Legs and Loops in Real Life

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Karlsruhe December 2007

Based on work done in collaboration with Stefano Actis, Christian Sturm and Sandro Uccirati

Outlines

(1, 2,)

1. *The standard model at two loop level*

   A probable decision about its truth is possible inductively by studying its success (verifiable consequences)

2. *How to reach it*

   A prospective case study, per aspera ad astra

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Loop calculus in a nutshell

Theorem

Any algorithm aimed at reducing the analytical complexity of a (multi - loop) Feynman diagram is generally bound to

- *replace the original integral with a sum of many simpler diagrams,*
- *introducing denominators that show zeros.*

Definition

An algorithm is *optimal* when

- there is a minimal number of terms,
- zeros of denominators correspond to solutions of Landau equations
- the nature of the singularities is not badly overestimated.

Sunny-side up

Progress

In the past years an enormous progress in the field of $2L$ integrals for massless $2 \to 2$ scattering; $gg \to gg$, $qg \to qg$ and $qQ \to qQ$ as well as Bhabha scattering.

Achievements

- basic $2L$ integrals have been evaluated
- e.g. analytic expressions for the two loop planar and non-planar box
- master integrals connected with the tensor integrals have been determined.

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zero or one

Impressive calculations (up to four loops) for zero or one kinematical variable, e.g. $g-2$, R, β-function

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Computations involving more than one kin. var. is a new art

Example

We would like to have $n = 4$ Green functions to all loop orders, from maximally supersymmetric YM amplitudes to real life it's a long way

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We would like to have $n=4$ Green functions to all loop orders, from maximally supersymmetric YM amplitudes to real life it's a long way

Main road

Step 1

reduce reducible integrals

Step 2

construct systems of IBP or Lorentz invariance identities

Step 3

reduce irreducible integrals to generalized scalar integrals

Step 4

solve systems of eqns in terms of MI

Step 5

evaluate MI, e.g. DE, MB representations, nested sums, etc.

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But, for the real problem

Loop integrals are not enough

$+$

assemblage of scattering amplitudes

$+$

infrared divergenges

$+$

collinear divergenges

$+$

numerical programs

Two-loop conceptual problems

WSTI vs LSZ

- Two loop *à la* LSZ
- The LSZ formalism is unambiguously defined only for stable particles, and it requires some care when external unstable particles appear

Unstable internal

Unphysical behaviors induced by self-energy insertions into 1 L diagrams; they signal the presence of an UP and are the consequence of a misleading organization of PT.

Around thresholds

These regions are not accessible with approximations, e.g. expansions.

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Technical problems I

Reduction to MI

Algebraic problem,

- Buchberger algorithm to construct Gröbner bases seems to be inefficient

New bases?

It remains

- to generalize to more than few scales
- to compute the MI

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New bases?

Technical problems II

Although

HTF (usually) have nice properties,

- expansions are often available with good properties of convergence
- the expansion parameter has the same cut of the function

where is the limit?

- One - loop, Nielsen - Goncharov
- Two - loop, one scale ($s = 0, m^2$ cuts) harmonic polylogarithms
- Two - loop, two scales ($s = 4\,m^2$ cuts) generalized harmonic polylogarithms
- next? New higher transcendental functions?

Part I

Future of $2L$ calc: exploratory case study

From modern $1\,L$ to $2\,L$

$1\,L$ in a nutshell

$$
\begin{aligned}
S_{n\,;\,N}(f) &= \frac{\mu^{\epsilon}}{i\,\pi^2}\int d^n q\,\frac{f\,(q\,,\,\{p\})}{\prod_{i=0,N-1}(i)}, \\
(i) &= (q+p_0+\cdots+p_i)^2+m_i^2 .
\end{aligned}
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&+ \sum_{ijk} d_{ijk}\,D_0(P_i^2,P_j^2,P_k^2)+R,
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The multi facets of QFT

Popular wisdom

- Tree is nirvana
- 1 L is limbo
- 2 L is samsara

1 L $\longrightarrow$

1 L will be nirvana when general consensus on reduction is reached

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Which is the most efficient way of computing the coefficients?

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Reduction at $2\,L$

Problem

At $2\,L$ reduction is different since irreducible scalar products are present

Master Integrals

One way or the other a basis of generalized scalar functions is selected (MI)

Which MI are present?

Some care should be payed in avoiding MIs that do not occur in the actual calculation. This fact is especially significant when the MI itself is divergent and the singularity must be extracted analytically

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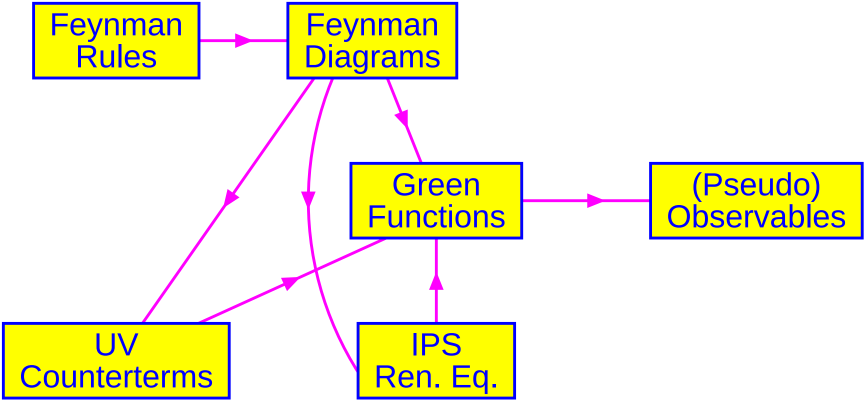
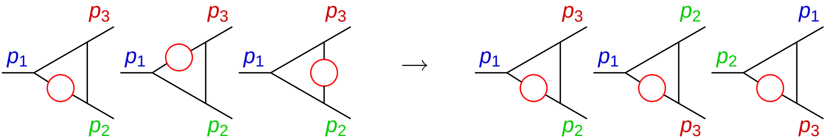
Part II

Renormalization Tree: embedded case study

flow-chart


Feynman Rules
Feynman Diagrams
Green Functions
(Pseudo) Observables
UV Counterterms
IPS Ren. Eq.

Group the diagrams into families



Steps

Combinatorial factors (Goldberg strategy)

Combine the topologies and the Feynman rules

Introduce projectors, compute the trace of Dirac matrices

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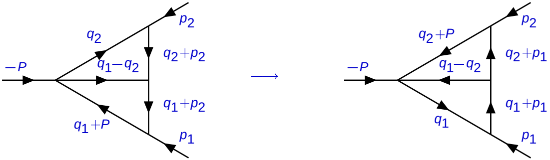
Reduction to Master Integrals

recursive algebraic reduction

$$\frac{2\,q.p}{(q^2+m^2)\,[(q+p)^2+M^2]} = \frac{1}{q^2+m^2} - \frac{1}{(q+p)^2+M^2} - \frac{p^2-m^2+M^2}{(q^2+m^2)\,[(q+p)^2+M^2]}$$

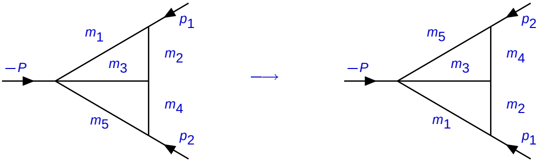
Mapping on a fixed standard routing for loop momenta



$$\begin{pmatrix} q_1 \to -q_1 - P \\ q_2 \to -q_2 - P \end{pmatrix}$$

Reduction to Master Integrals

Symmetrization

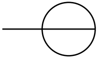


$$\begin{pmatrix} q_1 \rightarrow -q_2 - P \\ q_2 \rightarrow -q_1 - P \end{pmatrix}$$

Reduction to Master Integrals

We end with integrals up to rank 2: 1-loop functions, 2-loop tadpoles (2)

T^A



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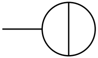
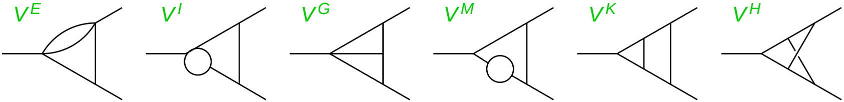


Reduction to Master Integrals

2-loop self-energies (4 topologies)

Reduction to Master Integrals

2-loop vertices (6 topologies)



Universal UV decomposition: I

One-loop integrals

Any one-loop integral f^1 can be decomposed as

$$f^1(\{l\}) = \sum_{k=-1}^{1} f^1(\{l\};k)\, F_k^1(x),$$

where $\{l\} \equiv$ external kinematical variables, masses of internal particles. x is some kinematical variable and the dependence on the dimensional regulator ϵ is entirely transferred to the universal UV factors,

$$F_{-1}^1(x) = \frac{1}{\epsilon} - \frac{1}{2}\Delta_{UV}(x) + \frac{1}{8}\Delta_{UV}^2(x)\,\epsilon,$$

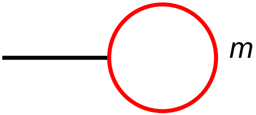
$$F_0^1(x) = 1 - \frac{1}{2}\Delta_{UV}(x)\,\epsilon, \qquad F_1^1(x) = \epsilon.$$

Universal UV decomposition: II

One-loop integrals

Because of overlapping divergencies we include $\mathcal{O}(\epsilon)$ terms in all one-loop results.

$$\Delta_{UV} = \gamma + \ln \pi + \ln \frac{M^2}{\mu^2}, \qquad \Delta_{UV}(x) = \Delta_{UV} - \ln \frac{M^2}{x},$$



$$= - 2\, F^{1}_{-1}(M^2) + \left(\ln \frac{m^2}{M^2} - 1 \right) F^{1}_{0}(M^2) + \cdots$$

Universal UV decomposition: III

Two-loop integrals

A generic two-loop integral f^2 can be written as

$$f^2(\{l\}) = \sum_{k=-2}^{0} f^2(\{l\};k)\, F_k^2(x).$$

Here the two-loop UV factors read as follows:

$$F^2_{-2}(x) = \frac{1}{\epsilon^2} - \frac{\Delta_{UV}(x)}{\epsilon} + \frac{1}{2}\Delta^2_{UV}(x),$$

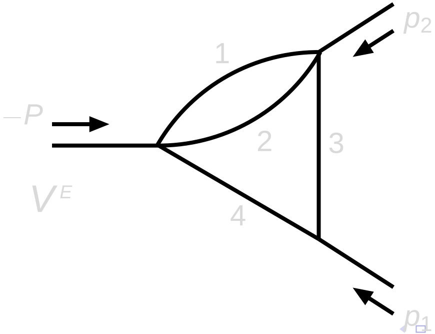
$$F^2_{-1}(x) = \frac{1}{\epsilon} - \Delta_{UV}(x), \quad F^2_0(x) = 1.$$

Note that the product of two one-loop integrals can be written through the same UV decomposition of a two-loop integral.

Universal UV decomposition: IV

Example

$$v_0^E(\cdots;-2) = -2,$$
$$v_0^E(p_2, P, \{m\}_{1234};-1) = -2\,b_0(1,1,p_1,\{m\}_{34};0)-1.$$



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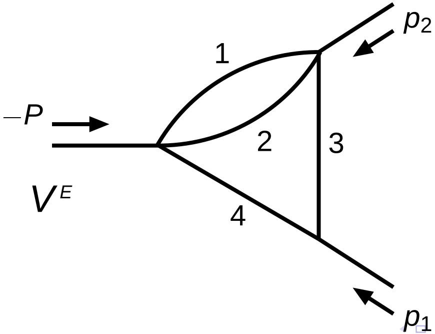


Multiplicative renormalization

c.t. not needed but useful

$$Z_i = 1 + \sum_{n=1}^{\infty} \left(\frac{g_R^2}{16\,\pi^2} \right)^n \delta Z_i^{(n)},$$

Example

masses, parameters

$$m = Z_m^{1/2}\, m_R\,,$$
$$p = Z_p\, p_R\,, \quad p = g\,, c_\theta\,, s_\theta$$

Example

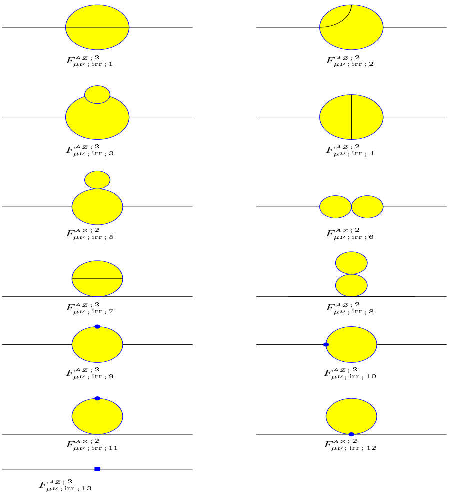
Fields, gauge parameters

$$Z_{\xi_{AZ}} = \sum_{n=1}^{\infty} \left(\frac{g_R^2}{16\,\pi^2} \right)^n \delta Z_{\xi_{AZ}}^{(n)}$$
$$\phi = Z_\phi^{1/2}\, \phi_R \quad \psi^{L,R} = Z_{\psi_{L,R}}^{1/2}\, \psi_R^{L,R}$$
$$A^\mu = Z_{AA}^{1/2}\, A_R^\mu + Z_{AZ}^{1/2}\, Z_R^\mu$$
$$Z_{AZ}^{1/2} = \sum_{n=1}^{\infty} \left(\frac{g_R^2}{16\,\pi^2} \right)^n \delta Z_{AZ}^{(n)}$$

FP ghost fields are not renormalized

Renormalization constants

$$G^{AZ\,;\,2}_{\mu\nu\,;\,\mathrm{irr}} =$$



$\overline{MS}$ and beyond I

$\overline{NMS}$ scheme: advancing renormalization theory

In the spirit of the UV decomposition we define a non-minimal ($\overline{NMS}$) subtraction scheme where

Definition

$$\begin{aligned}
\texttt{1 loop} &\rightarrow \quad \delta Z_i^{(1)} = \Delta Z_i^{(1)} F^1_{-1}(M_R^2), \\
\texttt{2 loops} &\rightarrow \quad \delta Z_i^{(2)} = \sum_{k=-2}^{-1} \Delta Z_{i\,;\,k}^{(2)} F^2_k(M_R^2),
\end{aligned}$$

$\overline{MS}$ and beyond II

counterterms

c.t. are fixed in order to remove order-by-order the poles at $\epsilon = 0$ for any Green function

Property of $\overline{NMS}$

The product of a one-loop c.t. with a one-loop diagram (i.e., a one-loop c.t. insertion) has the same UV decomposition of a two-loop function thus simplifying two-loop renormalized Green functions

The $\overline{NMS}$ scheme has the virtue of respecting the universal UV decomposition

The two facets of renormalization

Step 1

promote bare quantities p to renormalized ones p_R

Step 2

- fix the c.t. at $1\,L$ $\equiv$ to remove the UV poles from all $1\,L$ GF;
- check that $2\,L$ GF develop local UV residues;
- fix the $2\,L$ c.t. to remove $2\,L$ local UV poles.

Finite renomalization

the absorption of UV poles into local c.t. does not exhaust the procedure; we have to connect p_R to POs, thus making the theory predictive.

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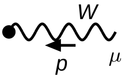
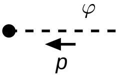
WST identities



$$-(2\pi)^4 i \; i p_\mu Z_W^{1/2} Z_{\xi_W}^{-1}$$



$$(2\pi)^4 i \; M Z_\varphi^{1/2} Z_M^{1/2} Z_{\xi_\varphi}$$

Figure: Sources related to the gauge-fixing functions $\mathcal{C}^{\pm}$

WST identities

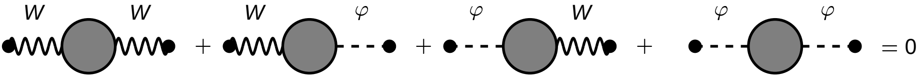


Figure: Doubly-contracted WST identity with two external $\mathcal{C}^{\pm}$ sources

Summary

▸▸ New

$SU(3) \otimes SU(2) \otimes U(1)$

We have been able to verify that the SM can be made (two-loop) UV finite by adding *local* c.t.

Generalization of $1\,L$

The well-known one-loop result that self-energies suffice in performing renormalization can be extended up to two loops.

Overlapping divergencies

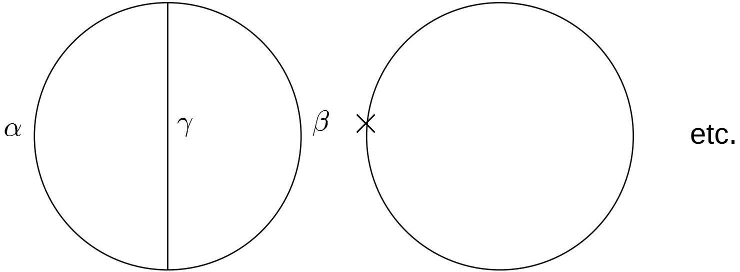


Figure: The arbitrary two-loop diagram $G_L^{\alpha\beta\gamma}$ and one of the associated subtraction sub-diagrams. Only in the sum we have cancellation of non-local residues

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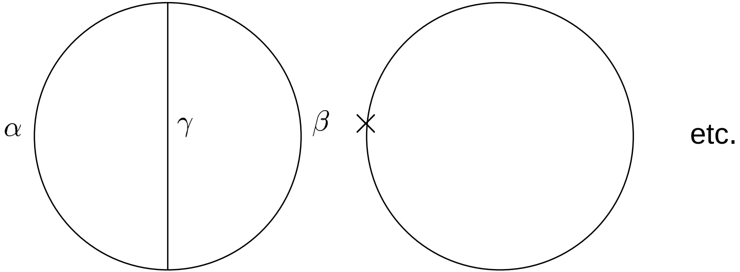

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Example: α

Renormalizing $g\, s_\theta$

In extracting α from Thomson scattering at zero momentum transfer we find four classes of two-loop diagrams:

(1, 2, 3, 4)

1. *irreducible two-loop vertices and wave-function factors, product of one-loop corrected vertices with one-loop wave-function factors;*
2. *one-loop vacuum polarization $\otimes$ one-loop vertices or one-loop wave-function factors;*
3. *irreducible two-loop AA, AZ, $A\phi^0$ transitions;*
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Consistency

A matter of life $\cdots$

You can't prove something by assuming it's true

We have verified that the non-vanishing contribution originates from III and IV only and, within these terms, only the reducible and irreducible *AA* transition survives.

Bluntly:

We have proven that the SM it's only slightly more complex than QED.

Example: G_F

Renormalizing g, M

In extracting the Fermi coupling constant from the muon lifetime all corrections to

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8\,M^2}\,(1 + \Delta g)$$

which do not originate from the *W* self-energy and that are UV (and IR) finite at one loop remain finite at two loops after one-loop renormalization (i.e. two-loop counterterms are not needed)

A sample of $\overline{MS}$ counterterms: I

an $\overline{MS}$ example

$$\delta Z_i^{(2)} \quad = \quad \left(\frac{1}{\epsilon} - \Delta_{UV}\right)\left(\frac{\Delta Z_{i;1}^{(2)}}{\epsilon} + \Delta Z_{i;2}^{(2)}\right) + \Delta_{UV}^2\,\Delta Z_{i;3}^{(2)}.$$

Higgs field renormalization

$$\Delta Z_{H\,;\,1}^{(2)} \quad = \quad 4 + \frac{43}{4}\frac{1}{c_\theta^4} + \frac{5}{4}\frac{x_t}{c_\theta^2} - \frac{37}{2}\frac{1}{c_\theta^2} - \frac{7}{2}x_t - \frac{9}{4}x_t^2 + 24x_t\frac{g_S^2}{g^2},$$

A sample of $\overline{MS}$ counterterms: II

Higgs field renormalization

$$\Delta Z^{(2)}_{H\,;\,2} = \frac{7}{6} - \frac{431}{96}\frac{1}{c_\theta^4} - \frac{85}{48}\frac{x_t}{c_\theta^2} + \frac{101}{12}\frac{1}{c_\theta^2} - \frac{25}{24}x_t + \frac{27}{16}x_t^2 - \frac{3}{32}x_H^2 - 10x_t\frac{g_S^2}{g^2},$$

$$\Delta Z^{(2)}_{H\,;\,3} = -2 + \frac{43}{16}\frac{1}{c_\theta^4} + \frac{5}{16}\frac{x_t}{c_\theta^2} - \frac{37}{8}\frac{1}{c_\theta^2} - \frac{7}{8}x_t - \frac{9}{16}x_t^2 + 6x_t\frac{g_S^2}{g^2}.$$

γ^5 in a nutshell

HVBM scheme

The HVBM scheme breaks all WST identities (so-called spurious or avoidable violations) which can be restored afterwards by introducing suitable ultraviolet finite counterterms. The procedure, however is lengthy and cumbersome.

pseudo-regularization of chiral theories

After considerable wrangling one is lead to the conclusion that the only sensible solution is the one proposed by Jegerlehner: n-dimensional γ-algebra with strictly anti-commuting γ^5 together with 4-dimensional treatment of the hard anomalies.

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About gauge independence I

Problem

$$\Sigma_{VV}(s,\xi) = \sum_{n=1}^{\infty} \Sigma_{VV}^{(n)}(s,\xi)\, g^{2n}$$

On mass shell

$$\Sigma_{VV}^{(1)}(s,\xi) = \Sigma_{VV\,;\,I}^{(1)}(s) + (s - M_V^2)\, \Phi_{VV}(s,\xi)$$

Nielsen identity

$$\frac{\partial}{\partial \xi} \Sigma_{VV}(s_P,\xi) = 0$$

$$s_P - M_V^2 + \Sigma_{VV}(s_P) = 0$$

Decomposition

$$\Sigma_{VV}^{(n)}(s,\xi) = \Sigma_{VV\,;\,I}^{(n)}(s) + \Sigma_{VV\,;\,\xi}^{(n)}(s,\xi)$$

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About gauge independence II

Solution

$$\Sigma^{(n)}_{VV\,;\,\xi}(s_P,\xi) \;=\; \Sigma^{(n-1)}_{VV\,;\,I}(s_P)\,\Phi_{VV}(s_P,\xi),$$

Theorem

As a result we can prove that all ξ-dependent parts cancel,

$$\Sigma_{VV}(s_P) \;=\; \sum_{n=1}^{\infty}\Sigma^{(n)}_{VV\,;\,I}(s_P)\,g^{2n}.$$

However, this example shows how an all-order relation should be carefully interpreted while working at some fixed order.

Renormalization equations

From ren. parameters to an IPS

$$G\left[M^2 - \frac{g^2}{16\,\pi^2}\,F_W(0)\right] = \frac{g^2}{8}$$

$$4\,\pi\,\alpha\left[1 - \frac{g^2}{16\,\pi^2}\,\Pi_{QQ}(0)\right] = g^2 s_\theta^2$$

Warning

One of our renormalization equations will always be of the type

$$s_V \quad = \quad M_V^2 - \frac{g^2}{16\,\pi^2}\,\Sigma_{VV}(s_V, M_V^2).$$

Options

(1, 2, 3,)

1. At $\mathcal{O}\left(g^4\right)$ in Σ_{VV} we keep $p^2 = -s_V$ with $\mathcal{O}\left(g^6\right)$ violation;
2. we replace Σ_{VV} with $\Sigma_{VV\,;\,I}$

$$\Sigma^{(2)}_{VV\,;\,I}(s_V) \;=\; \Sigma^{(2)}_{VV}(s_V, 1) - \Sigma^{(1)}_{VV\,;\,I}(s_V)\,\Phi_{VV}(s_V, 1).$$

3. we expand,

$$\begin{aligned} s_V \;=\;& M^2_V - \frac{g^2}{16\,\pi^2}\,\Sigma^{(1)}_{VV}(M^2_V, M^2_V) \\ &- \left(\frac{g^2}{16\,\pi^2}\right)^2 \Big[\Sigma^{(2)}_{VV}(M^2_V, M^2_V) \\ &- \Sigma^{(1)}_{VV}(M^2_V, M^2_V)\,\Sigma^{(1)}_{VV\,;\,p}(M^2_V, M^2_V)\Big] \end{aligned}$$

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Finite renormalization in running couplings

(intermediate) renormalized theory

$$\frac{1}{g^2(s)} = \frac{1}{g^2} - \frac{1}{16\,\pi^2}\,\Pi^{(1)}_{3Q}(s) - \frac{g^2}{(16\,\pi^2)^2}\,\Pi^{(2)}_{3Q}(s).$$

theory in terms of POs

$$\frac{1}{g^2(s)} = \frac{1}{8\,G\,\mu_W^2} - \frac{1}{16\,\pi^2\,\mu_W^2}\,\delta g^{(1)} - \frac{G}{32\,\pi^4}\,\delta g^{(2)},$$

$$\delta g^{(n)} = \mu_W^2\,\Pi^{(n)}_{3Q}(s) + \tilde{F}^{(n)}_W(s_W).$$

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Dressed propagators

From finite order

$$\Delta^{(n)}(p^2) \quad = \quad \Delta^{(0)}(p^2)\left[1 - \Delta^{(0)}(p^2)\,\Sigma^{(n)}\left(p^2\,,\;\Delta^{(n-1)}(p^2)\right)\right]^{-1},$$

To all orders

$$\overline{\Delta}(p^2) \quad = \quad \lim_{n\to\infty} \Delta^{(n)}(p^2),$$

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Properties: I

all orders

We assume that a dressed propagator obeys Källen - Lehmann representation; using the relation

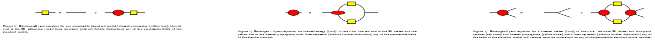
$$\begin{aligned} \mathrm{Im}\,\overline{\Delta}(p^2) &= \mathrm{Im}\,\Sigma \left[\left(p^2 + m^2 - \mathrm{Re}\,\Sigma \right)^2 + (\mathrm{Im}\,\Sigma)^2 \right]^{-1} \\ &= \pi\,\rho(-p^2), \end{aligned}$$

Källen - Lehmann

the Källen - Lehmann representation follows:

$$\overline{\Delta}(p^2) = \int_0^\infty ds\, \frac{\rho(s)}{p^2 + s - i\,\delta}.$$

Properties: II



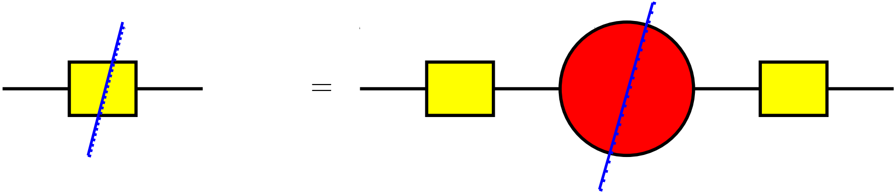


Figure: Cutting equation for a dressed propagator; the red circle is the (all-orders) cut self-energy.

Figure 1: Schwinger-Dyson equation for any permissible (standard model) dressed propagator (yellow box); the red oval is the SD self-energy; solid lines represent (without further distinction) any of the permissible fields of the standard model.

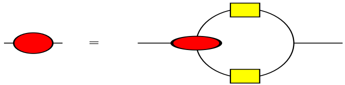


Figure 1: Schwinger - Dyson equation for the self-energy (l.h.s). In the r.h.s. the red oval is the SD vertex and the yellow box is the dressed propagator; solid lines represent (without further distinction) any of the permissible fields of the standard model.

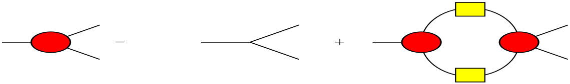


Figure 1: Schwinger-Dyson equation for a dressed vertex (l.h.s); in the r.h.s. we have SD three- and four-point vertices (red ovals) and dressed propagators (yellow boxes); solid lines represent (without further distinction) any of the fields of the standard model and vertices must be understood as any of the permissible standard model vertices.

Unitarity

all orders

- Unitarity follows if we add all possible ways in which a diagram with given topology can be cut in two.
- The shaded line separates S from $S^{\dagger}$.
- For a stable particle the cut line, proportional to $\overline{\Delta}^{+}$, contains a pole term $2\,i\,\pi\,\theta(p_0)\,\delta(p^2+m^2)$,
- whereas there is no such contribution for an unstable particle.

Theorem

- *We express $\mathrm{Im}\,\Sigma$ in terms of cut self-energy diagrams and repeat the procedure ad libidum, therefore proving that unstable particles contribute to the unitarity of the $S-$matrix only via their stable decay products.*

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WSTI with dressed propagators/vertices

- We assume that WST identities hold at any fixed order in perturbation theory for diagrams that contain bare propagators and vertices;
- they again form dressed propagators and vertices when summed.

but

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Finite renormalization with unstable particles

Give up unitarity (a quizzical approach)

Use

$$m^2 = s_m + \Sigma(s_m);$$

At one loop

$m^2 \to s_m$ everywhere;

At two loops

- $2\,L$ no Σ insertions: $m^2 = s_m$;
- $1\,L$: $m^2 = s_m + \Sigma(s_m)$ and the factor

$$\frac{\Sigma(s) - \Sigma(s_m)}{s - s_m},$$

expanded to first order;

- vertices: $m^2 = s_m$ in $2\,L$, $m^2 = s_m + \Sigma(s_m)$ in $1\,L$

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Part III

Computing MI

Beyond Nielsen - Goncharov

New Approach

New integral representations for diagrams

Theorem

Diagrams $\equiv$

$$\int dC_k(\{x\})\frac{1}{A}\ln\left(1+\frac{A}{B}\right) \quad \text{or} \quad \int dC_k(\{x\})\frac{1}{A}\,\mathrm{Li}_n\left(\frac{A}{B}\right)$$

where A, B are multivariate polynomials in the Feynman parameters. One-(Two-) loop diagrams are always reducible to combinations of integrals of this type where the usual monomials that appear in the integral representation of Nielsen - Goncharov generalized polylogarithms are replaced by multivariate polynomials of arbitrary degree.

Example

General C_0: definitions

$$
\begin{aligned}
C_0 &= \int dS_2\, V^{-1-\epsilon/2}(x_1,x_2),\\
V(x_1,x_2) &= x^t H\, x + 2\, K^t\, x + L = Q(x_1,x_2) + B,\\
H_{ij} &= -p_i \cdot p_j, \quad L = m_1^2,\\
K_1 &= \frac{1}{2}(p_1 \cdot p_1 + m_2^2 - m_1^2),\\
K_2 &= \frac{1}{2}(P \cdot P - p_1 \cdot p_1 + m_3^2 - m_2^2),
\end{aligned}
$$

General C_0: result

$$C_0 = \frac{1}{2}\sum_{i=0}^{2}(X_i - X_{i+1}) \times \int_0^1 \frac{dx}{Q(\widehat{i\; i+1})} \ln\left(1 + \frac{Q(\widehat{i\; i+1})}{B}\right)$$

$$Q(\widehat{0\;1}) = Q(1,x),\; Q(\widehat{1\;2}) = Q(x,x),\; Q(\widehat{2\;3}) = Q(x,0)$$

$$X^t = -K^t H^{-1},\; X_0 = 1,\; X_3 = 0$$

Basics

Define

$$\mathcal{L}_n(z) = z^n L_n(z) = z^n \int dC_n \left(\prod_{i=1}^{n} y_i\right)^{n-1} \left[1 + \prod_{j=1}^{n} y_j\, z\right]^{-n}$$

$$= \left(\frac{z}{n}\right)^n {}_{n+1}F_n\left((n)_{n+1}\,;\,(n+1)_n\,;\,-z\right),$$

$$\mathcal{L}_1(z) = -S_{0,1}(-z),$$

$$\mathcal{L}_2(z) = S_{0,1}(-z) - S_{1,1}(-z),$$

$$\mathcal{L}_3(z) = -\frac{1}{2}S_{0,1}(-z) + \frac{3}{2}S_{1,1}(-z) - S_{2,1}(-z),$$

Problem

- For any quadratic form in n-variables

$$V(x) \;=\; (x-X)^t \; H \; (x-X) + B = Q(x) + B,$$

- we want to compute

$$I(n,\mu) \;=\; \int dC_n \, V^{-\mu} = \int dC_n \left[Q(x) + B \right]^{-\mu}.$$

Definition

- Consider the operator

$$\mathcal{P} \;=\; (x-X)^t \; \partial, \quad \text{satisfying} \quad \mathcal{P}\, Q = 2\, Q$$

Solution

Introduce MT of y -shifte quadratic

$$J(\beta,\mu) \;=\; \int_0^1 dy\, y^{\beta-1}\, W^{-\mu}(y), \quad W(y) = Q(x)\,y + B.$$

Use integration-by-parts

$$\left(\frac{1}{2}\mathcal{P} - y\,\partial_y\right) W^{-\mu} = 0 \quad \to \quad V^{-\mu} = \left(\beta + \frac{1}{2}\mathcal{P}\right) J(\beta,\mu),$$

$$I(n,\mu) \;=\; \int dC_n \left(\beta + \frac{1}{2}\mathcal{P}\right) J(\beta,\mu),$$

Further definitions

Define

$$\begin{aligned} f([x]) &= f(x_1\,,\cdots,\,x_n), \\ f({}_i[x]) &= f(x_1\,,\cdots,\,x_i=0\,,\,x_n), \\ f([x]_i) &= f(x_1\,,\cdots,\,x_i=1\,,\,x_n), \\ \int dC_n &= \int_0^1 \prod_{i=1}^{n} dx_i\,, \qquad \int dC_{n,j} = \int_0^1 \prod_{i=1,i\neq j}^{n} dx_i\,. \end{aligned}$$

Results I

Example

- For $\mu = 1$ it is convenient to choose $\beta = 1$, to obtain

$$\begin{aligned} I(n,1) &= \left(\frac{n}{2} - 1\right) \int dC_n \, L_1([x]) \\ &- \frac{1}{2} \sum_{i=1}^{n} \int dC_{n,i} \left\{ X_i \, L_1({}_i[x]) - (1 - X_i) \, L_1([x]_i) \right\} \end{aligned}$$

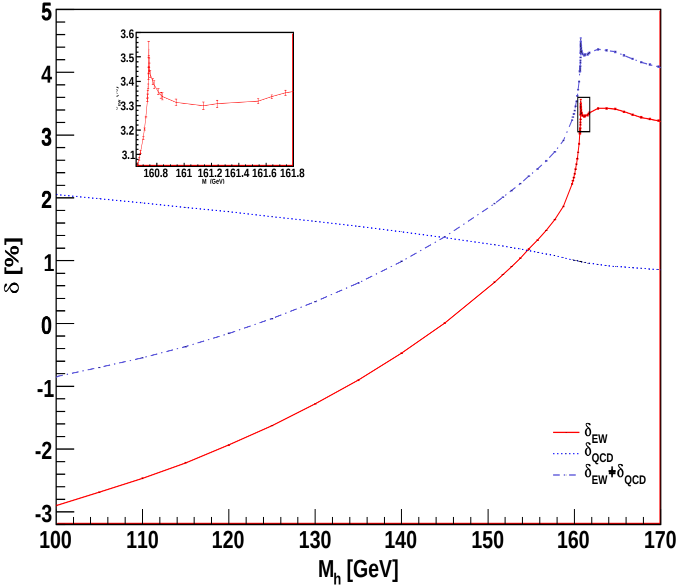
Results II

Example

- For $\mu = 2$ it is more convenient to write

$$V^{-2} = \left(2 + \frac{1}{2}\mathcal{P}\right)^2 J(2,2) = \left(2 + \frac{1}{2}\mathcal{P}\right)^2 L_2.$$

- integration-by-parts follows
- additional work (along the same lines) is needed to deal with surface terms ...



5
4
3
2
1
0
-1
-2
-3
δ [%]
100
110
120
130
140
150
160
170
M_h [GeV]
3.6
3.5
3.4
3.3
3.2
3.1
160.8
161
161.2
161.4
161.6
161.8
M (GeV)
δ_EW
δ_QCD
δ_EW+δ_QCD

$1/\alpha_{\overline{MS}}(s)$

$m_t = 174.3\,\mathrm{GeV}$	$M_H = 150\,\mathrm{GeV}$		
$\sqrt{s}$ [GeV]	M_Z	200	500
one-loop	128.104	127.734	127.305
two-loop	128.040	127.831	127.586
%	-0.05	$+0.08$	$+0.22$
$m_t = 174.3\,\mathrm{GeV}$	$M_H = 300\,\mathrm{GeV}$		
$\sqrt{s}$ [GeV]	M_Z	200	500
one-loop	128.104	127.734	127.305
two-loop	128.046	127.689	127.272
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m_t, M_H [Gev]	$1L + 2L$	$2L/1L$ perturbative only
169.3, 150	126.774(3)	14.64 %
174.3, 150	126.688(2)	16.27 %
179.3, 150	126.598(3)	17.97 %
169.3, 300	127.300(3)	4.35 %
174.3, 300	127.313(2)	1.51 %
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