

# Theory from LEP to LHC a semihistorical review

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Pseudo-observables: from LEP to LHC, 9–10 April 2015  
CERN



to old friends: Chiara, Dima, Günter, Manel, Martin, Wolfgang

to new friends: Andrè, Michael

..... and to the enemies that didn't let me down and made me continue





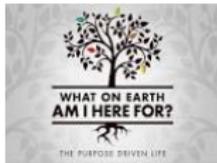
## of the POs

TH  To give a conventional, QFT-compatible, definition of non-existing quantities

EXP  To avoid having to redo the analysis if theory changes

☞ Of course, EXPs could stick to fiducial observables

☞ Of course, Run II could show NP at the screen level



## *Fiducial answers*

- ☞ ATLAS/CMS should publish their fiducial cross sections (this was not the case at Lep), “fiducial” and “pseudo” are alternative but not antithetic
- ☞ ATLAS/CMS will discover the anti-Higgs<sup>1</sup> (opening the road for Higgsogenesis), kryptonite<sup>2</sup> etc. Does that change the issue?

I don't think so. Studying SM deviations or trying to understand how the Higgs also interacts with dark matter requires understanding SM/BSM couplings/properties that are universal and not volume dependent.

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<sup>1</sup>Tulin and Servant, PRL

<sup>2</sup>The Adventures of Superman radio show, June 1943



Here there is more

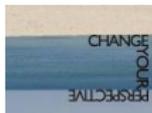
☞ EFT rules and can do everything

☞ this is all obvious after LEP

Well, ☞ what about kryptonite at screen level? I still would like to have a number for anti-Higgs couplings

☞ not at all, LEP and LHC are so different,





At LEP we had the SM with one missing ingredient, therefore the strategy was:

- ☞ Test the SM hypothesis versus  $M_H$ 
  - a) Fit FOs to derive  $M_H, \alpha_s(M_Z)$  etc.
  - b) Introduces POs, fit them, compute them, fit  $M_H, \alpha_s(M_Z)$  etc.

At LHC the SM is complete, therefore the strategy is:

- ☞ Study SM-deviations, which requires a bigger environment, e.g. EFT (for the whole set of processes)

- ① Can we reach the Holy Grail, Model Independence, so that EXPs do not have to repeat the analysis for every new calculation?
- ② Are POs incompatible with EFT when studying SM deviations? Are they incompatible with BSM models when studying evidence of NP?
- ① Not completely, even at LEP analyses were done and redone



- ② Not at all,

POs cut-away. Consider a process with one resonant contribution (due to particle X); the amplitude will be

$$\mathcal{A}(i \rightarrow f) = \frac{V_{\text{indices}}^i V_{\text{indices}}^f}{s - M_X^2} + \text{NR}$$

We would like to define  $\Gamma(\mathbf{X} \rightarrow i(j))$ , despite the fact that  $\mathcal{A}$  will be tortured within some fiducial volume.

- ① First replace  $M_X^2$  with the complex pole  $\mathbf{s}_X$
- ② Next, realize that (being a gauge theory) only the pole, the residue and the rest of the Laurent expansion are gauge invariant
- ③ integrate over the whole phase space, at virtuality  $= \mathbf{s}_X(M_X^2)$  and get the “peak cross-section” in terms of  $\Gamma_{\mathbf{X}}(i), \Gamma_{\mathbf{X}}(j)$  and  $\Gamma_{\mathbf{X}}^{\text{tot}}$

## A schetchy example

$$A = \frac{V_i(\mathbf{s}, \mathbf{s}_H, \xi, \dots) V_f(\mathbf{s}, \mathbf{s}_H, \xi, \dots)}{\mathbf{s} - \mathbf{s}_H} + B(\mathbf{s}, \xi, \dots)$$

$$V_{i,f}(\mathbf{s}, \mathbf{s}_H, \xi, \dots) = V_i^{\text{inv}}(\mathbf{s}, \mathbf{s}, \dots) + (\mathbf{s} - \mathbf{s}_H) \Delta V_{i,f}(\mathbf{s}, \mathbf{s}_H, \xi, \dots)$$

where  $\mathbf{s}_H$  is the  $\mathbf{H}$  complex pole,  $\mathbf{s}$  the  $\mathbf{H}$  virtuality,  $\xi$  the gauge parameter(s) and where ... represent other invariants

$$A = A_S + A_B \quad A_S = \frac{V_i^{\text{inv}} V_f^{\text{inv}}}{\mathbf{s} - \mathbf{s}_H}$$

$$\begin{aligned} \text{FO} &= \int_{\text{cut}} d\Phi \sum_{\text{spin}} |A_S + A_B|^2 = \int_{\text{cut}} d\Phi \sum_{\text{spin}} |A_S|^2 + \text{FO}_{\text{rest}} \\ &= \int d\Phi \sum_{\text{spin}} |A_S|^2 + \left( \int_{\text{cut}} - \int \right) d\Phi \sum_{\text{spin}} |A_S|^2 + \text{FO}_{\text{rest}} \\ &= \text{PO} + \text{rest} \end{aligned}$$



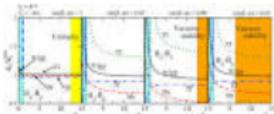
For each process compute the full answer within fiducial volumes

Another language: something is decaying into something else (on-shell) further decaying etc.

Can we make it rigorous while keeping the total intact ?

Yes, it's PO!

Nobody will memorize what  $\kappa_{ijk}^{XYZ}$  is, but will remember what an asymmetry is (even when "spoiled" enough to become a PO). Let's keep  $\kappa$  as a tool to (partly) get the UV-completion



or



PO is the language which the deaf can hear and the blind can see



At Lep1 the peak hadronic and leptonic cross-sections are defined by

$$\sigma_h^0 = 12\pi \frac{\Gamma_e \Gamma_h}{M_Z^2 \Gamma_Z^2} \quad \sigma_l^0 = 12\pi \frac{\Gamma_e \Gamma_l}{M_Z^2 \Gamma_Z^2}$$

where  $\Gamma_Z$  is the total decay width of the **Z** boson, i.e, the sum of all partial decay widths.

☞ But LHC is different (similar to Lep2), there are many more resonances around.

What will happen when theory changes (e.g. new higher order included)? Consider primordial POs: the  $\kappa$ -framework.

- ☞ The  $\kappa$ -framework, as seen from the point of view of EFT, allows you to deform both  $S$  and  $B$  in a consistent way. All “dynamical” parts are SM induced and they are deformed by constant  $\kappa$ -parameters, e.g.

$$\rho_H^{\gamma Z} = \mathcal{A}(H \rightarrow \gamma Z) = \kappa_W^{\gamma Z} \mathcal{A}_W^{(4)} + \kappa_t^{\gamma Z} \mathcal{A}_t^{(4)} + \kappa_b^{\gamma Z} \mathcal{A}_b^{(4)} + i g g_6 \frac{M_H^2}{M_W} a_{AZ} \\ + a_{\phi D} \mathcal{A}_W^{\text{NF}} + \sum_{f=t,b} \left( a_{\phi q}^{(3)} - a_{\phi q}^{(1)} - a_{\phi f} \right) \mathcal{A}_f^{\text{NF}}$$

If the calculation is at some given order and the  $\kappa$ -parameters have been fitted, then apply the “new”  $K$ -factor and derive the updated ( $\kappa$ ) deviation.  $\kappa_{\text{new}} = \kappa_{\text{old}} / K$

Of course, this cannot be trivially extended to PDFs or to QED/QCD final state radiation etc.

This means that (understating the problem) we face a decomposition

$$\mathbf{FO} = \mathbf{PO} \oplus \mathbf{T}_{\text{remnant}}$$

and the choice of PO must be such that  $\mathbf{T}_{\text{remnant}}$  is not a source of large errors due to bias (as using a phonebook to select participants in a survey). For example, as more terms are added to  $\mathbf{T}_{\text{remnant}}$ , the greater the resulting model's complexity will be. This represents a severe constraint on our “conventional” choice of POs.

Enough with the future



Back to the Past

By popular demand: POs at Lep (theory), a short guided tour to one historical venue



## POS at LEP

Ideally, one would like to combine the results of the LEP experiments at the level of the measured cross-sections and asymmetries - a goal that has never been achieved because of the intrinsic complexity, given the large number of measurements with different cuts and the complicated structure of the experimental covariance matrices relating their errors

- ☛ The practical attitude of the experiments was to stay with a *Model-Independent* fit, i.e. from FOs  $\rightarrow$  POs ( $\oplus$  a SM remnant) for each experiment, and these sets of POs are averaged
- ☞ The result of this procedure are best values for POs. The extraction of Lagrangian parameters,  $M_Z, M_t, M_H, \alpha_s(M_Z^2)$  and  $\alpha(M_Z^2)$ , was based on the LEP-averaged POs.

## RULES at Lep

- ① All QED initial state corrections and QED+QCD final state corrections are de-convoluted.
  - ☞ The rationale for the de-convolution is based on the fact that all experiments use different kinematic cuts and selection criteria, while an objective requirement is put forward by the scientific community for having universal results anchored to the  $Z$  peak.
  - ☞ Assuming a structure function representation for the initial fermions, in turn, allows us to de-convolute the measurements and to access the hard scattering at the nominal peak.
- ② The transition from FO's to PO's involves certain assumptions that reflect our understanding of QED effects but, within those assumptions, there is a well-defined mathematical procedure.

Insofar as this procedure is an explicitly specified and mathematically meaningful transformation, the PO's possess the status of observability.

③ We begin with the predicted amplitude, dressed by the weak loop corrections, and use the fact that in the Standard Model (SM) there are several effects, such as

- a) the imaginary parts or
- b) the  $\gamma$ - $Z$  interference or
- c) the pure QED background,

that have a usually negligible influence on the line shape.

④ Therefore, PO's are determined by fitting FO's but we will have some ingredients that are still taken from the SM, making the model-independent results dependent upon the SM.

- 5 In this way the exact (de-convoluted) cross-section is successively reduced to a  $Z$ -resonance.

It is a modification of a pure Breit-Wigner resonance because of the  $s$ -dependent width:

$$\sigma_{\bar{f}f}(s) = \sigma_0^{\bar{f}f} \frac{s^2 \Gamma_Z^2}{(s - M_Z^2)^2 + s^2 \Gamma_Z^2 / M_Z^2} \quad \sigma_0^{\bar{f}f} = \frac{12\pi}{M_Z^2} \frac{\Gamma_e \Gamma_f}{\Gamma_Z^2}$$

- ☞ The partial widths are computed by including all we know about loop corrections

- ⑥ Since we perform QED de-convolution we must have the QED uncertainty fully under control and, therefore, we need an estimate of the neglected higher-order QED effects.

They become particularly relevant if the effect is energy dependent, a fit to cross-sections switching from one QED radiator to another

☞ would indeed change not only  $\sigma_h$  but also  $M_Z$  and  $\Gamma_Z$

☞ This is needed to evaluate the error on the electroweak fits.

## CONVENTIONS at Lep

- 1 The quantity  $\sigma_h = \sum \sigma_0^{\bar{f}f}$  is the de-convoluted hadronic peak cross-section, which by definition includes only the  $Z$  exchange
  - 2 For the de-convoluted forward-backward asymmetry, typically only the  $Z$  exchange is included and
    - initial and final state QED corrections plus
    - the eventual final state QCD corrections
- ☞ are assumed to be subtracted from the experimental data

Evolution during those years (e-Print: hep-ph/9803425)

The main motivation for upgrading precision calculations around the Z-resonance with the programs TOPAZ0 and ZFITTER and for making public their description is a reflection of questions frequently asked by the experimental community:

- *A complete definition of lineshape and asymmetry pseudo observables (POs), together with the residual SM dependence in model-independent fits, is needed. This includes a description on what is actually taken from the SM*
- *Both codes calculate POs. A definition of these POs is needed, showing that TOPAZ0 and ZFITTER use the same definition so that any discrepancy is really a measure of missing higher-order corrections*

## Understanding Theory at Lep

- ① The structure of the amplitude is unique, but implies the introduction of complex-valued form factors that depend on the two Mandelstam variables:  $\mathbf{s}, \mathbf{t}$ ; the dependence on  $\mathbf{t}$  is due to the weak box diagrams.

The separation into insertions for the  $\gamma$  exchange and for the  $\mathbf{Z}$  exchange is lost

- ② The weak boxes are non-resonant (i.e.  $\sim \mathbf{s} - M_Z^2$ ) insertions to the electroweak form factors

At the  $\mathbf{Z}$ -resonance, the one-loop weak  $\mathbf{W}\mathbf{W}$  and  $\mathbf{Z}\mathbf{Z}$  box terms are small, with a

✌ relative contribution  $\leq 10^{-4}$

- ③ Full factorization is re-established by ignoring in addition, the other non-resonant loop contributions, such as
- the bosonic insertions to the photon propagator, and
  - photon-fermion vertex corrections

All the ignored terms are of the order  $\mathcal{O}(\alpha\Gamma_Z/M_Z)$ .

- ④ The factorization is the result of a variety of approximations that are

✌ valid at the **Z**-resonance to the accuracy needed ✌

and that are indispensable in order to relate the PO's to actually measured quantities

- ☞ In any complete calculation it is possible to control the numerical influence of all these approximations



## Th strategy

Within the context of the SM the FOs are described in terms of some set of amplitudes

$$\mathbf{A}_{\text{SM}} = \mathbf{A}_Y + \mathbf{A}_Z + \text{non-factorizable},$$

$$\sigma(\hat{s}) = \int dz H_{\text{in}}(z, \hat{s}) H_{\text{fin}}(z, \hat{s}) \hat{\sigma}(z, \hat{s})$$

One needs to specify  $\mathbf{M}_Z$ , the (remaining) relevant SM parameters for the SM-complement,

$$\text{FO} = \text{PO} + \overline{\text{SM}}$$

## PO - ology

The explicit formulae for the  $Z\bar{f}f$  vertex are

$$\rho_Z^f \gamma^\mu \left[ \left( I_f^{(3)} + i a_L \right) \gamma_+ - 2 Q_f \kappa_Z^f \sin^2 \theta + i a_Q \right] = \gamma^\mu \left( \mathcal{G}_V^f + \mathcal{G}_A^f \gamma^5 \right)$$

where  $\gamma_+ = 1 + \gamma^5$  and  $a_{Q,L}$  are the SM imaginary parts.

By definition, the total and partial widths of the  $Z$  boson include also QED and QCD corrections.

## Relations among POs

$$\Gamma_f \equiv \Gamma(Z \rightarrow \bar{f}f) = 4c_f \Gamma_0 \left( \left| \mathcal{G}_V^f \right|^2 R_V^f + \left| \mathcal{G}_A^f \right|^2 R_A^f \right) + \Delta_{\text{EW/QCD}}$$

where  $c_f = 1$  or  $3$  for leptons or quarks and  $R_{V,A}^f$  describe the final state QED and QCD corrections and take into account the fermion mass. The last term,

$$\Delta_{\text{EW/QCD}} = \Gamma_{\text{EW/QCD}}^{(2)} - \frac{\alpha_s}{\pi} \Gamma_{\text{EW}}^{(1)}$$

accounts for the non-factorizable corrections.

The standard partial width,  $\Gamma_0$ , is

$$\Gamma_0 = \frac{G_F M_Z^3}{24\sqrt{2}\pi} = 82.945(7) \text{ MeV}$$

✓ The peak hadronic and leptonic cross-sections are defined by

$$\sigma_h^0 = 12\pi \frac{\Gamma_e \Gamma_h}{M_Z^2 \Gamma_Z^2} \quad \sigma_l^0 = 12\pi \frac{\Gamma_e \Gamma_l}{M_Z^2 \Gamma_Z^2}$$

where  $\Gamma_Z$  is the total decay width of the  $Z$  boson, i.e, the sum of all partial decay widths.

✓ The effective electroweak mixing angles (*effective sinuses*) are always defined by

$$4|Q_f| \sin^2 \theta_{\text{eff}}^f = 1 - \frac{\text{Re } \mathcal{G}_V^f}{\text{Re } \mathcal{G}_A^f} = 1 - \frac{g_V^f}{g_A^f}$$

✓ where we define

$$g_V^f = \text{Re } \mathcal{G}_V^f \quad g_A^f = \text{Re } \mathcal{G}_A^f$$

The logo for MHOUs (Munich High Energy Physics) is a teal square with a thin orange border. The text 'MHOUs' is written in black, sans-serif font in the center. At the bottom right of the teal square, there is a small, faint URL 'http://www.mho.us'.

## MHOUs

- Run over all options (e.g. due to missing NNLO corrections) and use
  - ① *central* for PO evaluated at the preferred setup
  - ② minus error for  $PO_{\text{central}} - \min_{\text{opt}} PO$
  - ③ plus error for  $\max_{\text{opt}} PO - PO_{\text{central}}$
- almost Bayesian (flat prior)

Observable	central	minus error	plus error	total exp. error	
$1/\alpha^{(5)}(M_Z)$	128.877	-	-		
$1/\alpha(M_Z)$	128.887	-	-		
$M_W$ [ GeV ]	80.3731	5.8 MeV	0.3 MeV	90 MeV	
$\sigma_{\text{had}}^0$ [ nb ]	41.4761	1.0 pb	1.6 pb	58 pb	
$\sigma_e^0$ [ nb ]	1.9995	0.17 pb	0.26 pb	3.5 pb	
$\Gamma_\nu$ [ MeV ]	167.207	0.017	0.001	0.10*	
$\Gamma_{e,l}$ [ MeV ]	83.983	0.010	0.0005		
$\Gamma_\mu$ [ MeV ]	83.983	0.010	0.0005		
$\Gamma_\tau$ [ MeV ]	83.793	0.010	0.0005		
$\Gamma_u$ [ MeV ]	300.129	0.047	0.013		
$\Gamma_d$ [ MeV ]	382.961	0.054	0.010		
$\Gamma_c$ [ MeV ]	300.069	0.047	0.013		
$\Gamma_b$ [ MeV ]	375.997	0.208	0.077		
$\Gamma_{\text{had}}$ [ GeV ]	1.74211	0.26 MeV	0.11 MeV		2.3 MeV*
$\Gamma_{\text{inv}}$ [ GeV ]	0.50162	0.05 MeV	0.002 MeV		1.8 MeV*
$\Gamma_Z$ [ GeV ]	2.49549	0.34 MeV	0.11 MeV	2.4 MeV	

**Table:** Theoretical uncertainties for POs from TOPAZ0. \*) assumes lepton universality

## To Summarize the Lep Strategy:

- ① One starts with the SM, which introduces complex-valued couplings, calculated to some order in perturbation theory
- ② Next we define  $g_V^f, g_A^f$  as the real parts of the effective couplings and  $\Gamma_f$  as the physical partial width absorbing all radiative corrections including the imaginary parts of the couplings and fermion mass effects
- ③ Furthermore,

$$R_q = \frac{\Gamma_q}{\Gamma_h} \quad R_l = \frac{\Gamma_h}{\Gamma_l}$$

for quarks and leptons, respectively.

In conclusion, the flow of the calculation requested by the experimental Collaborations is:

- pick the Lagrangian parameters  $\mathbf{M}_t, \mathbf{M}_H$  etc. for the explicit calculation of the residual SM-dependent part
- perform the SM initialisation of everything, such as imaginary parts etc. giving, among other things, the complement  $\overline{\mathbf{SM}}$
- select  $\mathbf{g}_V^f, \mathbf{g}_A^f$
- perform a SM-like calculation of  $\Gamma_f$ , but using arbitrary values for  $\mathbf{g}_V^f, \mathbf{g}_A^f$ , and only the rest, namely

$$R_V^f, \quad R_A^f, \quad \Delta_{\text{EW/QCD}}, \quad \text{Im } \mathcal{G}_V^f, \quad \text{Im } \mathcal{G}_A^f$$

from the SM.

- ☞ Therefore, the expression for  $FO = FO(PO)$ , at arbitrary  $\hat{\mathbf{s}}$ , requires a careful examination and should be better understood as  $FO = FO(PO, \overline{SM})$  that is, for example:

$$\sigma = \sigma \left( R_1, A_{FB}^{0,1}, \dots \rightarrow g_V^f, g_A^f \rightarrow \rho_f, \sin^2 \theta_{\text{eff}}^f; \text{residual SM} \right)$$

As long as the procedure does not violate gauge invariance there is nothing wrong with the calculations.

- ☞ Another approach exists, extraction of lagrangian parameters directly from the FOs, which are not (of course) raw data but rather educated manipulations of raw data, e.g. distributions defined for some simplified setup.

will set the  $s'/s$  cut to 0.8 for all extrapolated quantities. More,

```
IND= 1
DMY= -1, D0
XPCUT= 0.8D0
CALL TCUTSET(IND, DMY, DMY, DMY, DMY, DMY, DMY, DMY, XPCUT, DMY)
```

will set the  $z_{\min}$  cut for initial pair production to  $z_{\min} = 0.8$ . Etc, etc, etc.

## 7 MI branch

If `OMODES, NE, CALC` then each quantity can be given as the sum of a Standard Model Independent (SMI) component  $\oplus$  its complement, Standard Model Dependent (SMD),  $\overline{\text{SM}}$ . 5 or 9 parameters are input, according to `ONPAR = 'FP', 'NP'`.

- For `ONPAR = 'FP'` 5 parameters are Input:  $\{\text{PO}\} = M_Z, \Gamma_Z, \sigma_{\text{h}}^0$ , the ratio  $R_t$  and the asymmetries  $A_{\text{FB}}^{0,t}$ .
- For `ONPAR = 'NP'` 9 parameters are Input:  $\{\text{PO}\} = M_Z, \Gamma_Z, \sigma_{\text{h}}^0$ , the ratios  $R_{e,\mu,\tau}$  and the asymmetries  $A_{\text{FB}}^{0,e,\mu,\tau}$ .

If one gives nine parameters, i.e. without assuming lepton universality, then the  $R$  are defined as the ratios of the physical partial width absorbing everything, also kinematic mass effects ( $\Gamma_\tau < \Gamma_\mu < \Gamma_e$ ); thus the SM calculation of  $R_\tau$  comes out 0.23% larger than  $R_e$  and  $R_\mu$ . *The*  $A_{\text{FB}}^{0,t}$  on the other hand, are all equal, as they are defined via the effective couplings where kinematic mass effects are no longer relevant.

- 1) Here `TOPAZO` will compute  $\sigma_{\text{had}}, \sigma_{e,\mu,\tau}$  and  $A_{\text{FB}}^{e,\mu,\tau}$  in terms of  $\{\text{PO}\}$  for an extrapolated setup where only an  $s'$ -cut is allowed. The  $e^+e^-$  channel is understood to be the  $s$ -channel alone, i.e. after  $t$ -channel subtraction.
- 2) Here `TOPAZO` will compute (in terms of  $\{\text{PO}\}$ )  $\sigma_{\text{had}}$  for an extrapolated setup where only an  $s'$ -cut is allowed. Instead, for  $\sigma_{e,\mu,\tau}$  and  $A_{\text{FB}}^{e,\mu,\tau}$  cuts are allowed. The  $e^+e^-$  channel can be  $s$ -channel alone or the complete  $s+t$ -channel. For the latter the  $s$ -channel component is computed starting from `PO` and  $t-t$  and  $s-t$  are part of the SM remnant.

In both cases the SMD part is included at fixed  $m_t, M_H$  and  $\alpha_S(M_Z^2)$ . For  $M_Z$  two alternatives are foreseen: either  $M_Z$  is kept fixed in the SM component while it is varied in the SMI component (`CFM = 'F1'`) or the constantly updated

# Synopsis

c. 1999

Part of the implementation of POs could be cumbersome. However, one has to live with the fact that - for practical reasons - the POs will be combined among the LEP experiments and survive forever. The cross-section and asymmetry measurements will probably be published by the experiments, but most likely no one will ever undertake the effort to combine them.

## POs versus Os in three sentences

from an old (30-SEP-1994) discussion with Manel Martinez

**What** the experimenters do is just collapsing (and/or transforming) some “primordial quantities” (say number of observed events in some pre-defined set-up) into some “secondary quantities” which we feel closer to the theoretical description of the phenomena.

In this step, if the number of quantities is reduced, this implies that some assumptions have been made on the behaviour of the primordial quantities. The validity of these assumptions is judged on statistical grounds. Within these assumptions (QED deconvolution, resonance approach, etc.) the secondary quantities are as “observable” as the first ones. **At** this point, let’s clarify that even the “primordial quantities”, are obtained through many assumptions (event classification, detector response, etc.) which, as in the previous case, can be judged just on statistical grounds.

**Therefore**, all the measurements are equally “observable” provided you endorse the conceptual description of the phenomena that they are supposed to quantify.

# Physics in the old days

the only non German



## CONCLUSIONS



Of course, there are other opinions . . . . . *Do not dwell in the past, do not dream of the future, concentrate the mind on the present moment* (Buddha)





# Backup Slides

Usually, 25 POs were introduced and discussed

- the mass of the W ( $M_W$ )
- the hadronic peak cross-section ( $\sigma_h$ )
- the partial leptonic and hadronic widths ( $\Gamma_f$ ,  $f = \nu, e, \mu, \tau, u, d, c, s, b$ )
- the total width ( $\Gamma_Z$ )
- the total hadronic width ( $\Gamma_h$ )
- the total invisible width ( $\Gamma_{inv}$ )
- various ratios ( $R_l, R_b, R_c$ )
- the asymmetries and polarization ( $A_{FB}^\mu, A_{LR}^e, A_{FB}^b, A_{FB}^c, P^\tau, P^b$ )
- effective sines ( $\sin^2 \theta_e, \sin^2 \theta_b$ )

The effective weak mixing angle is definable, in principle, for all fermions but we know that the largest difference will be in

$$\sin^2 \theta_{\text{eff}}^b - \sin^2 \theta_{\text{eff}}^e$$

due to large flavor dependent corrections. However, only  $\sin^2 \theta_{\text{eff}}^e$  is usually reported, which permits the following definition:

$$\sin^2 \theta_{\text{eff}}^1 = \sin^2 \theta_{\text{eff}}^e$$

by definition, the total and partial widths of the Z boson include final state QED and QCD radiation. Moreover,

$$\Gamma_h = \sum_{q \neq t} \Gamma_q \quad \Gamma_{inv} = \Gamma_Z - \Gamma_h - \sum_l \Gamma_l$$
$$R_l = \frac{\Gamma_h}{\Gamma_e} \quad R_{b,c} = \frac{\Gamma_{b,c}}{\Gamma_h}$$

☞ In our calculations we assumed, indeed, that  $\Gamma_{inv} = 3\Gamma_\nu$ .

Since peak asymmetries and polarizations do not contain, by definition, QED and QCD corrections and they will only refer to pure Z exchange, then they are just simple combinations of effective Z couplings:

$$A_{\text{FB}}^f = \frac{3}{4} \mathcal{A}^e \mathcal{A}^f \quad A_{\text{LR}}^e = \mathcal{A}^e \quad P^f = -\mathcal{A}^f \quad P_{\text{FB}}(\tau) = -\frac{3}{4} \mathcal{A}^e$$

$$\mathcal{A}^f = 2 \frac{g_V^f g_A^f}{(g_V^f)^2 + (g_A^f)^2}$$

$$A_{\text{FB}}^f = \frac{3}{4} \frac{\sigma_{\text{VA}}}{\sigma_{\text{T}}}$$

$$\frac{d\sigma_f}{d\Omega} = \frac{\alpha^2}{4s} N_f^c \beta_f \left[ (1 + c^2) \mathcal{F}_1(s) + 4\mu_f^2 (1 - c^2) \mathcal{F}_2(s) + 2\beta_f c \mathcal{F}_3(s) \right]$$

where  $c = \cos \theta$  is the cosine of the scattering angle and  $\beta_f^2 = 1 - 4\mu_f^2$  with  $\mu_f^2 = m_f^2/s$ .

The energy dependence is confined in the  $\mathcal{F}$ -functions

$$\mathcal{F}_1(\mathbf{s}) = Q_e^2 Q_f^2 + 2 Q_e Q_f g_V^e g_V^f \operatorname{Re} \chi(\mathbf{s})$$

$$+ \left[ (g_V^e)^2 + (g_A^e)^2 \right] \left[ (g_V^f)^2 + (g_A^f)^2 - 4 \mu_f^2 \right] |\chi(\mathbf{s})|^2,$$

$$\mathcal{F}_2(\mathbf{s}) = Q_e^2 Q_f^2 + 2 Q_e Q_f g_V^e g_V^f \operatorname{Re} \chi(\mathbf{s})$$

$$+ \left[ (g_V^e)^2 + (g_A^e)^2 \right] (g_V^f)^2 |\chi(\mathbf{s})|^2,$$

$$\mathcal{F}_3(\mathbf{s}) = 2 Q_e Q_f g_A^e g_A^f \operatorname{Re} \chi(\mathbf{s}) + 4 g_V^e g_V^f g_A^e g_A^f |\chi(\mathbf{s})|^2$$

$\chi$  is the reduced  $\gamma/\mathbf{Z}$  propagator ratio

$$\begin{aligned}\mathcal{F}_1(s) &= \frac{3}{4} \frac{s}{\pi \alpha^2} (\sigma_{VV} + \beta_f^2 \sigma_{AA}) \\ \mathcal{F}_2(s) &= \frac{3}{4} \frac{s}{\pi \alpha^2} \sigma_{VV} \\ \beta_f \mathcal{F}_3(s) &= \frac{3}{4} \frac{s}{\pi \alpha^2} \sigma_{VA}\end{aligned}$$

An example: starting from  $M_Z, \Gamma_Z, R_{e,\mu,\tau}$  and  $A_{\text{FB}}^{0,e,\mu,\tau}$  we first obtain

$$\Gamma_e = M_Z \Gamma_Z \left[ \frac{\sigma_{\text{had}}^0}{12 \pi R_e} \right]^{1/2}$$

$$\Gamma_h = M_Z \Gamma_Z \left[ \frac{\sigma_{\text{had}}^0 R_e}{12 \pi} \right]^{1/2}$$

$$\mathcal{A}_e = \frac{2}{\sqrt{3}} \sqrt{A_{\text{FB}}^{0,e}} \quad \text{and} \quad \gamma = \frac{G_F M_Z^3}{6 \sqrt{2} \pi}$$

☛ we subtract QED radiation

$$\Gamma_e^0 = \frac{\Gamma_e}{1 + \frac{3}{4} \frac{\alpha(M_Z^2)}{\pi}}$$

and get

$$\sin^2 \theta_{\text{eff}}^e = \frac{1}{4} \left( 1 + \frac{\sqrt{1 - \mathcal{A}_e^2} - 1}{\mathcal{A}_e} \right)$$

$$\rho_e = \frac{\Gamma_e^0}{\gamma} \left[ \left( \frac{1}{2} - 2 \sin^2 \theta_{\text{eff}}^e \right)^2 + \frac{1}{4} + (\text{Im } \mathcal{G}_V^e)^2 + (\text{Im } \mathcal{G}_A^e)^2 \right]^{-1}$$

$$\mathcal{A}_f = \frac{4}{3} \frac{A_{\text{FB}}^{0,f}}{\mathcal{A}_e}$$

☞ we further obtain

$$\sin^2 \theta_{\text{eff}}^f = \frac{1}{4|Q_f|} \left( 1 + \frac{\sqrt{1 - \mathcal{A}_f^2 - 1}}{\mathcal{A}_f} \right)$$
$$\rho_f = \frac{\Gamma_f^0}{\gamma} \left[ \left( \frac{1}{2} - 2|Q_f| \sin^2 \theta_{\text{eff}}^f \right)^2 + \frac{1}{4} + \sum_{i=V,A} \left( \text{Im } \mathcal{G}_i^f \right)^2 \right]^{-1}$$

where  $f = \mu, \tau$ . For quarks one should remember to subtract first non-factorizable terms and then to distinguish between  $R_V^f$  and  $R_A^f$ .

With electrons in the final state there is an additional complication. In the experimental analyses, either the full Bhabha cross-section is used or merely the  $s$ -channel contributions. The latter are obtained through  $t$ -channel subtraction; that is, by subtracting the non- $s$ -channel contributions that involve  $t$ -channel gauge–boson exchange

- ☛ The subtraction procedure is aimed at reducing the full large-angle Bhabha scattering to a simple annihilation process. The  $s$ – $t$  and  $t$ – $t$  contributions are subtracted from the data by using one of the many available calculations.
- ☛ All this makes it mandatory to assess the theoretical precision of the available calculations for different channels and for energies up to  $\sim 3$  GeV away from the resonance.

Table 1: Summary of measurements and theoretical predictions. First row is TOPAZO (GMS), second row ZFITTER (OMS).

$M_H$ (GeV)		65	300	1000
	Measurement with Total Error			
a) LEP				
$\Gamma_Z$ [GeV]	$2.4939 \pm 0.0024$	2.4974 2.4975	2.4934 2.4935	2.4884 2.4886
$\sigma_h^0$ [nb]	$41.491 \pm 0.058$	41.471 41.473	41.474 41.475	41.479 41.479
$R_e$	$20.765 \pm 0.026$	20.754 20.752	20.740 20.739	20.737 20.727
$A_{FB}^e$	$0.01683 \pm 0.00096$	0.0167 0.0166	0.0153 0.0152	0.0142 0.0141
$\mathcal{A}^e$	$0.1479 \pm 0.0051$			
$\mathcal{A}^\tau$	$0.1431 \pm 0.0045$	0.14872	0.14255	0.13732
$\sin^2 \theta_{\text{eff}}^{\text{lept}} ((Q_{Fn}))$	$0.2321 \pm 0.0010$	0.14872 0.23128 0.23131	0.14255 0.23207 0.23209	0.13732 0.23274 0.23275
$\sin^2 \theta_{\text{eff}}^{\text{lept}}$	$0.23189 \pm 0.00024$	0.23128 0.23131	0.23207 0.23209	0.23274 0.23275
$M_W$ [GeV]	$80.37 \pm 0.09$	80.410 80.409	80.312 80.312	80.217 80.219
(b) LEP/SLC				
$R_b$	$0.21656 \pm 0.00074$	0.21575 0.21573	0.21577 0.21578	0.21575 0.21579
$R_c$	$0.1735 \pm 0.0044$	0.17230 0.17230	0.17226 0.17225	0.17223 0.17222
$A_{FB}^b$	$0.0990 \pm 0.0021$	0.10445 0.10426	0.09994 0.09988	0.09612 0.09618
$A_{FB}^c$	$0.0709 \pm 0.0044$	0.07475 0.07458	0.07133 0.07119	0.06844 0.06834
$\mathcal{A}^{fb}$	$0.867 \pm 0.035$	0.93477	0.93428	0.93387
$\mathcal{A}^c$	$0.647 \pm 0.040$	0.66862	0.66590	0.66360
(b) SLC				
$\sin^2 \theta_{\text{eff}}^{\text{lept}} (A_{LR})$	$0.23109 \pm 0.00029$	0.23128 0.23131	0.23207 0.23209	0.23274 0.23275
(c) $\nu N$				
$\sin^2 \theta_W$	$0.2255 \pm 0.0021$	0.22239 0.22241	0.22429 0.22429	0.22612 0.22610
(d) $p\bar{p}$ and $\nu N$				
$M_W$ [GeV]	$80.41 \pm 0.09$	80.410 80.409	80.312 80.312	80.217 80.219

In any comparison between theoretical predictions and experimental measurements it is mandatory to assess the theoretical precision of the available calculations

- ☛ Only then can a de-convolution procedure be safely attempted

In this context it is important to note that the main conceptual difference between different approaches in the Bhabha channel lies in the implementation of the non-leading-log QED corrections. All available examples are based on the structure-function method for calculating the (dominant) leading-log corrections

$$\propto \left(\frac{\alpha}{\pi} L\right)^n \quad \text{with } n = 1, 2 \text{ and } L = \ln(s/m_e^2)$$

- ☛ When it comes to the non-leading-log corrections, different authors use different approaches

Table 2: Comparison of TOPAZO (T) and ALIBABA (A) for the cross-section (in pb) and the forward-backward asymmetry. The TOPAZO results do not include initial-state pair production. The full Bhabha results are indicated by the superscript “s + t”, the s-channel contributions by “s”. The quantity  $\delta$  stands for the relative deviation 100%  $\cdot (T - A)/T$ . The input parameters can be found in the text.

	LEP 1 energy in GeV					
	88.45	89.45	90.20	91.19	91.95	93.00
maximum acollinearity angle: 10°						
$\sigma^{s+t}$ (T)	457.08	644.86	912.06	1185.70	873.50	476.64
(A)	457.71	644.78	911.43	1184.59	876.40	480.23
( $\delta$ )	-0.14%	+0.01%	+0.07%	+0.09%	-0.33%	-0.75%
$A_{FB}^{s+t}$ (T)	+0.4448	+0.3411	+0.2492	+0.1386	+0.1008	+0.1298
(A)	+0.4454	+0.3409	+0.2489	+0.1389	+0.1020	+0.1315
(T-A)	-0.0006	+0.0002	+0.0003	-0.0003	-0.0012	-0.0017
$\sigma^s$ (T)	172.94	331.55	590.93	994.27	820.80	461.49
(A)	173.60	332.09	590.72	991.93	821.13	463.35
( $\delta$ )	-0.38%	-0.16%	+0.04%	+0.24%	-0.04%	-0.40%
$A_{FB}^s$ (T)	-0.2202	-0.1380	-0.0761	+0.0004	+0.0487	+0.0980
(A)	-0.2209	-0.1386	-0.0774	-0.0008	+0.0485	+0.0977
(T-A)	+0.0007	+0.0006	+0.0013	+0.0012	+0.0002	+0.0003
maximum acollinearity angle: 25°						
$\sigma^{s+t}$ (T)	485.17	674.89	945.00	1221.13	905.25	503.79
(A)	484.05	673.91	944.73	1220.49	907.15	504.73
( $\delta$ )	+0.23%	+0.15%	+0.03%	+0.05%	-0.21%	-0.19%
$A_{FB}^{s+t}$ (T)	+0.4605	+0.3554	+0.2613	+0.1501	+0.1175	+0.1584
(A)	+0.4576	+0.3521	+0.2596	+0.1484	+0.1173	+0.1580
(T-A)	+0.0029	+0.0033	+0.0017	+0.0017	+0.0002	+0.0004
$\sigma^s$ (T)	176.31	336.84	599.25	1007.03	831.43	468.16
(A)	177.43	338.44	601.29	1008.13	833.29	469.57
( $\delta$ )	-0.64%	-0.48%	-0.34%	-0.11%	-0.22%	-0.30%
$A_{FB}^s$ (T)	-0.2235	-0.1404	-0.0777	-0.0004	+0.0480	+0.0967
(A)	-0.2227	-0.1406	-0.0777	-0.0007	+0.0481	+0.0976
(T-A)	-0.0008	+0.0002	+0.0000	+0.0003	-0.0001	-0.0009

## De-convolution

- ① Single-de-convolution (SD), giving the kernel cross-sections without initial state QED radiation, but including all final-state correction factors.
- ② Double-de-convolution (DD), giving the kernel cross-sections without initial- and final-state QED radiation and without any final-state QCD radiation. There is an additional level, to be called DDD, and the difference between DD and DDD deserves a word of comment. The improvement upon naive electroweak/QCD factorization contains two effects, the FTJR correction which gives the leading two-loop answer for the  $\bar{b}b$ -channel and the CKHSS correction which gives the correct answer for the remaining mixed corrections in all quark channels.  
  
In DD-mode FTJR and CKHSS corrections are kept while in DDD-mode they are excluded. This option allows us to keep under control the implementation of the new CKHSS correction.
- ③ DDD with only  $\mathbf{Z}-\mathbf{Z}$  exchange (DDZ), weak boxes are not included,
- ④ DDD with only  $\mathbf{Z}\oplus\gamma$  (DDZG), i.e. no  $\mathbf{Z}-\gamma$  interference and weak boxes are not included.



Checking ...

Note the following relation between the pseudo-observable  $R_b$  and the ratio of DD cross-sections

$$R_b = \frac{\sigma_b}{\sigma_{\text{had}}} \Big|_{\sqrt{s}=M_Z} - \begin{cases} 0.00146 & \text{TOPAZ0} \\ 0.00146 & \text{ZFITTER} \end{cases}$$

- ☛ The difference reflects the SM-remnant effect, since the ratio of RO cross-sections has  $\gamma$ -exchange, imaginary parts, ..., and (substantially negligible) weak boxes.

	LEP 1 energy in GeV				
	$M_Z - 3$	$M_Z - 1.8$	$M_Z$	$M_Z + 1.8$	$M_Z + 3$
$\sigma_\mu$ [ nb] No lms	0.29996	0.65713	2.00343	0.65855	0.31045
$\sigma_\mu$ [ nb]	0.29999	0.65718	2.00341	0.65856	0.31047
Diff.[pb]	+0.03	+0.05	-0.02	+0.0	+0.02
$\sigma_{\text{had}}$ [ nb] No lms	5.78583	12.94061	39.93848	13.02635	6.05322
$\sigma_{\text{had}}$ [ nb]	5.78492	12.93841	39.92967	13.02421	6.05233
Diff.[pb]	-0.91	-2.20	-8.81	-2.14	-0.89
$A_{\text{FB}}^\mu$ No lms	-0.26311	-0.15181	0.01598	0.17364	0.26858
$A_{\text{FB}}^\mu$	-0.26170	-0.15037	0.01745	0.17510	0.27002
Diff.	+0.00141	+0.00144	+0.00147	+0.00146	+0.00144

**Table:** TOPAZ0 comparison of DD (completely de-convoluted) RO with/without imaginary parts (lms) in couplings and form factors.

## Autopsy:

*Should we retreat from a metaphysics of fits entirely?*

Another approach exists, extraction of lagrangian parameters directly from the **RO**, which are not (of course) raw data but rather educated manipulations of raw data, e.g. distributions defined for some simplified setup.

I have nothing to say about this first part, theorists should do theory and experimentalists should do experiments.

I have been asking so many times to produce this kind of fits that at the end I managed to get some answer. This goes back to 1999:

Changes in DELPHI fits ('93-95 data)

**SM** parameters directly from **RO** or through **PO**.

- The largest change was observed in  $M_Z$ : it amounts to 1.2 MeV or roughly 1/3 of their error (25% of common exp. error for Aleph);
- For  $m_t$  there was no observed change;
- $\log_{10}(M_H)$  changed by roughly 10% of its error, and  $\alpha_s$  by roughly 15%.

It has been tested how the results on **SM** parameters differ between

a **SM** fit to the measured **ROs** (to say 'direct fit') and

a **SM** fit to the **POs** which themselves are derived in an MI fit to the measured **ROs**.

The observed differences in fitted central values in the ALEPH case are

- 20% of the fitted error for  $M_Z$ ,
- 5% on  $\alpha_s$  and
- < 1% on  $m_t$  and  $\log_{10}(M_H/\text{GeV})$ .

L3 '99

SM fit to PO (POs having been fitted to L3 ROs)

$$\begin{aligned}1/\alpha &= 128.9000 \pm .0874 \\ \alpha_s &= .12614 \pm .00579 \\ M_Z &= 91.18917 \pm .00308 \\ m_t &= 175.70 \pm 4.83 \\ M_H &= 30.0 \pm 38.1\end{aligned}$$

SM fit to RO (from L3)

$$\begin{aligned}1/\alpha &= 128.9006 \pm .0886 \\ \alpha_s &= .12581 \pm .00583 \\ M_Z &= 91.18927 \pm .00310 \\ m_t &= 175.64 \pm 4.83 \\ M_H &= 29.8 \pm 39.0\end{aligned}$$

This is some shift but hardly explains the present paroxysmal attack of distress and tell us that for all that we know the RO - branches of our codes have been working properly.

How about Theoretical agreement/disagreement<sup>†</sup>?

$\sigma$ s generated with TOPAZ0 at seven energy points, with errors assigned such as to represent the experimental statistics, plus correlated luminosity errors, were run through an MI fit with ZFITTER. The agreement of the parameters is remarkable:

TOPAZ0 **PO** values

$$\begin{aligned} M_Z &= 91.1867, & \Gamma_Z &= 2.4955, & \sigma_{\text{had}}^0 &= 41.476 \\ m_t &= 173.8, & M_H &= 100.0, & \alpha_S &= 0.119, & 1/\alpha &= 128.878 \end{aligned}$$

Parameters fitted with ZFITTER 5.20 from TOPAZ0 cross sections:

$$M_Z = 91.1866, \quad \Gamma_Z = 2.4956, \quad \sigma_{\text{had}}^0 = 41.476$$

<sup>†</sup> It is converted into a covariance matrix of  $T_{\text{error}}$