NLO SMEFT, MPE and POs
a global view

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This short note is about why NLO SMEFT + POs[1], and partly about

- ✗ how NLO[2]
- ✗ what NLO[3]
- ✗ how POs and MPE[4] without beating around the bush

fuel for more work to come ... nothing more

uncovered, recoverable material here ▸ go

[1] What can be said at all should be said clearly and whereof one cannot speak thereof one must be silent (Wittgenstein)

[2] Covered in “ATLAS Higgs (N)NLO MC and Tools Workshop for LHC RUN-2”, https://indico.cern.ch/event/345455/, see also https://indico.desy.de/conferenceDisplay.py?confId=476

[3] same as above

[4] Covered in “Pseudo-observables: from LEP to LHC”, https://indico.cern.ch/event/373667/

How/what NLO?

- ✓ Start with Warsaw basis, full set, write down Lagrangian and Feynman rules ■
- ✓ Normalize the quadratic part of the Lagrangian and pay due attention to the FP ghost sector ■
- ✓ Compute (all) self-energies (up to one $\mathscr{O}_{\mathbf{dim=6}}$ insertion), write down counterterms, make self-energies UV finite ■
- ✓ Compute the set of processes you like/want (don't forget non-SM topologies), mix Wilson coefficients to make them UV finite, check closure under renormalization ■
- ✓ Perform finite renormalization, selecting a scheme (better the $G_{\mathbf{F}}$-scheme), introduce wave-function factors, get the answer ■
- ✓ Start making approximations now (if you like), e.g. neglecting operators etc.■

How/what NLO? (cont.)

- ✓ Transform the answer in terms of κ-shifted SM sub-amplitudes and non SM factorizable sub-amplitudes ■
- ✓ Derive κ-parameters in terms of Wilson coefficients ■
- ✓ Write Pseudo-Observables in terms of κ-parameters ■
- ✓ Decide about strategy for including EWPD ■
- ✓ Claim you invented the whole procedure □

NLO is like biking, you learn it when you are a kid

■ Fade Out ■ Round House ■ Fast Pace □ Coked Pistol

How/what NLO? 

- ✓ Are there some pieces that contain the dominant NLO effects?
- ✓ It depends on the TH bias:

   For EFT purists there is no model independent EFT statement on some operators being big and other small

  Remember, logarithms are not large, constants matter too

- ✓ which could be easily incorporated in other calculations/tools? (Well, Well, Well, its certainly a compelling provocative exciting to think about idea)

How/what NLO? 

✓ NLO SMEFT availability? From arXiv:1505.03706

① Counterterms (SM fields and parameters): all

② Mixing: those entries related to $\mathbf{H \to \gamma\gamma, Z\gamma, ZZ, WW}$

③ Self-energies, complete and at $\boldsymbol{p^2 = 0}$: all

④ Amplitudes, sub-amplitudes (both SM and non-factorizable, full PTG + LG scenario)

❶ $\mathbf{H \to \gamma\gamma}$ ❷ $\mathbf{H \to Z\gamma}$ ❸ $\mathbf{H \to ZZ, WW}$[5] ❹ $\mathbf{H \to \bar{f}f, gg \to H}$
(the latter available, although not public)

⑤ EWPD, $\boldsymbol{M_W}$, $\mathbf{T}$-parameter; $\mathbf{Z \to \bar{f}f}$ available, although not public.

[5] Green's functions in well-defined kinematic limit, i.e. residue of the poles after extracting the parts which are 1P reducible, which means MPE

No NP yet?

A study of SM-deviations: here the reference process is

$$\mathbf{H} \to \gamma\gamma$$

✓ κ-approach: write the amplitude as

$$\mathbf{A} \quad = \quad \sum_{i=\mathrm{t,b,W}} \kappa_i \mathscr{A}^i + \kappa_c$$

$\mathscr{A}^{\mathrm{t}}$ being the SM t-loop etc. The contact term (which is the LO SMEFT) is given by κ_c. Furthermore

$$\kappa_i = 1 + \Delta\kappa_i \qquad i \neq c$$

✓ For the sake of simplicity assume

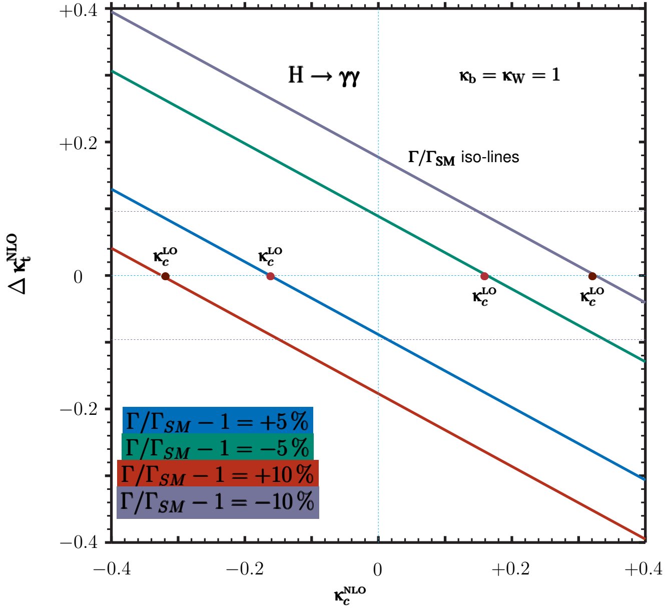
$$\kappa_b = \kappa_W = 1 \quad \left(\kappa_W^{\text{exp}} = 0.95^{+0.14}_{-0.13}\ \textit{ATLAS}\ \ 0.96^{+35}_{-16}\ \textit{CMS}\right)$$

and compute

$$✎ \quad \kappa_\gamma \mapsto \mathbf{R} = \Gamma\left(\kappa_t\,,\,\kappa_c\right)/\Gamma_{SM} - 1 \quad [\%]$$

In LO SMEFT κ_c is non-zero and $\kappa_t = 1$ [6]. You measure a deviation and you get a value for κ_c. However, at NLO $\Delta\kappa_t$ is non zero and you get a degeneracy. The interpretation in terms of κ_c^{LO} or in terms of $\{\kappa_c^{\text{NLO}}, \Delta\kappa_t^{\text{NLO}}\}$ could be rather different.

[6] Certainly true in the linear realization



$$\Gamma\left(\Delta\kappa_t\,,\,\kappa_c\right) = \left(42.29 - 23.87\,\Delta\kappa_t - 13.01\,\kappa_c\right)\,\frac{G_F\alpha^2}{128\sqrt{2}\pi^3}\,M_H^3$$

Fitting is not interpreting

Of course, depending on what you measure, the corresponding interpretation could tell us that the required kappas or Wilson coefficients are too large to allow for a meaningful interpretation in terms of a weakly coupled UV completion[7]

hi-tech center Caveat: SMEFT interpretation should include LO SMEFT and (at least) RGE modified predictions (arXiv:1301.2588); furthermore, full one-loop SMEFT gives you (new) logarithmic and constant terms that are not small compared to the one from RGE, see arXiv:1505.02646, arXiv:1505.03706

For interpretations other than weakly coupled renormalizable, see arXiv:1305.0017
EFT purist: there is no model independent EFT statement on some operators being big and other small (arXiv:1305.0017)

[7] Simpler theories are preferable to more complex ones because they are better testable and falsifiable

Going interpretational

$$\mathrm{A}_{\mathrm{SMEFT}} = \frac{g^2 s_\theta^2}{8\pi^2}\Big[\sum_{i=\mathrm{t,b,w}} \kappa_i \mathscr{A}^i + \frac{g_6}{g^2 s_\theta^2}\frac{M_{\mathrm{H}}^2}{M_{\mathrm{W}}^2} 8\pi^2 a_{\mathrm{AA}}\Big]$$

✓ **Assumption**: use arXiv:1505.03706, work in the Einhorn-Wudka PTG scenario (arXiv:1307.0478), adopt Warsaw basis (arXiv:1008.4884)

① LO SMEFT: $\kappa_i = 1$ and a_{AA} is scaled by $1/16\pi^2$ being LG

② NLO PTG-SMEFT: $\kappa_i \neq 1$ but only PTG operators inserted in loops (non-factorizable terms absent), a_{AA} scaled as above

At NLO, $\Delta\kappa = g_6\,\rho$ and $a_{\mathrm{AA}} = s_\theta^2\, a_{\phi\mathrm{W}} + c_\theta^2\, a_{\phi\mathrm{B}} + s_\theta c_\theta\, a_{\phi\mathrm{WB}}$

$$\mathscr{A}_{\mathrm{SMEFT}} = \sum_{i=\mathrm{t,b,w}} (1 + g_6\,\rho_i)\,\mathscr{A}^i + g_c\, a_{\mathrm{AA}}$$

Warsaw basis

$$g_6^{-1} = \sqrt{2}\, G_F \Lambda^2$$

$$g_c = \frac{1}{2}\frac{g_6}{g^2 s_\theta^2}\frac{M_H^2}{M_W^2}$$

$$\rho_t = -\frac{1}{2}\left[a_{\phi D} - 2 s_\theta^2 (a_{t\phi} + 2 a_{\phi\square})\right]\frac{1}{s_\theta^2}$$

$$\rho_b = -\frac{1}{2}\left[a_{\phi D} + 2 s_\theta^2 (a_{b\phi} - 2 a_{\phi\square})\right]\frac{1}{s_\theta^2}$$

$$\rho_W = -\frac{1}{2}\left[a_{\phi D} - 4 s_\theta^2 a_{\phi\square}\right]\frac{1}{s_\theta^2}$$

$$\Gamma_{SMEFT} = \frac{\alpha^2 G_F M_H^3}{32\sqrt{2}\pi^3}\frac{M_W^4}{M_H^4}\,|\mathscr{A}_{SMEFT}|^2 \qquad \Gamma_{SM} = \Gamma_{SMEFT}\Big|_{\Delta\kappa_i=0,\,\kappa_c=0}$$

Relaxing the PTG assumption introduces non-factorizable sub-amplitudes proportional to $a_{tW}, a_{tB}, a_{bW}, a_{bB}, a_{\phi W}, a_{\phi B}, a_{\phi WB}$ with a mixing among $\{a_{\phi W}, a_{\phi B}, a_{\phi WB}\}$. Meanwhile, renormalization has made one-loop SMEFT finite, e.g. in the G_F-scheme, with a residual μ_R-dependence

✓ Demonstration strategy:

① Allow each Wilson coefficient to vary in the interval $\mathbf{I_2} = [-2, +2]$ (naturalness[8]; put $\Lambda = 3\ \mathit{TeV}$, conventional point)

② LO: generate points from $\mathbf{I_2}$ for a_{AA} with uniform probability and calculate $\mathbf{R_{LO}}$

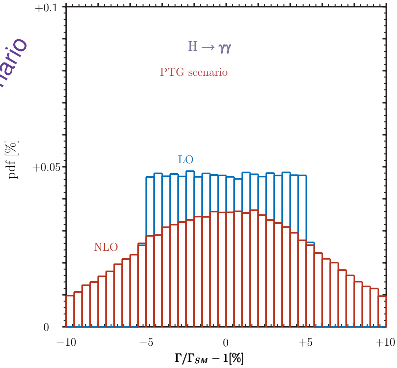
② NLO: generate points from $\mathbf{I_2^5}$ for $\{a_{\phi D}, a_{\phi \Box}, a_{t\phi}, a_{b\phi}, a_{AA}\}$ with uniform probability and calculate $\mathbf{R_{NLO}}$

③ Calculate the $\mathbf{R}$ pdf

N.B. $|\, a_{AA} \,| < 1$ is equivalent to $|\, g_c\, a_{AA} \,| < 8.6\, 10^{-2}$

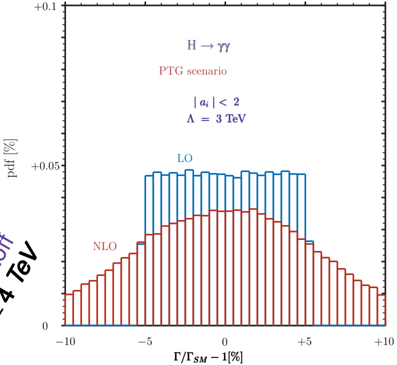
[8]Disregarding TH bias for the sign (Sect. D of arXiv:0907.5413)

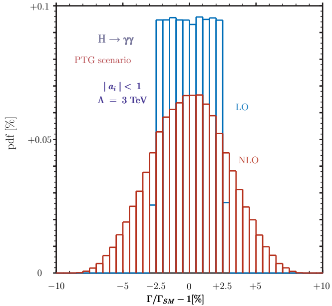
Benchmark scenario

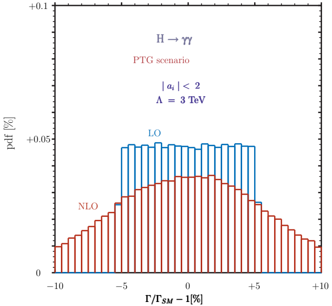


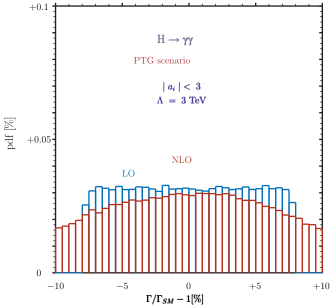
Changing the cutoff

$\Lambda = 4$ TeV









Changing the interval

$\mathcal{G}$

The inversion problem ad usum insane graphi There are correlations among different observables, and constraints too, e.g.

$$\Delta\kappa_b^{HAZ} - \Delta\kappa_t^{HAZ} = \Delta\kappa_b^{HAA} - \Delta\kappa_t^{HAA}$$

$$c_\theta^2\,\Delta\kappa_W^{HAZ} + \left(\frac{3}{2} + 2\,c_\theta^2\right)\Delta\kappa_t^{HAZ} = \left(\frac{3}{2} + 2\,c_\theta^2\right)\Delta\kappa_t^{HAA} - \left(\frac{1}{2} + 3\,c_\theta^2\right)\Delta\kappa_W^{HAA}$$

$$a_{t\phi} = \frac{1}{2s_\theta^2}\,a_{\phi D} - 2\,a_{\phi\Box} + \Delta\kappa_t^{HAA}$$

$$a_{b\phi} = -\frac{1}{2s_\theta^2}\,a_{\phi D} + 2\,a_{\phi\Box} - \Delta\kappa_b^{HAA}$$

$$a_{\phi\Box} = \frac{1}{4s_\theta^2}\,a_{\phi D} + \frac{1}{2}\,\Delta\kappa_W^{HAA}$$

$$2\,c_\theta^2\,a_{\phi D} = s_\theta^2\left(\Delta\kappa_b^{HAZ} - \Delta\kappa_b^{HAA}\right)$$

Conclusions:

① For the SMEFT, (almost) regardless of the κ_c, to have more than 5% deviation (at $\Lambda = 3\ TeV$) you have to go NLO, or unnatural[9] (Wilson coefficients not $\mathcal{O}(1)$)

② The LO, NLO distributions are different, therefore interpretation is different, how to reweight once your analysis was LO interpreted? It all depends on the new central value for $\kappa_\gamma^{\mathbf{exp}}$

presently ATLAS: $a_{\rm AA}^{\rm LO} = +3.79^{+5.31}_{-6.06}$ CMS: $a_{\rm AA}^{\rm LO} = -5.31^{+4.93}_{-4.55}$

naive dimensional estimate $a_{\rm AA} \approx 1$

③ *Chi ha avuto, ha avuto, ha avuto ... chi ha dato, ha dato, ha dato ... scurdammoce o ppassato*
Those who've taken, taken, taken ... Those who've given, given, given ... Let's forget about the past

[9]from the point of view of a weakly coupled UV completion

interpretation: POs à la LEP

https://indico.cern.ch/event/373667/ arXiv:1504.04018

$$\mathrm{H} \to \gamma\gamma\,(\gamma \mathrm{Z}) \quad \mapsto \quad \rho_{\mathrm{H}}^{\gamma\gamma(\mathrm{Z})}\,\frac{p_1 \cdot p_2\, g^{\mu\nu} - p_2^{\mu} p_1^{\nu}}{M_{\mathrm{H}}}$$

via MPE $$\mathrm{H} \to \mathrm{VV} \quad \mapsto \quad \rho_{\mathrm{H}}^{\mathrm{V}} \left(M_{\mathrm{H}}\, g^{\mu\nu} + \frac{\mathscr{G}_{\mathrm{L}}^{\mathrm{V}}}{M_{\mathrm{H}}}\, p_2^{\mu}\, p_1^{\nu} \right)$$

$$\mathrm{H} \to \overline{\mathrm{b}}\mathrm{b} \quad \mapsto \quad \rho_{\mathrm{H}}^{\mathrm{b}}\, \overline{\mathrm{u}}\, \mathrm{v}$$

etc. Production? Analyticity and crossing symmetry

a middle way language

wolf, goat, and cabbage

POs (container) at LHC: summary table

① external layer[a] (similar to LEP $\sigma_{\mathrm{f}}^{\mathrm{peak}}$)

$$(\sum_{\mathrm{f}})\,\Gamma_{\mathrm{Vff}} \quad A_{\mathrm{FB}}^{\mathrm{ZZ}} \quad N_{\mathrm{off}}^{41} \quad \text{etc}$$

not as trivial as NWA or truncated MPE

② intermediate layer (similar to LEP $g_{\mathrm{V\,A}}^{\mathrm{e}}$)

$$\rho_{\mathrm{H}}^{\mathrm{V}} \quad \mathcal{G}_{\mathrm{L}}^{\mathrm{V}} \quad \rho_{\mathrm{H}}^{\gamma\gamma},\, \rho_{\mathrm{H}}^{\gamma\mathrm{Z}} \quad \rho_{\mathrm{H}}^{\mathrm{f}}$$

③ internal layer: the kappas

$$\kappa_{\mathrm{f}}^{\gamma\gamma} \quad \kappa_{\mathrm{W}}^{\gamma\gamma} \quad \kappa_{i}^{\gamma\gamma\,\mathrm{NF}} \quad \text{etc}$$

④ innermost layer: Wilson coeff. or non-SM parameters in BSM (e.g. $\alpha, \beta, M_{\mathrm{sb}}$ etc. in THDMs)

more

uni-

ver-

sal

[a] where kinematics cannot be manipulated

Everything changes and nothing remains still ... and ... you cannot step twice into the same stream (Heraclitus)

Return

Appendix C. Dimension-Six Basis Operators for the SM[22].

Einhorn, Wudka

Grzadkowski, Iskrzynski, Misiak, Rosiek

X^3 (LG)		φ^6 and $\varphi^4 D^2$ (PTG)		$\psi^2\varphi^3$ (PTG)	
Q_G	$f^{ABC} G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	Q_φ	$(\varphi^\dagger\varphi)^3$	$Q_{e\varphi}$	$(\varphi^\dagger\varphi)(\bar l_p e_r \varphi)$
$Q_{\widetilde G}$	$f^{ABC} \widetilde G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	$Q_{\varphi\Box}$	$(\varphi^\dagger\varphi)\Box(\varphi^\dagger\varphi)$	$Q_{u\varphi}$	$(\varphi^\dagger\varphi)(\bar q_p u_r \widetilde\varphi)$
Q_W	$\varepsilon^{IJK} W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$	$Q_{\varphi D}$	$\left(\varphi^\dagger D^\mu\varphi\right)^\star \left(\varphi^\dagger D_\mu\varphi\right)$	$Q_{d\varphi}$	$(\varphi^\dagger\varphi)(\bar q_p d_r \varphi)$
$Q_{\widetilde W}$	$\varepsilon^{IJK} \widetilde W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$				
$X^2\varphi^2$ (LG)		$\psi^2 X\varphi$ (LG)		$\psi^2\varphi^2 D$ (PTG)	
$Q_{\varphi G}$	$\varphi^\dagger\varphi\, G^A_{\mu\nu} G^{A\mu\nu}$	Q_{eW}	$(\bar l_p \sigma^{\mu\nu} e_r) \tau^I \varphi W_{\mu\nu}^I$	$Q_{\varphi l}^{(1)}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar l_p \gamma^\mu l_r)$
$Q_{\varphi\widetilde G}$	$\varphi^\dagger\varphi\, \widetilde G^A_{\mu\nu} G^{A\mu\nu}$	Q_{eB}	$(\bar l_p \sigma^{\mu\nu} e_r) \varphi B_{\mu\nu}$	$Q_{\varphi l}^{(3)}$	$(\varphi^\dagger i\overleftrightarrow{D}^I_\mu \varphi)(\bar l_p \tau^I \gamma^\mu l_r)$
$Q_{\varphi W}$	$\varphi^\dagger\varphi\, W^I_{\mu\nu} W^{I\mu\nu}$	Q_{uG}	$(\bar q_p \sigma^{\mu\nu} T^A u_r) \widetilde\varphi\, G_{\mu\nu}^A$	$Q_{\varphi e}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar e_p \gamma^\mu e_r)$
$Q_{\varphi\widetilde W}$	$\varphi^\dagger\varphi\, \widetilde W^I_{\mu\nu} W^{I\mu\nu}$	Q_{uW}	$(\bar q_p \sigma^{\mu\nu} u_r) \tau^I \widetilde\varphi\, W_{\mu\nu}^I$	$Q_{\varphi q}^{(1)}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar q_p \gamma^\mu q_r)$
$Q_{\varphi B}$	$\varphi^\dagger\varphi\, B_{\mu\nu} B^{\mu\nu}$	Q_{uB}	$(\bar q_p \sigma^{\mu\nu} u_r) \widetilde\varphi\, B_{\mu\nu}$	$Q_{\varphi q}^{(3)}$	$(\varphi^\dagger i\overleftrightarrow{D}^I_\mu \varphi)(\bar q_p \tau^I \gamma^\mu q_r)$
$Q_{\varphi\widetilde B}$	$\varphi^\dagger\varphi\, \widetilde B_{\mu\nu} B^{\mu\nu}$	Q_{dG}	$(\bar q_p \sigma^{\mu\nu} T^A d_r) \varphi\, G_{\mu\nu}^A$	$Q_{\varphi u}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar u_p \gamma^\mu u_r)$
$Q_{\varphi WB}$	$\varphi^\dagger\tau^I\varphi\, W^I_{\mu\nu} B^{\mu\nu}$	Q_{dW}	$(\bar q_p \sigma^{\mu\nu} d_r) \tau^I \varphi\, W_{\mu\nu}^I$	$Q_{\varphi d}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar d_p \gamma^\mu d_r)$
$Q_{\varphi\widetilde WB}$	$\varphi^\dagger\tau^I\varphi\, \widetilde W^I_{\mu\nu} B^{\mu\nu}$	Q_{dB}	$(\bar q_p \sigma^{\mu\nu} d_r) \varphi\, B_{\mu\nu}$	$Q_{\varphi ud}$	$i(\widetilde\varphi^\dagger D_\mu \varphi)(\bar u_p \gamma^\mu d_r)$

Table C.1: Dimension-six operators other than the four-fermion ones.

[22] These tables are taken from [5], by permission of the authors.

SMEFT evolution

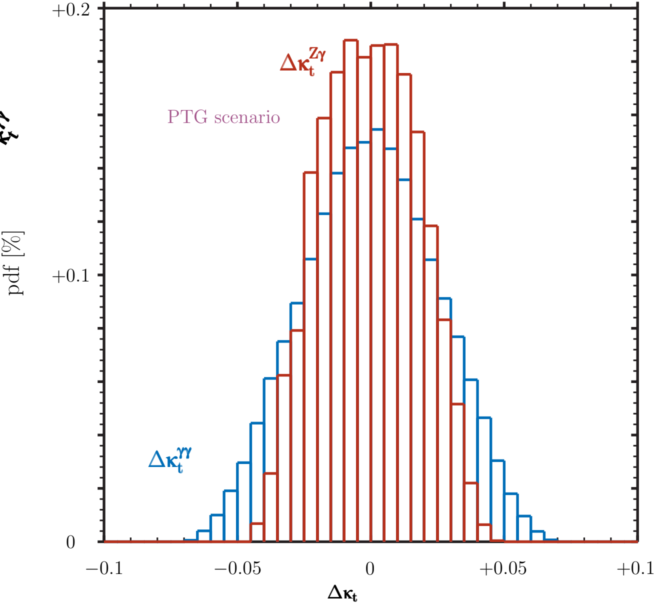
LO $\mathscr{A}^{\mathrm{SMEFT}} = \mathscr{A}^{\mathrm{SM}} + a_i$, where $a_i \in \mathbf{V}_6$ and $\mathbf{V}_6$ is the set of $\dim = 6$ Wilson coefficients

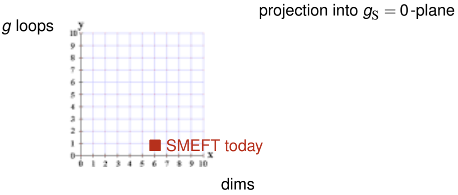
RGE $a_i \to \mathrm{Z}_{ij}(\mathrm{L})\, a^j$, where $\mathrm{L} = \ln(\Lambda / M_{\mathrm{H}})$ and $i, j \in \mathbf{H}_6 \subset \mathbf{V}_6$

NLO $\mathscr{A}^{\mathrm{SMEFT}} = \mathscr{A}^{\mathrm{SM}} + \mathscr{A}_k\,(\mathrm{L}, \mathrm{const})\; a^k$, where $k \in \mathbf{S}_6$ and $\mathbf{H}_6 \subset \mathbf{S}_6 \subset \mathbf{V}_6$

$\mathscr{G}$

$\kappa_t^{Z\gamma} \neq \kappa_t^{\gamma\gamma}$





① Each loop = multiply by g^2 (g is the $SU(2)$ coupling constant)

② Each $\dim + 2$ = multiply by $g_6 = 1/(G_F \Lambda^2)$

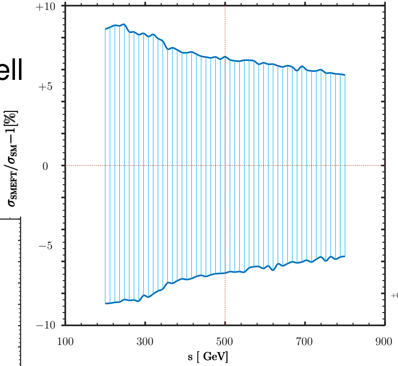
③ Warning: when squaring the amplitude respect the order in powers of g and of g_6

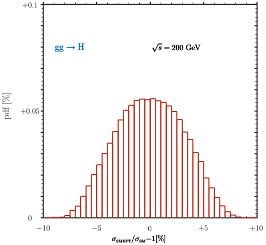
④ be carefull with Λ or you will claim NP simply because you are missing 2 loops SM.

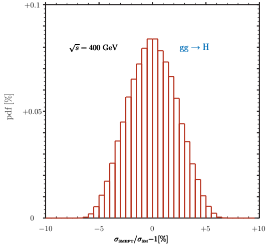
gg → H off-shell

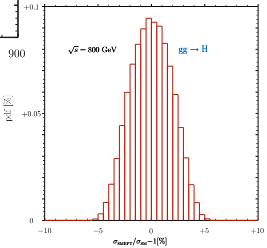
$|\,a_i\,| \in [-1\,,+1]$

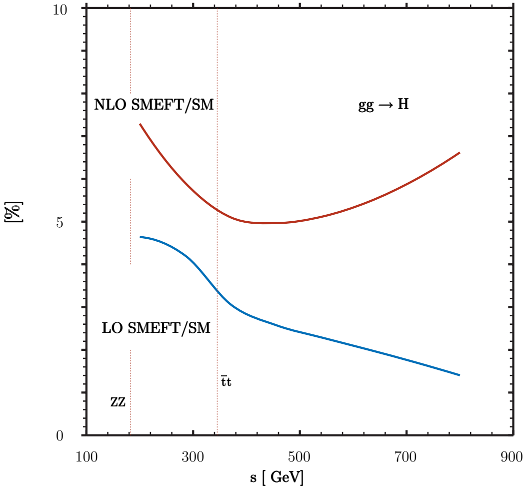
$\Lambda = 3\ TeV$











Another reason to go NLO

The contact term is real . . .

$$a_i = 1,\ \forall i$$
$$\Lambda = 3\ TeV$$

Scenarios for understanding SM deviations in (especially tails of) distributions:

A use SMEFT and stop where you have to stop, it is an honest assessment of our ignorance

B improve SMEFT with $\mathrm{dim} = 8$ (but this will not be enough)

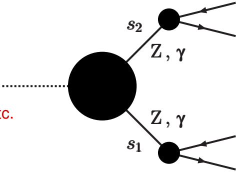
C use the kappa–BSM-parameters connection, i.e. replace SMEFT with BSM models, especially in the tails, optimally matching to SMEFT at lower scales

D introduce binned POs

Multi Pole Expansion

In any process, the residues of the poles (starting from maximal degree) are numbers.
The non-resonant part is a multivariate function and requires some basis.
That is to say, residue of the poles can be POs by themselves, expressing them in terms of other objects is an operation the can be postponed.
The very end of the chain, no poles left, requires (almost) model independent SMEFT or model dependent BSM.
Numerically speaking, it depends on the impact of the non-resonant part wich is small in gluon-fusion (ggF) but not in Vector Boson Scattering (VBS).

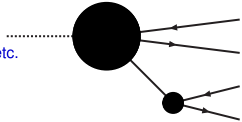
MPE: crab expansion



$\Gamma(\mathrm{H} \to \mathrm{ZZ})$ etc.

$$\frac{\mathscr{A}_{\mathrm{DR}}(s_1, s_2; \dots)}{(s_1 - s_Z)(s_2 - s_Z)} = \frac{\mathscr{A}_{\mathrm{DR}}(s_Z, s_Z; \dots)}{(s_1 - s_Z)(s_2 - s_Z)} + \frac{\mathscr{A}^{(2)}_{\mathrm{DR}}(s_Z, s_2; \dots)}{s_1 - s_Z}$$

$$\dots + \mathscr{A}^{\mathrm{rest}}_{\mathrm{DR}}(s_1, s_2; \dots)$$

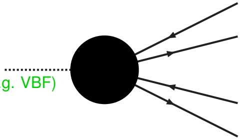


$\Gamma(\mathrm{H} \to \bar{\mathrm{f}}\mathrm{f}\gamma)$ etc.

$$\frac{\mathscr{A}_{\mathrm{SR}}(s_1; \dots)}{s_1 - s_Z} = \frac{\mathscr{A}_{\mathrm{SR}}(s_Z; \dots)}{s_1 - s_Z} + \mathscr{A}^{\mathrm{rest}}_{\mathrm{SR}}(s_1; \dots)$$

remember LEP

$$\sigma^{\mathrm{peak}}_{\mathrm{f}} = 12\pi \frac{\Gamma_{\mathrm{e}}\Gamma_{\mathrm{f}}}{M_Z^2 \Gamma_Z^2}$$



the difficult part (e.g. VBF)

$$\mathscr{A}_{\mathrm{NR}}(\dots)$$

$$+ \; (\mathrm{Z} \to \gamma)$$

directly POs

residue of poles ⇒ one number ⇐ interpretation: $\kappa \times$ sub-amplitudes

non-resonant ⇒ NAN ⇐ $\kappa \times$ sub-amplitudes needed even before interpretation

or dense binning in (say) p_{T} ⇐ interpretation: $\kappa \times$ sub-amplitudes (C used to “interpret” D!)

Going “far” off-shell. The κ-framework for BSM models (Singlet, THDMs, etc). Here THDM type I.

$$
\begin{aligned}
\mathscr{A}_{\mathrm{h}\to\gamma\gamma}(s) \quad \mapsto \quad & i\,\frac{g^2 s_\theta^2}{8\,\pi^2}\,\left(p_1\cdot p_2\,g^{\mu\nu} - p_2^\mu p_1^\nu\right) \\
\times \quad & \Big\{ \frac{\cos\alpha}{\sin\beta} \sum_{\mathrm{f}} \mathscr{A}_{\mathrm{f}}^{\mathrm{SM}} - \sin(\alpha-\beta)\,\mathscr{A}_{\mathrm{bos}}^{\mathrm{SM}} \\
+ \quad & \Big[\left(M_{\mathrm{sb}}^2 + s\right) \cos(\alpha-\beta)\cos 2\beta \\
- \quad & \left(2\,M_{\mathrm{sb}}^2 + s + 2\,M_{\mathrm{H}^+}^2\right) \sin(\alpha-\beta)\sin 2\beta \Big]\,\mathscr{A}_{\mathrm{H}^+}^{\mathrm{SM}} \Big\}
\end{aligned}
$$

where M_{sb} is the Z_2 soft-breaking scale, $\mathrm{h}(\mathrm{H})$ are the light(heavy) scalar Higg bosons. The h virtuality is s. The coeff are κs, $\mathscr{A}_{\mathrm{H}^+}^{\mathrm{SM}}$ is the “resolved” H^+ loop, becoming the contact term of SMEFT in the limit $M_{\mathrm{H}^+} \to \infty$.

The dual role of MPE

① Poles and their residues are intimately related to the gauge invariant splitting of the amplitude (Nielsen identities)

② Residues of poles (eventually after integration over residual variables) can be interpreted as POs (factorization)

Gauge invariant splitting is not the same as "factorization" of the process into sub-processes, indeed

Phase space factorization requires the pole to be inside the physical region

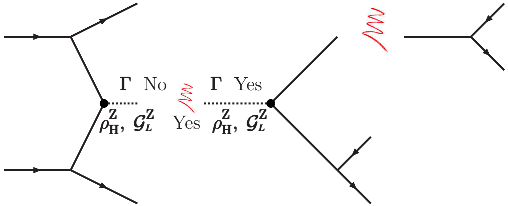
$$\Delta = \frac{1}{(s-M^2)^2 + \Gamma^2 M^2} = \frac{\pi}{M\Gamma}\,\delta\left(s-M^2\right) + \mathrm{PV}\left[\frac{1}{(s-M^2)^2}\right]$$

$$d\Phi_n(P,p_1\dots p_n) = \frac{1}{2\pi}\,dQ^2\,d\Phi_{n-j+1}\left(P,Q,p_{j+1}\dots p_n\right)\,d\Phi_j\left(Q,p_1\dots p_j\right)$$

To "complete" the decay ($d\Phi_j$) we need the δ-function …

The δ-part of the resonant propagator opens the line

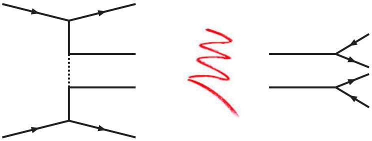
the right cut



$$\sigma(qq \to \bar{f} f \bar{f}' f' jj) \xmapsto{PO} \sigma(qq \to Hjj) \otimes \Gamma(H \to Z\bar{f}f) \otimes \Gamma(Z \to \bar{f}' f')$$

The δ-part of the resonant propagator opens the line

t-channel propagators cannot be cut



$$\sigma(qq \to \bar{f} f \bar{f}' f' jj) \stackrel{PO}{\longmapsto} \sigma(qq \to ZZjj) \otimes \Gamma(Z \to \bar{f} f) \otimes \Gamma(Z \to \bar{f}' f')$$

External and intermediate layers are complementary

but not always interchangeable

$\mathscr{G}$

Factorizing into "physical" sub-processes (external POs): fine points

❶ Process: $\mathscr{A} = \mathscr{A}^{(1)}_{\mu}\,\Delta_{\mu\nu}(p)\,\mathscr{A}^{(2)}_{\nu}$

❷ Replace: $\Delta_{\mu\nu} \to \frac{1}{s-s_c}\sum_{\lambda} \varepsilon_{\mu}(p,\lambda)\,\varepsilon^{*}_{\nu}(p,\lambda)$

❸ Obtain

$$| \mathscr{A} |^2 = \frac{1}{| s - s_c |^2} \Big| \Big[\mathscr{A}^{(1)} \cdot \varepsilon \Big] \Big[\mathscr{A}^{(2)} \cdot \varepsilon^{*} \Big] \Big|^2$$

❹ Extract the δ from the propagator, factorize phase space … but you don't have what you need, i.e.

$$\sum_{\lambda} \Big| \mathscr{A}^{(1)} \cdot \varepsilon(p,\lambda) \Big|^2 \sum_{\sigma} \Big| \mathscr{A}^{(2)} \cdot \varepsilon(p,\sigma) \Big|^2$$

Factorization continued

❺ *iff* cuts are not introduced, the interference terms among different helicities oscillate over the phase space and drop out

❻ MPE or "asymptotic expansion" means that no NWA is performed but, instead, the phase space decompostion obtains by using the two parts in the propagator expansion.

① The δ-term is what we need to reconstruct (external) POs

② the PV-term gives the remainder

Since the problem is extracting pseudo-observables, analytic continuation is performed only after integrating over residual variables.

It is an error to believe that rigour is the enemy of simplicity. On the contrary we find it confirmed by numerous examples that the rigorous method is at the same time the simpler and the more easily comprehended

Not only decay (cf. arXiv:1502.02990)

$$\mathrm{u}(p_1)+\mathrm{u}(p_2)\to\mathrm{u}(p_3)+\mathrm{e}^-(p_4)+\mathrm{e}^+(p_5)+\mu^-(p_6)+\mu^+(p_7)+\mathrm{u}(p_8) \quad \text{LO SMEFT}$$

$$\begin{aligned}
J^\mu_\pm(p_i\,,p_j) &= \overline{u}(p_i)\,\gamma^\mu\,\gamma_\pm\,u(p_j)\\
\mathscr{A}^{\mathrm{TR}}_{\mathrm{LO}} &= \Big[J^\mu_-(p_4\,,p_5)\,(1-v_{\mathrm{l}})+J^\mu_+(p_4\,,p_5)\,(1+v_{\mathrm{l}})\Big]\\
&\times \Big[J^-_\mu(p_6\,,p_7)\,(1-v_{\mathrm{l}})+J^+_\mu(p_6\,,p_7)\,(1+v_{\mathrm{l}})\Big]\\
&\times \Big[J^\nu_-(p_3\,,p_2)\,(1-v_{\mathrm{u}})+J^\nu_+(p_3\,,p_2)\,(1+v_{\mathrm{u}})\Big]\\
&\times \Big[J^-_\nu(p_8\,,p_1)\,(1-v_{\mathrm{u}})+J^+_\nu(p_8\,,p_1)\,(1+v_{\mathrm{u}})\Big]\\
\Delta^{-1}_\Phi(p) &= p^2+M^2_\Phi\\
\mathscr{A}^{\mathrm{TR}}_{\mathrm{SMEFT}} &= \frac{g^6}{4096}\,\Delta_{\mathrm{H}}(q_1+q_2)\prod_{i=1,4}\Delta_{\mathrm{Z}}(q_i)\,\frac{M^2_{\mathrm{W}}}{c^8_\theta}\,\kappa_{\mathrm{LO}}\,\mathscr{A}^{\mathrm{TR}}_{\mathrm{LO}}+g^6\,g_6\,\mathscr{A}^{\mathrm{TR;nf}}_{\mathrm{SMEFT}}\\
\Delta\kappa_{\mathrm{LO}} &= 2\,a_{\phi\,\square}+\frac{2\,M^2_{\mathrm{Z}}-2\,M^2_{\mathrm{H}}+q_1\cdot q_2+q_2\cdot q_2}{M^2_{\mathrm{W}}}\,c^2_\theta\,a_{\mathrm{ZZ}}
\end{aligned}$$

$$q_1=p_8-p_1,\; q_2=p_3-p_2,\; q_3=p_4+p_5,\; q_4=p_6+p_7$$

𝒢