), A_{F}^{ZZ}, A_{FB}^{eW} etc., or $\rho_{H}^{V}, \mathcal{G}_{L}^{V}$ etc.
- ✗ Derive Wilson coefficients or BSM Lagrangian parameters
- ✗ Publish the full list of FOs (with modern rivet technology) and POs à la Lep (LHC legacy)

⓭ POs as a way to “compress” results. LHC legacy.

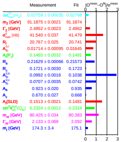
For each process compute the full answer within fiducial volumes

Another language: something is decaying into something else (on-shell) further decaying etc. Can we make it rigorous while keeping the total intact ? Yes, it’s PO!

Nobody will memorize what κ_{ijk}^{XYZ} is, but will remember what an asymmetry is (even when “spoiled” enough to become a PO). Let’s keep κ as a tool to (partly) get the UV-completion



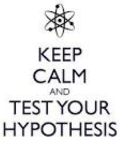
or



	Measurement	Fit
$\Delta\alpha^{(5)}_{had}(m_Z)$	0.02758 ± 0.00035	0.02768
m_Z [GeV]	91.1875 ± 0.0021	91.1874
Γ_Z [GeV]	2.4952 ± 0.0023	2.4962
σ^0_{had} [nb]	41.540 ± 0.037	41.479
R_l	20.767 ± 0.025	20.741
$A^{0,l}_{fb}$	0.01714 ± 0.00095	0.01645
$A_l(P_\tau)$	0.1465 ± 0.0032	0.1481
R_b	0.21629 ± 0.00066	0.21573
R_c	0.1721 ± 0.0030	0.1723
$A^{0,b}_{fb}$	0.0992 ± 0.0016	0.1038
$A^{0,c}_{fb}$	0.0707 ± 0.0035	0.0742
A_b	0.923 ± 0.020	0.935
A_c	0.670 ± 0.027	0.668
A_l(SLD)	0.1513 ± 0.0021	0.1481
$\sin^2\theta^{lept}_{eff}(Q_{fb})$	0.2324 ± 0.0012	0.2314
m_W [GeV]	80.425 ± 0.034	80.383
Γ_W [GeV]	2.133 ± 0.069	2.092
m_t [GeV]	174.3 ± 3.4	175.1

PO is the language which the deaf can hear and the blind can see

⓮ Beyond the SM, from the predictive (SM) phase to the "partially predictive (fitting)" one.



HEP phases

- PREDICTIVE phase: in any (strictly) renormalizable theory with $\boldsymbol{n}$ parameters you need to match $\boldsymbol{n}$ data points, the $(\boldsymbol{n}+1)$th calculation is a prediction, e.g. as doable in the SM
- FITTING (approximate predictive) phase: there are $(\boldsymbol{N_6}+\boldsymbol{N_8}+\cdots=\infty)$ renormalized Wilson coefficients that have to be fitted, e.g. measuring SM deformations due to a single $\mathscr{O}^{(6)}$ insertion ($\boldsymbol{N_6}$ enough for per mille accuracy)

⓯ TH uncertainties, not only QCD

☞ EW already discussed

☛ QCD? Well, Well, if faith can move mountains . . .

Summary on scale variation

- Choice of scale is a genuine ambiguity
- But size of scale variation knows little about physics, only about coefficients of the series
- Scale variation doesn't correctly handle case when coefficients grow large.

Can one do better? Possibly, e.g. by supplementing scale variation uncertainties with information on growth of coefficients (à la David–Passarino, maybe with simplifications)

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G. Salam https://indico.cern.ch/event/366472/

From Lep Jamboree 2003

Ontology: the Blue Band

The most celebrate figure of the LEP era: the blue-band.

I remember a meeting at Cern where I proposed to produce theoretical results with a $\Box$, reflecting our lack of knowledge of missing higher order corrections, instead of dimensionless $\circ$. There was an immediate consensus in the community. This is the progenitor of the blue-band.

This band was intended to

honestly show our degree of ignorance and,

several times, it was repeated that it should be used and interpreted with great care.

Actually there is no definition of *theoretical error* (only of theoretical stupidity) and one should not attach to it any meaning more deep than

modelling & selecting a set of options and
see how large is the band,

If it is too large then we better do a new calculation in that direction. If it is small yet it does not mean that we should take it as a rigorous bound.

Just remember, once you're over the hill you begin to pick up speed
(Arthur Schopenhauer)

Thank you for your attention