hep–ph/9909472

An effective-Lagrangian approach for unstable particles

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1. Introduction

$\boxed{\textbf{Physical observables are gauge invariant}}$

$\boxed{\textbf{Correct predictions should be gauge invariant}}$

∘ Born	all diagrams, subsets
∘ loops	pinch technique, bkg-field-method,...
∘ resummation	**this talk**

- **Unstable particles**

∘ Dyson resummation → mixing α-orders → gauge breaking
(e.g. Breit-Wigner)

∘ **wrong predictions**

- **Approaches**

∘ leading-pole-approx (Γ/M)

∘ fermion-loop-scheme (αN_f)

∘ Ward identities

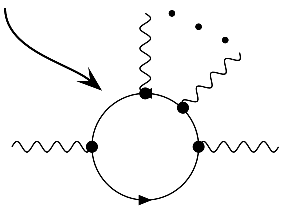
2. Simple example: $SU(N)$ + fermions

$$\mathcal{L} = -\frac{1}{2}\,\mathrm{Tr}\,[\boldsymbol{F}_{\mu\nu}\,\boldsymbol{F}^{\mu\nu}] + \bar{\psi}\,(i\,\not{D} - m)\,\psi$$

- effective action

$$i\,\mathcal{S}_{\text{eff}} = -\frac{i}{2}\int dx\,\mathrm{Tr}\,[\boldsymbol{F}_{\mu\nu}\,\boldsymbol{F}^{\mu\nu}] + \mathbf{Tr}\,\ln(-\not{D} - i\,m)$$

$$\mathbf{Tr}\,\ln\Big(1 + \frac{g}{i\,\not{\partial} - m}\,\not{\boldsymbol{A}}\Big) = \sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n}\,\mathbf{Tr}\,\Big(\frac{g}{i\,\not{\partial} - m}\,\not{\boldsymbol{A}}\Big)^{n}$$



∘ sum is gauge invariant
∘ each term $\to$ not gauge invariant

Fermion-Loop-Scheme

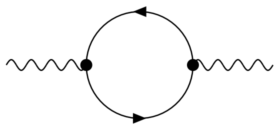
$$-\frac{1}{2}\,\mathbf{Tr}\,\Big(\frac{g}{i\,\not{\partial} - m}\,\not{\boldsymbol{A}}\Big)^{2} =$$

$$= -\frac{g^2}{2}\int dx\,dy\;\underbrace{\mathrm{Tr}\,[\boldsymbol{A}_\mu(\boldsymbol{x})\,\boldsymbol{A}_\nu(\boldsymbol{y})]}\;\;\underbrace{\mathrm{Tr}\Big[\boldsymbol{S}_{\mathrm{F}}(\boldsymbol{y}-\boldsymbol{x})\,\gamma^{\mu}\,\boldsymbol{S}_{\mathrm{F}}(\boldsymbol{x}-\boldsymbol{y})\,\gamma^{\nu}\Big]}$$

naive Dyson resummation

nonlocal

not gauge invariant



$\boldsymbol{\Sigma}^{\mu\nu}(\boldsymbol{x}-\boldsymbol{y})$

- $\boxed{\textbf{Can one make it gauge invariant?}}$

2. Simple example: $SU(N)$ + fermions

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n} \,\mathbf{Tr}\left(\frac{g}{i\not\partial - m}\,\not{\boldsymbol{A}}\right)^{n}$$

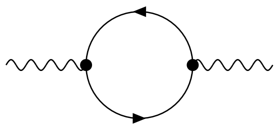
- meaning of the terms (n=2):

$$-\frac{1}{2}\,\mathbf{Tr}\left(\frac{g}{i\not\partial - m}\,\not{\boldsymbol{A}}\right)^{2} =$$

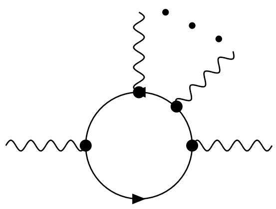
$$= -\frac{g^2}{2}\int dx\, dy\; \mathbf{Tr}\left[\,\frac{1}{i\not\partial - m}\,\not{\boldsymbol{A}}\;\cdot\;|\,x><x\,|\;\cdot\;\frac{1}{i\not\partial - m}\,\not{\boldsymbol{A}}\;\cdot\;|\,y><y\,|\,\right] =$$

$$= -\frac{g^2}{2}\int dx\, dy\;\mathrm{Tr}\left[\,\underbrace{<y\,|\,\frac{1}{i\not\partial - m}\,|\,x>}_{S_{\mathrm{F}}(y-x)}\cdot\not{\boldsymbol{A}}(x)\;\cdot\;\underbrace{<x\,|\,\frac{1}{i\not\partial - m}\,|\,y>}_{S_{\mathrm{F}}(x-y)}\cdot\not{\boldsymbol{A}}(y)\,\right]$$

$$: \quad -\frac{g^2}{2}\int dx\, dy\;\underbrace{\mathrm{Tr}\left[S_{\mathrm{F}}(y-x)\,\not{\boldsymbol{A}}(x) S_{\mathrm{F}}(x-y)\,\not{\boldsymbol{A}}(y)\right]}$$



- n-th term :



3. Unstable fermions: $SU(N)$ + fermions

J. Terning,
Phys. Rev. **D44** (1991) 887.

$$\mathcal{S} = \int dx \;\; \bar\psi \, (i \, \partial\!\!\!/ + g \, A\!\!\!/ - m) \, \psi$$

- **Path exponential**

$$\boldsymbol{U(x,y) \;\; = \;\; \mathrm{P}\exp\Big[- ig \int_x^y A_\mu(\omega) \, d\omega^\mu\Big]}$$

$$\boldsymbol{U(x,y) \to \mathrm{G}(x) \, U(x,y) \, \mathrm{G}^{-1}(y)}$$

gauge transform

◦ path

$$\delta(x-y) \int_x^y \boldsymbol{A}_\mu \, d\omega^\mu = 0 \qquad\qquad \partial^\mu_y \int_x^y \boldsymbol{A}_\nu \, d\omega^\nu = \boldsymbol{A}^\mu(y)$$

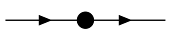
- equivalent form

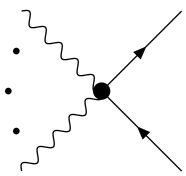
$$\mathcal{S} = \int dx \, dy \;\; \delta(x-y) \;\; \bar\psi(x) \; (i \, \partial\!\!\!/_y - m) \; U(x,y) \; \psi(y)$$

- nonlocal analogue

$$\mathcal{S}_{\rm NL} = \int dx \, dy \;\; \Sigma_{\rm NL}(x-y) \; \bar\psi(x) \;\; U(x,y) \;\; \psi(y)$$

- Np functions

 : $[p\!\!\!/ - m + \Sigma_{\rm NL}(p^2)]$



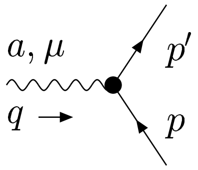
$\boxed{\textbf{higher-point functions} \sim \Sigma_{\rm NL}}$

easy to calculate

3. Unstable fermions: $SU(N)$ + fermions

- higher-point functions

$$\mathcal{S} = \int dx\, dy \;\; \delta(x-y) \;\; \bar{\psi}(x) \; (i\,\partial\!\!\!/_y - m) \; U(x,y) \; \psi(y)$$



$: \;\; ig\, \mathrm{T}^a\, \gamma^\mu$

- nonlocal analogue

$$\mathcal{S}_{\rm NL} = \int dx\, dy \;\; \Sigma_{\rm NL}(x-y) \;\; \bar{\psi}(x) \;\; U(x,y) \;\; \psi(y) = \sum_{n=0}^{\infty} \frac{1}{n!}\, \mathcal{S}^{(n)}_{\rm NL}\; \Sigma^{(n)}_{\rm NL}(0)$$

$$\mathcal{S}^{(n)}_{\rm NL} = \int dx\, dy\, \delta(x-y)\, \bar{\psi}(x)\, (-\partial_y^2)^n \;\; \mathrm{P}\exp\left[- ig \int_x^y \boldsymbol{A}_\mu\, d\omega^\mu\right] \;\; \psi(y)$$

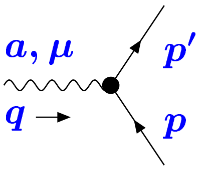
- ◦ recursion relation

$$\mathcal{A}^\mu_n = -\,i \int d\xi\, d\tau\, \delta(\xi-\tau)\, e^{ip'\cdot\xi}\, (-\partial_\tau^2)^n\, e^{-ip\cdot\tau} \int_\xi^\tau e^{-iq\cdot\omega}\, d\omega^\mu$$

$$\mathcal{A}^\mu_n = p^2 \mathcal{A}^\mu_{n-1} + (2p+q)^\mu\, (p+q)^{2n-2} \qquad\qquad \mathcal{A}^\mu_0 = 0$$

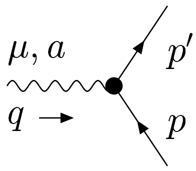
$$\mathcal{A}^\mu_n = (2p+q)^\mu\, \frac{(q+p)^{2n} - p^{2n}}{(q+p)^2 - p^2}$$

- $\boxed{\textbf{vertex}}$

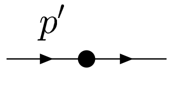


$: \;\; \boldsymbol{ig}\, \mathrm{T}^a\, \dfrac{(\boldsymbol{p}+\boldsymbol{p}')^\mu}{\boldsymbol{p}'^2 - \boldsymbol{p}^2} \left[\Sigma_{\rm NL}(\boldsymbol{p}'^2) - \Sigma_{\rm NL}(\boldsymbol{p}^2)\right]$

- Ward identity $\quad q_\mu \cdot$



$= \;\; \mathrm{T}^a \;\; \Big[$



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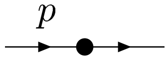


$\Big]$

4. Unstable bosons: $\boldsymbol{SU(N)}$ + fermions

- effective action

$$i\,\mathcal{S}_{\text{eff}} = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n}\,\mathbf{Tr}\left(\frac{g}{i\,\not{\partial} - m}\,\not{A}\right)^{n}$$

Fermion-Loop-Scheme

- naive Dyson resummation

$$-\frac{1}{2}\,\mathbf{Tr}\left(\frac{g}{i\,\not{\partial} - m}\,\not{A}\right)^{2} = -\frac{g^2}{2}\int dx\,dy\;\underbrace{\mathrm{Tr}\,[\boldsymbol{A_\mu(x)A_\nu(y)}]}\;\;\Sigma^{\mu\nu}(x-y)$$

$\boxed{\textbf{Can one make it gauge invariant?}}$

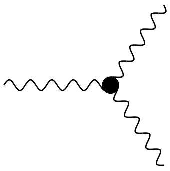
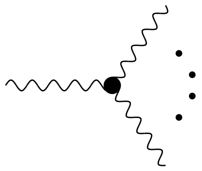
$$\boldsymbol{\mathcal{S}}_{\text{NL}} = -\frac{\mathbf{1}}{\mathbf{2}}\int \boldsymbol{dx\,dy}\;\boldsymbol{\Sigma}_{\text{NL}}\boldsymbol{(x-y)}\;\mathrm{Tr}\,[\boldsymbol{U(y,x)\,F_{\mu\nu}(x)\,U(x,y)\,F^{\mu\nu}(y)}]$$

$$U_{xy} \to G_x U_{xy} G_y^{-1}$$
$$F_x \to G_x F_x G_x^{-1}$$

$\boxed{\textbf{physical input}}$

- Np function

 $:\quad (q^\mu q^\nu - q^2 g^{\mu\nu})\,[1 + \boldsymbol{\Sigma}_{\text{NL}}(q_1^2)]$

  $\boxed{\textbf{are simple}}$

5. Standard Model: broken $SU(3)_C \times SU(2)_L \times U(1)_Y$

- unstable: gauge bosons, Higgs and top-quark

- $\boxed{\textbf{gauge bosons}}$

$$\mathcal{L}^{\mathrm{YM}} = -\frac{1}{2}\,\mathrm{Tr}\,\boldsymbol{F}^2 - \frac{1}{4}\,B^2$$

- <u>nonlocalize:</u> $(F \cdot F), (B \cdot F)$ and $(B \cdot B)$

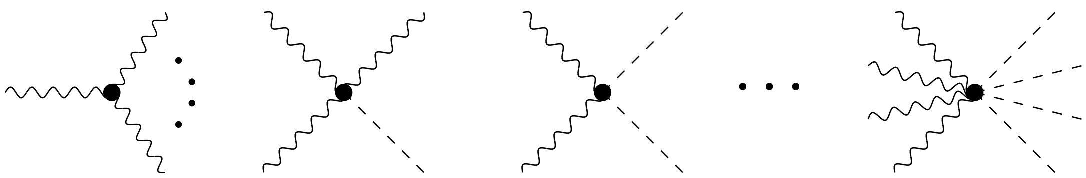
$$\begin{aligned} \mathcal{S}^{\mathrm{YM}}_{\mathrm{NL}} &= -\frac{1}{4}\int dx\,dy \;\; \Sigma_1(x-y) \;\; B_{\mu\nu}(x) \;\cdot\; B^{\mu\nu}(y) \\ &\quad -\frac{1}{2}\int dx\,dy \;\; \Sigma_2(x-y) \;\; \mathrm{Tr}\,[U_2(y,x)\,\boldsymbol{F}_{\mu\nu}(x)\,U_2(x,y)\,\boldsymbol{F}^{\mu\nu}(y)]. \end{aligned}$$

∘ 2 self-energies (unbroken, no Higgs)
∘ **need 4** self-energy (broken, **Higgs**)

∘ **need a field with non-zero expectation value: Higgs**

$$\begin{aligned} \mathcal{S}^{\Phi}_{\mathrm{NL}} &= -\frac{g_1 g_2}{2M_W^2}\int dx\,dy \;\; \Sigma_3(x-y) \;\; [\Phi^\dagger\,\boldsymbol{F}_{\mu\nu}\,\Phi](x) \;\cdot\; B^{\mu\nu}(y) \\ &\quad -\frac{g_2^4}{4M_W^4}\int dx\,dy \;\; \Sigma_4(x-y) \;\; [\Phi^\dagger\,\boldsymbol{F}_{\mu\nu}\,\Phi](x) \;\cdot\; [\Phi^\dagger\,\boldsymbol{F}^{\mu\nu}\,\Phi](y) \end{aligned}$$

- Np funcions:  $\sim \;\; (q^\mu q^\nu - q^2 g^{\mu\nu})$



5. Standard Model: broken $SU(3)_C \times SU(2)_L \times U(1)_Y$

- **Higgs boson**

$$\mathcal{L}^H(x) = \Big(D_\mu \Phi(x)\Big)^\dagger \Big(D^\mu \Phi(x)\Big) + \mu^2 \left[\Phi^\dagger(x)\,\Phi(x)\right] - \frac{\lambda}{4}\left[\Phi^\dagger(x)\,\Phi(x)\right]^2,$$

- nonlocalize:

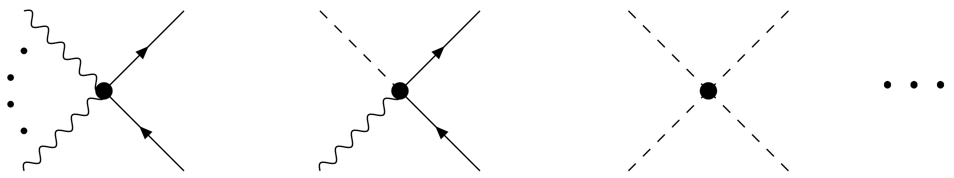
$$\mathcal{S}^H_{\text{NL}} = \frac{1}{2v^2} \int dx\, dy \;\; \Sigma_H(x-y) \;\; \left(\left|\Phi(x)\right|^2 - \frac{v^2}{2}\right) \;\cdot\; \left(\left|\Phi(y)\right|^2 - \frac{v^2}{2}\right)$$

- **Top-quark** **[Also QCD]**

$$\mathcal{L}^t_{\text{Yu}}(x) = -\, f_t\, \overline{Q}_L(x)\, \tilde{\Phi}(x)\, t_R(x) + \text{h.c.}$$

- nonlocalize:

$$\mathcal{S}^t_{\text{NL}} = \frac{\sqrt{2}}{v} \int dx\, dy\, \Sigma_t(x-y) \left\{ \left[\, \overline{Q}_L(x)\, \tilde{\Phi}(x) \,\right] \;\cdot\; U_1(x,y)\, U_3(x,y) \;\cdot\; t_R(y) + \text{h.c.} \right\}$$



∘ Note: gluon, Z, γ

5. Standard Model

- **Simple example**

∘ $\boxed{\text{Higgs boson}}$

$$\mathcal{S}^H_{\rm NL} = \frac{1}{2v^2}\int dx\,dy\;\;\Sigma_H(x-y)\;\;\left(|\Phi(x)|^2 - \frac{v^2}{2}\right)\;\cdot\;\left(|\Phi(y)|^2 - \frac{v^2}{2}\right)$$

$$\boldsymbol{\Sigma_H(x-y) = \Sigma_H\delta(x-y) \qquad M_H^2 \to M_H^2 - \Sigma_H}$$

∘ $\boxed{\text{top-quark}}$

$$\mathcal{S}^t_{\rm NL} = \frac{\sqrt{2}}{v}\int dx\,dy\,\Sigma_t(x-y)\left\{[\,\overline{Q}_L(x)\,\tilde{\Phi}(x)\,]\,U_1(x,y)\,U_3(x,y)\,t_R(y) + \text{h.c.}\right\}$$

$$\boldsymbol{\Sigma_t(x-y) = \Sigma_t\delta(x-y) \qquad m_t \to m_t - \Sigma_t}$$

∘ $\boxed{\text{gauge boson}}$

$$\begin{aligned}\mathcal{S}^{\rm YM}_{\rm NL} \;&=\; -\frac{1}{4}\int dx\,dy\;\;\Sigma_1(x-y)\;\;B_{\mu\nu}(x)\;\cdot\;B^{\mu\nu}(y)\\ &\quad\; -\frac{1}{2}\int dx\,dy\;\;\Sigma_2(x-y)\;\;\text{Tr}\,[U_2(y,x)\,\boldsymbol{F}_{\mu\nu}(x)\,U_2(x,y)\,\boldsymbol{F}^{\mu\nu}(y)].\end{aligned}$$

$$\Sigma_{1,2}(x-y) = \Sigma_{1,2}\delta(x-y) \qquad\qquad \Sigma_{3,4}(x-y) = 0$$

$$\mathcal{L}^{\rm YM} + \mathcal{L}^{\rm YM}_{\rm NL} = -\frac{1}{2}\,(1+\Sigma_2)\,\text{Tr}\,[\boldsymbol{F}^2] - \frac{1}{4}\,(1+\Sigma_1)\,B^2$$

∘ rescaling

$$W = \frac{W'}{\sqrt{1+\Sigma_2}}, \quad g_2 = g_2'\sqrt{1+\Sigma_2} \quad \text{and} \quad B = \frac{B'}{\sqrt{1+\Sigma_1}}, \quad g_1 = g_1'\sqrt{1+\Sigma_1}$$

$$\boldsymbol{M_W^2(\sim g_2^2) \to \frac{M_W^2}{1+\Sigma_W}, \qquad M_Z^2 \to \frac{M_Z^2}{1+\Sigma_Z} \qquad c_w^2 \to c_w^2\,\frac{1+\Sigma_Z}{1+\Sigma_W}}$$

A.Denner et al.
*hep–ph/***9904472**

6. Conclusions

- $\boxed{\textbf{gauge invariant calculations with unstable particles}}$
- **via effective actions**
- **via modified Feynman rules**

- **construction of a minimal solution of a system of Ward identities**

- **Comparison with FLS**

◦ Vertices are simpler

◦ Σ_{NL} — free input (also boson contributions)

- unstable: gauge bosons
  Higgs boson
  (top-)quarks

- simple solution: $\Sigma_i(x-y) = \Sigma_i \cdot \delta(x-y)$

◦ if $\Sigma_{3,4} = \Sigma_H = \Sigma_t = 0 \;\; \rightarrow$ complex mass/coupling scheme (A.Denner et al)

- we calculated vertices explicitly for

◦ $e^+e^-/q\bar{q}/gg \rightarrow 4f\gamma$

◦ $e^+e^-/q\bar{q}/gg \rightarrow 4fg$

◦ $e^+e^-/q\bar{q}/gg \rightarrow 6f...$

- easy to use for more complicated processes