Loops post-Lep: Renaissance in Radiative Corrections?

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RECENT ADVANCES IN PARTICLE AND ASTROPARTICLE PHYSICS

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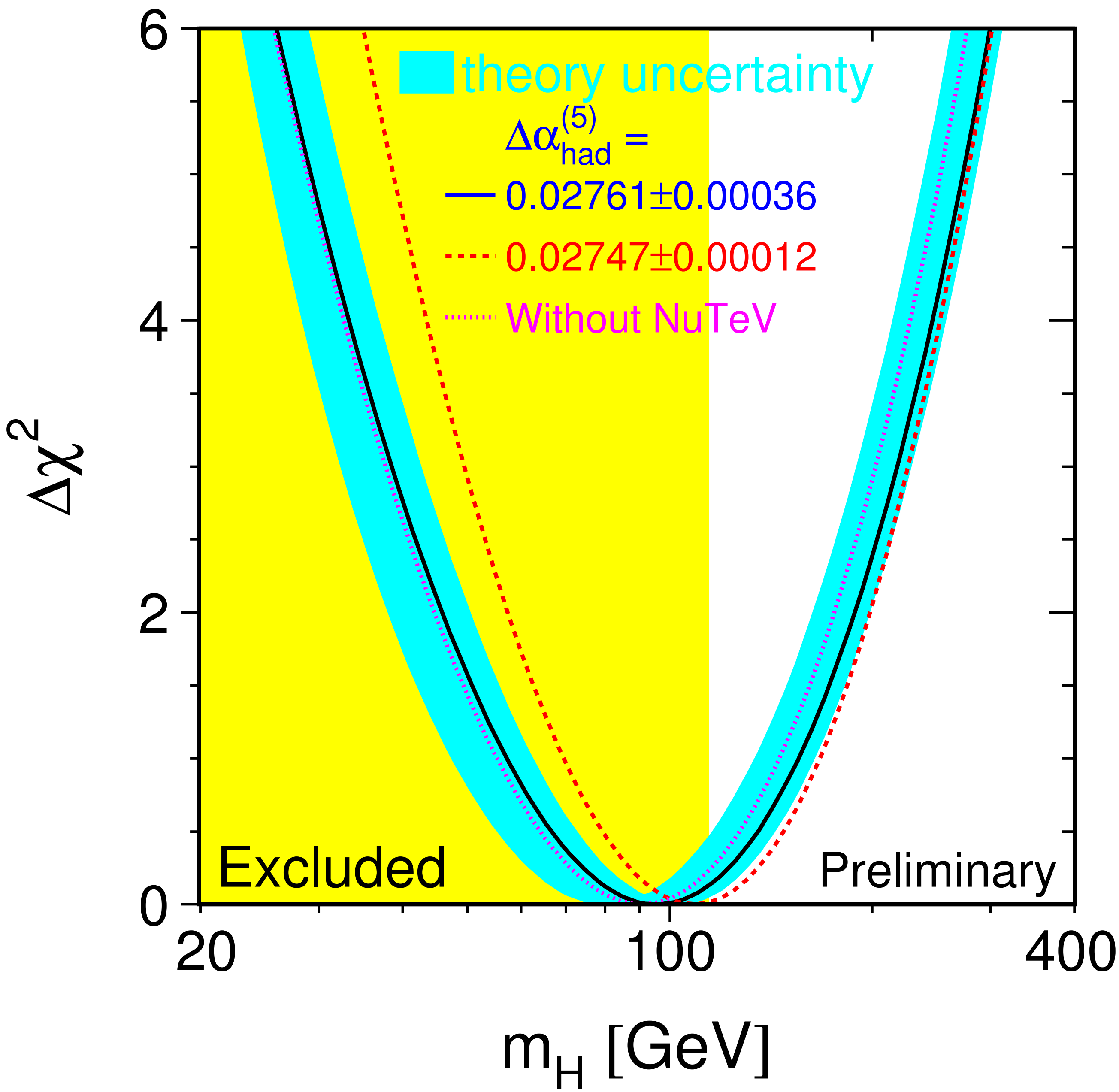
Experimental Progress

- The Exp. progress made in the 1990's is remarkable in many ways. With the right experiments we can reveal the fine details of high energy phenomena.

- After LEP Shutdown we known that physics at the microscopic scale has a basis as rationale as chemistry.

- Everything moves, not by magic, but because there is a mechanism at work.

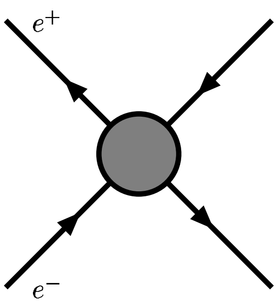
- Everything is encouraging for the major unsolved aspects of the breaking of electroweak gauge symmetry.



The blue band **a.d. 2003** **The most celebrated figure of LEP era**

STATUS of RC around LEP shutdown

PCP Report hep-ph/9902452



one-loop	$\Longrightarrow$	ALL	+ **re-summations**
two-loop	$\Longrightarrow$	LO + NLO	
IS QED		SF-method (or equivalent)	
FS QED		available	
IS $\otimes$ FS QED			

Fine Points in QED for $2 \to 2$

s-channel

$\mathcal{O}\left(\alpha^2 L^n\right), \ n = 0, 1, 2$ (Calculations), $\mathcal{O}\left(\alpha^3 L^3\right)$ important for Z lineshape

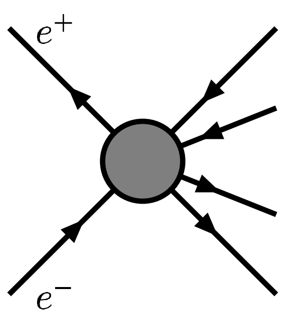
Differences and uncertainties amount to at most ± 0.1 MeV on M_Z and Γ_Z and $\pm 0.01\%$ on σ_h^0 (MIZA, TOPAZ0 and ZFITTER) LEP EWWG/LS 2000-01

non-annihilation

(Bhabha) both SF and PS have been analyzed, uncertainty estimated to be **0.061%** (BHLUMI)

FULL two-loop EW needed for GigaZ ($10^9 Z$ events),

Numerical evaluation of FD?



$e^+e^- \to 4\,\mathbf{f}$ **Born** $\Longrightarrow$ **ALL processes**

$\mathcal{O}(\alpha)$ **EW** $\Longrightarrow$ **only in DPA for** WW **signal**

$e^+e^- \to 4\mathrm{f} + \gamma$ $\Longrightarrow$ **ALL**

hep-ph/0005309

Fine Points in QED for $2 \to 4$

for $e^+e^- \to WW \to 4\,f \;\to$ **see DPA**

for generic $e^+e^- \to 4\,f \;\to$ **s – channel SF** $\to$ *i.e.* **LL approximation**

♢ **the latter strictly applicable only if ISR can be separated (may lead to excess of ISR)**

Mistakes & Incompleteness: Babel's tower

- Definition of mass for unstable particles; we *insisted* on using OMS masses in a situation where we had to use,

  complex poles

  This is, perhaps, not a big deal with M_Z (input parameter) but it is a mess – now that we start having two-loop results – for M_W (a derived quantity);
- Most likely, SA - codes could now be replaced by MC (with the same speed);
- QED would be even better: better treatment for

  SF, also for processes non-annihilation-dominated,

  for IS/FS interference, etc;
- the whole procedure of de-convolution could be changed or made more efficient like the (in)famous and well-known t-channel subtraction for Bhabha.
- signal defined by cuts and not by diagrams

LEP RC Rating:

- ⋆⋆⋆ FS QED
- ⋆⋆⋆ M_H-dependence of D - observables
- ⋆⋆ C - observables, extrapolated setup
- ⋆⋆ M_H-dependence of C - observables
- ⋆ C - observables, realistic setup
- ⋆ C - observables, MI approach

Rating to be understood as compared to exp. error

Road # 1; many particles, (almost) no loops, MonteCarlo

- The MC simulation of hard multiparticle final states at hadronic colliders is a very important issue.
- Multijet final states characterize a large class of important phenomena both within and beyond the SM. They can originate from
  - ⋆ hard QCD radiation processes,
  - ⋆ decay of SM massive particles (W, Z, top quark, Higgs boson),
  - ⋆ decay of heavy supersymmetric or more exotic particles.
- Thanks to recent efforts by different groups, several multiparton event generators based on exact matrix elements are now available.
- They generate samples of unweighted events which can be passed to the parton shower-based MC programs to go from the partons to the real final-state hadrons.
- The matching between a LO multijet event generator and the parton shower MC suffers from the serious problems of double-counting and dependence on the parton-level cuts.

continued...

- In general there are two different approaches to simulate multijet final states:

- the first one consists in generating the simplest possible final state by means of matrix elements and producing additional jets by parton showering

  - ⋆ HERWIG,
  - ⋆ ISAJET,
  - ⋆ PYTHIA

- This procedure works well in the soft/collinear regions but fails to describe configurations with several widely separated jets.

- A complementary strategy is to generate high-multiplicity partonic final states by means of exact matrix elements and eventually pass the generated events to further showering.

continued...

- **Even if it is a leading-order (LO) approach, this procedure can become very difficult because of the complexity of the matrix element calculations with many external legs and of the phase-space integration.**
- **Recently there has been extensive activity in developing several parton-level Monte Carlo (MC) event generators, such as**
  - ⋆ ACERMC, ALPGEN, AMEGIC++, CompHEP, GRACE, HELAC/PHEGAS/JETI, MADEVENT.

$$W + N \textbf{ jets}, \; Z/\gamma^* + N \textbf{ jets}. \; N \leq 6,$$

$$W Q\bar{Q} + N \textbf{ jets}, \; Z/\gamma^* Q\bar{Q} + N \textbf{ jets} \; (Q = c, b, t), \; N \leq 4,$$

$$W + c + N \textbf{ jets}, \; N \leq 5,$$

$$n\; W + m\; Z + p\; \textbf{Higgs} + N \textbf{ jets}, \; n + m + p \leq 8, \; N \leq 3,$$

$$m\; \gamma + N \textbf{ jets}, \; N + m \leq 8 \textbf{ and } N \leq 6,$$

$$Q\bar{Q} + N \textbf{ jets}, \; (Q = c, b, t), \; N \leq 6,$$

$$Q\bar{Q}Q'\bar{Q}' + N \textbf{ jets}, \; (Q, Q' = b, t), \; N \leq 4,$$

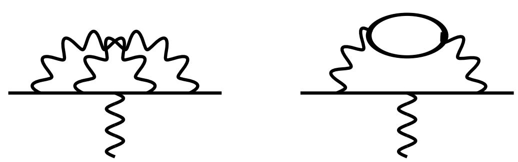
$$N \textbf{ jets}, \; N \leq 6,$$

$$Q\bar{Q}H + N \textbf{ jets}, \; (Q = b, t), \; N \leq 4.$$

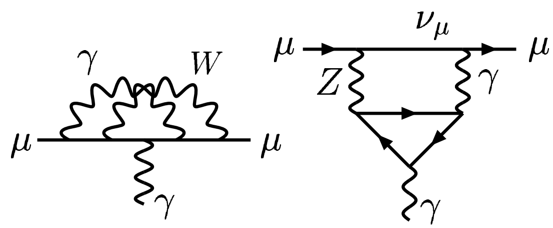
Road # 2; fewer particles, many loops

Feynman diagrams with up to four external lines contribute to many important physical quantities. They are rather complicated mathematical objects because they depend on many variables:

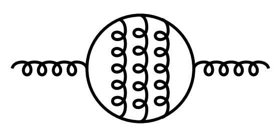
- ⋆ internal masses,
- ⋆ Mandelstam variables and
- ⋆ squares of external momenta.



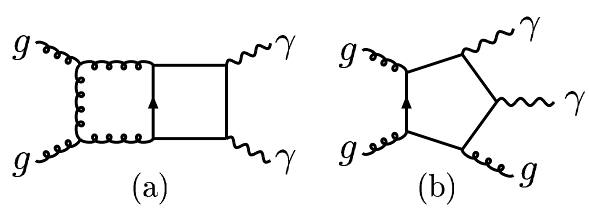
Examples of two-loop QED contributions to muon g-2.



Two of the many two-loop EW Feynman diagrams contributing to the muon g-2.

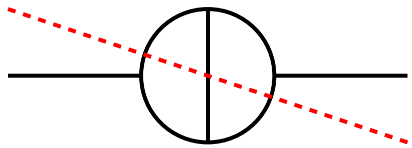


One of the many Feynman diagrams appearing in the computation of the four-loop QCD beta function.



Sample NLO diagrams contributing to gluon fusion into two photons

Why a difficult problem?



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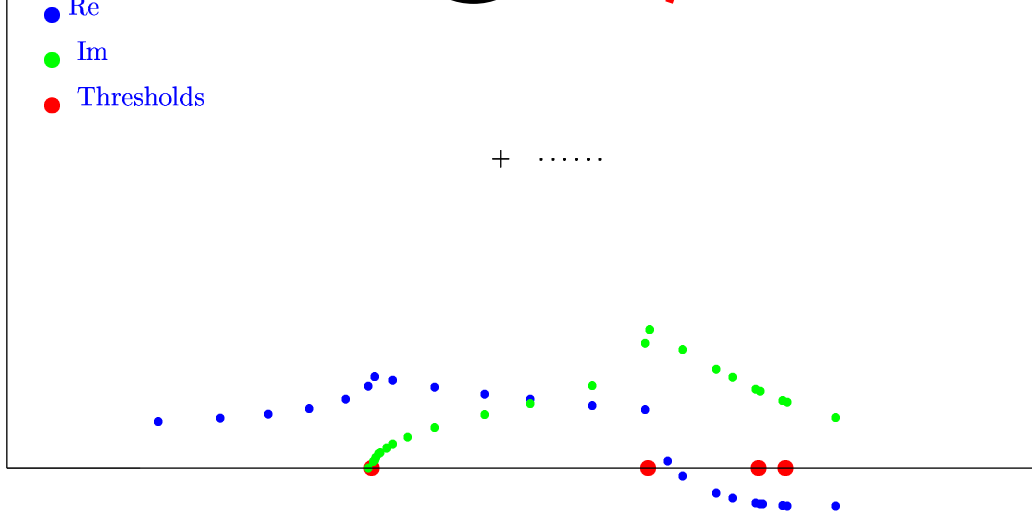
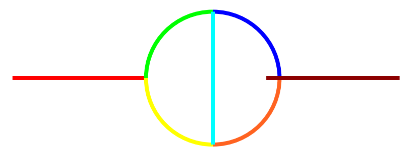


Numerica

↓

By TOPSIDE.F



Techniques I: Almost all available analytical results correspond to massless diagrams.

□ The problem of the evaluation of Feynman integrals is usually decomposed into two parts:

P_1 reduction of general Feynman integrals to the set of master integrals (which cannot be simplified further) and

P_2 the evaluation of these master integrals.

A standard tool to solve the first part of this problem is the method of integration by parts when

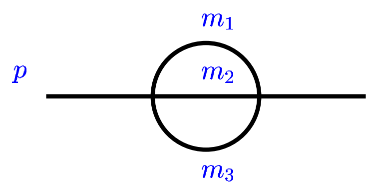
⋆ one writes down identities obtained by putting to zero various integrals of derivatives of the general integrand connected with the given graph and

⋆ tries to solve a resulting system of equations to obtain recurrence relations that express Feynman integrals with general integer powers of the propagators through the master integrals.

continued...

- It is believed that sooner or later we shall achieve the limit in analytical evaluation of Feynman integrals so that we shall be forced to proceed only numerically.
- However the dramatic recent progress in the field of analytical evaluation of Feynman integrals shows that we have not yet exhausted our abilities in this direction. Indeed, several powerful methods were developed during the last years. To calculate the master integrals one can apply
  - ⋆ the technique of MB integration (V.A. Smirnov) and the method of differential equat (A.V. Kotikov, T. Gehrmann and E. Remiddi);
  - ⋆ to construct appropriate recursive algorithms one can use recently developed methods based on shifting dimension (O.V. Tarasov),
  - ⋆ and differential equations (T. Gehrmann and E. Remiddi),
  - ⋆ as well as a method based on non-recursive solutions of recurrence relations (P.A. Baikov)
- One can also hope that new analytical results can be obtained for many other classes of Feynman integrals depending on many scales.

Differential equations for FD



In the equal mass limit the two-loop sunrise has two MIs, which in the usual d-continuous regularization scheme can be written as

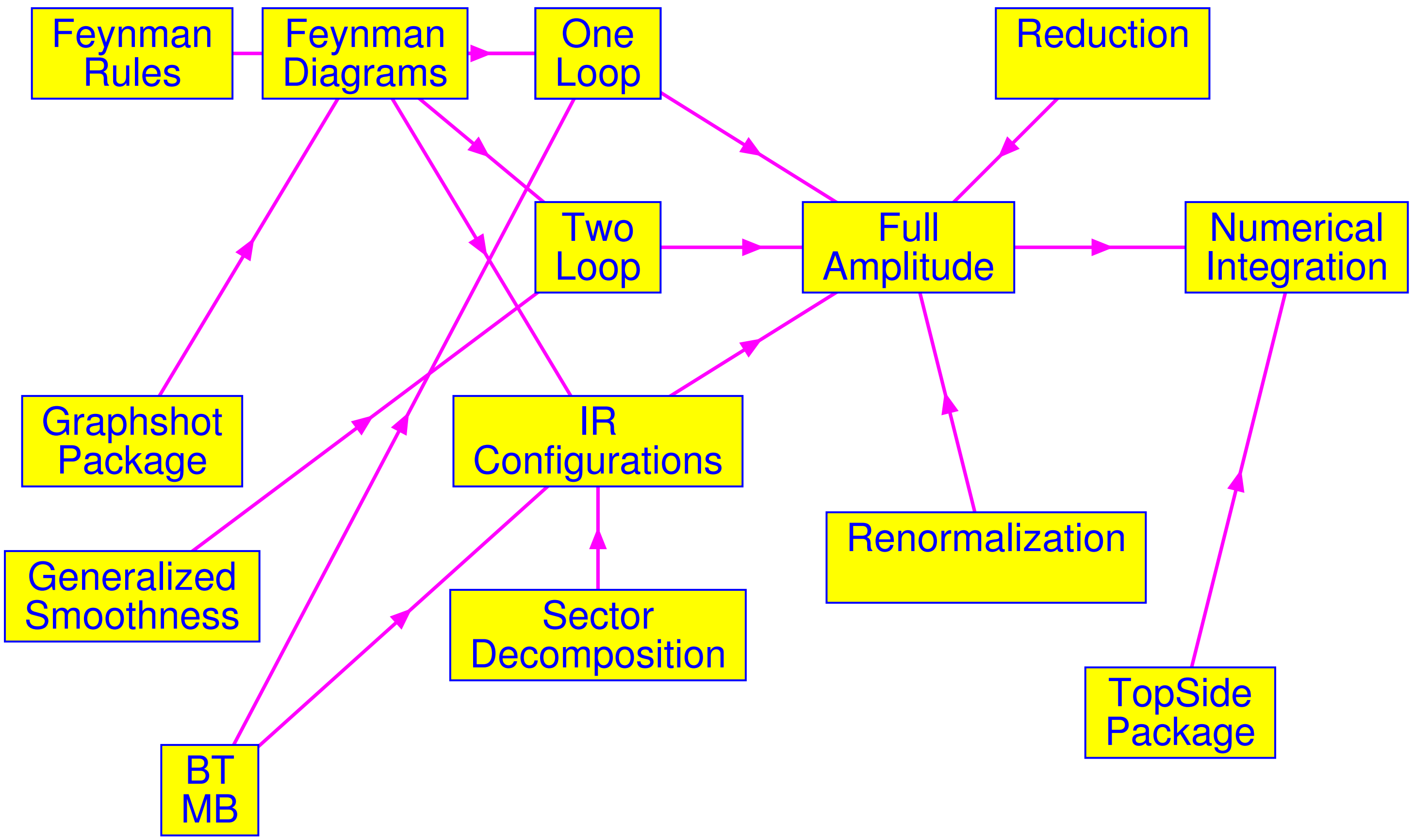
$$S(1,1,1;p^2) = \frac{1}{\Gamma^2\left(3-\frac{n}{2}\right)}\int\frac{d^nq_1}{4\pi^{\frac{n}{2}}}\int\frac{d^nq_2}{4\pi^{\frac{n}{2}}}\frac{1}{(q_1^2+1)(q_2^2+1)[(p-q_1-q_2)^2+1]} ,$$

$$S(2,1,1;p^2) = \frac{1}{\Gamma^2\left(3-\frac{n}{2}\right)}\int\frac{d^nq_1}{4\pi^{\frac{n}{2}}}\int\frac{d^nq_2}{4\pi^{\frac{n}{2}}}\frac{1}{(q_1^2+1)^2(q_2^2+1)[(p-q_1-q_2)^2+1]} .$$

$$\{z(z+1)(z+9)\frac{d^2}{dz^2}+\frac{1}{2}[(12-3n)z^2+10(6-n)z+9n]\frac{d}{dz} + \frac{1}{2}(n-3)[(n-4)z-n-4]\}\,S(1,1,1) = \frac{3}{2}\frac{1}{(n-4)^2} .$$

$$S_1(2,1,1) = \frac{1}{3}[-(n-3)+z\frac{d}{dz}]S(1,1,1) ,$$

Techniques II : the numerical way



Basics: the problem made explicit

No matter how long you turn it around the problem in any realistic multi-loop, multi-leg calculation is connected with a simple fact:

$$\text{(non-)scalar } G = \frac{(\text{something}\,;\,\int \text{something})^{\dagger}}{f(\text{parameters})},$$

$$\dagger\text{something} \equiv \text{HTF or a smooth integrand}.$$

Examples of f are

- Gram determinants in the standard tensor reduction;
- denominators in the IBP - techmique.

Optimal case for $f = 0$:

$\boxed{f = 0} \equiv$ solution(s) of Landau equations

Nature of singularity NOT overestimated $\boxed{f^{-1}}$ instead of $\boxed{f^{-1/2}, \ln f}$

An algorithm for all one-loop diagrams

Any one-loop Feynman diagram G, irrespective of the number N ofvertices, can be expressed as

$$G = \int_S dx\, Q(x)\, V^{\mu}(x),$$

where the integration region is $x_j \geq 0$, $\sum_j x_j \leq 1$, with $j = 1, \dots, N-1$, and $V(x)$ is a quadratic polynomial in x,

$$V(x) = x^t\, H\, x + 2\, K^t\, x + L,$$

and $Q(x)$ is also a polynomial that accounts for parametrized tensor integrals. The solution to the problem of determining the polynomial $\mathcal{P}$ is as follows:

$$\mathcal{P} = 1 - \frac{(x - X)^t\, \partial_x}{2\,(\mu + 1)},$$
$$X^t = -\, K^t\, H^{-1}, \qquad B = L - K^t\, H^{-1}\, K,$$

where the matrix H is symmetric.

Example: one-loop three-point C_0

$$C_0 = \frac{1}{B_3}\{\frac{1}{2} + \int_0^1 dx_1\,[\int_0^{x_1} dx_2\,\ln\,V(x_1,x_2) - \frac{1}{2}\sum_{i=0}^{2}(X_i - X_{i+1})\,\ln V(i\,\widehat{i+1})]\},$$

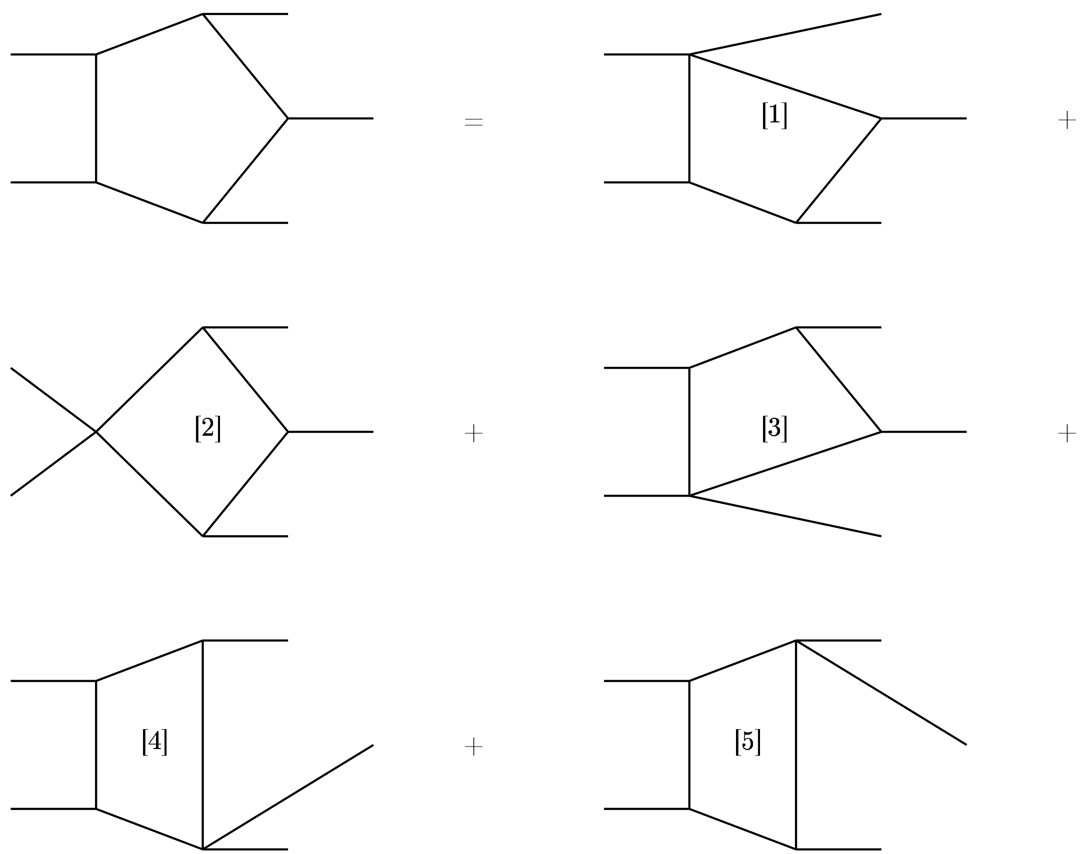
$$V(\widehat{0\,1}) = V(1,x_1), \quad V(\widehat{1\,2}) = V(x_1,x_1), \quad V(\widehat{2\,3}) = V(x_1,0),$$

Example: one-loop three-point C_0 at $\mathcal{O}(\epsilon)$

$$C_0(n) = C_0(4) + \epsilon\,C_0^{(1)} + \mathcal{O}\left(\epsilon^2\right),$$

$$B_3\,C_0^{(1)} = -\frac{1}{2}\int dS_2\,[\ln V(x_1,x_2) + \frac{1}{2}\ln^2 V(x_1,x_2)] + \frac{1}{8}\int dS_1\,\sum_{i=0}^{2}(X_i - X_{i+1})\,\ln^2 V(i\,\widehat{i+1}),$$

Diagrammatical representation of the BT algorithm for the pentagon. The symbol $[i]$ denotes multiplication of the corresponding box by a factor $w_i/(4\,B_5)$



IBP techniques

- For one-loop diagrams IBPI can be written as

$$\int d^n q \, \frac{\partial}{\partial q_\mu} \left[v_\mu \, F(q, p, m_1 \cdots) \right] \; = \; 0,$$

where $v = q, p_1 \cdots, p_N$, N being the number of independent external momenta. By careful examination of the IBPI one can show that all one-loop diagrams can be reduced to a limited set of MI. Here we would like to point out one drawback of the solution. Consider, for instance, the following identity,

$$\begin{aligned} B_0(1, 2\,;\, p, m_1, m_2) \; &= \; \frac{1}{\lambda(-p^2, m_1^2, m_2^2)} [(n-3)\,(m_1^2 - m_2^2 - p^2)\, B_0(1, 1\,;\, p, m_1, m_2) \\ &+ \; (n-2)\, A_0(m_1) - (n-2)\, \frac{p^2 + m_1^2 + m_2^2}{2\, m_2^2}\, A_0(m_2)], \end{aligned} \qquad (1)$$

The factor in front of the square bracket is exactly the BT-factor associated with the diagram;

- from a general analysis we know that at the normal threshold the leading behavior of $B_0(1, 2)$ is $\lambda^{-1/2}$, so that the reduction to MI apparently overestimates the singular behavior;

continued...

- **of course by a careful examination of the square bracket one can derive the right expansion at threshold but the result, as it stands, is a source of cancellations/instabilities.**

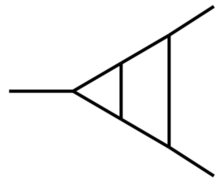
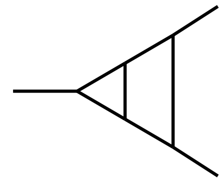
- **For two-loops we write**

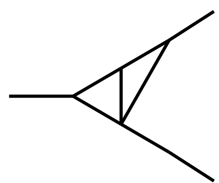
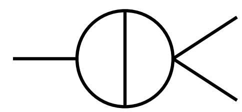
$$\int d^nq_1\, d^nq_2\, \frac{\partial}{\partial a_\mu}\left[b^\mu\, F(q_1, q_2, \{p\}, m_1 \cdots)\right] \;=\; 0,$$

$$a_\mu \;=\; q_{i\mu}, \quad b_\mu = q_{i\mu}, p_{1\mu} \cdots, p_{N\mu},$$

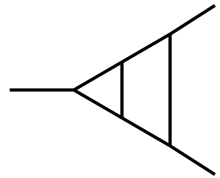
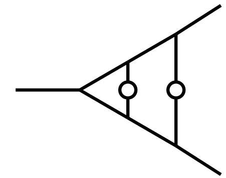
where $i = 1, 2$ and N is the number of independent external momenta.

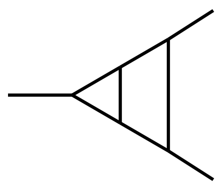
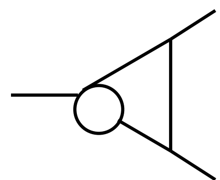
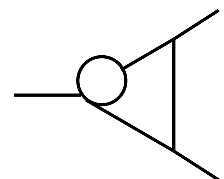
- **Again, using IBPI, arbitrary two-loop integrals can be written in terms of a restricted number of MI. The problem remains in the explicit evaluation of the MI.**

$q_2 \cdot p_1 \;\otimes$  $= \;-\frac{1}{2}\, l_{145}$ 

$-\frac{1}{2}$  $+ \;\frac{1}{2}$ 

Diagrammatic interpretation of the reduction induced by a reducible scalar product. Here $l_{145} = p_1^2 - m_4^2 + m_5^2$, while the symbol $\otimes$ denotes insertion of a scalar product into the numerator of the diagram.

$q_1 \cdot p_1 \;\otimes$  $= \dfrac{p_1^2\, p_2^2 - (p_1 \cdot p_2)^2}{P^2}\, \omega^2$ 

$-\dfrac{p_1 \cdot P}{2\, P^2} \Big[l_{P12}$  $-$  $+$  $\Big]$

Diagrammatic interpretation **of the reduction induced by an irreducible scalar product. In the first diagram of the RHS non-canonical powers -2 in propagators are explicitly indicated by a circle and the space-time dimension is $6-\epsilon$. Here $l_{P12} = P^2 - m_1^2 + m_2^2$ and $\omega = \mu^2/\pi$ where μ i s the unit of mass. The symbol $\otimes$ denotes insertion of a scalar product into the numerator of the diagram.**

General approach for numerical evaluation of arbitrary FD

$$G = \sum \left[\frac{1}{B_G} \int_S dx\, \mathcal{G}(x) \right],$$

- x is a vector of Feynman parameters,
- S is some simplex,
- $\mathcal{G}$ is an integrable function (in the limit $\epsilon \to 0_+$) and
- B_G is a function of masses and external momenta whose zeros correspond to true singularities of G, if any.

Smoothness requires that the kernel in and its first N derivatives should be continuous functions with N as large as possible. However, in most cases we will be satisfied with absolute convergence, e.g. logarithmic singularities of the kernel. This is particularly true around the zeros of B_G where the large number of terms obtained by requiring continuous derivatives of higher order leads to large numerical cancellations.

A GENERIC (scalar) N_L - Legs, n_l - Loops diagram

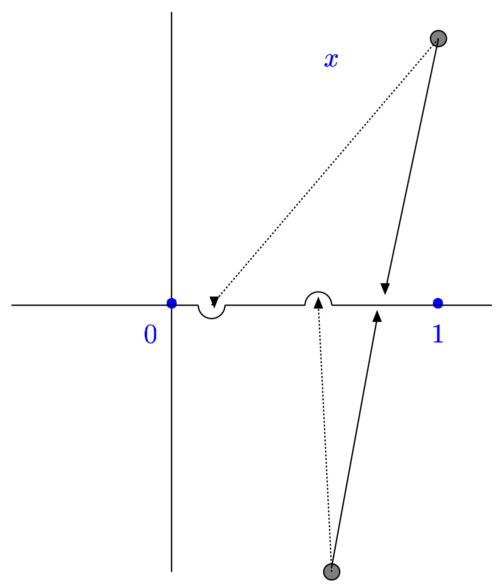
$$G \; = \; \int_S \prod_{i=1}^{N_l - 1} dx_i \, I_G(\{x_i\}),$$

The parametric function I_G has not a definite sign, i.e. poles inside the integration region that hinder the numerical integration. Example:

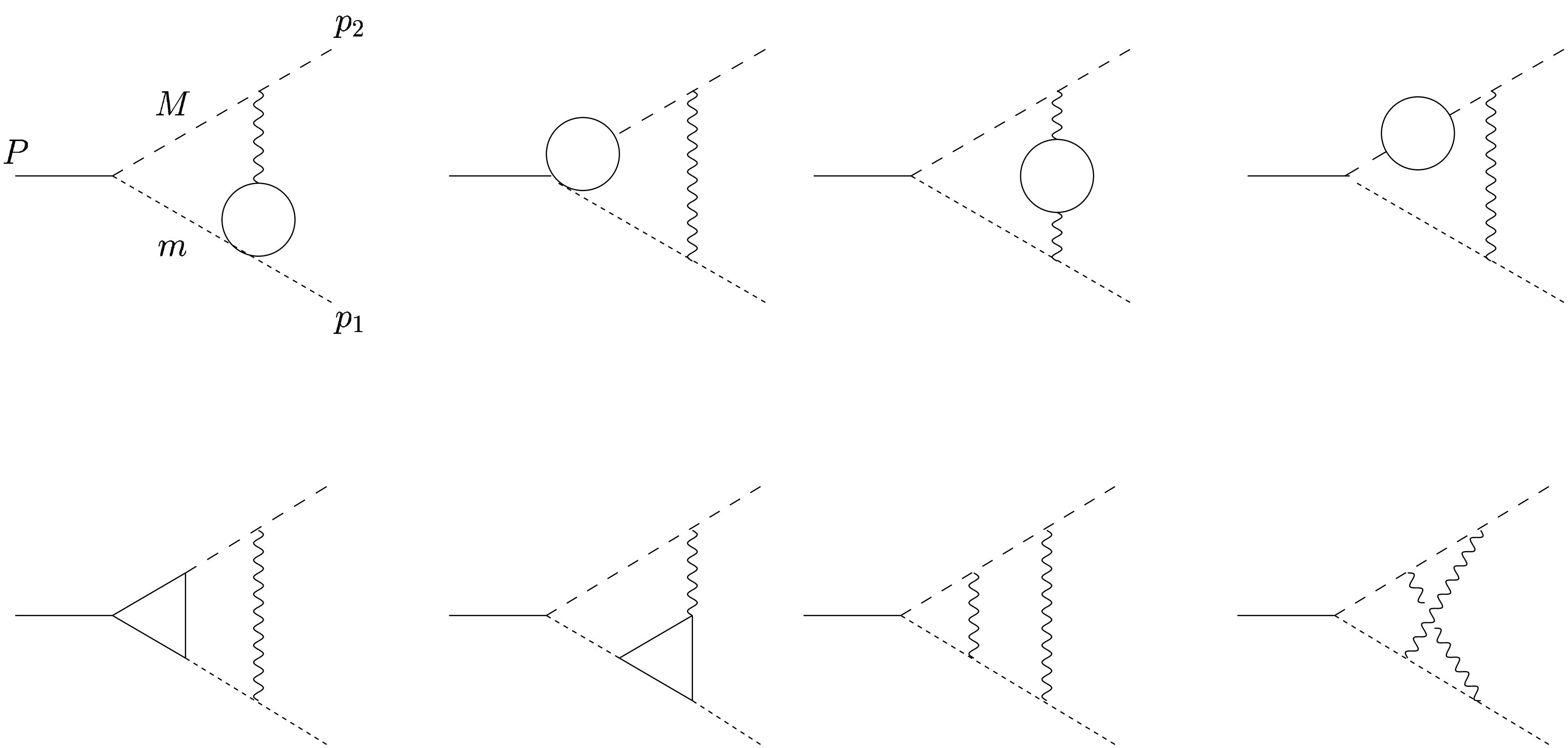
$$I(\alpha) \; = \; \int_0^1 dx \, [\chi_B(x) - i\,\epsilon]^{-\alpha} \qquad \chi_B(x) = a\,x^2 + b\,x + c.$$

When the roots of $\chi_B - i\,\epsilon = 0$ are real and $\in [0,1]$ the integral can be computed by distorting the integration contour unless a pinch will occur; in this case the integral is singular.

pinch singularity



Something difficult: IR configurations



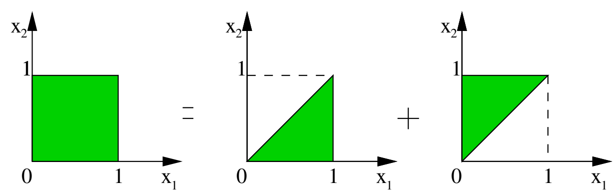
Sector decomposition

In multi-loop diagrams the IR singularities are often overlapping. Procedure:

- Place the singular points at the hedge of the parameters space and remap variables to the unit cube. Example:

$$I = \int_0^1 dx\, dy\, P^{-2-\epsilon}(x,y), \quad P(x,y) = a\,(1-y)\,x^2 + b\,y$$

- Decompose the integration domain



$$\int_0^1 dx \int_0^1 dy \;=\; \int_0^1 dx \int_0^x dy + \int_0^1 dy \int_0^y dx$$

- **Remap the variables to the unit cube**

$$I \;=\; \int_0^1 dx\,dy\,x\,P^{-2-\epsilon}(x,xy) + \int_0^1 dx\,dy\,y\,P^{-2-\epsilon}(xy,y),$$

- **Factorize the variables**

$$I \;=\; \int_0^1 dx\,dy\,\Big[x^{-1-\epsilon}\,P_1^{-2-\epsilon}(x,y) + y^{-1-\epsilon}\,P_2^{-2-\epsilon}(x,y)\Big],$$

$$P_1 \;=\; a\,(1-x\,y)\,x + b\,y, \qquad P_2 = a\,(1-y)\,x^2 + b$$

- **Iterate the procedure until all polynomials are free from zeros.**
- **Perform a Taylor expansion in the factorized variables and integrate to extract the IR poles:**

$$I_2 \;=\; -\frac{\mathbf{1}}{\epsilon}\int_0^1 dx\,P_2^{-2-\epsilon}(x,0) + \int_0^1 dx\int_0^1 dy\,\frac{P_2^{-2}(x,y) - P_2^{-2}(x,0)}{y}.$$

- **If $\mathbf{a},\mathbf{b} > \mathbf{0}$ we can integrate numerically**

Ranking Two-Loop Diagrams

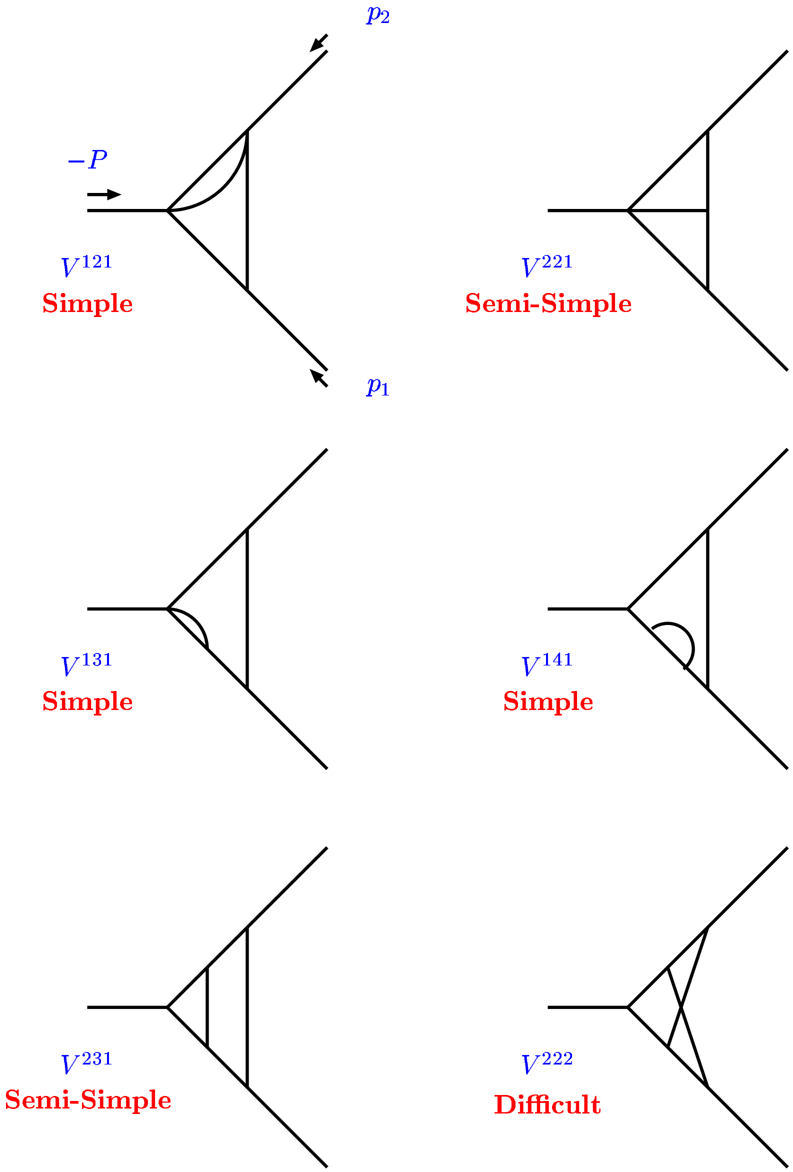
- Two - point: all Simple
- Three - Point: see next figure

Simple BT suffices

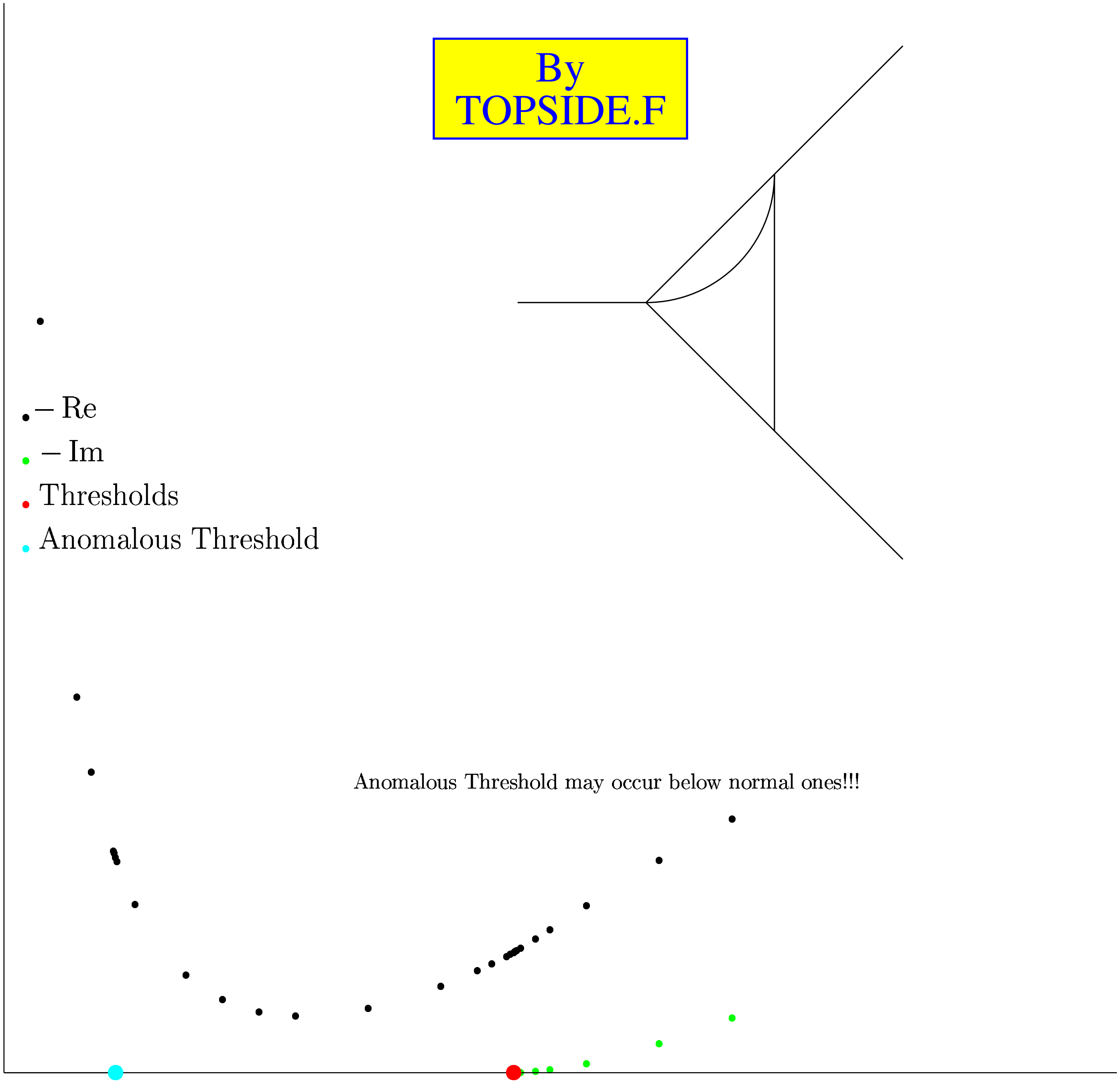
Semi-Simple BT not enough, new kernels required for the smothness algorithm

Difficult Numerical differentiation required

Vertices

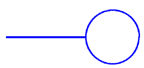

p_2
$-P$
V^{121}
Simple
p_1
V^{221}
Semi-Simple
V^{131}
Simple
V^{141}
Simple
V^{231}
Semi-Simple
V^{222}
Difficult

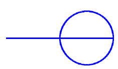
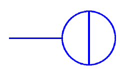
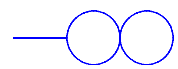
By TOPSIDE.F

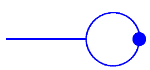

– Re
– Im
Thresholds
Anomalous Threshold
Anomalous Threshold may occur below normal ones!!!

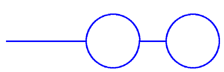
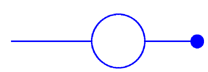
Complexity in renormalization

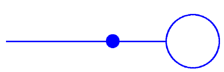
$T_0 :$  **(1)** $+$

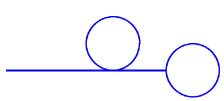
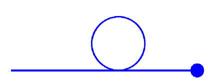
$T_1 :$  **(1/2)** $+$

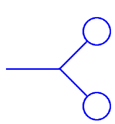
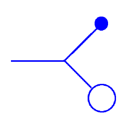
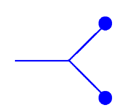
$T_2 :$  **(1/6)** $+$  **(1/4)** $+$  **(1/4)** $+$

$T_3 :$  **(1/2)** $+$

$T_4 :$  **(1/4)** $+$  **(1/2)** $+$

$T_5 :$  **(1/2)** $+$  **(1)** $+$

$T_6 :$  **(1/4)** $+$  **(1/2)** $+$

$T_7 :$  **(1/8)** $+$  **(1/2)** $+$  **(1/2).**