Offshellness and EFT

strategies to measure the UV completion

Giampiero Passarino

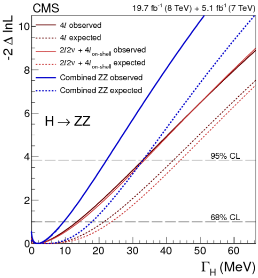
Dipartimento di Fisica Teorica, Università di Torino, Italy
INFN, Sezione di Torino, Italy

Padova, 3 December, 2014

Chiara Mariotti, André David and Michael Duehrssen

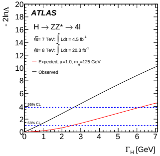
CMS-HIG-14-002, ATLAS-CONF-2014-042

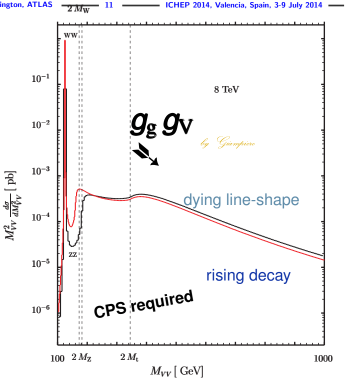
Facts of live



Direct Higgs width measurement

- *N.B.: see earlier talk in this session for indirect width measurement.*
- Analytical m_{4l} (non-relativistic Breit-Wigner) model convoluted with detector resolution with width Γ_H (m_H and μ free parameters) ($\Gamma_H = 4$ MeV at 125 GeV)
- Analysis assumes no interference with background processes
- $H \to ZZ^* \to 4l$:
  - Event-by-event modelling of detector resolution
  - Per-lepton resolution functions use sums of 2(3) Gaussians for muons (electrons)
  - Validated by fitting mass peak for $Z \to 4l$ using convolution of detector response with BW for Z mass
  - 95% CL: $\Gamma_H < 2.6$ GeV (exp. limit 3.5 GeV for $\mu = 1.7$, 6.2 GeV for $\mu = 1$)
- $H \to \gamma\gamma$:
  - 95% CL: $\Gamma_H < 5.0$ GeV (expected limit 6.2 GeV for $\mu = 1$)



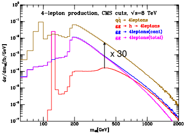


The big picture @ 8TeV

- Peak at Z mass due to singly resonant diagrams.
- Interference is an important effect.
- Destructive at large mass, as expected.
- With the standard model width, $H, challenging to see enhancement/deficit due to Higgs channel.

$p_{T,\mu} > 5$ GeV, $|\eta_\mu| < 2.4$,
$p_{T,e} > 7$ GeV, $|\eta_e| < 2.5$,
$m_{ll} > 4$ GeV, $m_{4\ell} > 100$ GeV.

CMS cuts
CMS PAS HIG-13-002



OFF – SHELL I

When a particle physicist describes something as "off mass-shell", they could be referring to a precise bit of quantum mechanics, or denouncing an unrealistic budget estimate, J. Butterworth

We define an **off-shell production cross-section** (for all channels) as follows:

$$\sigma^{\text{prop}}_{ij\to\text{all}} = \frac{1}{\pi}\,\sigma_{ij\to\mathrm{H}}\,\frac{s^2}{\left|s - s_{\mathrm{H}}\right|^2}\,\frac{\Gamma^{\text{tot}}_{\mathrm{H}}}{\sqrt{s}}$$

☞ When the cross-section $ij \to \mathrm{H}$ refers to an off-shell Higgs boson the choice of the QCD scales should be made according to the virtuality and not to a fixed value. Therefore, for the PDFs and $\sigma_{ij\to\mathrm{H+X}}$ one should select $\mu_{\mathrm{F}}^2 = \mu_{\mathrm{R}}^2 = z\,s/4$ ($z\,s$ being the invariant mass of the detectable final state).

We define an **off-shell production cross-section** (for all channels) as follows:

$$\sigma^{\rm prop}_{ij\to {\rm all}} = \frac{1}{\pi}\,\sigma_{ij\to {\rm H}}\,\frac{s^2}{\left|s - s_{\rm H}\right|^2}\,\frac{\Gamma^{\rm tot}_{\rm H}}{\sqrt{s}}$$

☞ When the cross-section $ij \to \mathrm{H}$ refers to an off-shell Higgs boson the choice of the QCD scales should be made according to the virtuality and not to a fixed value. Therefore, for the PDFs and $\sigma_{ij\to \mathrm{H+X}}$ one should select $\mu_F^2 = \mu_R^2 = zs/4$ (zs being the invariant mass of the detectable final state).

OFF – SHELL II

If you come out of your shell, you become more interested in other people and more willing to talk and take part in social activities
Cambridge Dictionaries

Let us consider the case of a *light Higgs boson; here, the common belief was that*

- the product of **on-shell production cross-section** (say in gluon-gluon fusion) and **branching ratios** reproduces the correct result to great accuracy. The expectation is based on the well-known result ($\Gamma_{\rm H} \ll M_{\rm H}$)

OFF

ON

$$\Delta_{\rm H} = \frac{1}{\left(s - M^2_{\rm H}\right)^2 + \Gamma^2_{\rm H} M^2_{\rm H}} = \frac{\pi}{M_{\rm H}\Gamma_{\rm H}}\,\delta\left(s - M^2_{\rm H}\right) + {\rm PV}\left[\frac{1}{\left(s - M^2_{\rm H}\right)^2}\right]$$

where **PV** denotes the principal value (understood as a distribution). Furthermore s is the Higgs virtuality and $M_{\rm H}$ and $\Gamma_{\rm H}$ should be understood as $M_{\rm H} = \mu_{\rm H}$ and $\Gamma_{\rm H} = \gamma_{\rm H}$ and not as the corresponding on-shell values. In more simple terms,

- the first term puts you on-shell and the second one gives you the off-shell tail
- $\Delta_{\rm H}$ is the Higgs propagator, there is no space for anything else in QFT (e.g. Breit-Wigner distributions).

Let us consider the case of a *light Higgs boson; here, the common belief was that*

☛ the product of **on-shell production cross-section** (say in gluon-gluon fusion) and **branching ratios** reproduces the correct result to great accuracy. The expectation is based on the well-known result ($\Gamma_H \ll M_H$)

$$\Delta_H = \frac{1}{\left(s - M_H^2\right)^2 + \Gamma_H^2 M_H^2} = \frac{\pi}{M_H \Gamma_H} \delta\left(s - M_H^2\right) + \mathrm{PV}\left[\frac{1}{\left(s - M_H^2\right)^2}\right]$$

where **PV** denotes the principal value (understood as a distribution). Furthermore s is the Higgs virtuality and M_H and Γ_H should be understood as $M_H = \mu_H$ and $\Gamma_H = \gamma_H$ and not as the corresponding on-shell values. In more simple terms,

☞ the first term puts you on-shell and the second one gives you the off-shell tail

☞ $\mathbf{\Delta_H}$ is the Higgs propagator, there is no space for anything else in QFT (e.g. Breit-Wigner distributions).

We define an **off-shell production cross-section** (for all channels) as follows:

$$\sigma^{\mathrm{prop}}_{ij\to \mathrm{all}} = \frac{1}{\pi} \sigma_{ij\to H} \frac{s^2}{\left|s - s_H\right|^2} \frac{\Gamma_H^{\mathrm{tot}}}{\sqrt{s}}$$

☞ When the cross-section $ij \to \mathbf{H}$ refers to an off-shell Higgs boson the choice of the QCD scales should be made according to the virtuality and not to a fixed value. Therefore, for the PDFs and $\sigma_{ij\to H+X}$ one should select $\mu_F^2 = \mu_R^2 = zs/4$ (zs being the invariant mass of the detectable final state).

OFF – SHELL III

A short History of beyond ZWA (don't try fixing something that is already broken in the first place)

① There is an enhanced Higgs tail **Kauer - Passarino (arXiv:1206.4803)**: away from the narrow peak the propagator and the off-shell H width behave like

$$\Delta_{\rm H} \approx \frac{1}{\left(M^2_{\rm VV} - \mu^2_{\rm H}\right)^2}, \qquad \frac{\Gamma_{\rm H\to VV}\left(M_{\rm VV}\right)}{M_{\rm VV}} \sim G_{\rm F}\, M^2_{\rm VV}$$

!

② Introduce the notion of **∞-degenerate** solutions for the Higgs couplings to SM particles **Dixon - Li (arXiv:1305.3854), Caola - Melnikov(arXiv:1307.4935)**

③ Observe that the enhanced tail is obviously $\gamma_{\rm H}$ -independent and that this could be exploited to constrain the Higgs width model-independently

④ Use a matrix element method (MEM) to construct a kinematic discriminant to sharpen the constraint **Campbell, Ellis and Williams (arXiv:1311.3589)**

A short History of beyond ZWA (don't try fixing something that is already broken in the first place)

① There is an enhanced Higgs tail **Kauer - Passarino (arXiv:1206.4803)**: away from the narrow peak the propagator and the off-shell H width behave like

$$\Delta_{\rm H} \approx \frac{1}{\left(M^2_{\rm VV} - \mu^2_{\rm H}\right)^2}, \qquad \frac{\Gamma_{\rm H\to VV}\left(M_{\rm VV}\right)}{M_{\rm VV}} \sim G_{\rm F}\, M^2_{\rm VV}$$

② Introduce the notion of **∞-degenerate** solutions for the Higgs couplings to SM particles **Dixon - Li (arXiv:1305.3854), Caola - Melnikov(arXiv:1307.4935)**

③ Observe that the enhanced tail is obviously $\gamma_{\rm H}$-independent and that this could be exploited to constrain the Higgs width model-independently

④ Use a matrix element method (MEM) to construct a kinematic discriminant to sharpen the constraint **Campbell, Ellis and Williams (arXiv:1311.3589)**

We define an **off-shell production cross-section** (for all channels) as follows:

$$\sigma^{\rm prop}_{ij\to {\rm all}} = \frac{1}{\pi}\,\sigma_{ij\to {\rm H}}\,\frac{s^2}{\left|s - s_{\rm H}\right|^2}\,\frac{\Gamma^{\rm tot}_{\rm H}}{\sqrt{s}}$$

☞ When the cross-section $ij \to \mathbf{H}$ refers to an off-shell Higgs boson the choice of the QCD scales should be made according to the virtuality and not to a fixed value. Therefore, for the PDFs and $\sigma_{ij\to \rm H+X}$ one should select $\mu^2_{\rm F} = \mu^2_{\rm R} = zs/4$ (zs being the invariant mass of the detectable final state).

Let us consider the case of a *light Higgs boson; here, the common belief was that*

☛ the product of **on-shell production cross-section** (say in gluon-gluon fusion) and **branching ratios** reproduces the correct result to great accuracy. The expectation is based on the well-known result ($\Gamma_{\rm H} \ll M_{\rm H}$)

$$\Delta_{\rm H} = \frac{1}{\left(s - M^2_{\rm H}\right)^2 + \Gamma^2_{\rm H} M^2_{\rm H}} = \frac{\pi}{M_{\rm H}\Gamma_{\rm H}}\,\delta\left(s - M^2_{\rm H}\right) + {\rm PV}\left[\frac{1}{\left(s - M^2_{\rm H}\right)^2}\right]$$

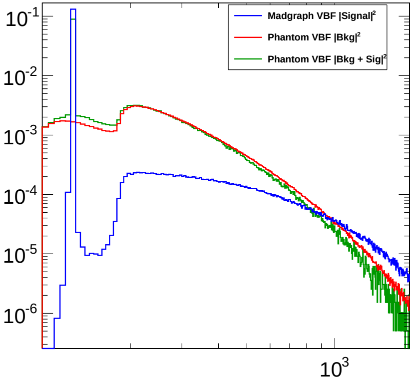
where **PV** denotes the principal value (understood as a distribution). Furthermore s is the Higgs virtuality and $M_{\rm H}$ and $\Gamma_{\rm H}$ should be understood as $M_{\rm H} = \mu_{\rm H}$ and $\Gamma_{\rm H} = \gamma_{\rm H}$ and not as the corresponding on-shell values. In more simple terms,

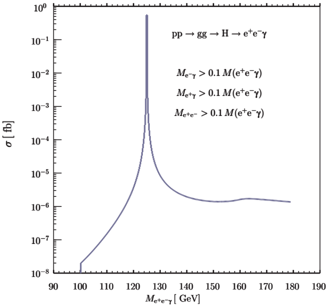
☞ the first term puts you on-shell and the second one gives you the off-shell tail

☞ $\mathbf{\Delta_H}$ is the Higgs propagator, there is no space for anything else in QFT (e.g. Breit-Wigner distributions).

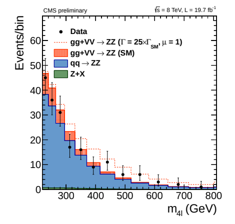
OFF – SHELL IV

Off-shellness forever





arXiv:1308.0422



We define an **off-shell production cross-section** (for all channels) as follows:

$$\sigma^{\rm prop}_{ij\to \rm all} = \frac{1}{\pi}\,\sigma_{ij\to \rm H}\,\frac{s^2}{\left|s-s_{\rm H}\right|^2}\,\frac{\Gamma^{\rm tot}_{\rm H}}{\sqrt{s}}$$

☞ When the cross-section $ij \to \mathbf{H}$ refers to an off-shell Higgs boson the choice of the QCD scales should be made according to the virtuality and not to a fixed value. Therefore, for the PDFs and $\sigma_{ij\to \rm H+X}$ one should select $\mu_F^2 = \mu_R^2 = zs/4$ (zs being the invariant mass of the detectable final state).

Let us consider the case of a *light Higgs boson; here, the common belief was that*

☛ the product of **on-shell production cross-section** (say in gluon-gluon fusion) and **branching ratios** reproduces the correct result to great accuracy. The expectation is based on the well-known result ($\Gamma_{\rm H} \ll M_{\rm H}$)

$$\Delta_{\rm H} = \frac{1}{\left(s-M_{\rm H}^2\right)^2 + \Gamma_{\rm H}^2 M_{\rm H}^2} = \frac{\pi}{M_{\rm H}\Gamma_{\rm H}}\,\delta\left(s-M_{\rm H}^2\right) + {\rm PV}\left[\frac{1}{\left(s-M_{\rm H}^2\right)^2}\right]$$

where **PV** denotes the principal value (understood as a distribution). Furthermore s is the Higgs virtuality and $M_{\rm H}$ and $\Gamma_{\rm H}$ should be understood as $M_{\rm H} - \mu_{\rm H}$ and $\Gamma_{\rm H} - \gamma_{\rm H}$ and not as the corresponding on-shell values. In more simple terms,

☞ the first term puts you on-shell and the second one gives you the off-shell tail

☞ $\Delta_{\rm H}$ is the Higgs propagator, there is no space for anything else in QFT (e.g. Breit-Wigner distributions).

A short History of beyond ZWA (don't try fixing something that is already broken in the first place)

① There is an enhanced Higgs tail **Kauer - Passarino (arXiv:1206.4803)**: away from the narrow peak the propagator and the off-shell H width behave like

$$\Delta_{\rm H} \approx \frac{1}{\left(M_{\rm VV}^2 - \mu_{\rm H}^2\right)^2}, \qquad \frac{\Gamma_{\rm H\to VV}\left(M_{\rm VV}\right)}{M_{\rm VV}} \sim G_F\,M_{\rm VV}^2$$

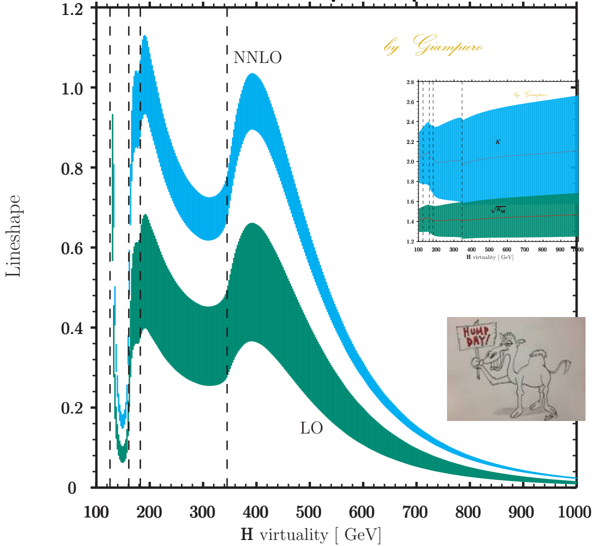
② Introduce the notion of **∞-degenerate** solutions for the Higgs couplings to SM particles **Dixon - Li (arXiv:1305.3854), Caola - Melnikov(arXiv:1307.4935)**

③ Observe that the enhanced tail is obviously $\gamma_{\rm H}$-independent and that this could be exploited to constrain the Higgs width model-independently

④ Use a matrix element method (MEM) to construct a kinematic discriminant to sharpen the constraint **Campbell, Ellis and Williams (arXiv:1311.3589)**

MHOU(THU)

The higher-order correction in gluon-gluon fusion have shown a huge $\mathbf{K}$-factor $\mathbf{K} = \sigma^{\mathrm{NNLO}}_{\mathrm{prod}} / \sigma^{\mathrm{LO}}_{\mathrm{prod}}$, $\sigma_{\mathrm{prod}} = \sigma_{\mathrm{gg}\to\mathrm{H}}$.

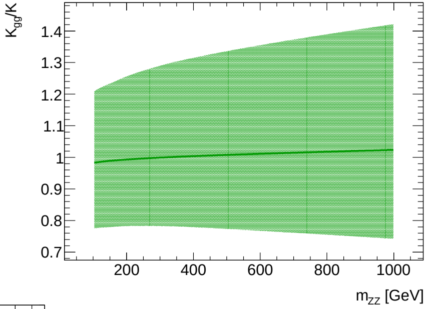


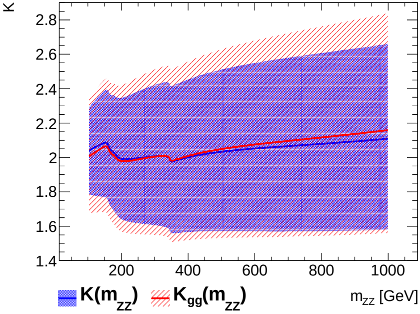
arXiv:1312.2397

Futher details

The ratio is K_{gg}/K with quadratically subtracted uncertainties of K from the uncertainty of K_{gg}

Assumption: the extra HO terms calculated in K give an uncorrelated extra MHO uncertainty of 20−30% which needs to be applied to K_{gg} on top of the correlated K MHOU

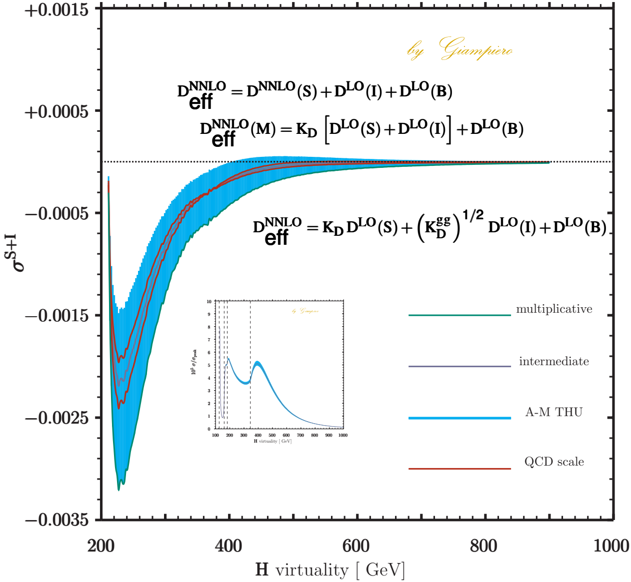




Courtesy of M. Duehrssen

subleading systematics on I or B alone are important as the leading systematics on I+B could cancel to some degree. Because of the negative I, 100% correlating is actually not conservative as this allows larger cancellations in S + I

❶ The *zero-knowledge* scenario



The *soft-knowledge* scenario: in a nutshell, one can ✎

$$\sigma = \sigma^{\rm LO} + \sigma^{\rm LO}\,\frac{\alpha_{\rm S}}{2\,\pi}\,[\text{universal} + \text{process dependent} + \text{reg}]$$

$$\rightsquigarrow \kappa(M_{\rm ZZ})\,\mathrm{S} + \kappa_{\rm gg}(M_{\rm ZZ})\left[\sqrt{\lambda}\,\mathrm{I} + \lambda\,\mathrm{B}\right]$$

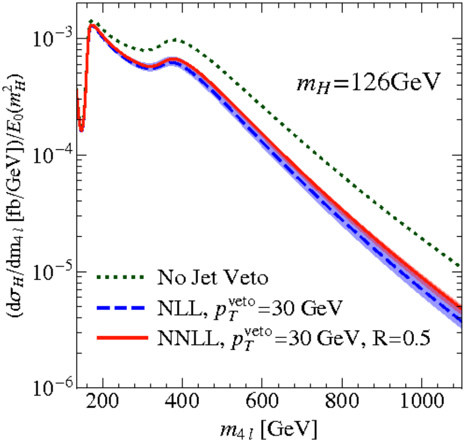
☛ where *universal* (the “ + ” distribution) gives the bulk of the result

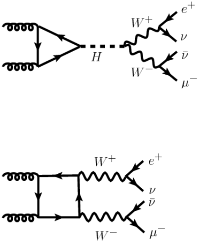
☛ while *process dependent* (the δ function) is known up to two loops for $\mathbf{S}$ but not for $\mathbf{B}$

☛ and *reg* is the regular part.

A possible strategy (arXiv:1304.3053) would be to use for $\mathbf{B}$ the same *process dependent* coefficients and allow for their variation within some ad hoc factor, e.g. $\lambda \in [1/2, 2]$.

WW? EXP: something like the p_T distribution is not part of the LO calculation at all.

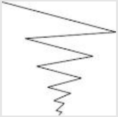




TH: **something is moving**

jet veto on far off-shell XS: arXiv:1405.5534

Questions



dialogue concerning the two chief world systems

Question about the deficit expected/observed (ATLAS + CMS): should we start worrying about TH drivenness of the deficit?

- The ATLAS off-shell measurement its basically spot on with the SM Compared to that CMS has a larger deficit in the off-shell high mass region
- Then, when combining with the on-shell measurement, ATLAS develops a deficit because on-shell μ for **ZZ** is $\approx$ **1.5**. On the other hand, for CMS the on-shell is $\approx$ **0.9**, so it actually goes back in the other direction.

In the end this does not allow to draw any physics conclusion

Nevertheless what is *observed* (*expected*) and what are the assumptions?

Definitions and *assumptions*

- the kosher *experimental* answer
  - ✓ EXPECTED $\mapsto$ generate Asimov dataset with $\mu_{\mathbf{VBF}} = \mu_{\mathbf{ggH}} = 1$, fit with floating $\mu_{\mathbf{VBF}}$ and $\mu_{\mathbf{ggH}}$
  - ✓ OBSERVED $\mapsto$ float $\mu_{\mathbf{VBF}}$ and $\mu_{\mathbf{ggH}}$
- the poor *theoretical* understanding
  - ✓ EXPECTED is what you get from a MC with $\mu = \mu_{\mathbf{hyp}}$
  - ✓ OBSERVED is what you get by fitting the data

Although I understand *questions* and *comments*

- ✓ What is wrong in plotting what you expect to the likelihood to look like when everything else is as expected in the SM?
- ✓ The post-fit expectation is a very important concept

Making on-shell hypothesis ($\mu_{\mathrm{OS}} = 1$ or $\mu_{\mathrm{OS}} = \mu_{\mathrm{obs}}$) is a consequence of assuming on-shell ∞-degeneracy, which is not realistic. Which BSM theory allows you to *fix* the on-shell and to *float* the off-shell?

Logic takes care of itself; all we have to do is to look and see how it does it

K *language*

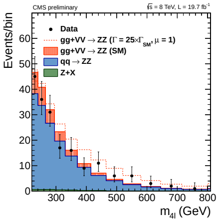
$$\sigma_{i\to \mathrm{H}\to f} = (\sigma\cdot \mathrm{BR}) = \frac{\sigma_i^{\mathrm{prod}}\,\Gamma_f}{\gamma_{\mathrm{H}}}$$

$$\sigma_{i\to \mathrm{H}\to f} \propto \frac{g_i^2\, g_f^2}{\gamma_{\mathrm{H}}} \qquad g_{i,f} = \xi\, g_{i,f}^{\mathrm{SM}} \quad \gamma_{\mathrm{H}} = \xi^4\, \gamma_{\mathrm{H}}^{\mathrm{SM}}$$

a consistent BSM interpretation?

On the whole, we have a constraint in the multidimensional κ-space

$$\kappa_{\mathrm{g}}^2 = \kappa_{\mathrm{g}}^2(\kappa_{\mathrm{t}}, \kappa_{\mathrm{b}}) \quad \kappa_{\mathrm{H}}^2 = \kappa_{\mathrm{H}}^2(\kappa_j, \forall j)$$



On-shell ∞-degeneracy
arXiv:1305.3854, 1307.4935, 1311.3589

The generalization is an ∞^2-degeneracy

$$g_i^2\, g_f^2 = \gamma_{\mathrm{H}}$$

$g_i \Longleftrightarrow \kappa_j$

$g_i(M_H)\, ?\, g_i(\sqrt{s})$

Only on the assumption of degeneracy one can prove that off-shell effects measure γ_{H}

Simplified version

$$\frac{\Gamma_{gg}}{\Gamma_{gg}^{\mathrm{SM}}(\mu_{\mathrm{H}})} = \frac{\kappa_{\mathrm{t}}^2\cdot\Gamma_{gg}^{\mathrm{tt}}(\mu_{\mathrm{H}}) + \kappa_{\mathrm{b}}^2\cdot\Gamma_{gg}^{\mathrm{bb}}(\mu_{\mathrm{H}}) + \kappa_{\mathrm{t}}\kappa_{\mathrm{b}}\cdot\Gamma_{gg}^{\mathrm{tb}}(\mu_{\mathrm{H}}}{\Gamma_{gg}^{\mathrm{tt}}(\mu_{\mathrm{H}}) + \Gamma_{gg}^{\mathrm{bb}}(\mu_{\mathrm{H}}) + \Gamma_{gg}^{\mathrm{tb}}(\mu_{\mathrm{H}})}$$

a combination of on-shell effects measuring

$g_i^2\, g_f^2 / \gamma_{\mathrm{H}}$

and off-shell effects measuring

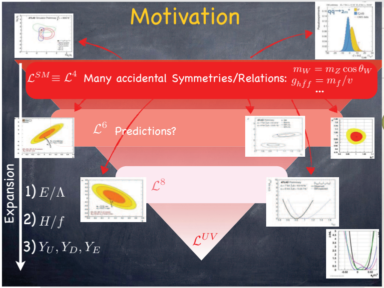
$g_i^2\, g_f^2$

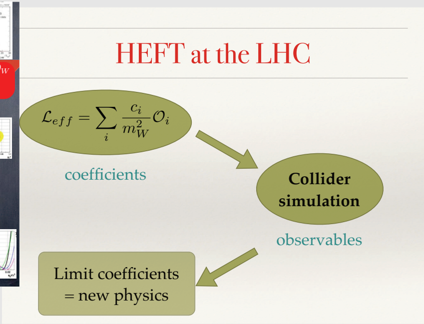
gives information on γ_{H}

without prejudices

original κ-language arXiv:1209.0040

HEFT *is needed*





- http://arxiv.org/abs/1405.0285
- http://arxiv.org/abs/1405.1925
- http://arxiv.org/abs/1406.1757
- http://arxiv.org/abs/1406.6338

light decoupling d.o.f. $\rightsquigarrow$ EFT not applicable

Having said that … no space left for annotations

Renormalization

FP-sector: handle with care

✓ Make finite all Green's functions

Schemes: remember β_{QED} in large m_e -limit

$$
\begin{aligned}
g &= g_{\text{ren}} \left[1 + \frac{g^2_{\text{ren}}}{16\,\pi^2} \left(dZ_g + g_6\, dZ^{(6)}_g \right) \frac{1}{\bar{\varepsilon}} \right] \\
M_{\mathrm{W}} &= M^{\text{ren}}_{\mathrm{W}} \left[1 + \frac{1}{2} \frac{g^2_{\text{ren}}}{16\,\pi^2} \left(dZ_{M_{\mathrm{W}}} + g_6\, dZ^{(6)}_{M_{\mathrm{W}}} \right) \frac{1}{\bar{\varepsilon}} \right]
\end{aligned}
$$

etc.

✓ Don't forget background

Oops! . . . $4f\,\mathcal{O}$ needed for $\mathbf{H} \to \bar{b}b$

$\mathbf{H} \to \gamma\gamma$ not finite

Wilson coefficients $\rightarrow$ W_i

$$
\begin{aligned}
W_i &= \sum_j Z^{\mathrm{wc}}_{ij}\, W^{\text{ren}}_j \\
Z^{\mathrm{wc}}_{ij} &= \delta_{ij} + \frac{g^2_{\text{ren}}}{16\,\pi^2}\, dZ^{\mathrm{wc}}_{ij}\, \frac{1}{\bar{\varepsilon}}
\end{aligned}
$$

$$\frac{1}{\bar{\varepsilon}} = \frac{2}{\varepsilon} - \gamma - \ln\pi + \ln\mu_R$$

Einhorn, Wudka

$\mathcal{O}$ is PTG

$\mathcal{O}$ is LG

Grzadkowski, Iskrzynski, Misiak, Rosiek

Appendix C. Dimension-Six Basis Operators for the SM[22].

X^3 (LG)		φ^6 and $\varphi^4 D^2$ (PTG)		$\psi^2\varphi^3$ (PTG)	
Q_G	$f^{ABC} G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	Q_φ	$(\varphi^\dagger\varphi)^3$	$Q_{e\varphi}$	$(\varphi^\dagger\varphi)(\bar l_p e_r \varphi)$
$Q_{\widetilde G}$	$f^{ABC} \widetilde G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	$Q_{\varphi\Box}$	$(\varphi^\dagger\varphi)\Box(\varphi^\dagger\varphi)$	$Q_{u\varphi}$	$(\varphi^\dagger\varphi)(\bar q_p u_r \widetilde\varphi)$
Q_W	$\varepsilon^{IJK} W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$	$Q_{\varphi D}$	$\left(\varphi^\dagger D^\mu\varphi\right)^\star \left(\varphi^\dagger D_\mu\varphi\right)$	$Q_{d\varphi}$	$(\varphi^\dagger\varphi)(\bar q_p d_r \varphi)$
$Q_{\widetilde W}$	$\varepsilon^{IJK} \widetilde W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$				

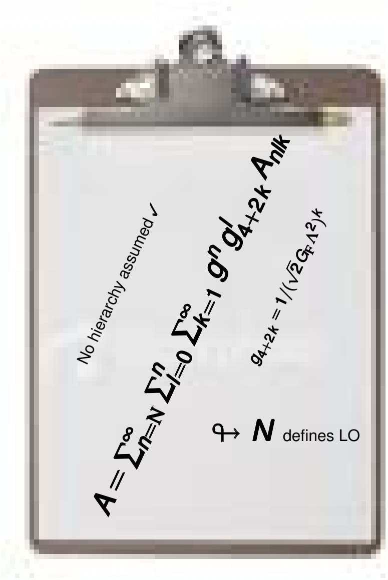
$X^2\varphi^2$ (LG)		$\psi^2 X\varphi$ (LG)		$\psi^2\varphi^2 D$ (PTG)	
$Q_{\varphi G}$	$\varphi^\dagger\varphi\, G_{\mu\nu}^A G^{A\mu\nu}$	Q_{eW}	$(\bar l_p \sigma^{\mu\nu} e_r)\tau^I \varphi W_{\mu\nu}^I$	$Q_{\varphi l}^{(1)}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar l_p \gamma^\mu l_r)$
$Q_{\varphi\widetilde G}$	$\varphi^\dagger\varphi\, \widetilde G_{\mu\nu}^A G^{A\mu\nu}$	Q_{eB}	$(\bar l_p \sigma^{\mu\nu} e_r)\varphi B_{\mu\nu}$	$Q_{\varphi l}^{(3)}$	$(\varphi^\dagger i\overleftrightarrow{D}^I_\mu \varphi)(\bar l_p \tau^I \gamma^\mu l_r)$
$Q_{\varphi W}$	$\varphi^\dagger\varphi\, W_{\mu\nu}^I W^{I\mu\nu}$	Q_{uG}	$(\bar q_p \sigma^{\mu\nu} T^A u_r)\widetilde\varphi\, G_{\mu\nu}^A$	$Q_{\varphi e}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar e_p \gamma^\mu e_r)$
$Q_{\varphi\widetilde W}$	$\varphi^\dagger\varphi\, \widetilde W_{\mu\nu}^I W^{I\mu\nu}$	Q_{uW}	$(\bar q_p \sigma^{\mu\nu} u_r)\tau^I \widetilde\varphi\, W_{\mu\nu}^I$	$Q_{\varphi q}^{(1)}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar q_p \gamma^\mu q_r)$
$Q_{\varphi B}$	$\varphi^\dagger\varphi\, B_{\mu\nu} B^{\mu\nu}$	Q_{uB}	$(\bar q_p \sigma^{\mu\nu} u_r)\widetilde\varphi\, B_{\mu\nu}$	$Q_{\varphi q}^{(3)}$	$(\varphi^\dagger i\overleftrightarrow{D}^I_\mu \varphi)(\bar q_p \tau^I \gamma^\mu q_r)$
$Q_{\varphi\widetilde B}$	$\varphi^\dagger\varphi\, \widetilde B_{\mu\nu} B^{\mu\nu}$	Q_{dG}	$(\bar q_p \sigma^{\mu\nu} T^A d_r)\varphi\, G_{\mu\nu}^A$	$Q_{\varphi u}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar u_p \gamma^\mu u_r)$
$Q_{\varphi WB}$	$\varphi^\dagger \tau^I \varphi\, W_{\mu\nu}^I B^{\mu\nu}$	Q_{dW}	$(\bar q_p \sigma^{\mu\nu} d_r)\tau^I \varphi\, W_{\mu\nu}^I$	$Q_{\varphi d}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar d_p \gamma^\mu d_r)$
$Q_{\varphi\widetilde WB}$	$\varphi^\dagger \tau^I \varphi\, \widetilde W_{\mu\nu}^I B^{\mu\nu}$	Q_{dB}	$(\bar q_p \sigma^{\mu\nu} d_r)\varphi\, B_{\mu\nu}$	$Q_{\varphi ud}$	$i(\widetilde\varphi^\dagger D_\mu \varphi)(\bar u_p \gamma^\mu d_r)$

Table C.1: Dimension-six operators other than the four-fermion ones.

[22] These tables are taken from [5], by permission of the authors.

✍ Effective Lagrangians cannot be blithely used without acknowledging implications of their choice
ex: non gauge-invariant, intended to be used in U-gauge
ex: $\mathbf{H} \to \mathbf{WW^*}$ is virtual $\mathbf{W}$ + something else, depending on the operator basis

Caveat

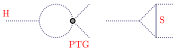


✍**PTG:** **T** - generated in at least one extension of SM

Note that for $\Lambda \approx 5$ *TeV* we have

$$1/(\sqrt{2}G_F\Lambda^2) \approx g^2/(4\pi),$$

i.e. ⤜ the contributions of $d=6$ operators are ≊ loop effects.
⤜ ⤜ For higher scales, loop contributions tend to be more important (⪸)



PTG - operators versus LG - operators, cf. Einhorn, Wudka, …

✍ It can be argued that (at LO) the basis operator should be chosen from among the **PTG operators**

✍ take $\mathcal{O}^{(6)}_{\rm LG}$, contract two lines, is ren of some $\mathcal{O}^{(4)}$ a SM vertex with $\mathcal{O}^{(6)}_{\rm PTG}$ required … same order

$1/\Lambda$ expansion → power-counting ✓

LG → low-energy analytic structure ✗

PROPOSITION: There are two ways of formulating HEFT

a) mass-dependent scheme(s) or **Wilsonian** HEFT

b) mass-independent scheme(s) or **Continuum** HEFT (CHEFT)

- only **a)** is conceptually consistent with the image of an EFT as a low-energy approximation to a high-energy theory
- however, inclusion of NLO corrections is only meaningful in **b)** since we cannot regularize with a cutoff and NLO requires regularization
    - There is an additional problem, CHEFT requires evolving our theory to lower scales until we get below the "heavy-mass" scale where we use $\mathscr{L} = \mathscr{L}_{\rm SM} + d\mathscr{L}$, $d\mathscr{L}$ encoding matching corrections at the boundary. Therefore, CHEFT does not integrate out heavy degrees of freedom but removes them compensating for by an appropriate matching calculation

☞ *Not quite the same as it is usually discussed (no theory approaching the boundary from above ...)* cf. low-energy SM, weak effects on $g-2$ etc.

$\dim \phi = d/2 - 1$
$\dim \mathcal{O}^d = N_\phi \dim \phi + N_{der}$
For $d \geq 3$ there is a finite number of relevant + marginal operators
For $d \geq 1$ there is a finite number of irrelevant operators
Sounds good for finite dependence on high-energy theory

This assumes that high-energy theory is weakly coupled

Dimensional arguments work for LO HEFT

In NLO HEFT scaling may break down, implying appeal to a particular renormalization scheme

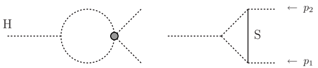
Ren. group should only be applied to EFTs that are nearly massless

Decoupling theorem fails for CHEFT, but, arguably this does not prevent them from supporting a well defined scheme, but decoupling must be inserted in the form of matching calculations (which we don't have . . .)

Match Feynman diagrams ∈ HEFT with corresponding **1**(light)**PI** diagrams ∈ high-energy theory
(and discover that Taylor-expanding is not always a good idea)

Symphony No. 8 in B minor

What do we lose without matching?



toy model: S dark Higgs field

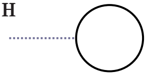
$$\mathscr{L} = \mathscr{L}_{\mathrm{SM}} - \frac{1}{2}\,\partial_\mu S\,\partial_\mu S - \frac{1}{2}\,M_{\mathrm{S}}^2\,S^2 + \mu_{\mathrm{S}}\,\Phi^\dagger\,\Phi\,S$$

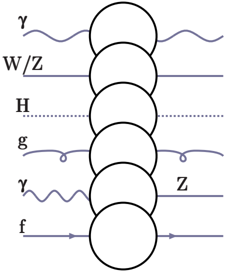
$$I_{\mathrm{eff}}^{\mathrm{DR}} = \frac{3}{4}\,g\,\frac{M_{\mathrm{H}}^2}{M_{\mathrm{W}}\Lambda^2}\left[\left(\frac{1}{2}\,s - 3\,M_{\mathrm{H}}^2\right)\left(\frac{1}{\bar{\varepsilon}} - \ln\frac{-s-i0}{\mu_{\mathrm{R}}^2}\right) + \quad \text{finite part}\right]$$

$$I_{\mathrm{full}} = -\frac{3}{2}\,g\,\frac{M_{\mathrm{H}}^2\mu_{\mathrm{S}}^2}{M_{\mathrm{W}}\,M_{\mathrm{S}}^2}\left[1 - \frac{1}{4}\,\frac{s}{M_{\mathrm{S}}^2} - \left(1 - \frac{1}{2}\,\frac{s}{M_{\mathrm{S}}^2}\right)\ln\frac{-s-i0}{M_{\mathrm{S}}^2} + \mathscr{O}\left(\frac{s^2}{M_{\mathrm{S}}^4}\right)\right]$$

full starts at $\mathscr{O}(\mu_{\mathrm{S}}^2/M_{\mathrm{S}}^2)$
eff starts at $\mathscr{O}(s/\Lambda^2)$
large mass expansion of **full** follows from Mellin-Barnes expansion and not from Taylor expansion

✓ Tadpoles $\mapsto$ $\boldsymbol{\beta_H}$

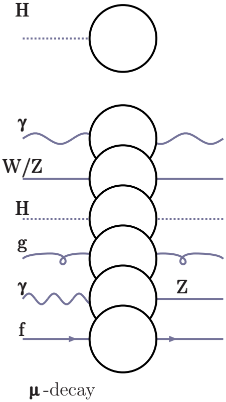




μ-decay

✓ Tadpoles $\mapsto$ $\beta_{\mathbf{H}}$

✓ $\Phi = Z_{\phi}^{1/2}\Phi_{\mathbf{R}}$ etc.

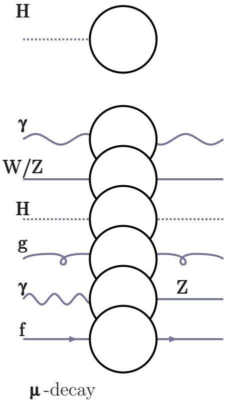


✓ Tadpoles $\mapsto$ $\beta_{\mathbf{H}}$

✓ $\Phi = Z_{\phi}^{1/2}\,\Phi_{\mathbf{R}}$ etc.

☛

$$Z_{\phi} = 1 + \frac{g^2}{16\,\pi^2}\left(\delta Z_{\phi}^{(4)} + g_6\,\delta Z_{\phi}^{(6)}\right)$$



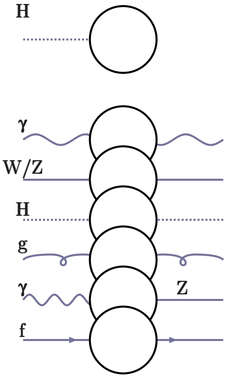
✓ Tadpoles $\mapsto$ β_{H}

✓ $\Phi = Z_{\phi}^{1/2}\,\Phi_{\mathrm{R}}$ etc.

☛

$$Z_{\phi} = 1 + \frac{g^2}{16\pi^2}\left(\delta Z_{\phi}^{(4)} + g_6\,\delta Z_{\phi}^{(6)}\right)$$

✓ Self-energies UV $\mathscr{O}^{(4)}, \mathscr{O}^{(6)}$ -finite



μ-decay

✓ Tadpoles $\mapsto$ β_{H}

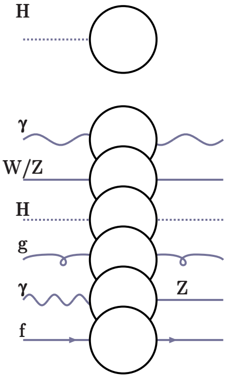
✓ $\Phi = Z_{\phi}^{1/2}\,\Phi_{\mathrm{R}}$ etc.

☛

$$Z_{\phi} = 1 + \frac{g^2}{16\pi^2}\left(\delta Z_{\phi}^{(4)} + g_6\,\delta Z_{\phi}^{(6)}\right)$$

✓ Self-energies UV $\mathscr{O}^{(4)}, \mathscr{O}^{(6)}$ -finite

☛ μ -decay

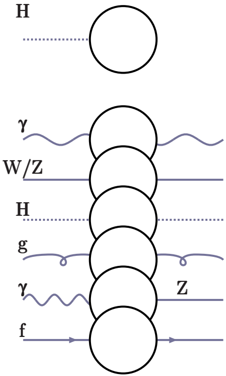


μ -decay

- ✓ Tadpoles $\mapsto$ β_{H}
- ✓ $\Phi = Z_{\phi}^{1/2}\,\Phi_{\mathrm{R}}$ etc.
- ☛

$$Z_{\phi} = 1 + \frac{g^2}{16\pi^2}\left(\delta Z_{\phi}^{(4)} + g_6\,\delta Z_{\phi}^{(6)}\right)$$

- ✓ Self-energies UV $\mathscr{O}^{(4)}, \mathscr{O}^{(6)}$ -finite
- ☛ μ -decay
- ✓ $g \to g_{\mathrm{R}}$



μ -decay

✓ Tadpoles $\mapsto$ β_{H}

✓ $\Phi = Z_{\phi}^{1/2}\,\Phi_{\mathrm{R}}$ etc.

☛

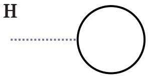
$$Z_{\phi} = 1 + \frac{g^2}{16\pi^2}\left(\delta Z_{\phi}^{(4)} + g_6\,\delta Z_{\phi}^{(6)}\right)$$

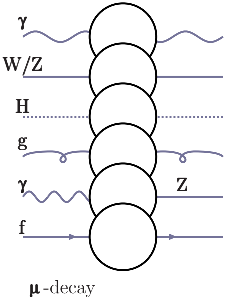
✓ Self-energies UV $\mathscr{O}^{(4)}, \mathscr{O}^{(6)}$ -finite

☛ μ -decay

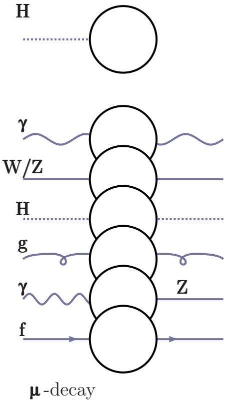
✓ $g \rightarrow g_{\mathrm{R}}$

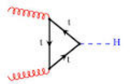
✓ Finite ren.





- ✓ Tadpoles $\mapsto$ β_{H}
- ✓ $\Phi = Z_\phi^{1/2}\,\Phi_{\mathrm{R}}$ etc.
- ☛ $$Z_\phi = 1 + \frac{g^2}{16\pi^2}\left(\delta Z_\phi^{(4)} + g_6\,\delta Z_\phi^{(6)}\right)$$
- ✓ Self-energies UV $\mathscr{O}^{(4)}, \mathscr{O}^{(6)}$ -finite
- ☛ μ -decay
- ✓ $g \to g_{\mathrm{R}}$
- ✓ Finite ren.
- ☛ $$M_{\mathrm{R}}^2 = M_{\mathrm{W}}^2\left[1 + \frac{g_{\mathrm{R}}^2}{16\pi^2}\left(\mathrm{Re}\,\Sigma_{\mathrm{WW}} - \delta Z_{\mathrm{M}}\right)\right]$$
- ✓ etc Propagators finite and μ_{R} -independent



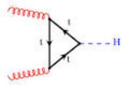


✓ requires $Z_{\mathrm{H}}, Z_{\mathrm{g}}, Z_{g}, Z_{g_{\mathrm{S}}}$

HEFT extension of ggF requires:

arXiv:1405.1925

v_{H} = Higgs virtuality



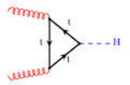
✓ requires $Z_{\mathbf{H}}, Z_{\mathbf{g}}, Z_{g}, Z_{g_{\mathbf{S}}}$

✓ It is $\mathcal{O}^{(4)}$ -finite but not $\mathcal{O}^{(6)}$ -finite

HEFT extension of ggF requires:

arXiv:1405.1925

$v_{\mathbf{H}}$ = Higgs virtuality



✓ requires $Z_{\mathrm{H}}, Z_{\mathrm{g}}, Z_{g}, Z_{g_S}$

✓ It is $\mathcal{O}^{(4)}$ -finite but not $\mathcal{O}^{(6)}$ -finite

✓ involves $a_{\phi D}$, $a_{\phi\Box}$, $a_{t\phi}$, $a_{b\phi}$, $a_{\phi W}$,

HEFT extension of ggF requires: $a_{\phi g}$, a_{tg}, a_{bg},

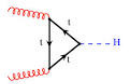
arXiv:1405.1925

v_{H} = Higgs virtuality

$$a_{tg} = W_1 \quad a_{bg} = W_2 \quad a_{\phi g} = W_3$$

$$a_{b\phi} + \frac{1}{4} a_{\phi D} - a_{\Phi W} - a_{\phi\Box} = W_4$$

$$a_{t\phi} - \frac{1}{4} a_{\phi D} + a_{\Phi W} + a_{\phi\Box} = W_5$$



✓ requires $Z_{\rm H}, Z_{\rm g}, Z_g, Z_{g_S}$

✓ It is $\mathcal{O}^{(4)}$ -finite but not $\mathcal{O}^{(6)}$ -finite

✓ involves $a_{\phi\rm D}$, $a_{\phi\Box}$, $a_{\rm t\phi}$, $a_{\rm b\phi}$, $a_{\phi\rm W}$, $a_{\phi\rm g}$, $a_{\rm tg}$, $a_{\rm bg}$,

HEFT extension of ggF requires:

arXiv:1405.1925

$v_{\rm H}$ = Higgs virtuality

$$a_{\rm tg} = W_1 \quad a_{\rm bg} = W_2 \quad a_{\phi\rm g} = W_3$$

$$a_{\rm b\phi} + \frac{1}{4}\, a_{\phi\rm D} - a_{\Phi\rm W} - a_{\phi\Box} = W_4$$

$$a_{\rm t\phi} - \frac{1}{4}\, a_{\phi\rm D} + a_{\Phi\rm W} + a_{\phi\Box} = W_5$$

✓ requires *extra* renormalization

$$W_i = \sum_j Z^{\rm mix}_{ij}\, W^{\rm R}_j\,(\mu_{\rm R})$$

$$Z^{\rm mix}_{ij} = \delta_{ij} + \frac{g\, g_{\rm S}}{16\,\pi^2}\,\delta Z^{\rm mix}_{ij}\,\frac{1}{\varepsilon}$$

$$\delta Z^{\rm mix}_{31(2)} = -\frac{1}{2\sqrt{2}}\,\frac{M_{\rm t(b)}}{M_{\rm W}}$$

✓ Define **gg** → **H** building blocks

$$\frac{8\,\pi^2}{i\,g_S^2}\,\frac{M_W}{M_q^2}\,A_q^{LO} \;=\; 2 - \left(4\,M_q^2 - v_H\right)\,C_0\left(-v_H,0,0\,;\,M_q,M_q,M_q\right)$$

$$\begin{aligned}\frac{32\,\pi^2}{i\,g_S^2}\,\frac{M_W^2}{M_q}\,A_q^{nf} \;&=\; 8\,M_q^4\,C_0\left(-v_H,0,0\,;\,M_q,M_q,M_q\right)\\ &+\; v_H\left[1 - B_0\left(-v_H\,;\,M_q,M_q\right)\right] - 4\,M_q^2\end{aligned}$$

✓ Define (process dependent) κ-factors

$$\kappa_{\mathrm{b}} = 1 + g_6 \left[\frac{1}{2} \frac{M_{\mathrm{b}}}{M_{\mathrm{W}}} W_2^{\mathrm{R}} - \frac{1}{\sqrt{2}} W_4^{\mathrm{R}} \right]$$

$$\kappa_{\mathrm{t}} = 1 + g_6 \left[\frac{1}{2} \frac{M_{\mathrm{t}}}{M_{\mathrm{W}}} W_1^{\mathrm{R}} - \frac{1}{\sqrt{2}} W_5^{\mathrm{R}} \right]$$

✓ Define (process dependent) κ-factors

$$\kappa_\mathrm{b} = 1 + g_6 \left[\frac{1}{2} \frac{M_\mathrm{b}}{M_\mathrm{W}} W_2^\mathrm{R} - \frac{1}{\sqrt{2}} W_4^\mathrm{R} \right]$$

$$\kappa_\mathrm{t} = 1 + g_6 \left[\frac{1}{2} \frac{M_\mathrm{t}}{M_\mathrm{W}} W_1^\mathrm{R} - \frac{1}{\sqrt{2}} W_5^\mathrm{R} \right]$$

✓ Obtain the **4+6** amplitude

$$\mathrm{A}^{(4+6)} = g \sum_{\mathrm{q=b,t}} \kappa_\mathrm{q} \, \mathrm{A}_\mathrm{q}^\mathrm{LO} + i \, \frac{g_6 \, g_\mathrm{S}}{\sqrt{2}} \frac{M_\mathrm{H}^2}{M_\mathrm{W}} W_3^\mathrm{R} + g_6 \, g \left[W_1^\mathrm{R} \, \mathrm{A}_\mathrm{t}^\mathrm{nf} + W_2^\mathrm{R} \, \mathrm{A}_\mathrm{b}^\mathrm{nf} \right]$$

✓ Define (process dependent) κ-factors

$$\kappa_b = 1 + g_6 \left[\frac{1}{2} \frac{M_b}{M_W} W_2^R - \frac{1}{\sqrt{2}} W_4^R \right]$$

$$\kappa_t = 1 + g_6 \left[\frac{1}{2} \frac{M_t}{M_W} W_1^R - \frac{1}{\sqrt{2}} W_5^R \right]$$

✓ Obtain the **4+6** amplitude

$$\mathrm{A}^{(4+6)} = g \sum_{q=b,t} \kappa_q \, \mathrm{A}_q^{LO} + i \, \frac{g_6 \, g_S}{\sqrt{2}} \frac{M_H^2}{M_W} W_3^R + g_6 \, g \left[W_1^R \, \mathrm{A}_t^{nf} + W_2^R \, \mathrm{A}_b^{nf} \right]$$

✓ Derive true relation

$$\mathrm{A}^{(4+6)} (gg \to H) = g_g \left(v_H \right) \mathrm{A}^{(4)} (gg \to H)$$

✓ Define (process dependent) κ-factors

$$\kappa_b = 1 + g_6 \left[\frac{1}{2} \frac{M_b}{M_W} W_2^R - \frac{1}{\sqrt{2}} W_4^R \right]$$

$$\kappa_t = 1 + g_6 \left[\frac{1}{2} \frac{M_t}{M_W} W_1^R - \frac{1}{\sqrt{2}} W_5^R \right]$$

✓ Obtain the **4+6** amplitude

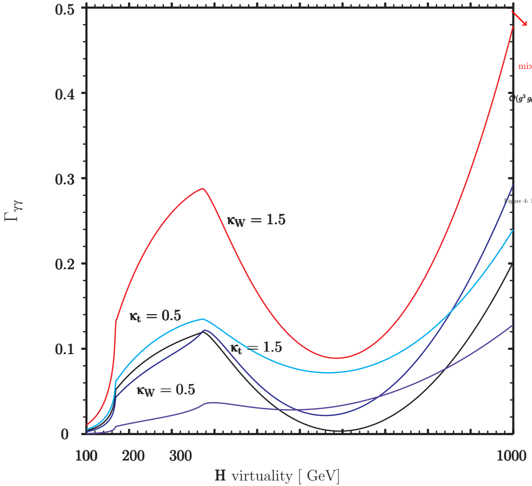
$$A^{(4+6)} = g \sum_{q=b,t} \kappa_q A_q^{LO} + i \frac{g_6 g_S}{\sqrt{2}} \frac{M_H^2}{M_W} W_3^R + g_6 g \left[W_1^R A_t^{nf} + W_2^R A_b^{nf} \right]$$

✓ Derive true relation

$$A^{(4+6)}(gg \to H) = g_g(v_H) A^{(4)}(gg \to H)$$

✓ Effective (running) scaling (g_i) is not a κ (constant) parameter (unless $\mathcal{O}^{(6)} = 0$ and $\kappa_b = \kappa_t$)

☛ Non-factorizable not included



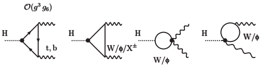


Figure 3: Example of one-loop SM diagrams with $\mathcal{O}$-insertions, contributing to the amplitude for $H \to \gamma\gamma$

$\searrow$ κ_f, A_{NF} $\searrow$ κ_W, A_{NF}

$\Uparrow$

mix under ren. with $\mathcal{O}(g\, g_6)$ $\nearrow$ A_{NF}

$\Downarrow$

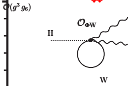


Figure 4: Example of one-loop $\mathcal{O}$-diagrams, contributing to the amplitude for $H \to \gamma\gamma$

✓ Background in HEFT? Consider $\mathbf{\bar{u}u \to ZZ}$

✓ Background in HEFT? Consider $\bar{\mathbf{u}}\mathbf{u} \to \mathbf{ZZ}$

✓ The following Wilson coefficients appear:

$$
\begin{aligned}
W_1 &= a_{\gamma\gamma} = s_\theta c_\theta\, a_{\Phi WB} + c_\theta^2\, a_{\phi B} + s_\theta^2\, a_{\phi W} \\
W_2 &= a_{ZZ} = -s_\theta c_\theta\, a_{\Phi WB} + s_\theta^2\, a_{\phi B} + c_\theta^2\, a_{\phi W} \\
W_3 &= a_{\gamma Z} = 2\, s_\theta\, c_\theta \left(a_{\phi W} - a_{\phi B} \right) + \left(c_\theta^2 - s_\theta^2 \right) a_{\Phi WB} \\
W_4 &= a_{\phi D} \\
W_5 &= a_{\phi q}^{(3)} + a_{\phi q}^{(1)} - a_{\phi u} \\
W_6 &= a_{\phi q}^{(3)} + a_{\phi q}^{(1)} + a_{\phi u}
\end{aligned}
$$

✓ Background in HEFT? Consider $\bar{\mathbf{u}}\mathbf{u} \to \mathbf{ZZ}$

✓ The following Wilson coefficients appear:

$$
\begin{aligned}
W_1 &= a_{\gamma\gamma} = s_\theta c_\theta\, a_{\Phi WB} + c_\theta^2\, a_{\phi B} + s_\theta^2\, a_{\phi W} \\
W_2 &= a_{ZZ} = -s_\theta c_\theta\, a_{\Phi WB} + s_\theta^2\, a_{\phi B} + c_\theta^2\, a_{\phi W} \\
W_3 &= a_{\gamma Z} = 2\, s_\theta\, c_\theta \left(a_{\phi W} - a_{\phi B} \right) + \left(c_\theta^2 - s_\theta^2 \right) a_{\Phi WB} \\
W_4 &= a_{\phi D} \\
W_5 &= a_{\phi q}^{(3)} + a_{\phi q}^{(1)} - a_{\phi u} \\
W_6 &= a_{\phi q}^{(3)} + a_{\phi q}^{(1)} + a_{\phi u}
\end{aligned}
$$

✓ Define

$$
\mathrm{A}^{\mathrm{LO}} = \frac{M_Z^4}{t^2} + \frac{M_Z^4}{u^2} - \frac{t}{u} - \frac{u}{t} - 4\,\frac{M_Z^2 s}{tu}
$$

✓ Obtain the result ($\bar{\mathbf{u}}\mathbf{u} \to \mathbf{ZZ}$)

$$\sum_{\text{spin}} \left| \mathrm{A}^{(4+6)} \right|^2 = g^4 \, \mathrm{A}^{\mathrm{LO}} \left[\mathrm{F}^{\mathrm{LO}}(s_\theta) + \frac{g_6}{\sqrt{2}} \sum_{i=1}^{6} \mathrm{F}^i(s_\theta) \, W_i \right]$$

✓ Obtain the result ($\bar{\mathbf{u}}\mathbf{u} \to \mathbf{ZZ}$)

$$\sum_{\text{spin}} \left| \mathrm{A}^{(4+6)} \right|^2 = g^4 \, \mathrm{A}^{\mathrm{LO}} \left[\mathrm{F}^{\mathrm{LO}} \left(s_\theta \right) + \frac{g_6}{\sqrt{2}} \sum_{i=1}^{6} \mathrm{F}^i \left(s_\theta \right) W_i \right]$$

✓ Background changes!

✓ Obtain the result ($\bar{\mathbf{u}}\mathbf{u} \to \mathbf{ZZ}$)

$$\sum_{\text{spin}} \left| \mathrm{A}^{(4+6)} \right|^2 = g^4 \, \mathrm{A}^{\mathrm{LO}} \left[\mathrm{F}^{\mathrm{LO}}(s_\theta) + \frac{g_6}{\sqrt{2}} \sum_{i=1}^{6} \mathrm{F}^i(s_\theta) \, W_i \right]$$

✓ Background changes!

✓ Note that

$$\begin{aligned} \mathrm{F}^{\mathrm{LO}} &\approx -0.57 \quad \mathrm{F}^1 \approx +2.18 \quad \mathrm{F}^2 \approx -3.31 \\ \mathrm{F}^3 &\approx +4.07 \quad \mathrm{F}^4 \approx -2.46 \quad \mathrm{F}^4 \approx -2.46 \quad \mathrm{F}^6 \approx -5.81 \end{aligned}$$

Increasing COMPLEXITY

✓ $\mathbf{H \to \gamma\gamma}$

① **3** LO amplitudes $\mathbf{A_t^{LO}, A_b^{LO}, A_W^{LO}}$, **3** $\boldsymbol{\kappa}$-factors

② **6** Wilson coefficients & non-factorizable amplitudes

Increasing COMPLEXITY

✓ $\mathbf{H \to \gamma\gamma}$

① **3** LO amplitudes $\mathbf{A_t^{LO}, A_b^{LO}, A_W^{LO}}$, **3** $\boldsymbol{\kappa}$-factors

② **6** Wilson coefficients & non-factorizable amplitudes

✓ $\mathbf{H \to ZZ}$

① **1** LO amplitude

② **6** NLO amplitudes, **6** $\boldsymbol{\kappa}$-factors

$$\delta^{\mu\nu} \sum_{i=\mathrm{t,b,B}} \mathrm{A}_{i,\mathrm{D}}^{\mathrm{NLO}} + p_2^{\mu} p_1^{\nu} \sum_{i=\mathrm{t,b,B}} \mathrm{A}_{i,\mathrm{P}}^{\mathrm{NLO}}$$

② **16** Wilson coefficients & non-factorizable amplitudes

Increasing COMPLEXITY

✓ $\mathbf{H \to \gamma\gamma}$

① **3** LO amplitudes $\mathbf{A^{LO}_t, A^{LO}_b, A^{LO}_W}$, **3** $\boldsymbol{\kappa}$-factors

② **6** Wilson coefficients & non-factorizable amplitudes

✓ $\mathbf{H \to ZZ}$

① **1** LO amplitude

② **6** NLO amplitudes, **6** $\boldsymbol{\kappa}$-factors

$$\delta^{\mu\nu} \sum_{i=\mathrm{t,b,B}} \mathrm{A}^{\mathrm{NLO}}_{i,\mathrm{D}} + p_2^{\mu} p_1^{\nu} \sum_{i=\mathrm{t,b,B}} \mathrm{A}^{\mathrm{NLO}}_{i,\mathrm{P}}$$

② **16** Wilson coefficients & non-factorizable amplitudes

✓ etc.

✎ $\boldsymbol{g}$ finite renormalization

$$g^2_{\text{exp}} = G^2 \left[1 + 2\,\frac{G^2}{16\,\pi^2} \left(\mathrm{d}G^{(4)} + g_6\,\mathrm{d}G^{(6)} \right) \right] \qquad G^2 = 4\,\sqrt{2}\,G_{\mathrm{F}}\,M^2_{\mathrm{W}}$$

✓ $\mathbf{d}\boldsymbol{G}^{(4,6)}$ from $\boldsymbol{\mu}$ -decay

✎ g finite renormalization

$$g^2_{\rm exp} \;=\; G^2\left[1+2\,\frac{G^2}{16\,\pi^2}\,\left(\mathrm{d}G^{(4)}+g_6\,\mathrm{d}G^{(6)}\right)\right] \qquad G^2=4\,\sqrt{2}\,G_{\rm F}\,M^2_{\rm W}$$

- ✓ $\mathbf{d}G^{(4,6)}$ from μ -decay
- ✓ Involving $\Sigma_{\rm WW}(0)$ (easy)

✎ ***g*** **finite renormalization**

$$g^2_{\text{exp}} = G^2\left[1 + 2\,\frac{G^2}{16\,\pi^2}\left(\mathrm{d}G^{(4)} + g_6\,\mathrm{d}G^{(6)}\right)\right] \qquad G^2 = 4\,\sqrt{2}\,G_{\mathrm{F}}\,M^2_{\mathrm{W}}$$

- ✓ $\mathbf{d}\boldsymbol{G}^{(\mathbf{4,6})}$ from $\boldsymbol{\mu}$-decay
- ✓ Involving $\boldsymbol{\Sigma}_{\mathbf{WW}}(\mathbf{0})$ (easy)
- ✗ and vertices & boxes (not easy with $\mathscr{O}^{(\mathbf{6})}$-insertions)

Fine points on PTG versus LG $\mathcal{O}^{(6)}$ operators

✓ Proposition: if we assume that the high-energy theory is

① weakly-coupled and

② renormalizable

Fine points on PTG versus LG $\mathcal{O}^{(6)}$ operators

✓ Proposition: if we assume that the high-energy theory is

① weakly-coupled and

② renormalizable

✓ it follows that the PTG/LG classification of arXiv:1307.0478 (used here) is correct.

Fine points on PTG versus LG $\mathcal{O}^{(6)}$ operators

- ✓ Proposition: if we assume that the high-energy theory is
  - ① weakly-coupled and
  - ② renormalizable
- ✓ it follows that the PTG/LG classification of arXiv:1307.0478 (used here) is correct.
- ✓ If we do not assume the above but work always in some EFT context (i.e.. also the next high-energy theory is EFT, possibly involving some strongly interacting theory) then classification changes, see Eqs. (A1-A2) of arXiv:1305.0017v2

The life and death of $\boldsymbol{\mu}_{\mathbf{R}}$

✓ $\boldsymbol{\gamma}$ bare propagator

$$
\begin{aligned}
\Delta_\gamma^{-1} &= -s - \frac{g^2}{16\,\pi^2}\,\Sigma_{\gamma\gamma}(s) \\
\Sigma_{\gamma\gamma}(s) &= \left(\mathrm{D}^{(4)} + g_6\,\mathrm{D}^{(6)}\right)\frac{1}{\bar{\varepsilon}} + \sum_{x \in \mathscr{X}} \left(\mathrm{L}_i^{(4)} + g_6\,\mathrm{L}_i^{(6)}\right) \ln\frac{x}{\mu_{\mathrm{R}}^2} + \Sigma_{\gamma\gamma}^{\mathrm{rest}} \\
\{\mathscr{X}\} &= \{s\,, m^2\,, m_0^2\,, m_{\mathrm{H}}^2\,, m_{\mathrm{t}}^2\,, m_{\mathrm{b}}^2\}
\end{aligned}
$$

The life and death of μ_{R}

✓ **γ** bare propagator

$$\Delta_\gamma^{-1} = -s - \frac{g^2}{16\,\pi^2}\,\Sigma_{\gamma\gamma}(s)$$

$$\Sigma_{\gamma\gamma}(s) = \left(\mathrm{D}^{(4)} + g_6\,\mathrm{D}^{(6)}\right)\frac{1}{\bar{\varepsilon}} + \sum_{x\in\mathscr{X}}\left(\mathrm{L}_i^{(4)} + g_6\,\mathrm{L}_i^{(6)}\right)\ln\frac{x}{\mu_{\mathrm{R}}^2} + \Sigma_{\gamma\gamma}^{\mathrm{rest}}$$

$$\{\mathscr{X}\} = \{s\,,\,m^2\,,\,m_0^2\,,\,m_{\mathrm{H}}^2\,,\,m_{\mathrm{t}}^2\,,\,m_{\mathrm{b}}^2\}$$

✓ **γ** renormalized propagator

$$\Delta_\gamma^{-1}\Big|_{\mathrm{ren}} = -Z_\gamma\,s - \frac{g^2}{16\,\pi^2}\,\Sigma_{\gamma\gamma}(s) = -s - \frac{g^2}{16\,\pi^2}\,\Sigma_{\gamma\gamma}^{\mathrm{ren}}(s)$$

The life and death of $\boldsymbol{\mu}_{\mathbf{R}}$

$$\Sigma_{\gamma\gamma}^{\text{ren}}(s) = \sum_{x \in \mathscr{X}} \left(\mathrm{L}_i^{(4)} + g_6 \, \mathrm{L}_i^{(6)} \right) \ln \frac{x}{\mu_{\mathrm{R}}^2} + \Sigma_{\gamma\gamma}^{\text{rest}}$$

✓ finite renormalization

$$\Sigma_{\gamma\gamma}^{\text{ren}}(s) = \Pi_{\gamma\gamma}^{\text{ren}}(s) \, s$$

$$\frac{\partial}{\partial \mu_{\mathrm{R}}} \left[\Pi_{\gamma\gamma}^{\text{ren}}(s) - \Pi_{\gamma\gamma}^{\text{ren}}(0) \right] = 0$$

The life and death of μ_R

$$\Sigma^{\text{ren}}_{\gamma\gamma}(s) \quad = \quad \sum_{x\in\mathscr{X}} \left(\mathrm{L}_i^{(4)} + g_6\,\mathrm{L}_i^{(6)}\right) \ln\frac{x}{\mu_\mathrm{R}^2} + \Sigma^{\text{rest}}_{\gamma\gamma}$$

✓ finite renormalization

$$\Sigma^{\text{ren}}_{\gamma\gamma}(s) \quad = \quad \Pi^{\text{ren}}_{\gamma\gamma}(s)\,s$$

$$\frac{\partial}{\partial\mu_\mathrm{R}}\left[\Pi^{\text{ren}}_{\gamma\gamma}(s) - \Pi^{\text{ren}}_{\gamma\gamma}(0)\right] \quad = \quad 0$$

✓ including $\mathscr{O}^{(6)}$ contribution. There is no μ_R problem when a subtraction point is available.

$\mathscr{O}^{(6)} \rightarrow \mathscr{O}^{(4)} \rightarrow$ field(parameter) redefinition

$$
\begin{aligned}
\mathscr{L} &= -\partial_\mu \mathrm{K}^\dagger \partial^\mu \mathrm{K} - \mu^2 \mathrm{K}^\dagger \mathrm{K} \\
&- \frac{1}{2}\lambda \left(\mathrm{K}^\dagger \mathrm{K}\right)^2 - \frac{1}{2} M_0^2 \phi_0^2 - M^2 \phi^+ \phi^- + g^2 \frac{a_\phi}{\Lambda^2} \left(\mathrm{K}^\dagger \mathrm{K}\right)^3 \\
&- g \frac{a_{\phi\Box}}{\Lambda^2} \mathrm{K}^\dagger \mathrm{K} \Box \mathrm{K}^\dagger \mathrm{K} - g \frac{a_{\phi\mathrm{D}}}{\Lambda^2} \left|\mathrm{K}^\dagger \partial^\mu \mathrm{K}\right|^2
\end{aligned}
$$

$$
\sqrt{2}\,\mathrm{K}_1 = \mathrm{H} + 2\frac{M}{g} + i\,\phi_0 \qquad\qquad \mathrm{K}_2 = i\,\phi^-
$$

Requires

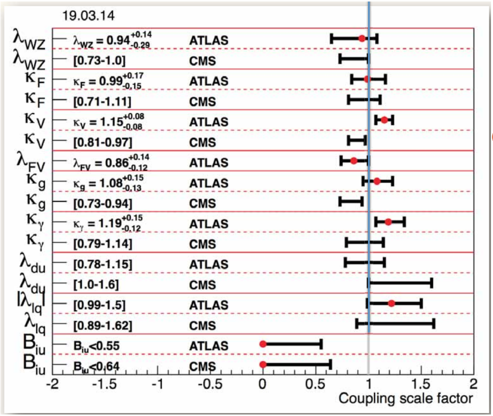
$$\mu^2 = \beta_{\mathrm{H}} - 2\,\frac{\lambda}{g^2}\,M^2 \qquad\qquad \lambda = \frac{1}{4}\,g^2\,\frac{M^2_{\mathrm{H}}}{M^2}$$

$$\mathrm{H} \;\; \to \;\; \left[1 - (a_{\phi\mathrm{D}} - 4\,a_{\phi\Box})\,\frac{M^2_{\mathrm{H}}}{g^2\Lambda^2}\right]\mathrm{H}$$

$$M_{\mathrm{H}} \;\; \to \;\; \left[1 + (a_{\phi\mathrm{D}} - 4\,a_{\phi\Box} + 24\,a_{\phi})\,\frac{M^2_{\mathrm{H}}}{g^2\Lambda^2}\right]M_{\mathrm{H}}$$

etc. with non-trivial effects on the $\boldsymbol{S}$-matrix

FUTURE (Moriod EW **2014**)



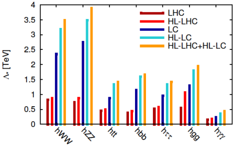


FIG. 2: Effective new physics scales Λ_* extracted from the Higgs coupling measurements collected in Table I. T for the loop-induced couplings to gluons and photons contain only the contribution of the contact terms, as t of the loop terms are already disentangled at the level of the input values Δ. (The ordering of the columns fro right corresponds to the legend from up to down.)

34

$$\mathscr{L} = \mathscr{L}_4 + \sum_{n>4} \sum_{i=1}^{N_n} \frac{a_i^n}{\Lambda^{n-4}} \mathcal{O}_i^{(d=n)}$$

TH is improving with NLO κ-language

NLO κ-language is NOT a simple scaling

Confusion is a word we have invented for an order which is not understood

Thank you for your attention

Backup Slides

Melius abundare quam deficere

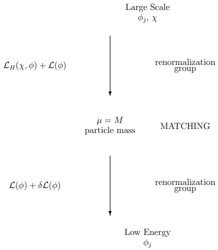


Figure 4: The general form of a matching calculation.

terms.

In this region, the physics is described by a set fields, χ, describing the heaviest particles, of mass M, and a set of light particle fields, ϕ, describing all the lighter particles. The Lagrangian has the form

$$\mathcal{L}_H(\chi, \phi) + \mathcal{L}(\phi) \,, \tag{3.15}$$

where $\mathcal{L}(\phi)$ contains all the terms that depend only on the light fields, and $\mathcal{L}_H(\chi, \phi)$ is everything else. You then evolve the theory down to lower scales. As long as no particle masses are encountered, this evolution is described by the renormalization group. However, when μ goes below the mass, M, of the heavy particles, you should change the effective theory to a new theory without the heavy particles. In the process, the parameters of the theory change, and new, nonrenormalizable interactions may be introduced. Thus the Lagrangian of the effective theory below M has the form

$$\mathcal{L}(\phi) + \delta\mathcal{L}(\phi) \,, \tag{3.16}$$

H wave function renormalization $1 - \frac{1}{2}\,\frac{g^2_{\rm exp}}{16\pi^2}\,\delta\mathscr{Z}_{\rm H}$

✎

$$
\begin{aligned}
\delta\mathscr{Z}^{(4)}_{\rm H} \quad = \quad & \frac{3}{2}\,\frac{M^2_{\rm t}}{M^2_{\rm W}}\,B^f_0\left(-M^2_{\rm H}\,;M_{\rm t},M_{\rm t}\right) + \frac{3}{2}\,\frac{M^2_{\rm b}}{M^2_{\rm W}}\,B^f_0\left(-M^2_{\rm H}\,;M_{\rm b},M_{\rm b}\right) \\
- \quad & B^f_0\left(-M^2_{\rm H}\,;M_{\rm W},M_{\rm W}\right) - 1/2\,\frac{1}{c^2_\theta}\,B^f_0\left(-M^2_{\rm H}\,;M_{\rm Z},M_{\rm Z}\right) \\
+ \quad & \frac{3}{2}\left(M^2_{\rm H} - 4\,M^2_{\rm t}\right)\frac{M^2_{\rm t}}{M^2_{\rm W}}\,B^p_0\left(-M^2_{\rm H}\,;M_{\rm t},M_{\rm t}\right) + \frac{3}{2}\left(M^2_{\rm H} - 4\,M^2_{\rm b}\right)\frac{M^2_{\rm b}}{M^2_{\rm W}}\,B^p_0\left(-M^2_{\rm H}\,;M_{\rm b},M_{\rm b}\right) \\
+ \quad & \frac{1}{4}\left(\frac{M^4_{\rm H}}{M^2_{\rm W}} - 4\,M^2_{\rm H} + 12\,M^2_{\rm W}\right)B^p_0\left(-M^2_{\rm H}\,;M_{\rm W},M_{\rm W}\right) + \frac{1}{8}\left(\frac{M^4_{\rm H}}{M^2_{\rm W}} - 4\,\frac{M^2_{\rm H}}{c^2_\theta} + 12\,\frac{M^2_{\rm Z}}{c^2_\theta}\right)B^p_0\left(-M^2_{\rm H}\,;M_{\rm Z},M_{\rm Z}\right) \\
+ \quad & \frac{9}{8}\,\frac{M^4_{\rm H}}{M^2_{\rm W}}\,B^p_0\left(-M^2_{\rm H}\,;M_{\rm H},M_{\rm H}\right)
\end{aligned}
$$

etc.

STU: (combination of) Wilson coefficients

$$W_1 = a_{\gamma\gamma} = s_\theta c_\theta \, a_{\Phi\mathrm{WB}} + c_\theta^2 \, a_{\phi\mathrm{B}} + s_\theta^2 \, a_{\phi\mathrm{W}}$$

$$W_2 = a_{\mathrm{ZZ}} = -s_\theta c_\theta \, a_{\Phi\mathrm{WB}} + s_\theta^2 \, a_{\phi\mathrm{B}} + c_\theta^2 \, a_{\phi\mathrm{W}}$$

$$W_3 = a_{\gamma\mathrm{Z}} = 2 \, s_\theta \, c_\theta \left(a_{\phi\mathrm{W}} - a_{\phi\mathrm{B}} \right) + \left(c_\theta^2 - s_\theta^2 \right) a_{\Phi\mathrm{WB}}$$

$W_4 = a_{\phi\mathrm{D}}$ $\qquad$ $W_5 = a_{\phi\Box}$

$W_6 = a_{\mathrm{bWB}}$ $\qquad$ $W_7 = a_{\mathrm{bBW}}$

$W_8 = a_{\mathrm{tWB}}$ $\qquad$ $W_9 = a_{\mathrm{tBW}}$

$W_{10} = a_{\mathrm{b}\phi}$ $\qquad$ $W_{11} = a_{\mathrm{t}\phi}$

$$a_{\mathrm{qW}} = s_\theta \, a_{\mathrm{qWB}} + c_\theta \, a_{\mathrm{qBW}} \qquad a_{\mathrm{qB}} = s_\theta \, a_{\mathrm{qBW}} - c_\theta \, a_{\mathrm{qWB}}$$

$$W_{12} = a_{\phi bA} \qquad W_{13} = a_{\phi bV}$$
$$W_{14} = a_{\phi tA} \qquad W_{15} = a_{\phi tV}$$

$$a_{\phi bV} = a_{\phi q}^{(3)} - a_{\phi b} - a_{\phi q}^{(1)} \qquad a_{\phi bA} = a_{\phi q}^{(3)} + a_{\phi b} - a_{\phi q}^{(1)}$$
$$a_{\phi tV} = a_{\phi q}^{(3)} - a_{\phi t} - a_{\phi q}^{(1)} \qquad a_{\phi tA} = a_{\phi q}^{(3)} + a_{\phi t} - a_{\phi q}^{(1)}$$

STU: building blocks $\boldsymbol{\gamma}-\boldsymbol{\gamma}$

$$\Sigma_{\gamma\gamma}(s) = \Pi_{\gamma\gamma}(s)\, s$$

$$\Pi_{\gamma\gamma}(s) = \frac{g^2 s_\theta^2}{16\,\pi^2}\,\Pi^{(4)}_{\gamma\gamma}(s) + \frac{g^2 g_6}{16\sqrt{2}\,\pi^2}\sum_{i=1}^{11}\Pi^{(6)}_{\gamma\gamma\,i}(s)\,W_i$$

$$\Pi^{(4)}_{\gamma\gamma}(0) = 3\,a_0^f\left(M_{\mathrm{W}}\right) + \frac{1}{9}\left[1 - 4\,a_0^f\left(M_{\mathrm{b}}\right) - 16\,a_0^f\left(M_{\mathrm{t}}\right)\right]$$

$$
\begin{aligned}
\Pi^{(6)}_{\gamma\gamma\,1}(0) &= -\left(1 - 8\,s^2_\theta + 2\,s^4_\theta\right) a^f_0\left(M_{\mathrm{W}}\right) \\
&- \frac{1}{2}\,\frac{M^2_{\mathrm{H}}}{M^2_{\mathrm{W}}}\,a^f_0\left(M_{\mathrm{H}}\right) - \frac{1}{2}\,\frac{1}{c^2_\theta}\,a^f_0\left(M_{\mathrm{Z}}\right) \\
&- \frac{4}{9}\,s^2_\theta\left[16\left(1 - \frac{1}{2}\,s^2_\theta\right) a^f_0\left(M_{\mathrm{t}}\right)\right. \\
&+ \left. 4\left(1 - \frac{1}{2}\,s^2_\theta\right) a^f_0\left(M_{\mathrm{b}}\right) + 17\left(1 - \frac{35}{34}\,s^2_\theta\right)\right] \\
\Pi^{(6)}_{\gamma\gamma\,2}(0) &= s_\theta\,c_\theta\left\{\frac{2}{9}\left[35 + 16\,a^f_0\left(M_{\mathrm{t}}\right) + 4\,a^f_0\left(M_{\mathrm{b}}\right)\right] - 2\,a^f_0\left(M_{\mathrm{W}}\right)\right\} \\
\Pi^{(6)}_{\gamma\gamma\,3}(0) &= s_\theta\,c_\theta\left\{4\left(1 - \frac{35}{18}\,c^2_\theta\right) + 4\left(1 - \frac{1}{2}\,s^2_\theta\right) a^f_0\left(M_{\mathrm{W}}\right)\right. \\
&- \left.\frac{8}{9}\,c^2_\theta\left[4\,a^f_0\left(M_{\mathrm{t}}\right) + a^f_0\left(M_{\mathrm{b}}\right)\right]\right\} \\
\Pi^{(6)}_{\gamma\gamma\,4}(0) &= c^2_\theta\left\{-\frac{3}{2}\,a^f_0\left(M_{\mathrm{W}}\right) + \frac{1}{18}\left[16\,a^f_0\left(M_{\mathrm{t}}\right) + 4\,a^f_0\left(M_{\mathrm{b}}\right) - 1\right]\right\} \\
\Pi^{(6)}_{\gamma\gamma\,6}(0) &= -2\,\frac{M^2_{\mathrm{b}}}{M^2_{\mathrm{W}}}\,s_\theta\left[a^f_0\left(M_{\mathrm{b}}\right) + 1\right] \\
\Pi^{(6)}_{\gamma\gamma\,8}(0) &= -4\left(c^2_\theta - s^2_\theta\right) s_\theta\,\frac{M^2_{\mathrm{b}}}{M^2_{\mathrm{W}}}\left[a^f_0\left(M_{\mathrm{t}}\right) + 1\right] \\
\Pi^{(6)}_{\gamma\gamma\,9}(0) &= 8\,s^2_\theta\,c_\theta\,\frac{M^2_{\mathrm{t}}}{M^2_{\mathrm{W}}}\left[a^f_0\left(M_{\mathrm{t}}\right) + 1\right]
\end{aligned}
$$

$\mathcal{STU}$: building blocks $\mathbf{Z}-\boldsymbol{\gamma}$

$$
\begin{aligned}
\Sigma_{Z\gamma}(s) &= \Pi_{Z\gamma}(s)\,s \\
\Pi_{Z\gamma}(s) &= \frac{g^2}{16\,\pi^2}\,\frac{s_\theta}{c_\theta}\,\Pi^{(4)}_{Z\gamma}(s) + \frac{g^2 g_6}{16\,\sqrt{2}\,\pi^2}\,\sum_{i=1}^{15}\Pi^{(6)}_{Z\gamma\,i}(s)\,W_i - \frac{g_6}{\sqrt{2}}\,W_3
\end{aligned}
$$

$$
\begin{aligned}
\Pi^{(4)}_{Z\gamma}(0) &= \frac{1}{6}\left(19 - 18\,s^2_\theta\right)\,a^f_0\,(M_W) - \frac{2}{9}\left(3 - 8\,s^2_\theta\right)\,a^f_0\,(M_t) \\
&- \frac{1}{9}\left(3 - 4\,s^2_\theta\right)\,a^f_0\,(M^b) + \frac{1}{18}\left(21 - 2\,s^2_\theta\right)
\end{aligned}
$$

$$
\begin{aligned}
\Pi^{(6)}_{Z\gamma\,1}(0) &= \frac{s_\theta}{c_\theta}\left[\frac{1}{3}\left(1+6\,c_\theta^4\right)a_0^f\left(M_{\rm W}\right)+\frac{4}{9}\left(5-8\,c_\theta^4\right)a_0^f\left(M_{\rm t}\right)\right.\\
&+ \left.\frac{2}{9}\left(1-4\,c_\theta^4\right)a_0^f\left(M_{\rm b}\right)-\frac{1}{9}\left(33-122\,s_\theta^2+70\,s_\theta^4\right)\right]\\
\Pi^{(6)}_{Z\gamma\,2}(0) &= s_\theta\,c_\theta\left[+2\left(3-c_\theta^2\right)a_0^f\left(M_{\rm W}\right)-\frac{32}{9}\,s_\theta^2\,a_0^f\left(M_{\rm t}\right)\right.\\
&- \left.\frac{8}{9}\,s_\theta^2\,a_0^f\left(M_{\rm b}\right)-\frac{2}{9}\left(8-35\,c_\theta^2\right)\right]\\
\Pi^{(6)}_{Z\gamma\,3}(0) &= -\frac{1}{18}\left(33-174\,s_\theta^2+140\,s_\theta^4\right)+\frac{1}{3}\left(2-9\,s_\theta^2+6\,s_\theta^4\right)a_0^f\left(M_{\rm W}\right)\\
&- \frac{1}{4}\,\frac{M_{\rm H}^2}{M_{\rm W}^2}\,a_0^f\left(M_{\rm H}\right)-\frac{1}{4}\,\frac{1}{c_\theta^2}\,a_0^f\left(M_{\rm Z}\right)-\frac{2}{9}\left(3-24\,s_\theta^2+16\,s_\theta^4\right)a_0^f\left(M_{\rm t}\right)-\frac{1}{9}\left(3-12\,s_\theta^2+8\,s_\theta^4\right)a_0^f\left(M_{\rm b}\right)\\
\Pi^{(6)}_{Z\gamma\,4}(0) &= \frac{1}{s_\theta\,c_\theta}\left[-\frac{1}{24}\left(19-56\,s_\theta^2+36\,s_\theta^4\right)a_0^f\left(M_{\rm W}\right)+\frac{1}{18}\left(3-24\,s_\theta^2+16\,s_\theta^4\right)a_0^f\left(M_{\rm t}\right)\right.\\
&+ \left.\frac{1}{36}\left(3-12\,s_\theta^2+8\,s_\theta^4\right)a_0^f\left(M_{\rm b}\right)-\frac{1}{72}\left(21+4\,s_\theta^4\right)\right]\\
\Pi^{(6)}_{Z\gamma\,6}(0) &= \frac{1}{4\,c_\theta}\,\frac{M_{\rm b}^2}{M_{\rm W}^2}\left(1-4\,c_\theta^2\right)\left[a_0^f\left(M_{\rm b}\right)-1\right]\\
\Pi^{(6)}_{Z\gamma\,7}(0) &= -\frac{M_{\rm b}^2}{M_{\rm W}^2}\,s_\theta^2\left[a_0^f\left(M_{\rm b}\right)+1\right]\\
\Pi^{(6)}_{Z\gamma\,8}(0) &= -\frac{1}{4\,c_\theta}\,\frac{M_{\rm t}^2}{M_{\rm W}^2}\left(5-34\,c_\theta^2+32\,c_\theta^4\right)\left[a_0^f\left(M_{\rm t}\right)-1\right]\\
\Pi^{(6)}_{Z\gamma\,9}(0) &= \frac{1}{2}\,s_\theta\,\frac{M_{\rm t}^2}{M_{\rm W}^2}\left(7-16\,s_\theta^2\right)\left[a_0^f\left(M_{\rm t}\right)+1\right]
\end{aligned}
$$

$$\Pi^{(6)}_{Z\gamma\,13}(0) = -\frac{2}{3}\,\frac{s_\theta}{c_\theta}\,\frac{M_b^2}{M_W^2}\left[a_0^f\left(M_b\right)+1\right]$$

$$\Pi^{(6)}_{Z\gamma\,15}(0) = -\frac{4}{3}\,\frac{s_\theta}{c_\theta}\,\frac{M_t^2}{M_W^2}\left[a_0^f\left(M_t\right)+1\right]$$

–

STU: building blocks $\mathbf{Z}$–$\mathbf{Z}$

$$
\begin{aligned}
\Sigma_{\mathrm{ZZ}}(s) &= \mathrm{S}_{\mathrm{ZZ}} + \Pi_{\mathrm{ZZ}}\, s + \mathscr{O}(s^2) \\
\mathrm{S}_{\mathrm{ZZ}} &= \frac{g^2}{16\,\pi^2\,c_\theta^2}\,\mathrm{S}^{(4)}_{\mathrm{ZZ}} + \frac{g^2 g_6}{16\,\sqrt{2}\,\pi^2}\sum_{i=1}^{15}\mathrm{S}^{(6)}_{\mathrm{ZZ}\,i}\,W_i \\
\Pi_{\mathrm{ZZ}} &= \frac{g^2}{16\,\pi^2\,c_\theta^2}\,\Pi^{(4)}_{\mathrm{ZZ}} + \frac{g^2 g_6}{16\,\sqrt{2}\,\pi^2}\sum_{i=1}^{15}\Pi^{(6)}_{\mathrm{ZZ}\,i}\,W_i
\end{aligned}
$$

$$
\begin{aligned}
\mathrm{S}^{(4)}_{\mathrm{ZZ}} \quad = \quad & \left(M_{\mathrm{Z}}^2 - \frac{1}{3}\, M_{\mathrm{H}}^2 + \frac{1}{12}\, \frac{M_{\mathrm{H}}^4}{M_{\mathrm{Z}}^2} \right) B_0^f \left(-M_{\mathrm{Z}}^2\, ; M_{\mathrm{H}}, M_{\mathrm{Z}} \right) \\
+ \quad & \frac{1}{18} \left[\left(7 - 16\, c_\theta^2 - 64\, c_\theta^2\, s_\theta^2 \right) M_{\mathrm{t}}^2 + \left(17 - 8\, c_\theta^2 - 32\, c_\theta^2\, s_\theta^2 \right) M_{\mathrm{Z}}^2 \right] B_0^f \left(-M_{\mathrm{Z}}^2\, ; M_{\mathrm{t}}, M_{\mathrm{t}} \right) \\
+ \quad & \frac{1}{18} \left[\left(5 + 4\, c_\theta^2 - 8\, c_\theta^2\, s_\theta^2 \right) M_{\mathrm{Z}}^2 - \left(17 - 8\, c_\theta^2 + 16\, c_\theta^2\, s_\theta^2 \right) M_{\mathrm{b}}^2 \right] B_0^f \left(-M_{\mathrm{Z}}^2\, ; M_{\mathrm{b}}, M_{\mathrm{b}} \right) \\
+ \quad & \frac{1}{12} \left[\left(1 - 20\, c_\theta^2 + 36\, c_\theta^2\, s_\theta^2 \right) M_{\mathrm{Z}}^2 - 16 \left(5 - 3\, s_\theta^2 \right) M_{\mathrm{Z}}^2\, c_\theta^6 \right] B_0^f \left(-M_{\mathrm{Z}}^2\, ; M_{\mathrm{W}}, M_{\mathrm{W}} \right) \\
+ \quad & \frac{1}{12} \left(M_{\mathrm{Z}}^4 - 2\, M_{\mathrm{H}}^2\, M_{\mathrm{Z}}^2 + M_{\mathrm{H}}^4 \right) B_0^p \left(0\, ; M_{\mathrm{H}}, M_{\mathrm{Z}} \right) + \frac{2}{3} \left(M_{\mathrm{Z}}^2 + \frac{M_{\mathrm{Z}}^4}{M_{\mathrm{H}}^2 - M_{\mathrm{Z}}^2} - \frac{3}{8}\, M_{\mathrm{H}}^2 + \frac{1}{8}\, \frac{M_{\mathrm{H}}^4}{M_{\mathrm{Z}}^2} \right) a_0^f \left(M_{\mathrm{H}} \right) \\
+ \quad & \frac{1}{4} \left(M_{\mathrm{Z}}^2 - \frac{8}{3}\, \frac{M_{\mathrm{Z}}^4}{M_{\mathrm{H}}^2 - M_{\mathrm{Z}}^2} - \frac{1}{3}\, M_{\mathrm{H}}^2 \right) a_0^f \left(M_{\mathrm{Z}} \right) - \frac{4}{27} \left(2 + c_\theta^2 - 5\, c_\theta^2\, s_\theta^2 \right) M_{\mathrm{Z}}^2 \\
\Pi^{(4)}_{\mathrm{ZZ}} \quad = \quad & \frac{5}{6} \left(M_{\mathrm{Z}}^2 - \frac{1}{5}\, M_{\mathrm{H}}^2 \right) B_0^p \left(0 ; M_{\mathrm{H}}, MZ \right) + \frac{1}{18} \left(7 - 16\, c_\theta^2 - 64\, c_\theta^2\, s_\theta^2 \right) M_{\mathrm{t}}^2\, B_0^p \left(0 ; M_{\mathrm{t}}, M_{\mathrm{t}} \right) \\
- \quad & \frac{1}{18} \left(17 - 8\, c_\theta^2 + 16\, c_\theta^2\, s_\theta^2 \right) M_{\mathrm{b}}^2\, B_0^p \left(0 ; M_{\mathrm{b}}, M_{\mathrm{b}} \right) + \frac{1}{3} \left[5\, M_{\mathrm{Z}}^2\, c_\theta^2 - 4 \left(5 - 3\, s_\theta^2 \right) M_{\mathrm{Z}}^2\, c_\theta^6 \right] B_0^p \left(0 ; M_{\mathrm{W}}, M_{\mathrm{W}} \right) \\
- \quad & \frac{1}{24} \left(M_{\mathrm{Z}}^4 - 2\, M_{\mathrm{H}}^2\, M_{\mathrm{Z}}^2 + M_{\mathrm{H}}^4 \right) B_0^s \left(0 ; M_{\mathrm{H}}, MZ \right) + \frac{1}{12} \left(1 + \frac{M_{\mathrm{Z}}^2}{M_{\mathrm{H}}^2 - M_{\mathrm{Z}}^2} \right) a_0^f \left(M_{\mathrm{H}} \right) \\
- \quad & \frac{1}{12}\, \frac{M_{\mathrm{Z}}^2}{M_{\mathrm{H}}^2 - M_{\mathrm{Z}}^2}\, a_0^f \left(M_{\mathrm{Z}} \right) + \frac{4}{27} \left(2 + c_\theta^2 - 5\, c_\theta^2\, s_\theta^2 \right)
\end{aligned}
$$


KEEP
CALM
TO
BE
CONTINUED

annotated DIAGRAMMATICA

$\mathcal{O}(g^3\, g_6)$

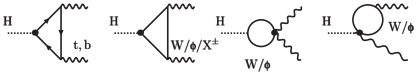


Figure 3: Example of one-loop SM diagrams with $\mathcal{O}$-insertions, contributing to the amplitude for H → γγ

↘ $\kappa_f\,,\ A_{NF}$ ↘ $\kappa_W\,,\ A_{NF}$

⇈

mix under ren. with $\mathcal{O}(g\, g_6)$ ↗ A_{NF}

⇊

$\mathcal{O}(g^3\, g_6)$

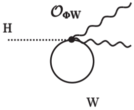


Figure 4: Example of one-loop $\mathcal{O}$-diagrams, contributing to the amplitude for H → γγ

EXAMPLE UV

H-propagator

$$\Delta_{\mathrm{H}}^{-1} = Z_{\mathrm{H}} \left(-s + Z_{m_{\mathrm{H}}} M_{\mathrm{H}}^2 \right) - \frac{1}{(2\,\pi)^4\, i} \Sigma_{\mathrm{HH}}$$

$$Z_{\mathrm{H}} = 1 + \frac{g_{\mathrm{R}}^2}{16\,\pi^2} \left(\delta Z_{\mathrm{H}}^{(4)} + g_6\, \delta Z_{\mathrm{H}}^{(6)} \right) \frac{1}{\bar{\varepsilon}}$$

$$\delta Z_{\mathrm{H}}^{(4)} = 16 \left[\frac{1}{288} \left(82 - \frac{16}{c_\theta^2} - 25\,\frac{s_\theta}{c_\theta} - 14\, s_\theta^2 - 14\, s_\theta\, c_\theta \right) - \frac{3}{32}\, \frac{m_{\mathrm{b}}^2 + m_{\mathrm{t}}^2}{M^2} \right]$$

$$\delta Z_{\mathrm{H}}^{(6)} = \frac{1}{6\,\sqrt{2}} \left[\frac{5}{c_\theta^2} + 12 - 18\, \frac{m_{\mathrm{b}}^2 + m_{\mathrm{t}}^2}{M^2} - 21\, \frac{m_{\mathrm{H}}^2}{M^2} \right] a_{\phi\Box}$$

$$+ \quad \text{etc}$$

EXAMPLE finite ren.

$$m_{\rm H}^2 = M_{\rm H}^2 \left[1 + \frac{g_{\rm R}^2}{16\,\pi^2} \left({\rm d}M_{\rm H}^{(4)} + g_6\,{\rm d}M_{\rm H}^{(6)} \right) \right]$$

$$\begin{aligned}
\frac{M_H^2}{16}\,{\rm d}M_{\rm H}^{(4)} \quad = \quad & \frac{1}{16}\,M_{\rm W}^2 \left(\frac{1}{c_\theta^4} + 2 \right) \\
- \quad & \frac{3}{32}\,\frac{M_{\rm t}^2}{M_{\rm W}^2} \left(M_{\rm H}^2 - 4\,M_{\rm t}^2 \right) B_0 \left(-M_{\rm H}^2\,;\,M_{\rm t}\,,M_{\rm t} \right) \\
- \quad & \frac{3}{32}\,\frac{M_{\rm b}^2}{M_{\rm W}^2} \left(M_{\rm H}^2 - 4\,M_{\rm b}^2 \right) B_0 \left(-M_{\rm H}^2\,;\,M_{\rm b}\,,M_{\rm b} \right) \\
- \quad & \frac{9}{128}\,\frac{M_{\rm H}^4}{M_{\rm W}^2}\,B_0 \left(-M_{\rm H}^2\,;\,M_{\rm H}\,,M_{\rm H} \right) \\
- \quad & \frac{1}{64} \left(\frac{M_{\rm H}^4}{M_{\rm W}^2} - 4\,M_{\rm H}^2 - 12\,M_{\rm W}^2 \right) B_0 \left(-M_{\rm H}^2\,;\,M_{\rm W}\,,M_{\rm W} \right) \\
- \quad & \frac{1}{128} \left(\frac{M_{\rm H}^4}{M_{\rm W}^2} - 4\,\frac{M_{\rm H}^2}{c_\theta^2} + 12\,\frac{M_{\rm W}^2}{c_\theta^4} \right) B_0 \left(-M_{\rm H}^2\,;\,M_{\rm Z}\,,M_{\rm Z} \right) \Big]
\end{aligned}$$

✓ SCALE dependence (no subtraction point)

✓ SCALE dependence (no subtraction point)

✓ Consider $\mathbf{H} \to \gamma\gamma$

$$
\begin{aligned}
Z_{ij}^{\text{mix}} &= \delta_{ij} + \frac{g_{\text{R}}^2}{16\,\pi^2}\left[\delta Z_{ij}^{\text{mix}}\,\frac{1}{\bar{\varepsilon}} + \Delta_{ij}\,\ln\frac{M_{\text{H}}^2}{\mu_{\text{R}}^2}\right] \\
W_1 &= a_{\gamma\gamma} = s_\theta\,c_\theta\;a_{\Phi\text{WB}} + c_\theta^2\;a_{\phi\text{B}} + s_\theta^2\;a_{\phi\text{W}} \\
M_{\text{W}}^2\,\Delta_{11} &= \frac{1}{4}\left[8\,s_\theta^2\left(2\,s_\theta^2 - c_\theta^2\right)M_{\text{W}}^2 + \left(4\,s_\theta^2\,c_\theta^2 - 5\right)M_{\text{H}}^2\right]
\end{aligned}
$$

- ✓ SCALE dependence (no subtraction point)
- ✓ Consider $\mathbf{H} \to \gamma\gamma$

$$
\begin{aligned}
Z^{\text{mix}}_{ij} &= \delta_{ij} + \frac{g^2_{\rm R}}{16\,\pi^2}\left[\delta Z^{\text{mix}}_{ij}\,\frac{1}{\bar{\varepsilon}} + \Delta_{ij}\,\ln\frac{M^2_{\rm H}}{\mu^2_{\rm R}}\right] \\
W_1 &= a_{\gamma\gamma} = s_\theta\,c_\theta\;a_{\Phi{\rm WB}} + c^2_\theta\;a_{\phi{\rm B}} + s^2_\theta\;a_{\phi{\rm W}} \\
M^2_{\rm W}\,\Delta_{11} &= \frac{1}{4}\left[8\,s^2_\theta\left(2\,s^2_\theta - c^2_\theta\right)M^2_{\rm W} + \left(4\,s^2_\theta\,c^2_\theta - 5\right)M^2_{\rm H}\right]
\end{aligned}
$$

- ✓ etc