June 9, 1999

Dear Jorgen, Reisaburo, Hywel, Martin, Marco and Giampiero:

In your "Proposal for the LEP2 MC workshop, 4F subgroup", on page 5, in the first paragraph, you have written the following:

" As was correctly pointed out during the first meeting, taking any radiative on-shell calculation and moving it to off shell final state particles is a dangerous thing; cancellation of divergences heavily depends on the on-shellness of particles. So, there is no CC03 $\mathcal{O}(\alpha)$ full calculation, there is an $\mathcal{O}(\alpha)$ $e^+e^- \to$ (on-shell)W^+W^- followed by a —???— transition to off-shell $W's$ towards CC03. This we need to discuss, deeply. Otherwise, if nothing new can be calculated in time for the workshop, it is still needed to estimate (guess?) say how accurate are things now - although tough without the explicit calculation."

We do not agree with the above statement, as we explain below in a more detail. The basic reason for disagreement is that in our opinion you have got the whole relation between "$\mathcal{O}(\alpha)$ full calculation" and "on-shell CC03" up-side-down. As a consequence some trivial things you regard as a serious problem and vice versa.

"As was correctly pointed out during the first meeting, taking any radiative on-shell calculation and moving it to off shell final state particles is a dangerous thing; cancellation of divergences heavily depends on the on-shellness of particles."

COMMENT: As we see, you have clearly in mind some kind of the "mapping" from (B) "$\mathcal{O}(\alpha)$on-shell CC03" towards (A) "$\mathcal{O}(\alpha)$ full calculation". This is the whole problem, one should not take the relation between the two as (i) "dressing up" or "upgrading" (B) to the level of (A) but rather as (ii) "projection" of the (A) into (B) within the double-pole approximation (DPA). All these words in double quotes are not defined very precisely, so let us explain them with examples. The bottom to top approach (i) can be realized in MC as follows: take MC generation *strictly* on-shell: $e^-e^+ \to W^+W^-(,\gamma)$ with subsequent decay $W^+ \to f_1, f_2(,\gamma)$ and $W^- \to f_3, f_4(,\gamma)$ with $s_1 = (p_1+p_2)^2 = M_W^2$ and $s_2 = (p_3+p_4)^2 = M_W^2$. Then in the next step we smear/generate s_1, s_2 according to Breit-Wigner and adjust the whole kinematics with some ad hoc prescription borrowed from PYTHIA. This scenario suffers from several awkward problems: first of all, although the whole phase space of the target process (A) $e^-e^+ \to f_1, f_2 f_3, f_4(,\gamma)$ is covered (even this will often not be true!) we introduced uncontrollable perturbation to the four(five)-body Lorentz invariant phase space. Take photon distributions: IR cancellations are satified by construction, but the photon spectra will get distorted and upgrading them BACK to the correct shape of the "$\mathcal{O}(\alpha)$ full calculation" looks rather hopeless. If we understand correctly, this is the nightmare which you have in mind and we agree fully that it is a real nightmare. But why to go into this kind of problems, if it is not really necessary! In our work, see Phys. Rev.D54 (1996) 5434, Phys. Lett. B417 (1998)

326, we have always seen the whole process of constructing practical calculation as top to bottom, (A) to (B), approach. One starts with the fermion-loop scheme off-shell and in the $\mathcal{O}(\alpha)$ part of the matrix element for CC03, we apply the double pole part of the respective $\mathcal{O}(\alpha)$ amplitude (this is known to be gauge invariant— see Beenakker et al.,preprints hep-ph/9811481, hep-ph/9902333, and references therein) and is accurate up to but not including $\mathcal{O}(\frac{\alpha}{\pi}\frac{\Gamma_W}{M_W})$. We approximate the residues of the double poles by their on-shell values in $\mathcal{O}(\alpha)$. Generally it involves a loss of accuracy at the level of $\mathcal{O}(\frac{\alpha}{\pi}\frac{\Gamma_W}{M_W})$. Notice that taking the residues of the poles does not necessarily mean replacing the two Breit-Wigners with the delta functions! This ultimate step can be done (getting back to (B)) but why shoot your own leg? Equally important: taking the (on-shell) residue means replacing $s_i \to M_W^2$ and $\Gamma_W \to 0$. But really nobody forces us to do it everywhere! We can do it in the places where we have to, we leave it unchanged wherever it can be done consistently, without violating gauge invariance and IR cancellations. For example in YFSWW calculation we do not set $s_i \to M_W^2$ in the YFS/Sudakov logarithm (of virtual corrections) and in hard photon eikonal S-factor. The emission of photons from the final W's is really implemented for W's with the different (variable) masses. (We use on-shell QED matrix element for FSR photon emission modified to accommodate for final particles of different masses, an easy modification). This works perfectly well, IR cancellations are perfectly OK. But let us stress as strongly as we can: *the above modification is not the approximation, quite on the contrary we remove one of many approximations on the way from (A) to (B), and as a result we get closer to (A).* Any further improvement in YFSWW will be also interpreted in the same way. Everybody is invited to follow our way!

Finally, let us add some additional remark on our past works and related papers: Applying the YFS exponentiation to this amplitude to arrive at the YFS exponentiated exact $\mathcal{O}(\alpha)$ correction to the DPA, with on-shell residues, for the production part of the CC03 process we claim is done in the YFSWW in the same spirit. If the final state BR's are taken from their best known values in a standard way, the only $\mathcal{O}(\alpha)$ effects not included in the respective total cross section are the non-factorizable effects, which are known to be at most of the order $\mathcal{O}(\frac{\alpha}{\pi}\frac{\Gamma_W}{M_W})$ — see again the preprints we just referenced by Beenakker et al. It follows that, as we have already emphasised in the paper Phys. Lett.B417(1998)326, the physical precision of the combined prediction from $\mathrm{YFSWW3}-1.11 \oplus \mathrm{KORALW1.42} \equiv \sigma(\mathrm{YFSWW3}-1.11)^{\mathcal{O}(\alpha)}_{exp} - \sigma_{Born}(CC03) + \sigma(\mathrm{KORALW1.42}) - \sigma_{Born}(CC03)|_{DPA}\delta_{ISR-LL}$ has a limiting physical precision at the level of $\mathcal{O}(\frac{\alpha}{\pi}\frac{\Gamma_W}{M_W})$. Beenakker et al. have arrived at a similar conclusion– see the aforementioned preprints of these authors also for an explicit illustration of the ambiguity involved in going from the off-shell point to an appropriate on-shell point (top to bottom). Here, the correction $\sigma_{Born}(CC03)|_{DPA}\delta_{ISR-LL}$ needs to be removed from KORALW1.42 because it is already included in the more complete $\mathcal{O}(\alpha)$ correction in YFSWW3-1.11 as well.

"So, there is no CC03 $\mathcal{O}(\alpha)$ full calculation,..."
COMMENT: This is true, but as we have just explained and as you can also see

in the papers by Beenakker et al., for the total cross section, we do not need the so-called full calculation to reach .5% precision.

"there is an $\mathcal{O}(\alpha)$ $e^+e^- \to (on-shell)W^+W^-$ followed by a —???— transition to off-shell $W's$ towards CC03. This we need to discuss, deeply."
<u>**COMMENT:**</u> You really have this backwards, as we have explained. Your question marks and further comments are simply misguided, in view of our explanation above.

"Otherwise, if nothing new can be calculated in time for the workshop, it is still needed to estimate(guess?) say how accurate are things now - although tough without the explicit calculation."
<u>**COMMENT:**</u> We have explained what our current physical precision is and that limit, $\mathcal{O}(\frac{\alpha}{\pi}\frac{\Gamma_W}{M_W})$ is well below .5% for the total cross section. Beenakker et al. have reached a similar conclusion by independent methods for their semi-analytical results. We need to verify our technical precision and write documentation – this is in progress. Moreover, we are implementing the exact $\mathcal{O}(\alpha)$ YFS exponentiated FSR in YFSWW3 so that we will have realistic finite p_T, $n(\gamma)$ radiative effects both for ISR and FSR– an initial version of this work, in the MC YFSWW3-1.12, UTHEP-98-0502, contributed to the 1998 Roch. Conf. and to appear, already features the $\mathcal{O}(\alpha^2)$ LL FSR. These corrections are calculated using the final state analogs of the methods used in YFSWW3-1.11. They open the possibility that we can realize a large portion of the various requests you have made in the remainder of the "Proposal..., 4F subgroup". We will return to these requests elsewhere.

Very truly yours,

Bennie F.L. Ward for all authors of KORALW and YFSWW