TESTS OF THE STANDARD MODEL AND DEFINITION OF ERRORS AT LEP

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Abstract

The most recent data from the four LEP Collaborations are used for a high precision test of the minimal standard model. Various results concerning the unknown parameters of the theory are discussed in order to perform a critical analysis of the conventions about different ways of presenting the fits to the data and the corresponding errors. In particular the strategy for discussing the expected range of variation of the Lagrangian parameters (m_t, α_s, m_H) as well as to the Z^0 parameters $(M_Z, \Gamma_Z, R_l$ etc...) is reexamined in the light of the increasing experimental precision.

Submitted to Nuclear Physics B

After the first three years of running at LEP the collected data are able to provide a check of the standard model to an accuracy of $\approx 1\%$ [1]. Among the major results we have the measurements of the Z^0 parameters [2, 3, 4] with an increasing precision which, by the end of the next run, will hopefully reach the following level [5]:

$$\Delta\Gamma_Z = 3\,\mathrm{MeV}, \qquad \Delta R_l = 0.03, \qquad \Delta\sigma_h = 0.08\,\mathrm{nb}.$$

In view of the intrinsic relevance connected with a detailed knowledge of the structure of the fundamental interactions we find rather important to have a clear understanding of the theoretical and experimental uncertainties involved. Our attempt to analyze the LEP data will involve the available published material and it is, by no means, antithetic to what the LEP experiments are doing. In particular, the aim of the present paper is not to perform an exhaustive and up-to-date analysis of LEP data, which remains the main goal of the LEP Collaborations, but instead it is to critically analyze the conventions about different ways of presenting the fits to the data and the corresponding errors.

First of all an unambiguous separation should be made among the various quantities reported by the LEP Collaborations. For a given channel the number of events, energy by energy, is an *observable* and the knowledge of the time integrated luminosity with the corresponding uncertainties allows to quote a cross section. Certainly the correct treatment of the experimental errors and of their correlations is a hard task and the only ones that can afford it without ambiguities at all are the Collaborations themselves. Nevertheless, since the LEP Collaborations suggest the error matrix approach as being a correct treatment of the statistical and systematic errors on a set of correlated experimental data, and for the purposes of this paper we follow in our analysis the procedure described in ref. [6]. In particular this means that the errors on the center-of-mass energy will not be taken into account. Ideally each cross section should be quoted with

1. a statistical error;

2. a point-to-point systematic uncertainty;

3. a normalization systematic error.

With these ingredients one can construct an $n \times n$ covariance matrix where the off-diagonal elements account for the fact that the experimental systematic errors on a set of n measured quantities are correlated.

On the contrary, the Z^0 total width and the partial widths are not by themselves *observables* but are usually derived by a model independent fit to the cross sections for $e^+e^- \rightarrow$ hadrons, $\mu^+\mu^-, \tau^+\tau^-$. Once this difference is accepted we may ask what is the exact or even the conventional definition of the associated error. First of all the theoretical cross sections are parameterized in terms of some effective Born approximation where $\Gamma_{_Z}$, etc... appear as free parameters. Let us consider a well defined example where the parameters to be fitted are

$$M_{_Z} \quad , \qquad \sigma^{pole}_{had}, \quad R_l = \frac{\Gamma_h}{\Gamma_l}, \quad \Gamma_{_Z}, \quad A^{pole}_{_{FB}}.$$

As well known, as a result of the fit we obtain the best values for the five parameters, the minimum of the χ^2 and a covariance (5×5) matrix. Notice that this matrix is an output of the full procedure and should not be confused with the input covariance matrix which correlates the systematic errors among the cross sections. At this point let ν be the number of fitted parameters whose joint confidence region one wish to display and let $\Delta\chi^2$ be such that the probability of a chi-square variable with ν degrees of freedom being less than $\Delta\chi^2$ is p. We take the 5×5 covariance matrix C and construct C_{proj}, corresponding to the ν parameters of interest. The equation for the elliptical boundary of the desired confidence region in the ν-dimensional subspace is

$$\Delta\chi^2 = \delta x^t \, C^{-1}_{proj} \, \delta x, \tag{1}$$

where $\delta x = (x_1 - \overline{x}_1, \dots, x_\nu - \overline{x}_\nu)^t$. In particular if $\nu = 1$ the projection of the higher-dimensional region bounded by $\Delta\chi^2 = 1$ is the formal standard error on that parameter.

There are two comments to be made. If we take the correlation matrix, as published for instance by the OPAL Collaboration (see the fourth paper in [7]), we find sizeable effects due to the off-diagonal elements, which reflect themselves in the regions of joint probability. To give an example let us consider $\nu = 2$, the two parameters of interest being $\Gamma_{_Z}$ and R_l. In this case we are interested in the region which contains the 68% of the probability distribution for $\Gamma_{_Z}$ and R_l jointly, which, as well known, is given by the $\Delta\chi^2 = 2.30$ ellipse. Therefore the range of variation for $\Gamma_{_Z}$ can be considerably larger than what we get from the formal standard error, as shown for instance in Fig. 1.

The second comment is that, in general, the minimum region is not sufficiently quadratic and therefore the covariance matrix is inadequate for describing the errors. What one should do is to construct explicitly the $\Delta\chi^2 = const$ curves (surfaces) using the full calculation and derive the parameter values that correspond to $\chi^2 = \chi^2_{min} + \Delta\chi^2$. In this way we usually end up with asymmetric errors which, for instance, are customarily reported by the LEP Collaborations for m_t, α_s and sometimes for m_H.

To illustrate further this point we have considered a 3 parameter fit (M_Z, m_t and α_s) to the lineshape and forward-backward asymmetry data for ALEPH (see the first paper in [7]). In Fig. 2 we report in the plane $m_t - \alpha_s$ the exact contours of constant $\Delta\chi^2$ with the corresponding ellipses derived under the assumption of a parabolic χ^2. The differences are, in some cases, not negligible. For the sake of completeness, Fig. 3 shows the exact contours at $\Delta\chi^2 = 1$ and 2.30 (a and b respectively) in the plane $m_t - \alpha_s$ for ALEPH (dotted line), DELPHI (dash-dotted line) L3 (solid line) and OPAL (dashed line) [7]. M_Z is fixed at the best value as quoted by each experiment and $m_H = 300$ GeV.

To give a further example let us consider a set of electroweak parameters as reported by the experimental Collaborations [8]:

$$
\begin{aligned}
\Gamma_Z &= 2.488 \pm 0.007\,\mathrm{GeV}, \quad \sigma^0 = 41.44 \pm 0.17\,\mathrm{nb}, \\
R_L &= 20.83 \pm 0.06, \quad A^0_{FB}(l) = 0.0164 \pm 0.0021, \\
M_W &= 79.91 \pm 0.39\,\mathrm{GeV}, \quad M_W/M_Z = 0.8807 \pm 0.0031, \\
\Gamma_b/\Gamma_h &= 0.220 \pm 0.003, \quad A^0_{FB}(b) = 0.098 \pm 0.009, \\
A^\tau_{pol} &= 0.142 \pm 0.017.
\end{aligned}
$$

We would like to make some prediction for the Higgs boson mass. The idea is to consider the $\Delta\chi^2 = const$ curves in the $m_t - m_H$ plane for fixed α_s and $M_Z = 91.187$ GeV. These curves are reported in Fig. 4, where we can compare $\Delta\chi^2 = 1$ with $\Delta\chi^2 = 2.30$ which give informations on the 68% of probability for m_t, m_H separately or jointly. From these curves we learn, first of all, that the standard model is compatible with the data at the 68% of probability for a considerably high Higgs boson mass, and that to conclude that relatively low Higgs boson masses are favored on the basis of the $\Delta\chi^2 = 1$ curve is misleading, since in this way the correlation with m_t is lost. Moreover we learn that a prediction for m_H is heavily correlated with the value of α_s and it is enough to consider $\alpha_s \approx 0.125$ for being well above that value of m_H where perturbation theory breaks down in the scalar Higgs sector. To use the projection of the $\Delta\chi^2 = 1$ curve for giving a qualitative estimate is more than legitimate, however the potential for new discoveries should not relay on it. The same analysis has also been performed experiment by experiment and using the cross section results [7]. M_Z is fixed at the best value as quoted by each experiment. The corresponding curves are shown in Figs. 5-8 and the situation changes, as far as the details are concerned, from experiment to experiment but the overall conclusion remains unaltered. There is a strong correlation with the value of α_s as it can be seen from our figures where $\alpha_s = 0.118$ and 0.138 are reported. These values for α_s are consistent with the central value according to [8] and assuming a total uncertainty of 0.010. For some experiment, notably DELPHI and L3, and low values of α_s the $\Delta\chi^2 = 1$ curve gives a relatively low upper bound on $m_{_H}$. However this is not true anymore if we move to 2.30 and in any case such low values of α_s are in disagreement with what the same experiments report. On the basis of the present analysis one can conclude that a heavy Higgs boson, as predicted in [9], is not excluded by LEP data.

To add evidence to our argument we remember that the current convention for a fit to m_t etc... is to quote the central values plus the asymmetric errors plus the variation in the central values due to a change in the Higgs boson mass between 60 GeV and 1 TeV. A part from the arbitrariness of this upper value we feel that, if needed, the Higgs boson mass should enter in the fit at the same level of the other unknown parameters of the standard model and therefore one should consider a four parameter fit to the data. Performing this exercise on ALEPH and OPAL data we found respectively $m_H \leq 1.1$ TeV and $m_H \leq 3.3$ TeV, while for the other experiments we did not attain convergence of the fitting procedure. Whereas our analysis on ALEPH data supports the convention above cited, the one on OPAL data does not.

The Z^0 parameters are not, strictly speaking, *observables* and the $\gamma - Z^0$ interference term is normally taken from the standard model. It is a well known fact that by varying m_t and $m_{_H}$ in the interference term one can reach a 1 MeV variation in the fitted value of $M_{_Z}$. On this basis we find equally plausible to adopt an alternative strategy, namely to analyze directly cross section and asymmetry data as described in [10]. To do this the code TOPAZ0 [11] has been used, in which the formulation [10] is implemented together with the most recent improvements in the field of radiative corrections [12].

We would like to start from the assumption of the validity of the standard model, after which the fundamental parameters are $M_{_Z}, m_t, m_{_H}$ and α_s. These parameters can be analyzed by fitting directly the cross sections, their range of variability can be computed and the corresponding values for the secondary parameters ($\Gamma_{_Z}, R_l$ etc...) can be obtained within the minimal standard model. Moreover a fit to a set of measured quantities, the *observables*, has the virtue that the uncertainties on these can be adequately treated as Gaussian. To summarize we feel on a more secure ground when fitting Lagrangian variables from the observables with respect to fitting Lagrangian parameters by using Standard Model formulas once the Z^0 parameters are extracted in a model independent analysis.

In the fitting procedure there are several important ingredients. First of all we include Bhabha scattering without the so called t-channel subtraction, at least for those experiments that publish the full data $\sigma^{s+t}(e)$. The proper treatment of the systematic errors is included in the error matrix approach. Indeed the relevance of systematic errors can be analyzed [6] by taking into account the off-diagonal elements of the covariance matrix. In brief we assume that the data published by the LEP Collaborations for the cross sections have an uncorrelated error, which is a quadratic sum, statistical $\oplus$ point-to-point systematic error. Moreover we assume a common normalization factor with variance around 1 given by the relative normalization error. Whenever different data sets within the same experiment are combined, e.g. the 1990 and the 1991 data, then the 1991 systematic errors (smaller than those of 1990) are treated as common uncertainties between the two sets of data. Our fits to the lineshape and forward-backward data include the 1990 $-$ 1991 results for ALEPH (89 dof) and for L3 (105 dof). For OPAL (56 dof) we have not included the 1990 measurements because of an intrinsic difficulty in reconstructing from the published data the inflating factor for the 1990 systematic errors. For DELPHI we have not included the 1990 results because to this date we don't feel confident in combining the data prior to the publication of their analysis.

Due to the behavior of the radiative corrections the fit is usually a 3 parameter fit (M_Z, m_t and α_s) with a fixed m_H of 300 GeV. Asymmetric errors on the parameters are obtained with standard and conventional techniques [13, 14]. For instance conditional errors can be obtained by keeping some of the parameters at their estimated values. For this suppose that the minimum value of the non-parabolic χ^2 has been successfully located somewhere. The error $\sigma(x_i)$ in the parameter value $\overline{x}_i$ corresponds to an increase in χ^2 by one unit from the minimum where only the parameter x_i is varied and all the others are kept fixed at their values at the minimum. Therefore the conditional errors are obtained with an intersection method, that however does not correspond to a search of the projection of the higher-dimensional (3-dimensional in our case) region on the axes (this we call the projection or tangential method). Certainly one may prefer to consider the errors for each parameter considered separately. For a non-parabolic χ^2 one may also deduce confidence intervals by looking at the increase in χ^2 by one unit above the minimum where $\chi^2(x_i)$ is computed by minimizing with respect to the rest of the parameters for each fixed value of x_i. Confidence intervals are assigned according to this χ^2_{min} curve and we have shown several examples of this technique in our previous papers [10]. Moreover the same results of the projection method are obtained whenever the χ^2_{min} curve has only one minimum[1].

Finally, as a by-product of the minimization procedure, we obtain an estimate of the variances of the regression coefficients. From that and from the

[1]We are grateful to A. Ballestrero for discussions on this point

knowledge of the χ^2_{min} we can study the various ellipses in the $m_t - \alpha_s$ plane, to be compared with the exact, non-parabolic, $\Delta\chi^2 = const$ curves. This is shown, for illustration, in Fig. 9 where we report the $\Delta\chi^2 = 2.30$ curves for the ALEPH results. In particular we can compare the limits on the two variables as obtained from the intersection with the straight lines passing through the $\Delta\chi^2 = 0$ point and as obtained by projecting the curve on the $m_t - \alpha_s$ axes.

There is an additional comment to be made at this point. Each theoretical cross section has a numerical error which, although very small, will in the end contribute to the total χ^2. This is totally irrelevant as far as the fully extrapolated s-channel quantities are concerned but including in the fit the leptons in a realistic setup implies that for each experiment the total χ^2, of the order of 100 for combined $1990 - 1991$ data, will be determined with an absolute error of about 0.05. We have verified that a very pessimistic estimate of a 0.1 absolute error on $\Delta\chi^2$ will translate into an uncertainty of about 5 GeV for m_t. We can make a more precise statement about the influence of the theoretical error on the determination of the unknown parameters of the standard model. For definiteness we have considered our predictions for L3 (electron channel) where the typical situation is as given in Tab. 1. The largest relative error for the lineshape into electrons is $\approx 0.2\%$ (away from the Z^0 peak) while the largest absolute error on the forward-backward asymmetry into electrons is ≈ 0.0025 (again away from the Z^0 peak), but usually and expecially around the resonance they are considerably smaller. The estimates of the error, as returned by the integration routines, can be considered as a sort of systematics and we have made an attempt in order to understand its possible influence. In the minimization procedure we have modified the residuals in the electron channel and the total χ^2 as follows

$$\chi^2 = \Delta^t C^{-1}\Delta + \left(\frac{\lambda - 1}{\Delta\lambda}\right)^2 + \left(\frac{\rho}{\Delta\rho}\right)^2, \tag{2}$$

where

$$\Delta_i(\text{electrons}) = \frac{\lambda\sigma^{th} - \sigma^{exp}}{\Delta\sigma^{exp}} \tag{3}$$

or

$$\Delta_i(\text{electrons}) = \frac{A^{th}_{FB} + \rho - A^{exp}_{FB}}{\Delta A^{exp}_{FB}}, \tag{4}$$

where we assumed conservatively $\Delta\lambda = \Delta\rho = 2 \times 10^{-3}$. The result of the fit and its modification with respect to a situation where the theoretical errors are not included is

		no theor. err.	$\to$	+theor. err.
χ^2/dof	$=$	0.949	$\to$	0.949,
M_Z	$=$	$91.193\,\text{GeV}$	$\to$	$91.193\,\text{GeV}$,
m_t	$=$	$81.4\,\text{GeV}$	$\to$	$80.7\,\text{GeV}$,
α_s	$=$	0.1431	$\to$	0.1431,
λ	$=$	1	$\to$	0.99972,
ρ	$=$	0	$\to$	0.59986×10^{-3}.

Therefore we may conclude that the situation is well under control even in the presence of a theoretical error due to the numerical integration over a realistic phase space.

In the end we are interested in presenting results for the secondary parameters and in identifying the correct definition for their errors. The covariance matrix is, in our opinion, totally inadequate for this purpose and we tend to attribute a scarce relevance to the variance

$$var\,(\Gamma_Z) = \sum_{i,j=1}^{3} \frac{\partial \Gamma_Z}{\partial x_i}\, C_{ij}\, \frac{\partial \Gamma_Z}{\partial x_j}, \tag{5}$$

where $x_1 = M_Z, x_2 = m_t$ and $x_3 = \alpha_s$, C_{ij} being the estimate of the covariance of $\overline{x}_i$ and $\overline{x}_j$

$$C_{ij} = covar\,(\overline{x}_i, \overline{x}_j)\,, \qquad C_{ii} = var\,(\overline{x}_i)\,. \tag{6}$$

Instead of considering the variance of the secondary parameters we prefer to adopt the following procedure. As a result of a 3 parameter fit we consider the region R_p corresponding to $p\%$ of probability distribution for the three parameters. For instance if $p = 0.68$ then we are discussing the region R_p bounded by $\Delta\chi^2 = 3.53$. Next we define the range of variation of the secondary parameter f as

$$f_{min} = f\,(\overline{x}_1, \overline{x}_2, \overline{x}_3)\,, \tag{7}$$

$$f_+ = \max_{x \in R_p} f, \qquad f_- = \min_{x \in R_p} f, \tag{8}$$

and therefore

$$f = f_{min}\,{}^{+\delta f^+}_{-\delta f^-}, \qquad \delta f^{\pm} = \mid f_{\pm} - f_{min} \mid\,. \tag{9}$$

The resulting *errors* are rather conservative estimates of the allowed variations since they turn out to be larger, in general, than the corresponding variances. Whenever the function f coincides with one of its arguments (primary parameters) we choose R_p as bounded by the $\Delta\chi^2 = 1$ curve in such a way that $f_\pm$

gives the projection of the pseudo-ellipsoid into the x_i axis. It is worth stressing that for secondary parameters $f_+ - f_-$ shows the expected range of variation of the parameter itself without reflecting the standard meaning of single parameter confidence level region. We have illustrated the general idea by taking the ALEPH data sample and a fixed value for M_Z, i.e $M_Z = 91.187\,\mathrm{GeV}$. In this case $p = 0.68$ is given by the region in the $m_t - \alpha_s$ plane bounded by the $\Delta\chi^2 = 2.30$ curve. This curve is reported in Fig. 10 together with the curves corresponding to the maxima and minima of Γ_Z, Γ_h and R_l. In a field-theoretical language we attribute a primary significance to the unknown Lagrangian parameters of the standard model and derive a region which reflects some content of probability that the standard model is consistent when the parameters are inside that region. Secondary parameters are those parameters which do not appear directly in the Lagrangian of the standard model and moreover are not obtainable as a consequence of a direct experimental observation but rather follow from a fit to some set of data. For these parameters we prefer to give the minimum and maximum values over the relevant region of the primary parameters. In any case the non-linearity of the model is such that the exact distribution of the various quantities is not known and the associated probabilities as deduced from a comparison with the chi-square distribution will only be approximately correct.

The outlined strategy has been followed in obtaining the results shown in Tab. 2-5. The best values for a 3-parameter fit (M_Z, m_t and α_s) to line-shape and asymmetry data are quoted (Tab. 2-5a) together with the conventional error estimate as given by the covariance matrix (cov.), the intersection method (int.) or the projection method (proj.). The data sample used are the most recently published by the four LEP experiments [7]. In general and within the errors we find a reasonable agreement between m_t as obtained from our global fit to the data and m_t as reported by the LEP Collaborations as a consequence of a model independent fit. Perhaps the largest difference in the central value is for L3 where we find $m_t = 72^{+59}_{\dots}$ GeV from a fit to the fully extrapolated observables or $m_t = 83^{+54}_{\dots}\,\mathrm{GeV}$ from a fit to the realistic observables vs a reported value of $132^{+44}_{-63}\,\mathrm{GeV}$. A closer study of the fit could be done by looking at the residuals, which directly measure the deviations between the experimental observables and the fitted values. The examination of the fit should be done in terms of the stretch functions or *pulls*. Given a $\chi^2 = \Delta^t C^{-1}\Delta$ with $\Delta_i = x_i - \eta_i, (i = 1, \dots, n)$ then the i-th *pull* can be expressed as

$$z_i = \frac{x_i - \eta_i}{\sqrt{\sigma^2(x_i) - \sigma^2(\eta_i)}}. \tag{10}$$

The *pull* z_i is anticipated to have a distribution which is fairly close to $N(0,1)$.

If, in a fit, we find that one of the z_i deviates very much from the others then the corresponding experimental point should be abandoned if it looks suspicious. For our case the lineshape into μ's has a *pull* of 3.9 at $\sqrt{s} = 91.222\,\mathrm{GeV}$ and of 2.8 at $\sqrt{s} = 93.228$. If the muons are excluded from the fit we obtain instead $m_t = 145^{+41}_{-56}\,\mathrm{GeV}$. In Tab. 2-5b the corresponding best values for the Z^0 parameters Γ_h, Γ_Z, R_l and σ_h^0 are reported together with the error estimate as given by the covariance matrix and by the projection method. As can be seen, different error estimates lead to almost equivalent results in the case of Lagrangian parameters (Tab. 2-5a) and the difference between intersection and projection errors (both relative to $\Delta\chi^2 = 1$) is, as usually, linked to the correlation among the parameters themselves. Perhaps one of the most unsatisfactory features of the standard model, as we know it now, is the presence of a relatively high number of unknowns and therefore a specified probability content for the joint confidence region in all these parameters requires a relatively large value of the constant a in $\Delta\chi^2 = a$. Confidence intervals for a subset of the unknown parameters are a quick and conventional way of conveying information, however a test for the presence of new physics should take into account the hypersurface circumscribing the joint confidence region and $a = 1$ gives a quickly decreasing probability content.

In the case of the Z^0 parameters the *errors* given by the projection method (i.e. their expected range of variation) are obtained by varying the Lagrangian parameters within the region bounded by $\Delta\chi^2 = 3.53$ (a much larger range of variation, corresponding to the 68% of joint probability for M_Z, m_t and α_s) and are almost a factor of 2 larger than the corresponding (symmetric, conventional) errors as obtained by means of the covariance matrix.

Tab. 6 a and b collect the weighted average over the four experiments for Lagrangian and Z^0 parameters respectively. The weighted average has been obtained by following the iterative procedure described in [15]. While in each entry the errors are more or less halved, the same relation between the various kinds of error estimates is maintained.

We are indebted with the DELPHI collaboration for having provided the '91 results on cross sections and asymmetries. We are also grateful to G. Zumerle for his continuous support, to M. Consoli for several stimulating discussions and to G. Pollarolo for his helpful assistance in numerical and graphical problems. INFN, Sezioni di Pavia e Torino, are acknowledged for having provided computer resources.

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Figure and Table Captions

Fig. 1 - The curves $\Delta\chi^2 = 1$ (solid line) and $\Delta\chi^2 = 2.30$ (dashed line) in the $(\Delta R_l, \Delta\Gamma_Z)$ plane. The region which contains the 68% of the probability distribution for R_l and Γ_Z jointly is bounded by the $\Delta\chi^2 = 2.30$ ellipse.

Fig. 2 - The exact contours of constant $\Delta\chi^2$ for $\Delta\chi^2 = 1$ (a) and 2.30 (b) in the $m_t - \alpha_s$ plane, to be compared with the corresponding ellipses as derived under the assumption of parabolic χ^2. ALEPH data sample.

Fig. 3 - The exact contours of constant $\Delta\chi^2$ for $\Delta\chi^2 = 1$ (a) and 2.30 (b) in the $m_t - \alpha_s$ plane, for ALEPH (dotted line), DELPHI (dash-dotted line), L3 (solid line) and OPAL (dashed line).

Fig. 4 - The contours at $\Delta\chi^2 = 2.30$ and 1 (a and b respectively) in the $m_t - \ln(m_H/M_Z)$ plane for $M_Z = 91.187$ GeV and $\alpha_s = 0.118$ (solid line), 0.128 (dashed line) and 0.138 (dotted line).

Fig. 5 - The contours at $\Delta\chi^2 = 1$ (dash-dotted line) and 2.30 (solid line) in the plane $m_t - \ln(m_H/M_Z)$ for $\alpha_s = 0.138$ (a) and 0.118 (b). ALEPH data sample.

Fig. 6 - The same as Fig. 5 for DELPHI data sample.

Fig. 7 - The same as Fig. 5 for L3 data sample.

Fig. 8 - The same as Fig. 5 for OPAL data sample.

Fig. 9 - $\Delta\chi^2 = 2.30$ contours in the plane $m_t - \alpha_s$ from covariance matrix (dash dotted line) and from full calculation (solid line). ALEPH data sample.

Fig. 10 - The $\Delta\chi^2 = 2.30$ contour in the plane $m_t - \alpha_s$ from full calculation (solid line) and the curves corresponding to the maxima and minima of Γ_Z, Γ_h and R_l. ALEPH data sample.

Tab. 1 - Our prediction for cross sections and forward-backward asymmetries in the electron channel for the L3 experimental setup together with the numerical errors coming from the integration over the realistic phase space.

Tab. 2a - The best values of a 3-parameter fit (M_Z, m_t and α_s) to line-shape and asymmetry data together with the errors estimate as given by the covariance matrix (cov.), the intersection (int.) and the projection (proj.) method. The dots indicate a lower value below open top production. ALEPH data sample.

Tab. 2b - The best values of the Z^0 parameters corresponding to the Lagrangian parameters of Tab. 2a together with the errors estimate as given by the covariance matrix and the projection method. ALEPH data sample.

Tab. 3a - The same as Tab. 2a for DELPHI data sample.

Tab. 3b - The same as Tab. 2b for DELPHI data sample.

Tab. 4a - The same as Tab. 2a for L3 data sample.

Tab. 4b - The same as Tab. 2b for L3 data sample.

Tab. 5a - The same as Tab. 2a for OPAL data sample.

Tab. 5b - The same as Tab. 2b for OPAL data sample.

Tab. 6a - Weighted average of the Lagrangian parameters of Tab. 2-5a.

Tab. 6b - Weighted average of the Z^0 parameters of Tab. 2-5b.