please add your institute also !!

Notes on two-fermion group activities

Contact persons: *M-N. Minard (A), A. Olchevski[b,e] (D), D. Bourilkov (L), M. Kobel (O), G Taylor ($\nu\bar{\nu}$), Z. Wąs[a,f] (Th),* **and***: A. Arbuzov, D. Bardin[b], P. Christova[b,d], J. Fujimoto, M. Grunewald, T. Ishikawa, M. Jack[c], S. Jadach, L. Kalinovskaya[b], Y. Kurihara, R. Mcpherson, Guido Montagna, Mauro Moretti, T. Munehisa Oreste Nicrosini, Fulvio Piccinini, W. Płaczek, S. Riemann[c], T. Riemann[c], Y. Shimizu, M. Skrzypek[f], S. Spagnolo, I. Tomalin, ...*

[a] CERN, Theory Division, CH-1211 Geneva 23, Switzerland,
[b]LNP, JINR, RU-141980 Dubna, Russia
[c]DESY, D-15738 Zeuthen, Germany
[d]Faculty of Physics, Bishop Preslavsky University, Shoumen, Bulgaria
[e]EP Division, CERN, CH-1211 Geneva 23, Switzerland
[f] Institute of Nuclear Physics, Cracow, Poland

Abstract

This note contains a preliminary discussion of the issues which are to be addressed by the 2-fermion group. New input or criticism would be most welcome. It contains a list of experimental and theoretical observables for difermion events, and provides estimates of their theoretical uncertainties, together with the one requested by experiments. Comparisons are made of predictions from different packages.

Technical Point: This file has the usepackage showkeys activated. For the normal printout form the appropriate line should be commented out. Then all information about the labels will disappear. At present it is printed for the sake of editing comfort.

Mar. 17 2000,

Contents

1. Introduction **6**

2. Observables **7**

2.1 Precision requirements . . . 8
2.2 General comments of observable definition . . . 9
2.3 Notation . . . 9
2.4 questions and answers are temporarily here . . . 10
2.41 ALEPH . . . 10
2.42 DELPHI . . . 11
2.43 *L3* . . . 11
2.44 OPAL . . . 11
2.5 Additional pair treatment . . . 11
2.51 ALEPH . . . 11
2.52 DELPHI . . . 11
2.53 *L3* . . . 12

2.54 OPAL 12
2.6 Realistic $e^+e^- \to q\bar{q}(\gamma)$ Observables 13
2.61 ALEPH 13
2.62 DELPHI 13
2.63 L3 14
2.64 OPAL 14
2.65 Numerical results 14
2.7 Idealized $e^+e^- \to q\bar{q}(\gamma)$ Observables 17
2.71 ALEPH 17
2.72 DELPHI 17
2.73 L3 17
2.74 OPAL 17
2.75 Numerical results 18
2.8 Realistic $e^+e^- \to e^+e^-(\gamma)$ Observables 23
2.81 ALEPH 23
2.82 DELPHI 23
2.83 L3 23
2.84 OPAL 23
2.85 Numerical results 24
2.9 Idealized $e^+e^- \to e^+e^-(\gamma)$ Observables 28
2.91 ALEPH 28
2.92 DELPHI 28
2.93 L3 28
2.94 OPAL 28
2.95 Numerical results 29
2.10 Realistic $e^+e^- \to \mu^+\mu^-(\gamma)$ Observables 34
2.101 ALEPH 34
2.102 DELPHI 34
2.103 L3 34
2.104 OPAL 34
2.105 Numerical results 34
2.11 Idealized $e^+e^- \to \mu^+\mu^-(\gamma)$Observables 37
2.111 ALEPH 37
2.112 DELPHI 37
2.113 L3 37
2.114 OPAL 37
2.115 Numerical results 38
2.12 Realistic $e^+e^- \to \tau^+\tau^-(\gamma)$Observables 47
2.121 ALEPH 47
2.122 DELPHI 47
2.123 L3 47

2.124 OPAL . . . 47
2.125 Numerical results . . . 48
2.13 Idealized $e^+e^- \to \tau^+\tau^-(\gamma)$ Observables . . . 51
2.131 ALEPH . . . 51
2.132 DELPHI . . . 51
2.133 L3 . . . 51
2.134 OPAL . . . 51
2.135 Numerical results . . . 52
2.14 $q\bar{q}\gamma$'s observables . . . 59
2.141 ALEPH . . . 59
2.142 DELPHI . . . 59
2.143 L3 . . . 59
2.144 OPAL . . . 59
2.145 Numerical results . . . 59
2.15 $\ell\bar{\ell}\gamma$'s observables . . . 61
2.151 ALEPH . . . 61
2.152 DELPHI . . . 61
2.153 L3 . . . 62
2.154 OPAL . . . 62
2.155 Numerical results . . . 64
2.16 $\nu\bar{\nu}\gamma$'s observables . . . 67
2.161 ALEPH . . . 67
2.162 DELPHI . . . 67
2.163 L3 . . . 67
2.164 OPAL . . . 68
2.165 Numerical results . . . 68

3. Conventions and translation guidelines **71**

4. Semi-analytical and Monte Carlo tools used **72**
4.1 presentation of the program MCdummy . . . 72
4.2 Presentation of the program BHWIDE . . . 73
4.3 Presentation of the program KORALZ . . . 75
4.4 Presentation of the program KK2f . . . 76
4.5 Presentation of the program LABSMC . . . 77
4.6 Presentation of the program grc$\nu\nu\gamma$ . . . 79
4.7 Presentation of the program NUNUGPV . . . 81
4.8 Presentation of the program ZFITTER with electroweak library DIZET . . . 83

5. Theoretical uncertainties of observables **84**
5.1 MCdummy . . . 84
5.2 BHWIDE . . . 85

5.3 grc$\nu\nu\gamma$ . . . 85
5.4 KORALZ . . . 85
5.5 KK2f . . . 85
5.6 LABSMC . . . 85
5.7 NUNUGPV . . . 85
5.8 ZFITTER . . . 85
5.9 Manual for authors . . . 85

6. Architecture of the calculations **86**
6.1 hadronic low energy vacuum polarisation . . . 87
6.2 QCD corrections for vacuum polarisation at higher energies . . . 88
6.3 electroweak libraries . . . 88
6.4 pairs as of M. Kobel but with some addups by M. Skrzypek . . . 88
6.41 Signal . . . 89
6.42 Background . . . 89
6.5 pairs -subsection to be removed . . . 91
6.6 special problems of hadronic final states . . . 93

7. Tests and comparisons **93**
7.1 hadronic low energy vacuum polarisation . . . 94
7.2 QCD corrections for vacuum polarisation at higher energies . . . 94
7.3 electroweak libraries . . . 94
7.4 pairs . . . 94
7.5 semi-analytical programs and Monte Carlos for idealized observables . . . 94
7.6 Monte Carlos for strong cut-offs . . . 94
7.7 report on final state hadronization . . . 94
7.8 workshop status . . . 94

8. Summary – what will remain to be done by th community later ... **94**

1. Introduction

It is probably fair to assume that the theoretical effort to describe difermion physics at LEP2 is now almost finished. The main goal of the difermion workshop should thus be to review our present knowledge of these theoretical predictions, collecting references from relevant papers and programs. It should estimate theoretical systematic errors for experimentally interesting quantities, etc. However, if important predictions are still missing, the present workshop should bring this to the attention of the theoretical community.

Our group will work closely with the LEP2 $f\bar{f}$ working group (coordinated by Pete Renton), which aims to develop the procedure for combining the results from the four LEP experiments. We should also interact with searches groups, for which Standard Model (SM) difermion processes are a source of background. It is natural that members of these working groups are best placed to formulate a list of practical observables for which theoretical predictions and their systematic errors are needed. They can also highlight cases where the theoretical issues need clarifying and estimate the relative urgency with which the different issues need to be addressed. Of course we should coordinate our efforts with other working groups of the workshop, in particular the 4-fermion as well as $\nu\bar{\nu}\gamma's$ working groups.

Two kinds of observables should be studied.

- Realistic observables which correspond more closely to the event selection cuts actually applied by experiments.
- Idealized theoretical quantities, such as the total cross sections or asymmetry with no cuts or very simple cuts. Such quantities are best suited for theoretically clear extraction of basic quantities such as particle masses or couplings. They also make it easier to average results from different experiments and allow a comparison with SM predictions from semi-analytical programs.

A list of such observables is given in **Section 2.**. The list of realistic observables may be long and semi-repetitive . (e.g., Observables with cuts differing slightly by numerical value of some parameters – angles, energies etc.) However, this is necessary, as the discussion of theoretical systematic errors often depends on particular kinematical cuts applied. It may also contain observables which are known to have good theoretical predictions and reliably estimated theoretical errors. Later, we can decide on how to group and simplify the list according to e.g., convenience for discussion of theoretical errors. However, it is important that from the very beginning the list should be complete and describe exhaustively the event selections of the various experiments. Section 2. will also describe why some observables are better than others from an experimental point of view. It will also identify for which variables it is most urgent to have theoretical input, e.g., because experimental errors are small compared with theoretical ones, or where experimental cuts have to be modified to exclude phase space regions poorly known from theory. The list of idealized variables must use simple cuts, which make sense experimentally and can also be implemented in semi-analytical programs.

One of the most important tools to achieve the aim of this workshop is comparisons between different available theoretical calculations. Before making such comparisons, one need a clear understanding of definitions of quantities and tuning of input parameters. **Section 3.** will therefore address the question of conventions and definitions in order to avoid such inconsistencies when using different programs. This chapter on definitions should also address the question of compatibility of LEP1 and LEP2 conventions and motivation of why they are chosen like that (gauge invariance, conventions of LEP1 published results, etc.). Eventually such section(s) could be part of general section of the whole report and could be taken over by the main editor.

In **Section 4.**, we plan to describe the various theoretical programs and give the basic ideas behind them, together with the list of main publications and indication of where the present versions of these programs can be taken from.

Section 5. will contain numerical results and present theoretical studies of systematic errors for the individual (or for groups of) observables presented in Section 2.. It should also assess theoretical

errors on the difference between idealized and experimental observables, to determine how reliably experiments can correct their results to the idealized case. The aim will be, of course, not to list all the predictions for each case but to provide numerical benchmark and concentrate on the discussion of possible systematic errors. In this discussion the breakdown of errors and understanding of applicability of different components is important.

The contributions to this section will be the responsibility of the individual authors of the various packages used in making predictions of difermion production. We should ensure that all such packages are considered, not only those already used by experimentalists, but also all other which are officially released.

The ideal item in this section would be just extended citation to published papers (preprints or completed entries in this book) of the form which may look like the following one: "Observables (*list of labels from Section 2.*) have theoretical systematic uncertainties of xx% as shown in [1]. These uncertainties are estimated with the help of the program [2], by varying the following assumptions/parameters .... Simmilarily on the basis of that calculations we expect that in the following cases: (*another list of labels from Section 2.*) systematic errors for predictions should not exceed the tag of xx %. Finally in the following cases: (*again another list of labels from Section 2.*) the predictions and their systematic errors can be calculated accordingly to scenario explained above, but we can not guess the numerical size of the outcome with enough certaintity.

We can not expect the ideal situation in case of all observables. We will need to adopt our approach, stimulate effort etc., accordingly to experimental importance of the particular observable, available theoretical results and manpower. In the cases where the theory error is unknown or possibly big, we will try to find other alternative observables with reliable/small errors.

Section 6. will provide numerical comparisons of predictions from different theory packages, comparisons for the observables listed in Section 2. .These comparisons will check the robustness of the error estimates provided in Section 5.. Eventual traps or potential misunderstandings due to different conventions will be presented here.

Finally, **Section 8.** will summarize the effort. In some cases we may find predictions to be insufficiently precise. In such cases stimulation for the future efforts can be generated.

2. Observables

From the preliminary discussion we find that at least the two classes of observables are to be discussed: those corresponding to realistic experimental cuts and those of an idealized nature. Furthermore, each of these may apply to either

1. 2 fermion final states
2. 2 fermion + visible photons (esp radiative return).

The perpendicular classification was of course on the basis of the level of the idealization of observable. It is thus rather reasonable to start with the creation first of individual entries in the list, and only later continue with ordering it.

==========

As it was pointed out, the first item should be discussed in close contact with the LEP2 ffbar subgroup of the LEP electroweak working group. It should help to facilitate the averaging procedure of the results of the four LEP experiments, and assess the theoretical and MC modelling errors for each experiment as well as their correlation in the average. It might even help to clearly define a benchmark 2-fermion "signal" for the averaging procedure in terms of s' definition, $\cos\theta$ definition, QED I/F interference, and pair radiation correction.

The 2f+photon final state (i.e. high pt photons) , mainly consisting of radiative return events, is essential to be understood for searches for excited leptons , decaying to gamma-ℓ and anomalous ZZ-γ

and γZ-γ couplings. Here the theory predictions , semi-analytical programs and MC generators agree worst with the data, especially for electrons. An improvement of these predictions and an assessment of remaining errors is crucial for obtaining valid limits (or discoveries) of new physics.

Precise realistic observables together with the required precision for each of the above categories are currently set up by the experimentalists.

In the definition of cuts we need to address the question of possible contamination of one process by another one. Often experimental cuts are specifically designed to eliminate such contaminations, but in practice some remnant contaminations may still stay. As simple example we can imagine the τ-predictions (case of the $\tau \to \mu$ decay) which can provide non-negligible contribution for the observable designed eg. for the measurement of μ-pair with ISR photons. As prediction for such τ-channel can be easily calculated we may expect the question on its size and related uncertainty.

More complicated is the case of 2-f versus 4-f definition, i.e. how to set the limit defining which pair radiation is a radiative correction to the 2f signal, and should be included in the measurement and the efficiency determination, and which is background, to be subtracted. To understand this interplay between 2-f and 4-f processes three groups have to work closely together: Our group, the LEP2 ffbar subgroup of the LEP electroweak working group and the 4-fermion subgroup of this MC workshop. Theoretically little effort has yet gone in this problem at LEP2, and there is urgent need for theorists to talk to experimentalists and vice versa, in order to define the work that has to be done (e.g. common exponentiation of pairs and photons, options to apply a cut on the pair mass) The outcome might even have some impact on LEP1 results, although there the effect is very much suppressed.

We may expect other similar situations, due e.g. of particle misidentiffications which we may need to include in our scheme of predictions and their systematic errors. There is no doubt that the higher precision required the more cut off details need to be included into observable definition. Also discussion of theoretical uncertainties will depend rather strongly on required precision.

2.1 Precision requirements

As we can see the observables can be divided into different classes of abstraction depending where they are placed on the spectrum from full experimental condition toward final idealized observables with QED corrections subtracted, 4π-detector etc. One of the most important ingredient of the observable definition is its precision tag. Obviously the higher precision the more complicated study of its theoretical uncertainties will be. Also more of the details of experimental cuts will be needed to get the answer on theoretical systematic errors. In cases when description will become extremely long a reference to experimental paper can be the only solution. For the convenience of editing we suggest that the references are directly taken from SLAC database including the bib-tex-reference-name!. This should be helpful in matching all into one long (or more of shorter) list(s). Every observable will enter as individual item in the tex-enumerate list.

The following rule of thumb with respect to the errors obtained for the data taken at $\sqrt{s} = 189$ GeV was suggested for estimating required precision in cases when there is no better scenario.

1. Experimental statistical error is decreased by factor of $\sqrt{12} \simeq 3.5$ with respect to present (summer 1999) one for the single collaboration. We still expect statistics to grow by factor of 3 and combination of all 4 experiments brings total statistics factor of 12 bigger.
2. Experimental systematic error can be expected to go down by a factor of 2 (may be 3) due to improvements and partial non-correlations among experiments.
3. The above estimates of experimental statistical and systematic error should be added in quadrature.
4. The required precision should be $1/3$ of that to assure that theoretical effects will not deteriorate experimental results (will not increase an error) by more than 10% .

The demand of maximum 10 % increase of final experimental overall error due to theoretical uncertainties is not an over-demand. It is equivalent to decrease in running time of experiments by 20 %

in cases when statistical error dominates. Similar 10% deterioration of an overall detection performance would occur only after 30 % decrease in data taking for measurements where statistical and systematic errors are similar in size. Having this in mind and also an enormous effort of so many peoples over so many years, not mentioning the costs may even lead to conclusion that our rule for precision requirements from theory is not enough strict.

In general as inclusive we understand cross section for $\sqrt{s'/s} > 0.1$ and exclusive for $\sqrt{s'/s} > 0.85$. For Bhabha Excl_1 denotes $|\cos\theta| < 0.9$ and Excl_2 denotes $|\cos\theta| < 0.7$ For asymmetries the (absolute) error tag can be obtained from the one of cross section multiplying it by $0.01\cdot\sqrt{2}$ and dropping the % symbol.

The following precision tags can be assumed if explicit numbers do not overrule them in the text:

1. cross sections for $q\bar{q}$ final states: incl. 0.11 %, excl. 0.23 %
2. cross sections for e^-e^+ final states: excl_1 0.13 %, excl_2 0.21 %
3. cross sections and asymmetries for $\mu^-\mu^+$ final states: incl. 0.41 %, excl. 0.53 %
4. cross sections and asymmetries for $\tau^-\tau^+$ final states: incl. 0.44 %, excl. 0.61 %
5. searches background cross sections from quarks and tagged hard photons 0.3 %
6. searches background cross sections from leptons and tagged hard photons:
   single photons 1.5 %, multiple photons 5%
7. searches background cross sections from neutrinos and tagged hard photons
   single photons 0.2 %, multiple photons 1%
   These numbers can serve as a starting point for definition of precision tags required from complete theoretical calculations. For separate ingredients such as interferences pair corrections etc, physical precision tag must be even more strict, to assure that their combination will not overcome the required tag. Finally the tag for technical tests must be set another factor of few smaller. This point will be extended later, at the moment let us assume that in general case of physical effect which is not-dominating overall theoretical uncertainty its individual uncertainty must be smaller at least by factor of 3 from required precision tag. Of course technical precision of its calculation must be even more strict. To enable any meaningful comparisons between different theoretical calculations it must be at least another factor of 5 or so smaller, that is to be at the level of 0.05 of listed above precisions tags required by experiments.

2.2 General comments of observable definition

In the following chapters the observables will be defined. For more details see respectively: ALEPH [3], DELPHI [4], L3 [5] OPAL [6–8] (11(γ)), [9] ($\ell\ell+\gamma$) [10, 11] ($\nu\bar{\nu}+\gamma$)

ALPUB183 DELPUB172 L3PUB172 OPAL2E172, OPALGAMG, OPALGAMNU172, OPALGAMNU183

Maing groups of observables are formed by physics processes, they are later divided into realistic and non-realistic ones corresponding to different stages of experimental analysic. Finally specific solutions adopted by experiments are placed in subsection. Some points which could be extracted from observables definition like glossary or approach to the extra pair are discussed at the beginning of the chapter to avoid unecessary repetition.

The purpose of the list of observables is to review all necessary conditions for calculation of theoretical predictions. More explanatory example are given later in the section.

2.3 Notation

In definition of observables we will use some short-hand notations to make it easier. Let me illustrate just a few of them.

- E_{cm}: centre of mass energy.
- θ_γ: The polar angle of particle γ with respect to the electron beam.
- $|\cos\theta_{f/\bar{f}}| < 0.9$: The polar angle of both particles f and $\bar{f}$ must satisfy the cut.
- E_γ: The energy of particle γ.

- $x_\gamma = 2E_\gamma/E_{cm}$: The energy fraction of particle γ.
- N_{trk}: Number of charged tracks in the event.
- a$col(e^+e^-)$: The acollinearity angle between particles e^+ and e^-.
- A_{FB}: forward-backward asymmetry constructed on the basis of final state charged particles.
- $M_{prop-}(\mu^+\mu^-)$: invariant mass of the s-channel propagator, with ISR/FSR interference subtracted.
- $M_{prop+}(\mu^+\mu^-)$: invariant mass of the s-channel propagator, with ISR/FSR interference not subtracted. In this case, the propagator mass is ambiguous for the interference contribution, so this part is actually evaluated using the $\mu^+\mu^-$ invariant mass excluding radiative photons.
- $M_{inv}(\mu^+\mu^-)$: invariant mass of μ^+ and μ^- - all other particles such as collinear photons are excluded. For realistic observables, this is determined from the reconstructed 4-momenta of the two particles.
- $M_{ang}(\mu^+\mu^-)$: invariant mass determined from the measured polar angles of particles μ^+ and μ^- with respect to the electron beam. This calculation is based on four-momentum conservation assuming that only one radiative photon is present. If no photon is seen in the detector, this radiative photon is assumed to go along the beam axis. If an energetic, isolated photon is seen in the detector, then its reconstructed polar angle is used.
- $M_{kine}(\mu^+\mu^-)$: invariant mass determined from a kinematic fit. This uses 4-momentum conservation to improve the mass estimate based on the reconstructed 4-momenta of the μ^+ and μ^-.
- s'$_{L3}$ for $\mu^+\mu^-$ or $\tau^+\tau^-$ from observed ISR photon (E more than 10 GeV, $|\cos\theta_\gamma| < 0.985$, separation to nearest fermion more than 10 degrees), *or* from fermion angles assuming photon escapes along the beam pipe
- s'$_{L3}$ for $q\bar{q}$ from observed ISR photon (E more than 10 GeV, $|\cos\theta_\gamma| < 0.985$, separation to nearest energy cluster more than 10 degrees), *or* from jet angles after kinematic fit
- s'$_{\rm OPAL}$ for $\rm q\overline{q}$: from a series of kinematic fits using observed ISR photons ($|\cos\theta_\gamma| < 0.985$, isolated with less than 1 GeV energy flow in a cone of half angle 0.2 rad around them) plus hadronic jets. The fits allow zero, one or two additional photons emitted close to the beam direction. s' is taken from the fit with the lowest number of extra photons giving an acceptable χ^2.

2.4 questions and answers are temporarily here

2.41 ALEPH

= question to ALEPH:

Not, at all! I'm still waiting for any response concerning ALEPH's "Idealized Observable". Let me repeat my question below:

==========

Hello,

I have a question concerning ALEPH's cuts for Bhabha. For realistic observables the agular cuts are defined for θ^* ((**Obs.** 19) and (**Obs.** 20)), while for idealized observables these cut are for the electron θ only ((**Obs.** 30) and (**Obs.** 31)), without a cut on the positron angle. Is this a misprint or something really intended?

Best regards, Wiesiek Placzek

Answer: You should read θ^* in both , the text is now corrected. (M.N Minard)

==========

Another questios is about treatment of extra fermion pairs in Bhabha. So far, there is only an anwer from L3. Do the other experiments use the same criteria?

Answer: Updates are present now in appropriate sections.

Without exact specifications of observables/cuts we cannot start our tests/cross-checks of theoretical programs as proposed by LEP2MC Workshop.

Best regads, Wiesiek

2.42 DELPHI

= question to DELPHI:

we are producing NUNUGPV numbers for nu nu gamma(s) observables and have a couple of questions concerning the signature single photon plus missing energy in the case of the DELPHI set of cuts:

1) In case a), b) and c) it is always specified a forward angular coverage. Is it intended that the cuts have to be extended simmetrically to the backward emisphere or not?

Answer is yes

2) After case c), it is stated "No additional photons with $E_\gamma > 0.8GeV$ and $\theta < 38$ mrad..."; we suspect that it should be $\theta > 38$ mrad, right?

Answer is yes

We will wait for your answer before going further. Thanks in advance for your reply, best regards

Fulvio, Guido and Oreste

2.43 L3

supplement for some of the L3 observables: The threshold for registration ((**Obs.** $\overline{\text{LT5}}$ 23)-(**Obs.** $\overline{\text{LT7}}$ 25)) is 15-20 GeV. The signal definition ((**Obs.** $\overline{\text{ILT5}}$ 34)-(**Obs.** $\overline{\text{ILT7}}$ 36)) now is at least 1 GeV for both e+e- /can be changed to say 20 GeV if dictated by theory arguments/

2.44 OPAL

2.5 Additional pair treatment

2.51 ALEPH

Apart from the cuts listed for realistic observables, no additional requirement is added to reject events with real or virtual pair emission, their contribution is taken in account for efficiency calculation. The main rejection to 4-fermions processed is due to invariant mass cuts associated the the topological cut on the event shape.
For a two-fermion final state are considered as backgrounds and their residual contribution estimated from Monte-Carlo four-fermion events for which both fermion pair invariant mass $\mathrm{f\overline{f}}$ or $\mathrm{f'\overline{f'}}$ are above 65 GeV. This exclude from the signal definition the 4-fermion arising from virtual Z or W contribution, but not the the contribution from virtual photon.
The above requirement are valid for hadronic, muon pairs and taus pairs.

Note that for ALEPH the pair contribution is kept both in realistic and idealized observables.

2.52 DELPHI

In case of idealized observables it is assumed that theoretical predictions should not include neither real nor virtual pair corrections. These being subtracted from the data with the help of the Monte Carlo.

The appropriate systematic error is to be discussed with the help of Monte Carlo program or programs. We need matching Monte Carlo predictions for idealized observables with pair corrections excluded as well as for realistic observables with pair corrections included.

We expect that in case of realistic observables implicit cuts on additional fermion pairs are given with the sufficient detail for DELPHI observables. Exception are $\nu\bar{\nu}\gamma$ observables, where we request no jets/leptons (ch) within our acceptance that is of energy $E_{ch} > 0.5$ GeV and $|\cos\theta_{ch}| < 0.97$.

2.53 L3

Let us define first the cut on the secondary pairs for idealized observables:

1. We assume that all the phase space for the secondary pairs is included and integrated over, however contribution form resonant diagrams from 4-fermion processes etc. must be subtracted from the data first by means of Monte Carlo.

For the secondary pair cut to be used for realistic L3 observables our aim was to describe it in a form as brief as possible. In particular we accept ambiguities which may lead to differences much smaller than precision tag. The cut off on the secondary pair is performed in the following way:

1. The invariant mass of the primary pair must be at least 60 GeV for qq, 75 GeV for mm/tt. The invariant mass is determined from acollinearity of primary pair and other trackcs under assumption that missing is only single photon (collinear to the beam). The primary pair means lepton same-flavour anti-lepton or two jets, of highest energies.
2. cut on invariant mass of the secondary pair to be smaller than invariant mass of the primary pair is not explicitly required, but it is in practice through the selection targets on the 2 highest energy jets to form the primary pair, seen in the detector.
3. The W and Z-pair production background is reduced by applying the following cuts. Semi-leptonic W-pair decays are rejected by requiring the transverse energy imbalance to be smaller than 0.3 $E_{\rm vis}$. The background from hadronic W(Z)-pair decays is reduced by rejecting events with at least four jets each with energy larger than 15 GeV. The jets are obtained using the JADE algorithm with a fixed jet resolution parameter $y_{\rm cut} = 0.01$. The remaining part of the 4-fermion processes due to W and Z contributions remain and must be subtracted later with the help of Monte Carlo of well established theoretical uncertainty.
4. for bhabhas there is no explicit cut on primary mass, but implicit cut on secondary pair is present through the acollinearity cut. The primary electrons are selected as the two highest in energy clusters which are matched to charged tracks. Note that in this way we allow the presence of photon(s) which is harder than one or both of the final state e+e-.
5. $M_{inv}(e^+e^-)$ ($L3$) and electron clusters definitions: In L3 electrons are considered dressed, absorbing photons (or extra pairs) in a half-cone 2.5 deg, all these can be seen as detector coverage is larger than used phase space.
6. our definition is not practical for primary pair being neutrinos. Then, instead we request veto cut, no visible charged energy deposits above 1 GeV and $\cos_{\theta_X} < 0.97$.

2.54 OPAL

To define our strategy for additional pair treatment let us recall main points from the ref. [6]$^{\texttt{OPAL2F172}}_{OPAL}$.

No additional cuts, apart those listed for the respective observables, are applied to reject events with pair emission. In general, we compare our measurements with analytical predictions including pair emission. This means that pair emission via virtual photons from both the initial and final state must be included in efficiency calculations, and be excluded from background estimates. In order to perform the separation, we ignore interference between s- and t-channel diagrams contributing to the same four-fermion final state, and generate separate Monte Carlo samples for the different diagrams for each final state. For a two-fermion final state $\mathrm{f\bar{f}}$ we then include as signal those four-fermion events arising from s-channel processes for which $m_{\mathrm{f\bar{f}}} > m_{\mathrm{f'\bar{f}'}}$, $m_{\mathrm{f'\bar{f}'}} < 70$ GeV and $m_{inv}(\mathrm{f\bar{f}})/\sqrt{s} > 0.1$ ($m_{inv}(\mathrm{f\bar{f}})/\sqrt{s} > 0.85$ in the non-radiative case). This kinematic classification closely models the desired

classification of $\mathrm{f\bar{f}f'\bar{f}'}$ in terms of intermediate bosons, in that pairs arising from virtual photons are generally included as signal whereas those arising from virtual Z bosons are not. All events arising from s-channel processes failing the above cuts, together with those arising from the t-channel process (Zee) and two-photon processes are regarded as background. Four-fermion processes involving WW or single W production are also background in all cases. The overall efficiency, ϵ, is calculated as

$$\epsilon = (1 - \frac{\sigma_{\mathrm{f\bar{f}f'\bar{f}'}}}{\sigma_{\mathrm{tot}}})\epsilon_{\mathrm{f\bar{f}}} + \frac{\sigma_{\mathrm{f\bar{f}f'\bar{f}'}}}{\sigma_{\mathrm{tot}}}\epsilon_{\mathrm{f\bar{f}f'\bar{f}'}} \quad (1)$$

where $\epsilon_{\mathrm{f\bar{f}}}$, $\epsilon_{\mathrm{f\bar{f}f'\bar{f}'}}$ are the efficiencies derived from the two-fermion and four-fermion signal Monte Carlo events respectively, $\sigma_{\mathrm{f\bar{f}f'\bar{f}'}}$ is the generated four-fermion cross-section, and σ_{tot} is the total cross-section from the analytical prediction (e.g. ZFITTER) including pair emission. Using this definition of efficiency, effects of cuts on soft pair emission in the four-fermion generator are correctly summed with vertex corrections involving virtual pairs. The inclusion of the four-fermion part of the signal produces negligible changes to the efficiencies for hadronic events and for lepton pairs with $\sqrt{s'_{OPAL}/s} > 0.85$. The efficiencies for lepton pairs with $\sqrt{s'_{OPAL}/s} > 0.10$ are decreased by about 0.5%.

The discussion in the above paragraph applies to hadronic, muon pair and tau pair final states. In the case of electron pairs, the situation is slightly different. In principle the t-channel process with a second fermion pair arising from the conversion of a virtual photon emitted from an initial- or final-state electron should be included as signal. As this process is not included in any program we use for comparison we simply ignore such events: they are not included as background as this would underestimate the cross-section.

2.6 Realistic $e^+e^- \to q\bar{q}(\gamma)$ Observables

2.61 ALEPH

1. **Inclusive Selection:** `Aleph1` Event clustered into jets with JADE algorithm until $(M_{jet-jet}/E_{cm})^2 > 0.008$. Low mass jets with high electro-magnetic energy fraction are assumed to be radiative photons. Non-photon jets then cluster together until only two left, corresponding to the $q\bar{q}$ system. Then
   `Aleph1a`
   (a) $M_{inv}(q\bar{q}) > 50$ GeV (excluding photon jets),
   `Aleph1b`
   (b) $M_{ang}(q\bar{q}) > 0.1E_{cm}$.
2. **Exclusive Selection:** `Aleph2` Same as inclusive selection, then following extra cuts,
   `Aleph2a`
   (a) $M_{inv}(q\bar{q}) > 0.7E_{cm}$ (excluding photon jets),
   `Aleph2b`
   (b) $M_{ang}(q\bar{q}) > 0.9E_{cm}$,
   `Aleph2c`
   (c) $|\cos\theta_{q/\bar{q}}| < 0.95$,
   `Aleph2d`
   (d) Event thrust > 0.85.

2.62 DELPHI

Events were retained if they contained at least 7 charged tracks and if the charged energy was greater than 15 % of the collision energy. In addition, the quantity $E_{rad} = \sqrt{E_F^2 + E_B^2}$, where E_F and E_B stand for the total energy seen in the Forward and Backward electro-magnetic calorimeters, was required to be less than 90 % of the beam energy. Each event was first reconstructed assuming it was made of two jets and the $M_{prop+}(q\bar{q})$ was computed from jet directions. When an isolated energetic photon was reconstructed in the electro-magnetic calorimeters the value of $M_{prop+}(q\bar{q})$ was computed from the measured photon energy.

3. **Inclusive Selection:** `Delphi1` $M_{prop+}(q\bar{q}) > 75\ GeV$
4. **Exclusive Selection:** `Delphi2` $M_{prop+}(q\bar{q})/E_{cm} > 0.85$

2.63 L3

Events are selected by restricting the visible energy, $E_{\rm vis}$, to $0.4 < E_{\rm vis}/\sqrt{s} < 2.0$. The longitudinal energy imbalance must satisfy $|E_{\rm long}|/E_{\rm vis} < 0.7$. These cuts account for a large reduction of the two-photon background. In order to reject background originating from lepton pair events, more than 18 calorimetric clusters with an energy larger than 300 MeV are requested. To reconstruct the effective centre-of-mass energy two different methods are used. For the first one, all events are reclustered into two jets using the JADE algorithm. A single ISR photon is assumed to be emitted along the beam axis and to result in a missing momentum vector. From the polar angles of the jets, θ_1 and θ_2, the photon energy can be estimated. The second method uses the clustered jets ($y_{\rm cut} = 0.01$) obtained using the JADE algorithm. A kinematic fit is performed on the jets and the missing four-momentum vector using different hypotheses for the emitted ISR photons. The missing energy is attributed to zero, one or two ISR photons. From the differences given by the two methods, systematic errors on the effective centre-of-mass energy reconstruction are calculated. When an isolated energetic photon (energy larger than 10 GeV) is detected, the energy and momentum of this photon are added to the undetected ISR photons.

5. **Inclusive Selection 1:** `LT1` $M_{prop-}(q\bar{q})/E_{cm} > 0.10$
6. **Inclusive Selection 2:** `LT2` $M_{prop-}(q\bar{q}) > 60\ GeV$
7. **Exclusive Selection:** `LT3` $M_{prop-}(q\bar{q})/E_{cm} > 0.85$

2.64 OPAL

Events are required to have at least 7 electromagnetic clusters and at least 5 tracks. The total energy deposited in the electromagnetic calorimeter has to be at least 14% of the centre-of-mass energy. The energy balance R_{bal} along the beam direction has to satisfy $R_{bal} \equiv \mid \Sigma(E_{clus} \cdot \cos\theta) \mid /\Sigma E_{clus} < 0.75$, where the sum runs over all clusters in the electromagnetic calorimeter, θ is the polar angle, and E_{clus} is the energy of each cluster. Events selected as W-pair candidates according to the criteria of [`OPALWW183`] are rejected.

8. **Inclusive Selection:** `OPAL1` $\sqrt{s'_{OPAL}/s} > 0.10$
9. **Exclusive Selection:** `OPAL2` $\sqrt{s'_{OPAL}/s} > 0.85$

2.65 Numerical results

Table 1: Numerical predictions from theoretical calculations of the following realistic observables in $q\bar{q}(\gamma)$ final states; cross sections. Warning: this is at work version, misprints are expected.

obs.	program	ene	value and error estimate

Table 2: Numerical predictions from theoretical calculations of the following realistic observables in $q\bar{q}(\gamma)$ final states; asymmetries. Warning: this is at work version, misprints are expected.

obs.	program	ene	value and error estimate

2.7 Idealized $e^+e^- \to q\bar{q}(\gamma)$ Observables

2.71 ALEPH

10. **Inclusive Selection:** `IAleph1` $M_{prop+}(q\bar{q})/E_{cm} > 0.1$. No additional pair cut$_{ALEPH}$.
11. **Exclusive Selection:** `IAleph2` $M_{prop+}(q\bar{q})/E_{cm} > 0.9$ and $|\cos\theta_q| < 0.95$. No additional pair cut$_{ALEPH}$.

2.72 DELPHI

12. **Inclusive Selection:** `IDelphi1` $M_{prop+}(q\bar{q})/E_{cm} > 0.1$ and $|\cos\theta_q| < 1.0$.
13. **Exclusive Selection:** `IDelphi2` $M_{prop+}(q\bar{q})/E_{cm} > 0.85$ and $|\cos\theta_q| < 1.0$.

2.73 L3

14. **Inclusive Selection 1:** `ILT1` $M_{prop-}(q\bar{q})/E_{cm} > 0.10$ and $|\cos\theta_q| < 1.0$.
15. **Inclusive Selection 2:** `ILT2` $M_{prop-}(q\bar{q}) > 60\ GeV$ and $|\cos\theta_q| < 1.0$.
16. **Exclusive Selection:** `ILT3` $M_{prop-}(q\bar{q})/E_{cm} > 0.85$ and $|\cos\theta_q| < 1.0$.

2.74 OPAL

17. **Inclusive Selection:** `IOpal1` $M_{prop-}(q\bar{q})/E_{cm} > 0.10$ and $|\cos\theta_q| < 1.0$.
18. **Exclusive Selection:** `IOpal2` $M_{prop-}(q\bar{q})/E_{cm} > 0.85$ and $|\cos\theta_q| < 1.0$.

Table 3: Numerical predictions from theoretical calculations of the following idealized observables in $q\bar{q}(\gamma)$ final states; cross sections. Warning: this is at work version, misprints are expected.

obs, IA1eph1	program	ene	value and error estimate
(**Obs.** I0) IA1eph1	KKMC 4.13	189.00	.9700E+02 ±.14E+00 ±.19E+01
(**Obs.** I0) IA1eph1	KKMC 4.13	200.00	.8393E+02 ±.11E+00 ±.17E+01
(**Obs.** I0) IA1eph2	KKMC 4.13	206.00	.7806E+02 ±.14E+00 ±.16E+01
(**Obs.** I1) IA1eph2	KKMC 4.13	189.00	.2302E+02 ±.10E+00 ±.46E+00
(**Obs.** I1) IA1eph2	KKMC 4.13	200.00	.1975E+02 ±.80E-01 ±.39E+00
(**Obs.** I1) IDelphi1	KKMC 4.13	206.00	.1838E+02 ±.10E+00 ±.37E+00
(**Obs.** I2) IDelphi1	KKMC 4.13	189.00	.9700E+02 ±.14E+00 ±.19E+01
(**Obs.** I2) IDelphi1	KKMC 4.13	200.00	.8393E+02 ±.11E+00 ±.17E+01
(**Obs.** I2) IDelphi2	KKMC 4.13	206.00	.7806E+02 ±.14E+00 ±.16E+01
(**Obs.** I3) IDelphi2	KKMC 4.13	189.00	.2505E+02 ±.11E+00 ±.50E+00
(**Obs.** I3) IDelphi2	KKMC 4.13	200.00	.2142E+02 ±.81E-01 ±.43E+00
(**Obs.** I3) ILT1	KKMC 4.13	206.00	.1989E+02 ±.11E+00 ±.40E+00
(**Obs.** I4) ILT1	KKMC 4.13	189.00	.9700E+02 ±.14E+00 ±.19E+01
(**Obs.** I4) ILT1	KKMC 4.13	200.00	.8393E+02 ±.11E+00 ±.17E+01
(**Obs.** I4) ILT2	KKMC 4.13	206.00	.7806E+02 ±.14E+00 ±.16E+01
(**Obs.** I5) ILT2	KKMC 4.13	189.00	.9533E+02 ±.14E+00 ±.19E+01
(**Obs.** I5) ILT2	KKMC 4.13	200.00	.8250E+02 ±.11E+00 ±.17E+01
(**Obs.** I5) ILT3	KKMC 4.13	206.00	.7676E+02 ±.14E+00 ±.15E+01
(**Obs.** I6) ILT3	KKMC 4.13	189.00	.2505E+02 ±.11E+00 ±.50E+00
(**Obs.** I6) ILT3	KKMC 4.13	200.00	.2142E+02 ±.81E-01 ±.43E+00
(**Obs.** I6) IOpal1	KKMC 4.13	206.00	.1989E+02 ±.11E+00 ±.40E+00
(**Obs.** I7) IOpal1	KKMC 4.13	189.00	.9700E+02 ±.14E+00 ±.19E+01
(**Obs.** I7) IOpal1	KKMC 4.13	200.00	.8393E+02 ±.11E+00 ±.17E+01
(**Obs.** I7) IOpal2	KKMC 4.13	206.00	.7806E+02 ±.14E+00 ±.16E+01
(**Obs.** I8)	KKMC 4.13	189.00	.2505E+02 ±.11E+00 ±.50E+00

Table 4: Numerical predictions from theoretical calculations of the following idealized observables in $q\bar{q}(\gamma)$ final states; cross sections. Warning: this is at work version, misprints are expected.

obs	program	ene	value and error estimate
(**Obs.** I8) IOpal2	KKMC 4.13	200.00	.2142E+02 ±.81E-01 ±.43E+00
(**Obs.** I8) IOpal2	KKMC 4.13	206.00	.1989E+02 ±.11E+00 ±.40E+00

Table 5: Numerical predictions from theoretical calculations of the following idealized observables in $q\bar{q}(\gamma)$ final states; asymmetries. Warning: this is at work version, misprints are expected.

obs.	program	ene	value and error estimate
(**Obs.** [0] tA1eph1)	KKMC 4.13	189.00	.2142E+00 ±.15E-02 ±.20E-01
(**Obs.** [0] tA1eph1)	KKMC 4.13	200.00	.2112E+00 ±.13E-02 ±.20E-01
(**Obs.** [0] tA1eph1)	KKMC 4.13	206.00	.2105E+00 ±.19E-02 ±.20E-01
(**Obs.** [1] tA1eph2)	KKMC 4.13	189.00	.5951E+00 ±.52E-02 ±.20E-01
(**Obs.** [1] tA1eph2)	KKMC 4.13	200.00	.5958E+00 ±.47E-02 ±.20E-01
(**Obs.** [1] tA1eph2)	KKMC 4.13	206.00	.5994E+00 ±.66E-02 ±.20E-01
(**Obs.** [2] tDelphi1)	KKMC 4.13	189.00	.2142E+00 ±.15E-02 ±.20E-01
(**Obs.** [2] tDelphi1)	KKMC 4.13	200.00	.2112E+00 ±.13E-02 ±.20E-01
(**Obs.** [2] tDelphi1)	KKMC 4.13	206.00	.2105E+00 ±.19E-02 ±.20E-01
(**Obs.** [3] tDelphi2)	KKMC 4.13	189.00	.5778E+00 ±.49E-02 ±.20E-01
(**Obs.** [3] tDelphi2)	KKMC 4.13	200.00	.5804E+00 ±.44E-02 ±.20E-01
(**Obs.** [3] tDelphi2)	KKMC 4.13	206.00	.5850E+00 ±.62E-02 ±.20E-01
(**Obs.** [4] tLT1)	KKMC 4.13	189.00	.2142E+00 ±.15E-02 ±.20E-01
(**Obs.** [4] tLT1)	KKMC 4.13	200.00	.2112E+00 ±.13E-02 ±.20E-01
(**Obs.** [4] tLT1)	KKMC 4.13	206.00	.2105E+00 ±.19E-02 ±.20E-01
(**Obs.** [5] tLT2)	KKMC 4.13	189.00	.2184E+00 ±.15E-02 ±.20E-01
(**Obs.** [5] tLT2)	KKMC 4.13	200.00	.2150E+00 ±.14E-02 ±.20E-01
(**Obs.** [5] tLT2)	KKMC 4.13	206.00	.2144E+00 ±.19E-02 ±.20E-01
(**Obs.** [6] tLT3)	KKMC 4.13	189.00	.5778E+00 ±.49E-02 ±.20E-01
(**Obs.** [6] tLT3)	KKMC 4.13	200.00	.5804E+00 ±.44E-02 ±.20E-01
(**Obs.** [6] tLT3)	KKMC 4.13	206.00	.5850E+00 ±.62E-02 ±.20E-01
(**Obs.** [7] tOpa11)	KKMC 4.13	189.00	.2142E+00 ±.15E-02 ±.20E-01
(**Obs.** [7] tOpa11)	KKMC 4.13	200.00	.2112E+00 ±.13E-02 ±.20E-01
(**Obs.** [7] tOpa11)	KKMC 4.13	206.00	.2105E+00 ±.19E-02 ±.20E-01
(**Obs.** [8] tOpa12)	KKMC 4.13	189.00	.5778E+00 ±.49E-02 ±.20E-01

Table 6: Numerical predictions from theoretical calculations of the following idealized observables in $q\bar{q}(\gamma)$ final states; asymmetries. Warning: this is at work version, misprints are expected.

obs.	program	ene	value and error estimate
(**Obs.** I8) I0pa12	KKMC 4.13	200.00	.5804E+00 ±.44E-02 ±.20E-01
(**Obs.** I8) I0pa12	KKMC 4.13	206.00	.5850E+00 ±.62E-02 ±.20E-01

2.8 Realistic $e^+e^- \to e^+e^-(\gamma)$ Observables

2.81 ALEPH

19. **Exclusive Selection 1:** `Aleph3` $2 \leq N_{trk} \leq 8$. Two of tracks must be identified as electrons, have $|\cos\theta| < 0.95$ and opposite charge. The scalar sum of their momenta should exceed $0.3E_{cm}$. The sum of their energies including photons within 20° of each tracks should exceed $0.4E_{cm}$. Then $M_{inv}(e^+e^-) > 80$ GeV (reconstructed from track momenta), $M_{ang}(e^+e^-) > 0.9E_{cm}$ (where radiative photons reconstructed as for $q\bar{q}$ selection), and $-0.9 < \cos\theta^*_{e^-} < 0.9$.
20. **Exclusive Selection 2:** `Aleph4` Same as exclusive selection 1, but $-0.9 < \cos\theta^*_{e^-} < 0.7$.

2.82 DELPHI

21. **Exclusive Selection:** `Delphi3` The electron and positron were required to be in the polar angle range $44^\circ < \theta < 136^\circ$ and the non-radiative events were selected by asking the acollinearity to be smaller than 20°.

2.83 L3

The electron and positron are required to be in the polar angle range $44^\circ < \theta < 136^\circ$ or $20^\circ < \theta < 160^\circ$ and non-radiative events are selected by asking the acollinearity to be smaller than 20° OR $M_{inv}(e^+e^-)/E_{cm} > 0.85$.

22. **Exclusive Selection 1:** `LT4` $M_{inv}(e^+e^-)/E_{cm} > 0.85$ and $|\cos\theta_{e^+/e^-}| < 0.71934$.
23. **Exclusive Selection 2:** `LT5` $\mathrm{a}col(e^+e^-) < 25deg.$ and $|\cos\theta_{e^+/e^-}| < 0.71934$.
24. **Exclusive Selection 3:** `LT6` $\mathrm{a}col(e^+e^-) < 25deg.$ and $|\cos\theta_{e^+/e^-}| < 0.94$.
25. **Inclusive Selection 1:** `LT7` $\mathrm{a}col(e^+e^-) < 120deg.$ and $|\cos\theta_{e^+/e^-}| < 0.71934$.
26. **Inclusive Selection 2:** `LT8` $M_{inv}(e^+e^-)/E_{cm} > 0.10$ and $|\cos\theta_{e^+/e^-}| < 0.94$.

2.84 OPAL

Events selected as electron pairs are required to have at least two and not more than eight clusters in the electromagnetic calorimeter, and not more than eight tracks in the central tracking chambers. At least two clusters must have an energy exceeding 20% of the beam energy, and the total energy deposited in the electromagnetic calorimeter must be at least 50% of the centre-of-mass energy. For the inclusive selection and the exclusive selection 1, at least two of the three highest energy clusters must each have an associated central detector track. If all three clusters have an associated track, the two highest energy clusters are chosen to be the electron and positron. For the large acceptance exclusiv selection 2, no requirement is placed on the associationof tracks to clusters, but the requirement on the total electromagnetic energy is increased to 70% of the centre-of-mass energy.

27. **Inclusive Selection :** `Opal3` $\mathrm{acol}(e^+e^-) < 170^\circ$ and $|\cos\theta_{e^+/e^-}| < 0.9$.
28. **Exclusive Selection 1:** `Opal4` $\mathrm{acol}(e^+e^-) < 10^\circ$ and $|\cos\theta_{e^-}| < 0.7$.
29. **Exclusive Selection 2:** `Opal5` $\mathrm{acol}(e^+e^-) < 10^\circ$ and $|\cos\theta_{e^+/e^-}| < 0.96$.

Table 7: Numerical predictions from theoretical calculations of the following realistic observables in $e^+e^-(\gamma)$ final states; cross sections. Warning: this is at work version, misprints are expected.

obs.	program	ene	value and error estimate
(**Obs.** Aleph3 19)	LABSMC	189.00	.1116E+00 ±.56E-04 ±.28E-03
(**Obs.** Aleph3 19)	LABSMC	200.00	.9990E-01 ±.50E-04 ±.25E-03
(**Obs.** Aleph3 19)	LABSMC	206.00	.9427E-01 ±.48E-04 ±.24E-03
(**Obs.** Aleph4 20)	LABSMC	189.00	.2246E-01 ±.25E-04 ±.56E-04
(**Obs.** Aleph4 20)	LABSMC	200.00	.2003E-01 ±.23E-04 ±.50E-04
(**Obs.** Aleph4 20)	LABSMC	206.00	.1885E-01 ±.21E-04 ±.47E-04
(**Obs.** Delphi3 21)	LABSMC	189.00	.2214E-01 ±.25E-04 ±.55E-04
(**Obs.** Delphi3 21)	LABSMC	200.00	.1970E-01 ±.22E-04 ±.49E-04
(**Obs.** Delphi3 21)	LABSMC	206.00	.1852E-01 ±.21E-04 ±.46E-04
(**Obs.** Lt4 22)	LABSMC	189.00	.2102E-01 ±.24E-04 ±.53E-04
(**Obs.** Lt4 22)	LABSMC	200.00	.1869E-01 ±.22E-04 ±.47E-04
(**Obs.** Lt4 22)	LABSMC	206.00	.1757E-01 ±.21E-04 ±.44E-04
(**Obs.** Lt5 23)	LABSMC	189.00	.2246E-01 ±.25E-04 ±.56E-04
(**Obs.** Lt5 23)	LABSMC	200.00	.1999E-01 ±.23E-04 ±.50E-04
(**Obs.** Lt5 23)	LABSMC	206.00	.1880E-01 ±.21E-04 ±.47E-04
(**Obs.** Lt6 24)	LABSMC	189.00	.1941E+00 ±.74E-04 ±.49E-03
(**Obs.** Lt6 24)	LABSMC	200.00	.1735E+00 ±.67E-04 ±.43E-03
(**Obs.** Lt6 24)	LABSMC	206.00	.1636E+00 ±.63E-04 ±.41E-03
(**Obs.** Lt7 25)	LABSMC	189.00	.2286E-01 ±.26E-04 ±.57E-04
(**Obs.** Lt7 25)	LABSMC	200.00	.2034E-01 ±.23E-04 ±.51E-04
(**Obs.** Lt7 25)	LABSMC	206.00	.1914E-01 ±.21E-04 ±.48E-04
(**Obs.** Lt8 26)	LABSMC	189.00	.1966E+00 ±.75E-04 ±.49E-03
(**Obs.** Lt8 26)	LABSMC	200.00	.1757E+00 ±.67E-04 ±.44E-03
(**Obs.** Lt8 26)	LABSMC	206.00	.1657E+00 ±.63E-04 ±.41E-03
(**Obs.** Opal3 27)	LABSMC	189.00	.1029E+00 ±.54E-04 ±.26E-03

Table 8: Numerical predictions from theoretical calculations of the following realistic observables in $e^+e^-(\gamma)$ final states; cross sections. Warning: this is at work version, misprints are expected.

obs.	program	ene	value and error estimate
(**Obs.** 27) Opal3	LABSMC	200.00	.9193E-01 ±.48E-04 ±.23E-03
(**Obs.** 27) Opal3	LABSMC	206.00	.8666E-01 ±.46E-04 ±.22E-03
(**Obs.** 28) Opal4	LABSMC	189.00	.1924E-01 ±.23E-04 ±.48E-04
(**Obs.** 28) Opal4	LABSMC	200.00	.1708E-01 ±.21E-04 ±.43E-04
(**Obs.** 28) Opal4	LABSMC	206.00	.1605E-01 ±.20E-04 ±.40E-04
(**Obs.** 29) Opal5	LABSMC	189.00	.2919E+00 ±.91E-04 ±.73E-03
(**Obs.** 29) Opal5	LABSMC	200.00	.2607E+00 ±.82E-04 ±.65E-03
(**Obs.** 29) Opal5	LABSMC	206.00	.2458E+00 ±.77E-04 ±.61E-03

Table 9: Numerical predictions from theoretical calculations of the following realistic observables in $e^+e^-(\gamma)$ final states; asymmetries. Warning: this is at work version, misprints are expected.

obs.	program	ene	value and error estimate

2.9 Idealized $e^+e^- \to e^+e^-(\gamma)$ Observables

2.91 ALEPH

30. **Exclusive Selection 1:** `IAleph3` $M_{inv}(e^+e^-)/E_{cm} > 0.9$ and $-0.9 < \cos\theta^*_{e^-} < 0.9$. No additional pair cut$_{ALEPH}$.
31. **Exclusive Selection 2:** `IAleph4` $M_{inv}(e^+e^-)/E_{cm} > 0.9$ and $-0.9 < \cos\theta^*_{e^-} < 0.7$. No additional pair cut$_{ALEPH}$.

2.92 DELPHI

32. **Exclusive Selection:** `IDelphi4` $M_{inv}(e^+e^-)/E_{cm} > 0.85$ and $|\cos\theta_{e^+/e^-}| < 0.7$.

2.93 L3

33. **Exclusive Selection 1:** `ILT4` $M_{inv}(e^+e^-)/E_{cm} > 0.85$ and $|\cos\theta_{e^+/e^-}| < 0.71934$.
34. **Exclusive Selection 2:** `ILT5` $\mathrm{a}col(e^+e^-) < 25deg.$ and $|\cos\theta_{e^+/e^-}| < 0.71934$.
35. **Exclusive Selection 3:** `ILT6` $\mathrm{a}col(e^+e^-) < 25deg.$ and $|\cos\theta_{e^+/e^-}| < 0.94$.
36. **Inclusive Selection 1:** `ILT7` $\mathrm{a}col(e^+e^-) < 120deg.$ and $|\cos\theta_{e^+/e^-}| < 0.71934$.
37. **Inclusive Selection 2:** `ILT8` $M_{inv}(e^+e^-)/E_{cm} > 0.10$ and $|\cos\theta_{e^+/e^-}| < 0.94$.

2.94 OPAL

38. **Inclusive Selection :** `IOpal3` $\mathrm{acol}(e^+e^-) < 170^\circ$ and $|\cos\theta_{e^+/e^-}| < 0.9$.
39. **Exclusive Selection 1:** `IOpal4` $\mathrm{acol}(e^+e^-) < 10^\circ$ and $|\cos\theta_{e^-}| < 0.7$.
40. **Exclusive Selection 2:** `IOpal5` $\mathrm{acol}(e^+e^-) < 10^\circ$ and $|\cos\theta_{e^+/e^-}| < 0.96$.

Table 10: Numerical predictions from theoretical calculations of the following idealized observables in $e^+e^-(\gamma)$ final states; cross sections. Warning: this is at work version, misprints are expected.

obs.	program	ene	value and error estimate
(**Obs.** 30) IA1eph3	LABSMC	189.00	.1117E+00 ±.56E-04 ±.28E-03
(**Obs.** 30) IA1eph3	BHWIDE	189.00	.8726E+02 ±.27E+00 ±.87E+00
(**Obs.** 30) IA1eph3	LABSMC	200.00	.9971E-01 ±.50E-04 ±.25E-03
(**Obs.** 30) IA1eph3	BHWIDE	200.00	.7808E+02 ±.24E+00 ±.78E+00
(**Obs.** 30) IA1eph3	LABSMC	206.00	.9398E-01 ±.48E-04 ±.23E-03
(**Obs.** 30) IA1eph3	BHWIDE	206.00	.7372E+02 ±.23E+00 ±.74E+00
(**Obs.** 31) IA1eph4	LABSMC	189.00	.1640E-01 ±.22E-04 ±.41E-04
(**Obs.** 31) IA1eph4	BHWIDE	189.00	.1781E+02 ±.10E+00 ±.87E+00
(**Obs.** 31) IA1eph4	LABSMC	200.00	.1454E-01 ±.19E-04 ±.36E-04
(**Obs.** 31) IA1eph4	BHWIDE	200.00	.1584E+02 ±.91E-01 ±.78E+00
(**Obs.** 31) IA1eph4	LABSMC	206.00	.1365E-01 ±.18E-04 ±.34E-04
(**Obs.** 31) IA1eph4	BHWIDE	206.00	.1499E+02 ±.86E-01 ±.74E+00
(**Obs.** 32) IDelphi4	LABSMC	189.00	.0000E+00 ±.00E+00 ±.00E+00
(**Obs.** 32) IDelphi4	BHWIDE	189.00	.1831E+02 ±.10E+00 ±.18E+00
(**Obs.** 32) IDelphi4	LABSMC	200.00	.0000E+00 ±.00E+00 ±.00E+00
(**Obs.** 32) IDelphi4	BHWIDE	200.00	.1630E+02 ±.91E-01 ±.16E+00
(**Obs.** 32) IDelphi4	LABSMC	206.00	.0000E+00 ±.00E+00 ±.00E+00
(**Obs.** 32) ILT4	BHWIDE	206.00	.1539E+02 ±.86E-01 ±.15E+00
(**Obs.** 33) ILT4	LABSMC	189.00	.1930E-01 ±.23E-04 ±.48E-04
(**Obs.** 33) ILT4	BHWIDE	189.00	.2183E+02 ±.11E+00 ±.22E+00
(**Obs.** 33) ILT4	LABSMC	200.00	.1715E-01 ±.21E-04 ±.43E-04
(**Obs.** 33) ILT4	BHWIDE	200.00	.1956E+02 ±.10E+00 ±.20E+00
(**Obs.** 33) ILT4	LABSMC	206.00	.1610E-01 ±.20E-04 ±.40E-04
(**Obs.** 33) ILT5	BHWIDE	206.00	.1849E+02 ±.95E-01 ±.18E+00
(**Obs.** 34)	LABSMC	189.00	.2252E-01 ±.25E-04 ±.56E-04

Table 11: Numerical predictions from theoretical calculations of the following idealized observables in $e^+e^-(\gamma)$ final states; cross sections. Warning: this is at work version, misprints are expected.

obs.	program	ene	value and error estimate
(**Obs.** 34) ILT5	BHWIDE	189.00	.2314E+02 ±.11E+00 ±.23E+00
(**Obs.** 34) ILT5	LABSMC	200.00	.2003E-01 ±.23E-04 ±.50E-04
(**Obs.** 34) ILT5	BHWIDE	200.00	.2074E+02 ±.10E+00 ±.21E+00
(**Obs.** 34) ILT5	LABSMC	206.00	.1884E-01 ±.21E-04 ±.47E-04
(**Obs.** 34) ILT5	BHWIDE	206.00	.1959E+02 ±.97E-01 ±.20E+00
(**Obs.** 35) ILT6	LABSMC	189.00	.1946E+00 ±.75E-04 ±.49E-03
(**Obs.** 35) ILT6	BHWIDE	189.00	.1978E+03 ±.40E+00 ±.20E+01
(**Obs.** 35) ILT6	LABSMC	200.00	.1739E+00 ±.67E-04 ±.43E-03
(**Obs.** 35) ILT6	BHWIDE	200.00	.1773E+03 ±.36E+00 ±.18E+01
(**Obs.** 35) ILT6	LABSMC	206.00	.1640E+00 ±.63E-04 ±.41E-03
(**Obs.** 35) ILT7	BHWIDE	206.00	.1671E+03 ±.34E+00 ±.17E+01
(**Obs.** 36) ILT7	LABSMC	189.00	.2292E-01 ±.26E-04 ±.57E-04
(**Obs.** 36) ILT7	BHWIDE	189.00	.2465E+02 ±.12E+00 ±.37E+00
(**Obs.** 36) ILT7	LABSMC	200.00	.2039E-01 ±.23E-04 ±.51E-04
(**Obs.** 36) ILT7	BHWIDE	200.00	.2199E+02 ±.11E+00 ±.33E+00
(**Obs.** 36) ILT7	LABSMC	206.00	.1918E-01 ±.21E-04 ±.48E-04
(**Obs.** 36) ILT8	BHWIDE	206.00	.2068E+02 ±.10E+00 ±.31E+00
(**Obs.** 37) ILT8	LABSMC	189.00	.1966E+00 ±.75E-04 ±.49E-03
(**Obs.** 37) ILT8	BHWIDE	189.00	.2067E+03 ±.42E+00 ±.31E+01
(**Obs.** 37) ILT8	LABSMC	200.00	.1757E+00 ±.67E-04 ±.44E-03
(**Obs.** 37) ILT8	BHWIDE	200.00	.1851E+03 ±.37E+00 ±.28E+01
(**Obs.** 37) ILT8	LABSMC	206.00	.1657E+00 ±.63E-04 ±.41E-03
(**Obs.** 37) ILT8	BHWIDE	206.00	.1742E+03 ±.35E+00 ±.26E+01
(**Obs.** 38) IOpal3	LABSMC	189.00	.1035E+00 ±.54E-04 ±.26E-03
(**Obs.** 38) IOpal3	BHWIDE	189.00	.1098E+03 ±.30E+00 ±.16E+01

Table 12: Numerical predictions from theoretical calculations of the following idealized observables in $e^+e^-(\gamma)$ final states; cross sections. Warning: this is at work version, misprints are expected.

obs.	program	ene	value and error estimate
(**Obs.** 38) IOpal3	LABSMC	200.00	.9246E-01 ±.49E-04 ±.23E-03
(**Obs.** 38) IOpal3	BHWIDE	200.00	.9826E+02 ±.27E+00 ±.15E+01
(**Obs.** 38) IOpal3	LABSMC	206.00	.8716E-01 ±.46E-04 ±.22E-03
(**Obs.** 38) IOpal3	BHWIDE	206.00	.9259E+02 ±.25E+00 ±.14E+01
(**Obs.** 39) IOpal4	LABSMC	189.00	.1852E-01 ±.23E-04 ±.46E-04
(**Obs.** 39) IOpal4	BHWIDE	189.00	.2016E+02 ±.11E+00 ±.20E+00
(**Obs.** 39) IOpal4	LABSMC	200.00	.1644E-01 ±.20E-04 ±.41E-04
(**Obs.** 39) IOpal4	BHWIDE	200.00	.1800E+02 ±.95E-01 ±.18E+00
(**Obs.** 39) IOpal4	LABSMC	206.00	.1545E-01 ±.19E-04 ±.39E-04
(**Obs.** 39) IOpal4	BHWIDE	206.00	.1702E+02 ±.90E-01 ±.17E+00
(**Obs.** 40) IOpal5	LABSMC	189.00	.3004E+00 ±.93E-04 ±.75E-03
(**Obs.** 40) IOpal5	BHWIDE	189.00	.3071E+03 ±.50E+00 ±.15E+01
(**Obs.** 40) IOpal5	LABSMC	200.00	.2684E+00 ±.83E-04 ±.67E-03
(**Obs.** 40) IOpal5	BHWIDE	200.00	.2748E+03 ±.44E+00 ±.14E+01
(**Obs.** 40) IOpal5	LABSMC	206.00	.2531E+00 ±.78E-04 ±.63E-03
(**Obs.** 40) IOpal5	BHWIDE	206.00	.2592E+03 ±.42E+00 ±.13E+01

Table 13: Numerical predictions from theoretical calculations of the following idealized observables in $e^+e^-(\gamma)$ final states; asymmetries. Warning: this is at work version, misprints are expected.

obs.	program	ene	value and error estimate

2.10 Realistic $e^+e^- \to \mu^+\mu^-(\gamma)$ Observables

2.101 ALEPH

41. **Inclusive Selection:** `Aleph5` $2 \leq N_{trk} \leq 8$. Two tracks must be identified as muons, have $p > 6$ GeV, $|\cos\theta| < 0.95$ and opposite charge. The scalar sum of their momenta should exceed 60 GeV. Photons are identified from jets clustered with JADE algorithm until $(M_{jet-jet}/E_{cm})^2 > 0.008$. Then $M_{ang}(\mu^+\mu^)/E_{cm} > 0.10$ (where radiative photons reconstructed as for $q\bar{q}$ selection).
42. **Exclusive Selection:** `Aleph6` Same as inclusive selection, then following extra cuts, $M_{ang}(\mu^+\mu^)/E_{cm} > 0.9E_{cm}$ (where radiative photons reconstructed as for $q\bar{q}$ selection) and $M_{inv}(\mu^+\mu^) > 0.74$ (excluding photons).

2.102 DELPHI

An event was required to have two identified muons in the polar angle range $20^\circ \leq \theta_\mu \leq 160^\circ$ and the highest muon momentum of at least $30\ GeV/c$. $M_{inv}(\mu^+\mu^)$ was calculated from a kinematical fit, where four different topologies were investigated for each event: i) no photon radiated, ii) one photon radiated along the beam line, iii) one seen and one unseen photon in any direction, iv) a single unseen photon in any direction. The seen photon fit was performed if a neutral energy deposit greater than $5\ GeV$ was measured in the electro-magnetic calorimeters.

43. **Inclusive Selection:** `Delphi4` $M_{inv}(\mu^+\mu^) > 75\ GeV$
44. **Exclusive Selection:** `Delphi5` $M_{inv}(\mu^+\mu^)/E_{cm} > 0.85$

2.103 L3

An event must have two identified muons in the polar angle range $20^\circ \leq \theta_\mu \leq 160^\circ$ and the highest muon momentum should exceed 35 GeV. The effective centre-of-mass energy for each event is determined assuming the emission of a single ISR photon. In case the photon is found in the detector it is required to have an energy, E_γ, larger than 15 GeV in the electro-magnetic calorimeter and an angular separation to the nearest muon of more than 10 degrees. Otherwise the photon is assumed to be emitted along the beam axis and its energy is calculated from the polar angles of the outgoing muons.

45. **Inclusive Selection:** `LT9` $M_{ang}(\mu^+\mu^) > 75\ GeV$
46. **Exclusive Selection:** `LT10` $M_{ang}(\mu^+\mu^)/E_{cm} > 0.85$

2.104 OPAL

$N_{trk} \geq 2$. A pair of tracks is taken as muon pair candidate, if both tracks are identified as muons, have $p > 6$ GeV, $|\cos\theta| < 0.95$ and are separated by 320 mrad in azimuthal angle ϕ. If more than one pair of tracks satisfies the above conditions, the pair with the largest scalar momentum sum is chosen. The event is rejected if more than one other track has a transverse momentum greater than 0.7 GeV. Finally, E_{vis}, defined as the scalar sum of the two muon momenta plus the energy of the highest energy cluster in the electromagnetic calorimeter, has to be larger than $0.35\sqrt{s} + 0.5M_z^2/\sqrt{s}$.

47. **Inclusive Selection:** `Opal6` $M_{ang}(\mu^+\mu^)/E_{cm} > 0.10$ and $M_{inv}(\mu^+\mu^-) > 70$ GeV, if $0.35\sqrt{s} < E_{vis} - 0.5M_z^2/\sqrt{s} < 0.75\sqrt{s}$.
48. **Exclusive Selection:** `Opal7` $M_{ang}(\mu^+\mu^)/E_{cm} > 0.85$ and $M_{inv}(\mu^+\mu^-) > \sqrt{M_Z^2 + 0.1s}$.

2.105 Numerical results

Table 14: Numerical predictions from theoretical calculations of the following realistic observables in $\mu^+\mu^-(\gamma)$ final states; cross sections. Warning: this is at work version, misprints are expected.

obs.	program	ene	value and error estimate

Table 15: Numerical predictions from theoretical calculations of the following realistic observables in $\mu^+\mu^-(\gamma)$ final states; asymmetries. Warning: this is at work version, misprints are expected.

obs.	program	ene	value and error estimate

2.11 Idealized $e^+e^- \to \mu^+\mu^-(\gamma)$Observables

2.111 ALEPH

49. **Inclusive Selection:** `IAleph5` $M_{inv}(\mu^+\mu^-)/E_{cm} > 0.1$ and $|\cos\theta_{\mu^-}| < 0.95$. No additional pair cut$_{ALEPH}$.
50. **Exclusive Selection:** `IAleph6` $M_{inv}(\mu^+\mu^-)/E_{cm} > 0.9$ and $|\cos\theta_{\mu^-}| < 0.95$. No additional pair cut$_{ALEPH}$.

2.112 DELPHI

51. **Inclusive Selection:** `IDelphi5` $M_{inv}(\mu^+\mu^-) > 75\ GeV$ and $|\cos\theta_{\mu^-}| < 0.95$.
52. **Exclusive Selection:** `IDelphi6` $M_{inv}(\mu^+\mu^-)/E_{cm} > 0.85$ and $|\cos\theta_{\mu^-}| < 0.95$.

2.113 L3

53. **Inclusive Selection:** `ILT9` $M_{inv}(\mu^+\mu^-) > 75\ GeV$ and $|\cos\theta_{\mu^-}| < 0.90$.
54. **Exclusive Selection 1:** `ILT10` $M_{inv}(\mu^+\mu^-)/E_{cm} > 0.85$ and $|\cos\theta_{\mu^-}| < 0.90$.
55. **Exclusive Selection 2:** `ILT11` $M_{inv}(\mu^+\mu^-)/E_{cm} > 0.85$ and $|\cos\theta_{\mu^-}| < 1.0$.

2.114 OPAL

56. **Inclusive Selection:** `IOpal6` $M_{prop-}(\mu^+\mu^-)/\sqrt{s} > 0.10$ and $|\cos\theta_{\mu^-}| < 0.95$.
57. **Inclusive Selection:** `IOpal7` $M_{prop-}(\mu^+\mu^-)/\sqrt{s} > 0.10$ and $|\cos\theta_{\mu^-}| < 1.00$.
58. **Exclusive Selection:** `IOpal8` $M_{prop-}(\mu^+\mu^-)/\sqrt{s} > 0.85$ and $|\cos\theta_{\mu^-}| < 0.95$.
59. **Exclusive Selection:** `IOpal9` $M_{prop-}(\mu^+\mu^-)/\sqrt{s} > 0.85$ and $|\cos\theta_{\mu^-}| < 1.00$.

Table 16: Numerical predictions from theoretical calculations of the following idealized observables in $\mu^+\mu^-(\gamma)$ final states; cross sections. Warning: this is at work version, misprints are expected.

obs.	program	ene	value and error estimate
(**Obs.** 49) IA1eph5	ZFITTER v6.23	189.00	.6861E+01 +.59E-04 −.25E-01
(**Obs.** 49) IA1eph5	ZFITTER v6.23p	189.00	.6862E+01 +.59E-04 −.25E-01
(**Obs.** 49) IA1eph5	KKMC 4.13	189.00	.6740E+01 ±.20E-01 ±.13E+00
(**Obs.** 49) IA1eph5	ZFITTER v6.23	200.00	.6012E+01 +.50E-04 −.23E-01
(**Obs.** 49) IA1eph5	ZFITTER v6.23p	200.00	.6013E+01 +.50E-04 −.23E-01
(**Obs.** 49) IA1eph5	KKMC 4.13	200.00	.5886E+01 ±.15E-01 ±.12E+00
(**Obs.** 49) IA1eph5	ZFITTER v6.23	206.00	.5618E+01 +.45E-04 −.22E-01
(**Obs.** 49) IA1eph5	ZFITTER v6.23p	206.00	.5619E+01 +.46E-04 −.22E-01
(**Obs.** 49) IA1eph5	KKMC 4.13	206.00	.5481E+01 ±.20E-01 ±.11E+00
(**Obs.** 50) IA1eph6	ZFITTER v6.23	189.00	.2667E+01 +.13E-02 −.15E-01
(**Obs.** 50) IA1eph6	ZFITTER v6.23p	189.00	.2668E+01 +.13E-02 −.15E-01
(**Obs.** 50) IA1eph6	KKMC 4.13	189.00	.2657E+01 ±.17E-01 ±.53E-01
(**Obs.** 50) IA1eph6	ZFITTER v6.23	200.00	.2351E+01 +.12E-02 −.13E-01
(**Obs.** 50) IA1eph6	ZFITTER v6.23p	200.00	.2352E+01 +.12E-02 −.13E-01
(**Obs.** 50) IA1eph6	KKMC 4.13	200.00	.2350E+01 ±.13E-01 ±.47E-01
(**Obs.** 50) IA1eph6	ZFITTER v6.23	206.00	.2202E+01 +.11E-02 −.12E-01
(**Obs.** 50) IA1eph6	ZFITTER v6.23p	206.00	.2203E+01 +.11E-02 −.13E-01
(**Obs.** 50) IA1eph6	KKMC 4.13	206.00	.2201E+01 ±.17E-01 ±.44E-01
(**Obs.** 51) IDelphi5	ZFITTER v6.23	189.00	.6295E+01 +.54E-04 −.18E-01
(**Obs.** 51) IDelphi5	ZFITTER v6.23p	189.00	.6296E+01 +.55E-04 −.18E-01
(**Obs.** 51) IDelphi5	KKMC 4.13	189.00	.6273E+01 ±.20E-01 ±.13E+00
(**Obs.** 51) IDelphi5	ZFITTER v6.23	200.00	.5527E+01 +.46E-04 −.16E-01
(**Obs.** 51) IDelphi5	ZFITTER v6.23p	200.00	.5529E+01 +.46E-04 −.16E-01
(**Obs.** 51) IDelphi5	KKMC 4.13	200.00	.5488E+01 ±.15E-01 ±.11E+00
(**Obs.** 51)	ZFITTER v6.23	206.00	.5170E+01 +.42E-04 −.16E-01

Table 17: Numerical predictions from theoretical calculations of the following idealized observables in $\mu^+\mu^-(\gamma)$ final states; cross sections. Warning: this is at work version, misprints are expected.

obs.	program	ene	value and error estimate
(**Obs.** 51) IDelphi5	ZFITTER v6.23p	206.00	.5172E+01 +.42E-04 −.16E-01
(**Obs.** 51) IDelphi5	KKMC 4.13	206.00	.5117E+01 ±.19E-01 ±.10E+00
(**Obs.** 52) IDelphi6	ZFITTER v6.23	189.00	.2843E+01 +.11E-02 −.12E-01
(**Obs.** 52) IDelphi6	ZFITTER v6.23p	189.00	.2845E+01 +.11E-02 −.12E-01
(**Obs.** 52) IDelphi6	KKMC 4.13	189.00	.2830E+01 ±.17E-01 ±.57E-01
(**Obs.** 52) IDelphi6	ZFITTER v6.23	200.00	.2507E+01 +.10E-02 −.10E-01
(**Obs.** 52) IDelphi6	ZFITTER v6.23p	200.00	.2508E+01 +.10E-02 −.11E-01
(**Obs.** 52) IDelphi6	KKMC 4.13	200.00	.2501E+01 ±.13E-01 ±.50E-01
(**Obs.** 52) IDelphi6	ZFITTER v6.23	206.00	.2350E+01 +.95E-03 −.99E-02
(**Obs.** 52) IDelphi6	ZFITTER v6.23p	206.00	.2351E+01 +.95E-03 −.10E-01
(**Obs.** 52) IDelphi6	KKMC 4.13	206.00	.2345E+01 ±.17E-01 ±.47E-01
(**Obs.** 53) ILT9	ZFITTER v6.23	189.00	.5770E+01 +.49E-04 −.18E-01
(**Obs.** 53) ILT9	ZFITTER v6.23p	189.00	.5771E+01 +.50E-04 −.18E-01
(**Obs.** 53) ILT9	KKMC 4.13	189.00	.5670E+01 ±.18E-01 ±.11E+00
(**Obs.** 53) ILT9	ZFITTER v6.23	200.00	.5055E+01 +.41E-04 −.17E-01
(**Obs.** 53) ILT9	ZFITTER v6.23p	200.00	.5056E+01 +.41E-04 −.17E-01
(**Obs.** 53) ILT9	KKMC 4.13	200.00	.4956E+01 ±.14E-01 ±.99E-01
(**Obs.** 53) ILT9	ZFITTER v6.23	206.00	.4723E+01 +.38E-04 −.16E-01
(**Obs.** 53) ILT9	ZFITTER v6.23p	206.00	.4724E+01 +.38E-04 −.16E-01
(**Obs.** 53) ILT9	KKMC 4.13	206.00	.4610E+01 ±.18E-01 ±.92E-01
(**Obs.** 54) ILT10	ZFITTER v6.23	189.00	.2623E+01 +.13E-02 −.11E-01
(**Obs.** 54) ILT10	ZFITTER v6.23p	189.00	.2624E+01 +.13E-02 −.11E-01
(**Obs.** 54) ILT10	KKMC 4.13	189.00	.2589E+01 ±.16E-01 ±.52E-01
(**Obs.** 54) ILT10	ZFITTER v6.23	200.00	.2312E+01 +.12E-02 −.97E-02
(**Obs.** 54)	ZFITTER v6.23p	200.00	.2313E+01 +.12E-02 −.97E-02

Table 18: Numerical predictions from theoretical calculations of the following idealized observables in $\mu^+\mu^-(\gamma)$ final states; cross sections. Warning: this is at work version, misprints are expected.

obs.	program	ene	value and error estimate
(**Obs.** 54) ILT10	KKMC 4.13	200.00	.2301E+01 ±.12E-01 ±.46E-01
(**Obs.** 54) ILT10	ZFITTER v6.23	206.00	.2166E+01 +.11E-02 −.92E-02
(**Obs.** 54) ILT10	ZFITTER v6.23p	206.00	.2167E+01 +.11E-02 −.92E-02
(**Obs.** 54) ILT10	KKMC 4.13	206.00	.2153E+01 ±.16E-01 ±.43E-01
(**Obs.** 55) ILT11	ZFITTER v6.23	189.00	.3073E+01 +.44E-03 −.13E-01
(**Obs.** 55) ILT11	ZFITTER v6.23p	189.00	.3075E+01 +.44E-03 −.13E-01
(**Obs.** 55) ILT11	KKMC 4.13	189.00	.3068E+01 ±.19E-01 ±.61E-01
(**Obs.** 55) ILT11	ZFITTER v6.23	200.00	.2711E+01 +.41E-03 −.11E-01
(**Obs.** 55) ILT11	ZFITTER v6.23p	200.00	.2712E+01 +.41E-03 −.11E-01
(**Obs.** 55) ILT11	KKMC 4.13	200.00	.2708E+01 ±.15E-01 ±.54E-01
(**Obs.** 55) ILT11	ZFITTER v6.23	206.00	.2541E+01 +.37E-03 −.11E-01
(**Obs.** 55) ILT11	ZFITTER v6.23p	206.00	.2542E+01 +.37E-03 −.11E-01
(**Obs.** 55) ILT11	KKMC 4.13	206.00	.2543E+01 ±.19E-01 ±.51E-01
(**Obs.** 56) IOpal6	ZFITTER v6.23	189.00	.6848E+01 +.59E-04 −.25E-01
(**Obs.** 56) IOpal6	ZFITTER v6.23p	189.00	.6850E+01 +.59E-04 −.25E-01
(**Obs.** 56) IOpal6	KKMC 4.13	189.00	.6740E+01 ±.20E-01 ±.13E+00
(**Obs.** 56) IOpal6	ZFITTER v6.23	200.00	.6001E+01 +.50E-04 −.23E-01
(**Obs.** 56) IOpal6	ZFITTER v6.23p	200.00	.6002E+01 +.50E-04 −.23E-01
(**Obs.** 56) IOpal6	KKMC 4.13	200.00	.5886E+01 ±.15E-01 ±.12E+00
(**Obs.** 56) IOpal6	ZFITTER v6.23	206.00	.5607E+01 +.46E-04 −.22E-01
(**Obs.** 56) IOpal6	ZFITTER v6.23p	206.00	.5609E+01 +.46E-04 −.22E-01
(**Obs.** 56) IOpal6	KKMC 4.13	206.00	.5481E+01 ±.20E-01 ±.11E+00
(**Obs.** 57) IOpal7	ZFITTER v6.23	189.00	.7746E+01 +.69E-04 −.31E-02
(**Obs.** 57) IOpal7	ZFITTER v6.23p	189.00	.7748E+01 +.69E-04 −.31E-02
(**Obs.** 57) IOpal7	KKMC 4.13	189.00	.7647E+01 ±.22E-01 ±.15E+00

Table 19: Numerical predictions from theoretical calculations of the following idealized observables in $\mu^+\mu^-(\gamma)$ final states; cross sections. Warning: this is at work version, misprints are expected.

obs.	program	ene	value and error estimate
(**Obs.** 57) [IOpal17]	ZFITTER v6.23	200.00	.6821E+01 +.61E-04 −.29E-02
(**Obs.** 57) [IOpal17]	ZFITTER v6.23p	200.00	.6823E+01 +.62E-04 −.29E-02
(**Obs.** 57) [IOpal17]	KKMC 4.13	200.00	.6703E+01 ±.17E-01 ±.13E+00
(**Obs.** 57) [IOpal17]	ZFITTER v6.23	206.00	.6390E+01 +.62E-04 −.28E-02
(**Obs.** 57) [IOpal17]	ZFITTER v6.23p	206.00	.6392E+01 +.62E-04 −.28E-02
(**Obs.** 57) [IOpal17]	KKMC 4.13	206.00	.6265E+01 ±.21E-01 ±.13E+00
(**Obs.** 58) [IOpal18]	ZFITTER v6.23	189.00	.2960E+01 +.15E-02 −.56E-04
(**Obs.** 58) [IOpal18]	ZFITTER v6.23p	189.00	.2961E+01 +.15E-02 −.56E-04
(**Obs.** 58) [IOpal18]	KKMC 4.13	189.00	.2830E+01 ±.17E-01 ±.57E-01
(**Obs.** 58) [IOpal18]	ZFITTER v6.23	200.00	.2612E+01 +.13E-02 −.48E-04
(**Obs.** 58) [IOpal18]	ZFITTER v6.23p	200.00	.2613E+01 +.13E-02 −.48E-04
(**Obs.** 58) [IOpal18]	KKMC 4.13	200.00	.2501E+01 ±.13E-01 ±.50E-01
(**Obs.** 58) [IOpal18]	ZFITTER v6.23	206.00	.2449E+01 +.12E-02 −.45E-04
(**Obs.** 58) [IOpal18]	ZFITTER v6.23p	206.00	.2450E+01 +.12E-02 −.44E-04
(**Obs.** 58) [IOpal18]	KKMC 4.13	206.00	.2345E+01 ±.17E-01 ±.47E-01
(**Obs.** 59) [IOpal19]	ZFITTER v6.23	189.00	.3191E+01 +.46E-03 −.59E-04
(**Obs.** 59) [IOpal19]	ZFITTER v6.23p	189.00	.3193E+01 +.46E-03 −.59E-04
(**Obs.** 59) [IOpal19]	KKMC 4.13	189.00	.3068E+01 ±.19E-01 ±.61E-01
(**Obs.** 59) [IOpal19]	ZFITTER v6.23	200.00	.2817E+01 +.42E-03 −.51E-04
(**Obs.** 59) [IOpal19]	ZFITTER v6.23p	200.00	.2818E+01 +.42E-03 −.51E-04
(**Obs.** 59) [IOpal19]	KKMC 4.13	200.00	.2708E+01 ±.15E-01 ±.54E-01
(**Obs.** 59) [IOpal19]	ZFITTER v6.23	206.00	.2641E+01 +.38E-03 −.48E-04
(**Obs.** 59) [IOpal19]	ZFITTER v6.23p	206.00	.2643E+01 +.38E-03 −.48E-04
(**Obs.** 59) [IOpal19]	KKMC 4.13	206.00	.2543E+01 ±.19E-01 ±.51E-01

Table 20: Numerical predictions from theoretical calculations of the following idealized observables in $\mu^+\mu^-(\gamma)$ final states; asymmetries. Warning: this is at work version, misprints are expected.

obs.	program	ene	value and error estimate
(**Obs.** 49) IAleph5	ZFITTER v6.23	189.00	.2903E+00 +.11E-02 −.13E-04
(**Obs.** 49) IAleph5	ZFITTER v6.23p	189.00	.2903E+00 +.11E-02 −.13E-04
(**Obs.** 49) IAleph5	KKMC 4.13	189.00	.2973E+00 ±.30E-02 ±.20E-01
(**Obs.** 49) IAleph5	ZFITTER v6.23	200.00	.2897E+00 +.12E-02 −.12E-04
(**Obs.** 49) IAleph5	ZFITTER v6.23p	200.00	.2897E+00 +.12E-02 −.12E-04
(**Obs.** 49) IAleph5	KKMC 4.13	200.00	.2963E+00 ±.27E-02 ±.20E-01
(**Obs.** 49) IAleph5	ZFITTER v6.23	206.00	.2893E+00 +.12E-02 −.11E-04
(**Obs.** 49) IAleph5	ZFITTER v6.23p	206.00	.2893E+00 +.12E-02 −.11E-04
(**Obs.** 49) IAleph5	KKMC 4.13	206.00	.2931E+00 ±.37E-02 ±.20E-01
(**Obs.** 50) IAleph6	ZFITTER v6.23	189.00	.5737E+00 +.92E-04 −.17E-03
(**Obs.** 50) IAleph6	ZFITTER v6.23p	189.00	.5736E+00 +.93E-04 −.17E-03
(**Obs.** 50) IAleph6	KKMC 4.13	189.00	.5676E+00 ±.73E-02 ±.20E-01
(**Obs.** 50) IAleph6	ZFITTER v6.23	200.00	.5638E+00 +.10E-03 −.17E-03
(**Obs.** 50) IAleph6	ZFITTER v6.23p	200.00	.5637E+00 +.10E-03 −.17E-03
(**Obs.** 50) IAleph6	KKMC 4.13	200.00	.5615E+00 ±.64E-02 ±.20E-01
(**Obs.** 50) IAleph6	ZFITTER v6.23	206.00	.5590E+00 +.11E-03 −.17E-03
(**Obs.** 50) IAleph6	ZFITTER v6.23p	206.00	.5589E+00 +.11E-03 −.17E-03
(**Obs.** 50) IAleph6	KKMC 4.13	206.00	.5525E+00 ±.88E-02 ±.20E-01
(**Obs.** 51) IDelphi5	ZFITTER v6.23	189.00	.3199E+00 +.97E-03 −.13E-04
(**Obs.** 51) IDelphi5	ZFITTER v6.23p	189.00	.3199E+00 +.97E-03 −.13E-04
(**Obs.** 51) IDelphi5	KKMC 4.13	189.00	.3203E+00 ±.33E-02 ±.20E-01
(**Obs.** 51) IDelphi5	ZFITTER v6.23	200.00	.3184E+00 +.10E-02 −.12E-04
(**Obs.** 51) IDelphi5	ZFITTER v6.23p	200.00	.3183E+00 +.10E-02 −.12E-04
(**Obs.** 51) IDelphi5	KKMC 4.13	200.00	.3178E+00 ±.29E-02 ±.20E-01
(**Obs.** 51)	ZFITTER v6.23	206.00	.3174E+00 +.10E-02 −.12E-04

Table 21: Numerical predictions from theoretical calculations of the following idealized observables in $\mu^+\mu^-(\gamma)$ final states; asymmetries. Warning: this is at work version, misprints are expected.

obs.	program	ene	value and error estimate
(**Obs.** 51) IDelphi5	ZFITTER v6.23p	206.00	.3174E+00 +.10E-02 −.12E-04
(**Obs.** 51) IDelphi5	KKMC 4.13	206.00	.3139E+00 ±.40E-02 ±.20E-01
(**Obs.** 52) IDelphi6	ZFITTER v6.23	189.00	.5696E+00 +.45E-04 −.11E-03
(**Obs.** 52) IDelphi6	ZFITTER v6.23p	189.00	.5695E+00 +.45E-04 −.11E-03
(**Obs.** 52) IDelphi6	KKMC 4.13	189.00	.5659E+00 ±.70E-02 ±.20E-01
(**Obs.** 52) IDelphi6	ZFITTER v6.23	200.00	.5593E+00 +.52E-04 −.11E-03
(**Obs.** 52) IDelphi6	ZFITTER v6.23p	200.00	.5592E+00 +.52E-04 −.11E-03
(**Obs.** 52) IDelphi6	KKMC 4.13	200.00	.5584E+00 ±.61E-02 ±.20E-01
(**Obs.** 52) IDelphi6	ZFITTER v6.23	206.00	.5544E+00 +.55E-04 −.11E-03
(**Obs.** 52) IDelphi6	ZFITTER v6.23p	206.00	.5543E+00 +.55E-04 −.11E-03
(**Obs.** 52) IDelphi6	KKMC 4.13	206.00	.5491E+00 ±.84E-02 ±.20E-01
(**Obs.** 53) ILT9	ZFITTER v6.23	189.00	.3112E+00 +.10E-02 −.11E-04
(**Obs.** 53) ILT9	ZFITTER v6.23p	189.00	.3112E+00 +.10E-02 −.11E-04
(**Obs.** 53) ILT9	KKMC 4.13	189.00	.3113E+00 ±.33E-02 ±.20E-01
(**Obs.** 53) ILT9	ZFITTER v6.23	200.00	.3096E+00 +.11E-02 −.10E-04
(**Obs.** 53) ILT9	ZFITTER v6.23p	200.00	.3096E+00 +.11E-02 −.10E-04
(**Obs.** 53) ILT9	KKMC 4.13	200.00	.3109E+00 ±.29E-02 ±.20E-01
(**Obs.** 53) ILT9	ZFITTER v6.23	206.00	.3086E+00 +.11E-02 −.98E-05
(**Obs.** 53) ILT9	ZFITTER v6.23p	206.00	.3086E+00 +.11E-02 −.99E-05
(**Obs.** 53) ILT9	KKMC 4.13	206.00	.3070E+00 ±.41E-02 ±.20E-01
(**Obs.** 54) ILT10	ZFITTER v6.23	189.00	.5525E+00 +.45E-04 −.16E-03
(**Obs.** 54) ILT10	ZFITTER v6.23p	189.00	.5524E+00 +.45E-04 −.16E-03
(**Obs.** 54) ILT10	KKMC 4.13	189.00	.5449E+00 ±.70E-02 ±.20E-01
(**Obs.** 54) ILT10	ZFITTER v6.23	200.00	.5425E+00 +.51E-04 −.16E-03
(**Obs.** 54)	ZFITTER v6.23p	200.00	.5424E+00 +.51E-04 −.16E-03

Table 22: Numerical predictions from theoretical calculations of the following idealized observables in $\mu^+\mu^-(\gamma)$ final states; asymmetries. Warning: this is at work version, misprints are expected.

obs.	program	ene	value and error estimate
(**Obs.** 54)$_{\texttt{ILT10}}$	KKMC 4.13	200.00	.5399E+00 ±.61E-02 ±.20E-01
(**Obs.** 54)$_{\texttt{ILT10}}$	ZFITTER v6.23	206.00	.5377E+00 +.53E-04 −.16E-03
(**Obs.** 54)$_{\texttt{ILT10}}$	ZFITTER v6.23p	206.00	.5376E+00 +.54E-04 −.16E-03
(**Obs.** 54)$_{\texttt{ILT10}}$	KKMC 4.13	206.00	.5301E+00 ±.83E-02 ±.20E-01
(**Obs.** 55)$_{\texttt{ILT11}}$	ZFITTER v6.23	189.00	.5875E+00 +.14E-03 −.52E-04
(**Obs.** 55)$_{\texttt{ILT11}}$	ZFITTER v6.23p	189.00	.5874E+00 +.14E-03 −.52E-04
(**Obs.** 55)$_{\texttt{ILT11}}$	KKMC 4.13	189.00	.5842E+00 ±.72E-02 ±.20E-01
(**Obs.** 55)$_{\texttt{ILT11}}$	ZFITTER v6.23	200.00	.5770E+00 +.15E-03 −.52E-04
(**Obs.** 55)$_{\texttt{ILT11}}$	ZFITTER v6.23p	200.00	.5769E+00 +.15E-03 −.52E-04
(**Obs.** 55)$_{\texttt{ILT11}}$	KKMC 4.13	200.00	.5757E+00 ±.62E-02 ±.20E-01
(**Obs.** 55)$_{\texttt{ILT11}}$	ZFITTER v6.23	206.00	.5720E+00 +.15E-03 −.52E-04
(**Obs.** 55)$_{\texttt{ILT11}}$	ZFITTER v6.23p	206.00	.5719E+00 +.15E-03 −.52E-04
(**Obs.** 55)$_{\texttt{ILT11}}$	KKMC 4.13	206.00	.5676E+00 ±.85E-02 ±.20E-01
(**Obs.** 56)$_{\texttt{IOpal6}}$	ZFITTER v6.23	189.00	.2866E+00 +.11E-02 −.13E-04
(**Obs.** 56)$_{\texttt{IOpal6}}$	ZFITTER v6.23p	189.00	.2866E+00 +.11E-02 −.13E-04
(**Obs.** 56)$_{\texttt{IOpal6}}$	KKMC 4.13	189.00	.2973E+00 ±.30E-02 ±.20E-01
(**Obs.** 56)$_{\texttt{IOpal6}}$	ZFITTER v6.23	200.00	.2859E+00 +.12E-02 −.12E-04
(**Obs.** 56)$_{\texttt{IOpal6}}$	ZFITTER v6.23p	200.00	.2859E+00 +.12E-02 −.12E-04
(**Obs.** 56)$_{\texttt{IOpal6}}$	KKMC 4.13	200.00	.2963E+00 ±.27E-02 ±.20E-01
(**Obs.** 56)$_{\texttt{IOpal6}}$	ZFITTER v6.23	206.00	.2854E+00 +.12E-02 −.11E-04
(**Obs.** 56)$_{\texttt{IOpal6}}$	ZFITTER v6.23p	206.00	.2854E+00 +.12E-02 −.11E-04
(**Obs.** 56)$_{\texttt{IOpal6}}$	KKMC 4.13	206.00	.2931E+00 ±.37E-02 ±.20E-01
(**Obs.** 57)$_{\texttt{IOpal7}}$	ZFITTER v6.23	189.00	.2779E+00 +.17E-03 −.14E-04
(**Obs.** 57)$_{\texttt{IOpal7}}$	ZFITTER v6.23p	189.00	.2779E+00 +.17E-03 −.14E-04
(**Obs.** 57)$_{\texttt{IOpal7}}$	KKMC 4.13	189.00	.2909E+00 ±.30E-02 ±.20E-01

Table 23: Numerical predictions from theoretical calculations of the following idealized observables in $\mu^+\mu^-(\gamma)$ final states; asymmetries. Warning: this is at work version, misprints are expected.

obs.	program	ene	value and error estimate
(**Obs.** 57) IOpal7	ZFITTER v6.23	200.00	.2764E+00 +.18E-03 −.14E-04
(**Obs.** 57) IOpal7	ZFITTER v6.23p	200.00	.2763E+00 +.18E-03 −.14E-04
(**Obs.** 57) IOpal7	KKMC 4.13	200.00	.2889E+00 ±.26E-02 ±.20E-01
(**Obs.** 57) IOpal7	ZFITTER v6.23	206.00	.2755E+00 +.18E-03 −.13E-04
(**Obs.** 57) IOpal7	ZFITTER v6.23p	206.00	.2754E+00 +.18E-03 −.13E-04
(**Obs.** 57) IOpal7	KKMC 4.13	206.00	.2862E+00 ±.36E-02 ±.20E-01
(**Obs.** 58) IOpal8	ZFITTER v6.23	189.00	.5539E+00 +.30E-05 −.15E-03
(**Obs.** 58) IOpal8	ZFITTER v6.23p	189.00	.5538E+00 +.30E-05 −.15E-03
(**Obs.** 58) IOpal8	KKMC 4.13	189.00	.5659E+00 ±.70E-02 ±.20E-01
(**Obs.** 58) IOpal8	ZFITTER v6.23	200.00	.5431E+00 +.28E-05 −.16E-03
(**Obs.** 58) IOpal8	ZFITTER v6.23p	200.00	.5430E+00 +.28E-05 −.16E-03
(**Obs.** 58) IOpal8	KKMC 4.13	200.00	.5584E+00 ±.61E-02 ±.20E-01
(**Obs.** 58) IOpal8	ZFITTER v6.23	206.00	.5379E+00 +.27E-05 −.15E-03
(**Obs.** 58) IOpal8	ZFITTER v6.23p	206.00	.5378E+00 +.27E-05 −.15E-03
(**Obs.** 58) IOpal8	KKMC 4.13	206.00	.5491E+00 ±.84E-02 ±.20E-01
(**Obs.** 59) IOpal9	ZFITTER v6.23	189.00	.5687E+00 +.93E-04 −.51E-04
(**Obs.** 59) IOpal9	ZFITTER v6.23p	189.00	.5686E+00 +.93E-04 −.51E-04
(**Obs.** 59) IOpal9	KKMC 4.13	189.00	.5842E+00 ±.72E-02 ±.20E-01
(**Obs.** 59) IOpal9	ZFITTER v6.23	200.00	.5576E+00 +.90E-04 −.50E-04
(**Obs.** 59) IOpal9	ZFITTER v6.23p	200.00	.5575E+00 +.90E-04 −.50E-04
(**Obs.** 59) IOpal9	KKMC 4.13	200.00	.5757E+00 ±.62E-02 ±.20E-01
(**Obs.** 59) IOpal9	ZFITTER v6.23	206.00	.5523E+00 +.92E-04 −.50E-04
(**Obs.** 59) IOpal9	ZFITTER v6.23p	206.00	.5522E+00 +.92E-04 −.50E-04
(**Obs.** 59) IOpal9	KKMC 4.13	206.00	.5676E+00 ±.85E-02 ±.20E-01

2.12 Realistic $e^+e^- \to \tau^+\tau^-(\gamma)$Observables

2.121 ALEPH

60. **Inclusive Selection:** `Aleph7` Two identified τ candidates with $M_{inv}(\tau^+\tau^-) > 25$ GeV and a$col(\tau^+\tau^-) < 250$ mrad in plane perpendicular to beam axis. Then $M_{ang}(\tau^+\tau^-) > 0.1E_{cm}$ (where radiative photons are reconstructed as for $q\bar{q}$ selection).
61. **Exclusive Selection:** `Aleph8` Same as inclusive selection, then following extra cut $M_{ang}(\tau^+\tau^-) > 0.9E_{cm}$ (where radiative photons reconstructed as for $q\bar{q}$ selection) and $|\cos\theta_{\tau^-}| < 0.95$.

2.122 DELPHI

The leading track in each hemisphere was required to lie in the polar angle range $|\cos\theta| < 0.94$, and the observed charged particle multiplicity was requested to be unity in one hemisphere and no more than five in the other. At least one of the leading tracks was required to have momentum greater than $0.025 \times \sqrt{s}$. Full description of the cuts can be found in [4]. The $M_{ang}(\tau^+\tau-)$ was calculated from fermion directions estimated by leading tracks.

62. **Inclusive Selection:** `Delphi6` $M_{ang}(\tau^+\tau-) > 75\ GeV$
63. **Exclusive Selection:** `Delphi7` $M_{ang}(\tau^+\tau-)/E_{cm} > 0.85$

2.123 L3

Taus are identified as narrow, low multiplicity jets, containing from one to five charged particles. Tau jets are formed by matching the energy depositions in the electro-magnetic and hadron calorimeters with tracks in the central tracker and the muon spectrometer. Two tau jets of at least 3.5 GeV are required to lie within the polar angular range $|\cos\theta| < 0.92$. To reject background from two photon processes the most energetic jet must have an energy larger than 20 GeV. The reconstruction of the effective centre-of-mass energy follows the procedure described in the muon subsection, using the polar angles of the two tau jets. Full description of the cuts can be found in [5].

64. **Inclusive Selection:** `LT11` $M_{ang}(\tau^+\tau-) > 75\ GeV$
65. **Exclusive Selection:** `LT12` $M_{ang}(\tau^+\tau-)/E_{cm} > 0.85$

2.124 OPAL

Tau-pair candidates are events with exactly two charged, low multiplicity cones with 35° half-angle.

- The Event is rejected if it was selected as a μ pair.
- $\mathrm{N_{trk}} < 7$, where $\mathrm{N_{trk}}$ is the number of tracks in the central detector.
- $\mathrm{N_{trk}} + \mathrm{N_{clus}} < 16$, where $\mathrm{N_{clus}}$ is the number of ECAL clusters.
- $|\cos\theta_{\tau^+\tau^-}| < 0.90$, where $|\cos\theta_\tau|$ is the cosine of the respective cone axis.
- $0.02 < \mathrm{R_{shw}} < 0.7$, where $\mathrm{R_{shw}}$ is the summed ECAL energy scaled by the centre of mass energy.
- $\mathrm{R_{trk}} < 0.8$, where $\mathrm{R_{trk}}$ is the scalar sum of track momenta scaled by the centre of mass energy.
- $\mathrm{R_{shw}} > 0.2$ or $\mathrm{R_{trk}} > 0.2$.
- $\mathrm{R_{shw}} + \mathrm{R_{trk}} < 1.05(1.10)$, for $|\cos\theta| > 0.7$ (< 0.7),where $|\cos\theta|$ is the average of the two tau cones.
- $|\cos\theta_{\mathrm{p^{ECAL}_{missing}}}| < 0.99$, where $\cos\theta_{\mathrm{p^{ECAL}_{missing}}}$ is the cosine of the direction of missing momentum calculated using the ECAL.
- $\mathrm{p_t^{ECAL}} > 0.015\sqrt{\mathrm{s}}$ where p_t^{ECAL} is the sum of the transverse ECAL energy.
- Events are rejected if $0.9 < \mathrm{E/p} < 1.1$ in both cones.

- Using the values of θ_τ, the expected energy of each lepton is calculated assuming that the final state consists only of two leptons plus a single unobserved photon along the beam direction. We then require that $0.02 < \sqrt{(X_{E1}^2 + X_{E2}^2)} < 0.8$, and $\sqrt{(X_{P1}^2 + X_{P2}^2)} < 0.8$, where $X_{E1,E2}$ are the total electromagnetic calorimeter energies in each tau cone normalized to the expected value calculated above, and $X_{P1,P2}$ are the scalar sums of track momenta in the two tau cones, also normalized to the expected values.
- $\theta_{\mathrm{acolinearity}} < 180^\circ - 2\tan^{-1}\left(\frac{2\mathrm{M_Z}\sqrt{\mathrm{s}}}{\mathrm{s}-\mathrm{M_Z^2}}\right) + 10^\circ$. This cut is placed 10° degrees above the expected radiative return peak.
- $\theta_{\mathrm{acoplanarity}} < 30^\circ$.

66. **Inclusive Selection:** `Opal8` $M_{ang}(\tau^+\tau^-)/\sqrt{s} > 0.10$.
67. **Exclusive Selection:** `Opal9` $M_{ang}(\tau^+\tau^-)/\sqrt{s} > 0.85$.

2.125 Numerical results

Table 24: Numerical predictions from theoretical calculations of the following realistic observables in $\tau^+\tau^-(\gamma)$ final states; cross sections. Warning: this is at work version, misprints are expected.

obs.	program	ene	value and error estimate

Table 25: Numerical predictions from theoretical calculations of the following realistic observables in $\tau^+\tau^-(\gamma)$ final states; asymmetries. Warning: this is at work version, misprints are expected.

obs.	program	ene	value and error estimate

2.13 Idealized $e^+e^- \to \tau^+\tau^-(\gamma)$ Observables

2.131 ALEPH

68. **Inclusive Selection:** `IAleph7` $M_{inv}(\tau^+\tau^-)/E_{cm} > 0.1$ and $|\cos\theta_{\tau^-}| < 0.95$. No additional pair cut$_{ALEPH}$.
69. **Exclusive Selection:** `IAleph8` $M_{inv}(\tau^+\tau^-)/E_{cm} > 0.9$ and $|\cos\theta_{\tau^-}| < 0.95$. No additional pair cut$_{ALEPH}$.

2.132 DELPHI

70. **Inclusive Selection:** `IDelphi7` $M_{inv}(\tau^+\tau^-) > 75\ GeV$ and $|\cos\theta_{\tau^-}| < 0.95$.
71. **Exclusive Selection:** `IDelphi8` $M_{inv}(\tau^+\tau^-)/E_{cm} > 0.85$ and $|\cos\theta_{\tau^-}| < 0.95$.

2.133 L3

72. **Inclusive Selection:** `ILT12` $M_{inv}(\tau^+\tau^-) > 75\ GeV$ and $|\cos\theta_{\tau^-}| < 0.92$.
73. **Exclusive Selection 1:** `ILT13` $M_{inv}(\tau^+\tau^-)/E_{cm} > 0.85$ and $|\cos\theta_{\tau^-}| < 0.92$.
74. **Exclusive Selection 2:** `ILT14` $M_{inv}(\tau^+\tau^-)/E_{cm} > 0.85$ and $|\cos\theta_{\tau^-}| < 1.0$.

2.134 OPAL

75. **Inclusive Selection 1:** `IOpal10` $M_{prop-}(\tau^+\tau^-)/\sqrt{s} > 0.10$ and $|\cos\theta_{\tau^-}| < 0.90$.
76. **Inclusive Selection 2:** `IOpal11` $M_{prop-}(\tau^+\tau^-)/\sqrt{s} > 0.10$ and $|\cos\theta_{\tau^-}| < 1.00$.
77. **Exclusive Selection:** `IOpal12` $M_{prop-}(\tau^+\tau^-)/\sqrt{s} > 0.85$ and $|\cos\theta_{\tau^-}| < 0.90$.
78. **Exclusive Selection:** `IOpal13` $M_{prop-}(\tau^+\tau^-)/\sqrt{s} > 0.85$ and $|\cos\theta_{\tau^-}| < 1.00$.

2.135 Numerical results

Table 26: Numerical predictions from theoretical calculations of the following idealized observables in $\tau^+\tau^-(\gamma)$ final states; cross sections. Warning: this is at work version, misprints are expected.

obs	program	ene	value and error estimate
(**Obs.** 68) tALeph7	ZFITTER v6.23	189.00	.6859E+01 +.59E-04 −.25E-01
(**Obs.** 68) tALeph7	KKMC 4.13	189.00	.6748E+01 ±.17E-01 ±.13E+00
(**Obs.** 68) tALeph7	ZFITTER v6.23	200.00	.6011E+01 +.49E-04 −.23E-01
(**Obs.** 68) tALeph7	KKMC 4.13	200.00	.5891E+01 ±.94E-02 ±.12E+00
(**Obs.** 68) tALeph7	ZFITTER v6.23	206.00	.5617E+01 +.45E-04 −.22E-01
(**Obs.** 68) tALeph7	KKMC 4.13	206.00	.5480E+01 ±.11E-01 ±.11E+00
(**Obs.** 69) tALeph8	ZFITTER v6.23	189.00	.2756E+01 +.14E-02 −.84E-02
(**Obs.** 69) tALeph8	KKMC 4.13	189.00	.2758E+01 ±.15E-01 ±.55E-01
(**Obs.** 69) tALeph8	ZFITTER v6.23	200.00	.2430E+01 +.12E-02 −.77E-02
(**Obs.** 69) tALeph8	KKMC 4.13	200.00	.2424E+01 ±.82E-02 ±.48E-01
(**Obs.** 69) tALeph8	ZFITTER v6.23	206.00	.2277E+01 +.12E-02 −.73E-02
(**Obs.** 69) tALeph8	KKMC 4.13	206.00	.2265E+01 ±.99E-02 ±.45E-01
(**Obs.** 70) tDelphi7	ZFITTER v6.23	189.00	.6359E+01 +.56E-04 −.18E-01
(**Obs.** 70) tDelphi7	KKMC 4.13	189.00	.6341E+01 ±.17E-01 ±.13E+00
(**Obs.** 70) tDelphi7	ZFITTER v6.23	200.00	.5582E+01 +.47E-04 −.17E-01
(**Obs.** 70) tDelphi7	KKMC 4.13	200.00	.5546E+01 ±.94E-02 ±.11E+00
(**Obs.** 70) tDelphi7	ZFITTER v6.23	206.00	.5221E+01 +.43E-04 −.16E-01
(**Obs.** 70) tDelphi7	KKMC 4.13	206.00	.5163E+01 ±.11E-01 ±.10E+00
(**Obs.** 71) tDelphi8	ZFITTER v6.23	189.00	.2915E+01 +.12E-02 −.65E-02
(**Obs.** 71) tDelphi8	KKMC 4.13	189.00	.2914E+01 ±.15E-01 ±.58E-01
(**Obs.** 71) tDelphi8	ZFITTER v6.23	200.00	.2571E+01 +.10E-02 −.59E-02
(**Obs.** 71) tDelphi8	KKMC 4.13	200.00	.2564E+01 ±.83E-02 ±.51E-01
(**Obs.** 71) tDelphi8	ZFITTER v6.23	206.00	.2409E+01 +.99E-03 −.56E-02
(**Obs.** 71) tDelphi8	KKMC 4.13	206.00	.2397E+01 ±.10E-01 ±.48E-01
(**Obs.** 72) tLT12	ZFITTER v6.23	189.00	.6057E+01 +.53E-04 −.19E-01

Table 27: Numerical predictions from theoretical calculations of the following idealized observables in $\tau^+\tau^-(\gamma)$ final states; cross sections. Warning: this is at work version, misprints are expected.

obs	program	ene	value and error estimate
(**Obs.** (2) tLLT12	KKMC 4.13	189.00	.5977E+01 ±.16E-01 ±.12E+00
(**Obs.** (2) tLLT12	ZFITTER v6.23	200.00	.5310E+01 +.44E-04 −.17E-01
(**Obs.** (2) tLLT12	KKMC 4.13	200.00	.5222E+01 ±.89E-02 ±.10E+00
(**Obs.** (2) tLLT12	ZFITTER v6.23	206.00	.4962E+01 +.40E-04 −.17E-01
(**Obs.** (2) tLLT13	KKMC 4.13	206.00	.4851E+01 ±.11E-01 ±.97E-01
(**Obs.** (3) tLLT13	ZFITTER v6.23	189.00	.2776E+01 +.14E-02 −.62E-02
(**Obs.** (3) tLLT13	KKMC 4.13	189.00	.2777E+01 ±.15E-01 ±.56E-01
(**Obs.** (3) tLLT13	ZFITTER v6.23	200.00	.2448E+01 +.12E-02 −.56E-02
(**Obs.** (3) tLLT13	KKMC 4.13	200.00	.2447E+01 ±.80E-02 ±.49E-01
(**Obs.** (3) tLLT13	ZFITTER v6.23	206.00	.2294E+01 +.12E-02 −.54E-02
(**Obs.** (3) tLLT14	KKMC 4.13	206.00	.2282E+01 ±.95E-02 ±.46E-01
(**Obs.** (4) tLLT14	ZFITTER v6.23	189.00	.3151E+01 +.46E-03 −.70E-02
(**Obs.** (4) tLLT14	KKMC 4.13	189.00	.3150E+01 ±.17E-01 ±.63E-01
(**Obs.** (4) tLLT14	ZFITTER v6.23	200.00	.2779E+01 +.41E-03 −.64E-02
(**Obs.** (4) tLLT14	KKMC 4.13	200.00	.2778E+01 ±.91E-02 ±.56E-01
(**Obs.** (4) tLLT14	ZFITTER v6.23	206.00	.2605E+01 +.39E-03 −.61E-02
(**Obs.** (4) tLLT14	KKMC 4.13	206.00	.2598E+01 ±.11E-01 ±.52E-01
(**Obs.** (5) tOpa110	ZFITTER v6.23	189.00	.6173E+01 +.52E-04 −.23E-01
(**Obs.** (5) tOpa110	KKMC 4.13	189.00	.6088E+01 ±.16E-01 ±.12E+00
(**Obs.** (5) tOpa110	ZFITTER v6.23	200.00	.5395E+01 +.43E-04 −.21E-01
(**Obs.** (5) tOpa110	KKMC 4.13	200.00	.5302E+01 ±.87E-02 ±.11E+00
(**Obs.** (5) tOpa110	ZFITTER v6.23	206.00	.5035E+01 +.39E-04 −.21E-01
(**Obs.** (5) tOpa111	KKMC 4.13	206.00	.4922E+01 ±.10E-01 ±.98E-01
(**Obs.** (6) tOpa111	ZFITTER v6.23	189.00	.7738E+01 +.68E-04 −.31E-02
(**Obs.** (6)	KKMC 4.13	189.00	.7635E+01 ±.19E-01 ±.15E+00

Table 28: Numerical predictions from theoretical calculations of the following idealized observables in $\tau^+\tau^-(\gamma)$ final states; cross sections. Warning: this is at work version, misprints are expected.

obs.	program	ene	value and error estimate
(**Obs.** 76) tOpa111	ZFITTER v6.23	200.00	.6814E+01 +.75E-04 −.29E-02
(**Obs.** 76) tOpa111	KKMC 4.13	200.00	.6702E+01 ±.10E-01 ±.13E+00
(**Obs.** 76) tOpa111	ZFITTER v6.23	206.00	.6384E+01 +.77E-04 −.28E-02
(**Obs.** 76) tOpa111	KKMC 4.13	206.00	.6254E+01 ±.12E-01 ±.13E+00
(**Obs.** 77) tOpa112	ZFITTER v6.23	189.00	.2739E+01 +.14E-02 −.52E-04
(**Obs.** 77) tOpa112	KKMC 4.13	189.00	.2926E+01 ±.14E-01 ±.59E-01
(**Obs.** 77) tOpa112	ZFITTER v6.23	200.00	.2416E+01 +.12E-02 −.45E-04
(**Obs.** 77) tOpa112	KKMC 4.13	200.00	.2568E+01 ±.79E-02 ±.51E-01
(**Obs.** 77) tOpa112	ZFITTER v6.23	206.00	.2265E+01 +.12E-02 −.41E-04
(**Obs.** 77) tOpa112	KKMC 4.13	206.00	.2393E+01 ±.94E-02 ±.48E-01
(**Obs.** 78) tOpa113	ZFITTER v6.23	189.00	.3191E+01 +.47E-03 −.59E-04
(**Obs.** 78) tOpa113	KKMC 4.13	189.00	.3412E+01 ±.17E-01 ±.68E-01
(**Obs.** 78) tOpa113	ZFITTER v6.23	200.00	.2817E+01 +.42E-03 −.51E-04
(**Obs.** 78) tOpa113	KKMC 4.13	200.00	.2998E+01 ±.92E-02 ±.60E-01
(**Obs.** 78) tOpa113	ZFITTER v6.23	206.00	.2641E+01 +.40E-03 −.48E-04
(**Obs.** 78)	KKMC 4.13	206.00	.2800E+01 ±.11E-01 ±.56E-01

Table 29: Numerical predictions from theoretical calculations of the following idealized observables in $\tau^+\tau^-(\gamma)$ final states; asymmetries. Warning: this is at work version, misprints are expected.

obs	program	ene	value and error estimate
(**Obs.** 68) tA1eph7	ZFITTER v6.23	189.00	.2903E+00 +.11E-02 −.13E-04
(**Obs.** 68) tA1eph7	KKMC 4.13	189.00	.2990E+00 ±.27E-02 ±.20E-01
(**Obs.** 68) tA1eph7	ZFITTER v6.23	200.00	.2897E+00 +.12E-02 −.12E-04
(**Obs.** 68) tA1eph7	KKMC 4.13	200.00	.2994E+00 ±.17E-02 ±.20E-01
(**Obs.** 68) tA1eph7	ZFITTER v6.23	206.00	.2893E+00 +.12E-02 −.11E-04
(**Obs.** 68) tA1eph7	KKMC 4.13	206.00	.2987E+00 ±.22E-02 ±.20E-01
(**Obs.** 69) tA1eph8	ZFITTER v6.23	189.00	.5730E+00 +.50E-04 −.17E-03
(**Obs.** 69) tA1eph8	KKMC 4.13	189.00	.5706E+00 ±.63E-02 ±.20E-01
(**Obs.** 69) tA1eph8	ZFITTER v6.23	200.00	.5631E+00 +.56E-04 −.17E-03
(**Obs.** 69) tA1eph8	KKMC 4.13	200.00	.5582E+00 ±.39E-02 ±.20E-01
(**Obs.** 69) tA1eph8	ZFITTER v6.23	206.00	.5583E+00 +.58E-04 −.17E-03
(**Obs.** 69) tA1eph8	KKMC 4.13	206.00	.5535E+00 ±.50E-02 ±.20E-01
(**Obs.** 70) tDelphi7	ZFITTER v6.23	189.00	.3175E+00 +.97E-03 −.13E-04
(**Obs.** 70) tDelphi7	KKMC 4.13	189.00	.3199E+00 ±.29E-02 ±.20E-01
(**Obs.** 70) tDelphi7	ZFITTER v6.23	200.00	.3160E+00 +.10E-02 −.12E-04
(**Obs.** 70) tDelphi7	KKMC 4.13	200.00	.3189E+00 ±.18E-02 ±.20E-01
(**Obs.** 70) tDelphi7	ZFITTER v6.23	206.00	.3151E+00 +.10E-02 −.12E-04
(**Obs.** 70) tDelphi7	KKMC 4.13	206.00	.3174E+00 ±.23E-02 ±.20E-01
(**Obs.** 71) tDelphi8	ZFITTER v6.23	189.00	.5692E+00 +.23E-04 −.11E-03
(**Obs.** 71) tDelphi8	KKMC 4.13	189.00	.5682E+00 ±.60E-02 ±.20E-01
(**Obs.** 71) tDelphi8	ZFITTER v6.23	200.00	.5589E+00 +.27E-04 −.11E-03
(**Obs.** 71) tDelphi8	KKMC 4.13	200.00	.5561E+00 ±.37E-02 ±.20E-01
(**Obs.** 71) tDelphi8	ZFITTER v6.23	206.00	.5539E+00 +.29E-04 −.11E-03
(**Obs.** 71) tDelphi8	KKMC 4.13	206.00	.5513E+00 ±.48E-02 ±.20E-01
(**Obs.** 72) tLT12	ZFITTER v6.23	189.00	.3114E+00 +.10E-02 −.12E-04

Table 30: Numerical predictions from theoretical calculations of the following idealized observables in $\tau^+\tau^-(\gamma)$ final states; asymmetries. Warning: this is at work version, misprints are expected.

obs.	program	ene	value and error estimate
(**Obs.** (2) $_{\texttt{tLLT1}_2}$	KKMC 4.13	189.00	.3168E+00 ±.29E-02 ±.20E-01
(**Obs.** (2) $_{\texttt{tLLT1}_2}$	ZFITTER v6.23	200.00	.3099E+00 +.11E-02 −.11E-04
(**Obs.** (2) $_{\texttt{tLLT1}_2}$	KKMC 4.13	200.00	.3165E+00 ±.18E-02 ±.20E-01
(**Obs.** (2) $_{\texttt{tLLT1}_2}$	ZFITTER v6.23	206.00	.3090E+00 +.11E-02 −.11E-04
(**Obs.** (2) $_{\texttt{tLLT1}_2}$	KKMC 4.13	206.00	.3145E+00 ±.23E-02 ±.20E-01
(**Obs.** (3) $_{\texttt{tLLT1}_3}$	ZFITTER v6.23	189.00	.5593E+00 +.24E-04 −.16E-03
(**Obs.** (3) $_{\texttt{tLLT1}_3}$	KKMC 4.13	189.00	.5577E+00 ±.60E-02 ±.20E-01
(**Obs.** (3) $_{\texttt{tLLT1}_3}$	ZFITTER v6.23	200.00	.5491E+00 +.28E-04 −.16E-03
(**Obs.** (3) $_{\texttt{tLLT1}_3}$	KKMC 4.13	200.00	.5466E+00 ±.37E-02 ±.20E-01
(**Obs.** (3) $_{\texttt{tLLT1}_3}$	ZFITTER v6.23	206.00	.5443E+00 +.29E-04 −.16E-03
(**Obs.** (3) $_{\texttt{tLLT1}_3}$	KKMC 4.13	206.00	.5406E+00 ±.47E-02 ±.20E-01
(**Obs.** (4) $_{\texttt{tLLT1}_4}$	ZFITTER v6.23	189.00	.5870E+00 +.11E-03 −.52E-04
(**Obs.** (4) $_{\texttt{tLLT1}_4}$	KKMC 4.13	189.00	.5847E+00 ±.61E-02 ±.20E-01
(**Obs.** (4) $_{\texttt{tLLT1}_4}$	ZFITTER v6.23	200.00	.5765E+00 +.12E-03 −.52E-04
(**Obs.** (4) $_{\texttt{tLLT1}_4}$	KKMC 4.13	200.00	.5735E+00 ±.38E-02 ±.20E-01
(**Obs.** (4) $_{\texttt{tLLT1}_4}$	ZFITTER v6.23	206.00	.5715E+00 +.12E-03 −.51E-04
(**Obs.** (4) $_{\texttt{tLLT1}_4}$	KKMC 4.13	206.00	.5682E+00 ±.49E-02 ±.20E-01
(**Obs.** (5) $_{\texttt{tOpa1l0}}$	ZFITTER v6.23	189.00	.2855E+00 +.11E-02 −.11E-04
(**Obs.** (5) $_{\texttt{tOpa1l0}}$	KKMC 4.13	189.00	.2976E+00 ±.27E-02 ±.20E-01
(**Obs.** (5) $_{\texttt{tOpa1l0}}$	ZFITTER v6.23	200.00	.2848E+00 +.12E-02 −.10E-04
(**Obs.** (5) $_{\texttt{tOpa1l0}}$	KKMC 4.13	200.00	.2986E+00 ±.17E-02 ±.20E-01
(**Obs.** (5) $_{\texttt{tOpa1l0}}$	ZFITTER v6.23	206.00	.2843E+00 +.12E-02 −.95E-05
(**Obs.** (5) $_{\texttt{tOpa1l0}}$	KKMC 4.13	206.00	.2975E+00 ±.22E-02 ±.20E-01
(**Obs.** (6) $_{\texttt{tOpa1l1}}$	ZFITTER v6.23	189.00	.2781E+00 +.17E-03 −.14E-04
(**Obs.** (6)	KKMC 4.13	189.00	.2921E+00 ±.26E-02 ±.20E-01

Table 31: Numerical predictions from theoretical calculations of the following idealized observables in $\tau^+\tau^-(\gamma)$ final states; asymmetries. Warning: this is at work version, misprints are expected.

obs.	program	ene	value and error estimate
(**Obs.** 76) tOpa111	ZFITTER v6.23	200.00	.2766E+00 +.18E-03 −.14E-04
(**Obs.** 76) tOpa111	KKMC 4.13	200.00	.2914E+00 ±.16E-02 ±.20E-01
(**Obs.** 76) tOpa111	ZFITTER v6.23	206.00	.2757E+00 +.18E-03 −.13E-04
(**Obs.** 76) tOpa111	KKMC 4.13	206.00	.2897E+00 ±.21E-02 ±.20E-01
(**Obs.** 77) tOpa112	ZFITTER v6.23	189.00	.5374E+00 +.29E-05 −.15E-03
(**Obs.** 77) tOpa112	KKMC 4.13	189.00	.5267E+00 ±.56E-02 ±.20E-01
(**Obs.** 77) tOpa112	ZFITTER v6.23	200.00	.5270E+00 +.27E-05 −.15E-03
(**Obs.** 77) tOpa112	KKMC 4.13	200.00	.5174E+00 ±.34E-02 ±.20E-01
(**Obs.** 77) tOpa112	ZFITTER v6.23	206.00	.5219E+00 +.26E-05 −.15E-03
(**Obs.** 77) tOpa112	KKMC 4.13	206.00	.5125E+00 ±.44E-02 ±.20E-01
(**Obs.** 78) tOpa113	ZFITTER v6.23	189.00	.5685E+00 +.91E-04 −.51E-04
(**Obs.** 78) tOpa113	KKMC 4.13	189.00	.5598E+00 ±.57E-02 ±.20E-01
(**Obs.** 78) tOpa113	ZFITTER v6.23	200.00	.5575E+00 +.90E-04 −.50E-04
(**Obs.** 78) tOpa113	KKMC 4.13	200.00	.5508E+00 ±.35E-02 ±.20E-01
(**Obs.** 78) tOpa113	ZFITTER v6.23	206.00	.5522E+00 +.90E-04 −.50E-04
(**Obs.** 78)	KKMC 4.13	206.00	.5463E+00 ±.45E-02 ±.20E-01

2.14 $q\overline{q}\gamma$'s observables

This section contains precision requirements for regions of phase space selected in searches for anomalous neutral gauge couplings (ZZγ,Z$\gamma\gamma$), especially for the visible $\gamma + Z \to \gamma + q\overline{q}$ final state. The prominent Standard Model process populating this region is the radiative return to the Z.

2.141 ALEPH

2.142 DELPHI

2.143 L3

79. **Total Cross-Section** `LT13`
    Hadronic events are selected by asking for more than 6 charged tracks and more than 11 calorimetric clusters. The transverse and longitudinal energy imbalances should be below 15 and 20 % respectively. A photon with $14^\circ < \theta_\gamma < 166^\circ$ and energy $80\ GeV < \sqrt{s - 2E_\gamma\sqrt{s}} < 110\ GeV$ is required.
    The precision tag is 0.3-0.4 %.

2.144 OPAL

80. **Total Cross-Section** `Opal10`
    Events are selected with two jets and one visible photon, fulfilling a 3C kinematic fit, allowing an additional photon of at most 5% of the beam energy to be radiated along the beam pipe. The relevant phase space is defined by
    (a) $E_\gamma > 50$ GeV
    (b) $15^\circ < \theta_\gamma < 165^\circ$
    (c) angle between photon and closest jet, $\alpha_{\gamma-jet} > 30^\circ$.
    The required precision tag for the radiative return cross-section in this phase space for a LEP-combined analysis is 0.3%. It assumes uncorrelated experimental systematics and is dominated by the experimental statistics. For fully correlated experimental systematics the precision tag is 0.4%. Currently, the predictions of the KK2f and PYTHIA generators in this phase space differ by 15%.

81. **Differential Cross-Section** `Opal11` Selected Events in the above phase space are subjected to a maximum likelihood fit in the differential distributions of E_γ, $\cos\theta_\gamma$, and $\cos\theta_q^\star$, which is the angle of the final state quark-jets in the Z restframe.
    Precision tag: The ratio between the rate of events within $|\cos\theta_\gamma| < 0.7$ and the boundary of the acceptance has to be known within about 1%. The discrepancy between KK2f and Pythia in the region $|\cos\theta_\gamma| < 0.7$ is currently at level of 30% (stat. significance 2σ).

2.145 Numerical results

Table 32: Numerical predictions from theoretical calculations of the following realistic observables in $q\bar{q}\gamma$ final states. Warning: this is at work version, misprints are expected.

obs.	program	ene	value and error estimate

2.15 $\ell\bar{\ell}\gamma$'s observables

This final state is primarily used to search for single (one photon) or pair (two photons) production of excited leptons. In principle, the radiative return peak in $M_{\ell\ell}$ can be rejected in these searches, which is not possible in the anomalous neutral coupling analyses (see above). In practice, however, experimental resolution and further (final state) radiation reduce the power of such an anticut. Thererfore both topologies (inlcuding and omitting the anticut) need to be studied.

2.151 ALEPH

Photons are required to have at least 15 GeV and $|\cos\theta_\gamma|$ <0.95. Leptons are also required to have $|\cos\theta_l|$ <0.95, they are identified from oppositely charged jet clustered from charged tracks (at most 3) with at least one jet above 5 GeV. The angle between the 2 jets being at least 30 degree and the photon candidate at least 10 degree from the charged jet.

- $\ell\bar{\ell}$ **plus one photon**
  Two leptons of same type are required accompagnied of at least one photon. The two most energetic leptons and the most energetic photon are considered.
  82. **ee**γ ALEPH-11 Two tracks are identified as electron and the sum of charged energy greater than 0.6*E_{cm}. Expected precision tag 1.4%
  83. $\mu\mu\gamma$ ALEPH-12 Two muon have to be identied and charged energy greater than $0.6 * E_{cm}$. Precision tag 1.5%
  84. $\tau\tau\gamma$ ALEPH-13 At least a 3 prong tau jet cancdidate in the event is required. Precision tag 1.7%
- $\ell\bar{\ell}$ **plus two photons**
  Two electron (or muon or tau) have to be identified and 2 photon jet identified Isolation cut on the 4 angles between photon and lepton required to be at least 20 degree . The 2 reconstructed lepton-gamma masses have to agree within 5 GeV/c**2
  85. **ee**$\gamma\gamma$ ALEPH-14 Precision tag 3 %.
  86. $\mu\mu\gamma\gamma$ ALEPH-15 Precision tag 5 %.
  87. $\tau\tau\gamma\gamma$ ALEPH-16 Precision tag 8 %.

2.152 DELPHI

Events with a visible energy above $0.2\sqrt{s}$ in the region $|cos\theta| < 0.9397$ and not more than 6 charged tracks are selected.

Photons are required to have an energy above 5 GeV and $|cos\theta| < 0.9848$. The total charged (neutral) energy in a 15° cone around the photon is required to be below 1 GeV (2 GeV).

Charged particles are clustered into jets. Jets are required to have $|cos\theta| < 0.9063$ and the most energetic to have an energy above 5 GeV.

In three- or four-body topologies the energies are rescaled by imposing energy momentum conservation and using the measured angles. An improved energy resolution is obtained. Compatibility between the measured and rescaled energies is required.

- $\ell\bar{\ell}$ **plus one photon**

  The energy of the photon is required to be above 10 GeV. Events with one photon and one or two jets in the final state are considered. This accounts for situations in which one of the leptons was lost in the beam pipe or has very low momentum (close to the kinematic limit).
  The expected backgrounds and thus the further selections and the efficiencies are very dependent on the lepton flavour.
  88. **ee**γ DELPHI8 At least one jet identified as an electron and the other not indentified as a muon, the most energetic jet with E¿10 GeV. One photon with $|cos\theta| < 0.7660$. If only one jet is found the angle between the jet and the photon must be in the between 100° and 179°.

Precision tag: 1.4

89. $\mu\mu\gamma$ DELPHI9 At least one jet identified as a muon and no jets indentified as electrons, the most energetic jet with E¿10 GeV.
    Precision tag: 1.6
90. $\tau\tau\gamma$ DELPHI10 The most energetic jet with E¿10 GeV. One photon with $|cos\theta| < 0.94$. Tau events are selected by requiring a difference between the measured and the rescaled energy of the jets, expected due to the presence of neutrinos. If only one jet is found the angle between the jet and the photon must be between 100° and 179°.
    Precision tag: 1.8

- $\ell\bar{\ell}$ **plus two photons** Two jets and two photons in the event. Compatibility between two reconstructed $\ell\gamma$ masses is required. The relevant quantity is the minimum of the electron-photon invariant mass differences ($\Delta m_{\ell\gamma}$).
  The statistical error of the selection clearly dominates.
  91. **ee**$\gamma\gamma$ DELPHI11 $\Delta m_{e\gamma} < 15$ GeV/c^2.
      Precision tag: 5
  92. $\mu\mu\gamma\gamma$ DELPHI12 $\Delta m_{\mu\gamma} < 10$ GeV/c^2
      Precision tag: 6
  93. $\tau\tau\gamma\gamma$ DELPHI13 $\Delta m_{\tau\gamma} < 10$ GeV/c^2
      Precision tag: 6

2.153 L3

Hadronic events are rejected by requiring less than 8 charged tracks. Photons are required to have at least 15 GeV and $|\cos\theta_\gamma|$ <0.97. Leptons are required to have the same flavor and $|\cos\theta_l|$ <0.94. They are identified from oppositely charged tracks/low multiplicity jets(for tau).

- $\ell\bar{\ell}$ **plus one photon**
  At least one lepton accompagnied by at most one hard photon. The second lepton may be in the beam pipe. The photon is required to have at least 20 GeV and $|\cos\theta_\gamma|$ <0.75. One mass (lγ) should be greater than 70 GeV.
  94. **ee**γ LT14 Expected precision tag 1.2%
  95. $\mu\mu\gamma$ LT15 Precision tag 1.5%
  96. $\tau\tau\gamma$ LT16 Precision tag 1.8%
- $\ell\bar{\ell}$ **plus two photons**
  Two electron (or muon or tau) and 2 photons seen. The photons are required to have at least 15 GeV and one should be in $|\cos\theta_\gamma|$ <0.75, the second in $|\cos\theta_\gamma|$ <0.94. The difference of the 2 reconstructed lepton-gamma masses should be below 10 GeV and their sum above 100 GeV.
  97. **ee**$\gamma\gamma$ LT17 Precision tag 4 %.
  98. $\mu\mu\gamma\gamma$ LT18 Precision tag 6 %.
  99. $\tau\tau\gamma\gamma$ LT19 Precision tag 6 %.

2.154 OPAL

Photon candidates are required to have an energy exceeding 5% of the beam energy, and have to lie within $|\cos\theta_\gamma| < 0.95$. Leptons are also identified within $|\cos\theta_\ell| < 0.95$ and are required to have $p_t > 1$ GeV. Photons, electrons, and muons are required to be isolated by at least 20 degree from the nearest charged track of momentum larger than 1 GeV.

- $\ell\bar{\ell}$ **plus one photon**
  There must be at least two identified leptons of the same type and at least one photon. The two most energetic leptons and the most energetic photon are used in the analysis. Their energy sum is called E_{vis} in the following. The precision tag assumes uncorrelated experimental systematics. Both, uncorrelated systematics, and expected statistical precision of the LEP combined result contribute to about equal parts to the precision tag. For correlated experimental systematics the required theoretical precision is loosened by about 0.9%.

100. **eeγ w/o rad return rejection** `Opal12`
   - $E_{vis} > 1.6 E_{\rm beam}$
   - $|\cos\theta_\gamma| < 0.70$
   - $|\cos\theta_e| < 0.70$ for at least one electron.

   Precision tag: 1.3%
101. **eeγ with rad return rejection** `Opal13`
   - $E_{vis} > 1.6 E_{\rm beam}$
   - $|\cos\theta_\gamma| < 0.70$
   - $|\cos\theta_e| < 0.70$ for at least one electron.
   - reject events with 85 GeV $< M_{ee} <$ 95 GeV

   Precision tag: 1.4%
102. **$\mu\mu\gamma$ w/o rad return rejection** `Opal14`
   - $E_{vis} > 1.6 E_{\rm beam}$

   Precision tag: 1.5%
103. **$\mu\mu\gamma$ with rad return rejection** `Opal15`
   - $E_{vis} > 1.6 E_{\rm beam}$
   - reject events with 85 GeV $< M_{\mu\mu} <$ 95 GeV

   Precision tag: 1.7%
104. **$\tau\tau\gamma$ w/o rad return rejection** `Opal16`
   - $0.8 E_{\rm beam} < E_{vis} > 1.9 E_{\rm beam}$

   Precision tag: 1.4%
105. **$\tau\tau\gamma$ with rad return rejection** `Opal17`
   - $0.8 E_{\rm beam} < E_{vis} < 1.9 E_{\rm beam}$
   - reject events with 85 GeV $< M_{\tau\tau} <$ 95 GeV

   Precision tag: 1.5%

- **$\ell\bar{\ell}$plus two photons**

There must be at least two identified leptons of the same type and at least two photons. The two most energetic leptons and the two most energetic photons are used in the analysis. Their energy sum is called E_{vis} in the following. The precision tag is dominated by the statistical error of the selection.

106. **ee$\gamma\gamma$ w/o rad return rejection** `Opal18`
   - $E_{vis} > 1.6 E_{\rm beam}$
   - minimum opening angle among all electron-photon combinations $|\cos\alpha^{e\gamma}_{\rm min}| < 0.90$.

   Precision tag: 3%
107. **ee$\gamma\gamma$ with rad return rejection** `Opal19`
   - $E_{vis} > 1.6 E_{\rm beam}$
   - minimum opening angle among all electron-photon combinations $|\cos\alpha^{e\gamma}_{\rm min}| < 0.90$.
   - reject events with 85 GeV $< M_{ee} <$ 95 GeV

   Precision tag: 3%
108. **$\mu\mu\gamma\gamma$ w/o rad return rejection** `Opal20`
   - $E_{vis} > 1.6 E_{\rm beam}$

   Precision tag: 5%
109. **$\mu\mu\gamma\gamma$ with rad return rejection** `Opal21`
   - $E_{vis} > 1.6 E_{\rm beam}$
   - reject events with 85 GeV $< M_{\mu\mu} <$ 95 GeV

   Precision tag: 6%
110. **$\tau\tau\gamma\gamma$ w/o rad return rejection** `Opal22`
   - $0.8 E_{\rm beam} < E_{vis} > 1.9 E_{\rm beam}$

   Precision tag: 5%
111. **$\tau\tau\gamma\gamma$ with rad return rejection** `Opal23`
   - $0.8 E_{\rm beam} < E_{vis} < 1.9 E_{\rm beam}$
   - reject events with 85 GeV $< M_{\tau\tau} <$ 95 GeV

   Precision tag: 6%

2.155 Numerical results

Table 33: Numerical predictions from theoretical calculations of the following realistic observables in $l^+l^-\gamma$ final states. Warning: this is at work version, misprints are expected.

obs.	program	ene	value and error estimate
(**Obs.** ALEPH-11 82)	LABSMC	189.00	.3674E-02 ±.10E-04 ±.92E-05
(**Obs.** ALEPH-11 82)	BHWIDE	189.00	.3551E+01 ±.70E-01 ±.00E+00
(**Obs.** ALEPH-11 82)	LABSMC	200.00	.3366E-02 ±.93E-05 ±.84E-05
(**Obs.** ALEPH-11 82)	BHWIDE	200.00	.3250E+01 ±.61E-01 ±.00E+00
(**Obs.** ALEPH-11 82)	LABSMC	206.00	.3237E-02 ±.88E-05 ±.81E-05
(**Obs.** ALEPH-11 82)	BHWIDE	206.00	.3039E+01 ±.53E-01 ±.00E+00
(**Obs.** DELPHI8 88)	LABSMC	189.00	.0000E+00 ±.00E+00 ±.00E+00
(**Obs.** DELPHI8 88)	LABSMC	200.00	.0000E+00 ±.00E+00 ±.00E+00
(**Obs.** DELPHI8 88)	LABSMC	206.00	.0000E+00 ±.00E+00 ±.00E+00
(**Obs.** LT14 94)	LABSMC	189.00	.0000E+00 ±.00E+00 ±.00E+00
(**Obs.** LT14 94)	LABSMC	200.00	.0000E+00 ±.00E+00 ±.00E+00
(**Obs.** LT14 94)	LABSMC	206.00	.0000E+00 ±.00E+00 ±.00E+00
(**Obs.** Opal12 100)	LABSMC	189.00	.6362E-03 ±.43E-05 ±.16E-05
(**Obs.** Opal12 100)	BHWIDE	189.00	.9375E+00 ±.45E-01 ±.00E+00
(**Obs.** Opal12 100)	LABSMC	200.00	.5573E-03 ±.38E-05 ±.14E-05
(**Obs.** Opal12 100)	BHWIDE	200.00	.7995E+00 ±.35E-01 ±.00E+00
(**Obs.** Opal12 100)	LABSMC	206.00	.5239E-03 ±.36E-05 ±.13E-05
(**Obs.** Opal12 100)	BHWIDE	206.00	.7102E+00 ±.28E-01 ±.00E+00
(**Obs.** Opal13 101)	LABSMC	189.00	.6163E-03 ±.42E-05 ±.15E-05
(**Obs.** Opal13 101)	BHWIDE	189.00	.8059E+00 ±.30E-01 ±.00E+00
(**Obs.** Opal13 101)	LABSMC	200.00	.5485E-03 ±.37E-05 ±.14E-05
(**Obs.** Opal13 101)	BHWIDE	200.00	.6653E+00 ±.24E-01 ±.00E+00
(**Obs.** Opal13 101)	LABSMC	206.00	.5164E-03 ±.35E-05 ±.13E-05
(**Obs.** Opal13 101)	BHWIDE	206.00	.6326E+00 ±.24E-01 ±.00E+00
(**Obs.** ALEPH-14 85)	BHWIDE	189.00	.1770E-01 ±.99E-02 ±.00E+00

Table 34: Numerical predictions from theoretical calculations of the following realistic observables in $l^+l^-\gamma$ final states. Warning: this is at work version, misprints are expected.

obs	program	ene	value and error estimate
(**Obs.** ALEPH-14 85)	BHWIDE	200.00	.3100E-02 ±.11E-02 ±.00E+00
(**Obs.** ALEPH-14 85)	BHWIDE	206.00	.2900E-02 ±.11E-02 ±.00E+00
(**Obs.** LT17 97)	BHWIDE	189.00	.1240E-01 ±.27E-02 ±.00E+00
(**Obs.** LT17 97)	BHWIDE	200.00	.1610E-01 ±.33E-02 ±.00E+00
(**Obs.** LT17 97)	BHWIDE	206.00	.1750E-01 ±.43E-02 ±.00E+00
(**Obs.** Opal18 I06)	BHWIDE	189.00	.1026E+00 ±.18E-01 ±.00E+00
(**Obs.** Opal18 I06)	BHWIDE	200.00	.6840E-01 ±.11E-01 ±.00E+00
(**Obs.** Opal18 I06)	BHWIDE	206.00	.5950E-01 ±.97E-02 ±.00E+00
(**Obs.** Opal19 I07)	BHWIDE	189.00	.9140E-01 ±.17E-01 ±.00E+00
(**Obs.** Opal19 I07)	BHWIDE	200.00	.6440E-01 ±.11E-01 ±.00E+00
(**Obs.** Opal19 I07)	BHWIDE	206.00	.5580E-01 ±.96E-02 ±.00E+00

2.16 $\nu\bar{\nu}\gamma$'s observables

The observables listed above do not include the cut to eliminate regions of phase space dominated by the radiative return to Z. Such events are usually selected by the cut-off on the mass of the invisible system, or, cut on photon energy in simpler, single photon case. For such extended observables the required precision tag should be simply scaled with the somewhat decreased statistics. In general we need however complete predictions on full multi-dimensional distributions of photon(s) energies and directions, with the precision up to the level of the one for indicated for the total cross section.

2.161 ALEPH

Nu1

112. Single photon and missing energy:
Exactly one photon with $|\cos\theta_\gamma| < 0.95$ and $P_t > 0.0375 * E_{cm}$. No other photon with $|\cos\theta_\gamma| < 0.9997$ and $E_\gamma > 1$ GeV. No additional pair cut$_{ALEPH}$. The required precision tag is 0.2%.

Nu2

113. Two or more photons and missing energy:
Two or more photons with $E_\gamma > 1$ GeV and $|\cos\theta_\gamma| < 0.95$. The transverse momentum of the multi-photon system must be such that $\Sigma P_t > 0.0375 * (E_{cm} - \Sigma E)$. No additional pair cut$_{ALEPH}$. No other photon with $|\cos\theta_\gamma|$ between 0.95 and 0.9997, and $E_\gamma > 1$ GeV. The required precision tag is 1%.

2.162 DELPHI

114. Single photon and missing energy:
Require exactly one observed photon with energy requirement varying with polar angle

Nu11

(a) Very forward (or backward): $3.8^o < \theta < 6.5^o$, $x_\gamma > 0.3$ or $x_\gamma > (9.2 - \theta)/9$

Nu12

(b) Forward (or backward): $12^o < \theta < 32^o$, $x_\gamma > 0.2$

Nu13

(c) Barrel: $45^o < \theta < 90^o$, $x_\gamma > 0.06$
No additional photons with $E_\gamma > 0.8$ GeV and $\theta > 38$ mrad unless they are within 3^o, 15^o and 20^o from the primary photon for the very forward, forward and barrel regions, respectively. The required precision tag is 0.2%.

115. Two or more photons and missing energy Nu14 :
Two or more photons with $E_\gamma > 0.05 * E_{beam}$ and $|\cos\theta_\gamma| < 0.985$, at least one of which has $|\cos\theta_\gamma| < 0.906$. No additional photon with $E_\gamma > 0.02 * E_{cm}$ and $|\cos\theta_\gamma|$ between 0.985 and 0.9994. The required precision tag is 1%.

2.163 L3

116. Single photon and missing energy:
Exactly one photon with $|\cos\theta_\gamma| < 0.97$ and:

Nu3

(a) energy and $P_t > 5GeV$ if $14^o < \theta$ /endcaps/

Nu4

(b) energy and $P_t > 1GeV$ if $43^o < \theta$ /barrel/
No other photon with $|\cos\theta_\gamma| < 0.9997$ and $E_\gamma > 10$ GeV.
The required precision tag is 0.2%.

117. Two or more photons and missing energy:
Two or more photons selected as above:

Nu5

(a) energy and $P_t > 5GeV$ if $14^o < \theta$ /endcaps/

Nu6

(b) energy and $P_t > 1GeV$ if $43^o < \theta$ /barrel/

The transverse momentum of the multi-photon system must be greater than 5 GeV and the a-collinearity greater than 2.5 deg.

No other photon with $|\cos\theta_\gamma| < 0.9997$ and $E_\gamma > 10$ GeV.

The required precision tag is 1%.

2.164 OPAL

118. Single photon and missing energy Nu7 :

The maximum energy photon must have $|\cos\theta_\gamma| < 0.9660$ and $P_t > 0.05 * E_{beam}$. There may be at most ONE additional photon with $E_\gamma > 0.3$ GeV and $|\cos\theta_\gamma| < 0.9848$. The required precision tag is 0.2%.

119. Two or more photons and missing energy:

Two or more photons with

Nu8

(a) $E_\gamma > 0.05 * E_{beam}$ and $|\cos\theta_\gamma| < 0.9660$,

Nu9

(b) $E_\gamma > 1.75$ GeV, $|\cos\theta_\gamma| < 0.8$ and the sum of their $P_t > 0.05 * E_{beam}$

Nu10

(c) $E_\gamma > 1.75$ GeV, $|\cos\theta_\gamma| < 0.966$ and the sum of their $P_t > 0.05 * E_{beam}$

The required precision tag is 1%.

2.165 Numerical results

Table 35: Numerical predictions from theoretical calculations of the following realistic observables in $\nu\bar{\nu}\gamma$ final states. Warning: this is at work version, misprints are expected.

obs.	program	ene	value and error estimate
(**Obs.** $\overline{\text{TI2}}^{\text{Nu1}}$)	NUNUGPV	183.00	.3517E+01 ±.21E-02 ±.00E+00
(**Obs.** $\overline{\text{TI2}}^{\text{Nu1}}$)	NUNUGPV	189.00	.3256E+01 ±.20E-02 ±.00E+00
(**Obs.** $\overline{\text{TI2}}^{\text{Nu1}}$)	NUNUGPV	200.00	.2871E+01 ±.18E-02 ±.00E+00
(**Obs.** $\overline{\text{TI2}}^{\text{Nu1}}$)	grcnunugam	200.00	.2792E+01 ±.20E-02 ±.00E+00
(**Obs.** $\overline{\text{TI4a}}^{\text{Nu11}}$)	NUNUGPV	183.00	.9606E+00 ±.80E-03 ±.00E+00
(**Obs.** $\overline{\text{TI4a}}^{\text{Nu11}}$)	NUNUGPV	189.00	.8895E+00 ±.90E-03 ±.00E+00
(**Obs.** $\overline{\text{TI4a}}^{\text{Nu11}}$)	NUNUGPV	200.00	.7812E+00 ±.80E-03 ±.00E+00
(**Obs.** $\overline{\text{TI4a}}^{\text{Nu11}}$)	grcnunugam	200.00	.7050E+00 ±.10E-02 ±.00E+00
(**Obs.** $\overline{\text{TI4b}}^{\text{Nu12}}$)	NUNUGPV	183.00	.1918E+01 ±.30E-02 ±.00E+00
(**Obs.** $\overline{\text{TI4b}}^{\text{Nu12}}$)	NUNUGPV	189.00	.1778E+01 ±.38E-02 ±.00E+00
(**Obs.** $\overline{\text{TI4b}}^{\text{Nu12}}$)	NUNUGPV	200.00	.1566E+01 ±.29E-02 ±.00E+00
(**Obs.** $\overline{\text{TI4b}}^{\text{Nu12}}$)	grcnunugam	200.00	.1530E+01 ±.10E-02 ±.00E+00
(**Obs.** $\overline{\text{TI4c}}^{\text{Nu13}}$)	NUNUGPV	183.00	.1858E+01 ±.28E-02 ±.00E+00
(**Obs.** $\overline{\text{TI4c}}^{\text{Nu13}}$)	NUNUGPV	189.00	.1724E+01 ±.19E-02 ±.00E+00
(**Obs.** $\overline{\text{TI4c}}^{\text{Nu13}}$)	NUNUGPV	200.00	.1528E+01 ±.16E-02 ±.00E+00
(**Obs.** $\overline{\text{TI4c}}^{\text{Nu13}}$)	grcnunugam	200.00	.1477E+01 ±.10E-02 ±.00E+00
(**Obs.** $\overline{\text{TI6a}}^{\text{Nu3}}$)	NUNUGPV	183.00	.4589E+01 ±.29E-02 ±.00E+00
(**Obs.** $\overline{\text{TI6a}}^{\text{Nu3}}$)	NUNUGPV	189.00	.4291E+01 ±.28E-02 ±.00E+00
(**Obs.** $\overline{\text{TI6a}}^{\text{Nu3}}$)	NUNUGPV	200.00	.3853E+01 ±.26E-02 ±.00E+00
(**Obs.** $\overline{\text{TI6a}}^{\text{Nu3}}$)	grcnunugam	200.00	.2091E+01 ±.20E-02 ±.00E+00
(**Obs.** $\overline{\text{TI6b}}^{\text{Nu4}}$)	NUNUGPV	183.00	.6544E+01 ±.61E-02 ±.00E+00
(**Obs.** $\overline{\text{TI6b}}^{\text{Nu4}}$)	NUNUGPV	189.00	.6195E+01 ±.58E-02 ±.00E+00
(**Obs.** $\overline{\text{TI6b}}^{\text{Nu4}}$)	NUNUGPV	200.00	.5697E+01 ±.54E-02 ±.00E+00
(**Obs.** $\overline{\text{TI6b}}^{\text{Nu4}}$)	grcnunugam	200.00	.2364E+01 ±.30E-02 ±.00E+00
(**Obs.** $\overline{\text{TI8}}^{\text{Nu7}}$)	NUNUGPV	183.00	.4971E+01 ±.15E-01 ±.00E+00

Table 36: Numerical predictions from theoretical calculations of the following realistic observables in $\nu\bar{\nu}\gamma$ final states. Warning: this is at work version, misprints are expected.

obs.	program	ene	value and error estimate
(**Obs.** $\overline{\text{Nu7}}$ I18)	NUNUGPV	189.00	.4614E+01 ±.14E-01 ±.00E+00
(**Obs.** $\overline{\text{Nu7}}$ I18)	NUNUGPV	200.00	.4071E+01 ±.10E-01 ±.00E+00
(**Obs.** $\overline{\text{Nu7}}$ I18)	grcnunugam	200.00	.4067E+01 ±.30E-02 ±.00E+00
(**Obs.** $\overline{\text{Nu2}}$ I13)	NUNUGPV	183.00	.2308E+00 ±.70E-03 ±.00E+00
(**Obs.** $\overline{\text{Nu2}}$ I13)	NUNUGPV	189.00	.2165E+00 ±.80E-03 ±.00E+00
(**Obs.** $\overline{\text{Nu2}}$ I13)	NUNUGPV	200.00	.1937E+00 ±.90E-03 ±.00E+00
(**Obs.** $\overline{\text{Nu14}}$ I15)	NUNUGPV	183.00	.1289E+00 ±.15E-03 ±.00E+00
(**Obs.** $\overline{\text{Nu14}}$ I15)	NUNUGPV	189.00	.1223E+00 ±.20E-03 ±.00E+00
(**Obs.** $\overline{\text{Nu14}}$ I15)	NUNUGPV	200.00	.1121E+00 ±.20E-03 ±.00E+00
(**Obs.** $\overline{\text{Nu5}}$ I17a)	NUNUGPV	183.00	.1257E+00 ±.90E-03 ±.00E+00
(**Obs.** $\overline{\text{Nu5}}$ I17a)	NUNUGPV	189.00	.1197E+00 ±.90E-03 ±.00E+00
(**Obs.** $\overline{\text{Nu5}}$ I17a)	NUNUGPV	200.00	.1070E+00 ±.90E-03 ±.00E+00
(**Obs.** $\overline{\text{Nu6}}$ I17b)	NUNUGPV	183.00	.3268E+00 ±.63E-02 ±.00E+00
(**Obs.** $\overline{\text{Nu6}}$ I17b)	NUNUGPV	189.00	.3113E+00 ±.77E-02 ±.00E+00
(**Obs.** $\overline{\text{Nu6}}$ I17b)	NUNUGPV	200.00	.2689E+00 ±.53E-02 ±.00E+00
(**Obs.** $\overline{\text{Nu8}}$ I19a)	NUNUGPV	183.00	.1811E+00 ±.30E-03 ±.00E+00
(**Obs.** $\overline{\text{Nu8}}$ I19a)	NUNUGPV	189.00	.1679E+00 ±.40E-03 ±.00E+00
(**Obs.** $\overline{\text{Nu8}}$ I19a)	NUNUGPV	200.00	.1472E+00 ±.40E-03 ±.00E+00
(**Obs.** $\overline{\text{Nu9}}$ I19b)	NUNUGPV	183.00	.5120E-01 ±.10E-03 ±.00E+00
(**Obs.** $\overline{\text{Nu9}}$ I19b)	NUNUGPV	189.00	.4740E-01 ±.10E-03 ±.00E+00
(**Obs.** $\overline{\text{Nu9}}$ I19b)	NUNUGPV	200.00	.4140E-01 ±.10E-03 ±.00E+00
(**Obs.** $\overline{\text{Nu10}}$ I19c)	NUNUGPV	183.00	.4984E-01 ±.90E-03 ±.00E+00
(**Obs.** $\overline{\text{Nu10}}$ I19c)	NUNUGPV	189.00	.4598E-01 ±.90E-03 ±.00E+00
(**Obs.** $\overline{\text{Nu10}}$ I19c)	NUNUGPV	200.00	.4020E-01 ±.10E-03 ±.00E+00

onventions

3. Conventions and translation guidelines

This section should be written later, during the time of fixing discrepancies, adjusting progam inputs to LEP 1 conventions etc.

program

4. Semi-analytical and Monte Carlo tools used

4.1 presentation of the program MCdummy

This part of the description must be included:

1. Author(s)
2. Program name and version, date of release
3. Can be obtained from ...
4. Reference to main description (an appendix in our book?)
5. Reference to example discussion of prediction and its systematic uncertainty (an appendix in our book?).
6. Status: adverticement, preliminary, complete-draft, final.

It should be followed by 1-2 pages description of the program content, applicability region its systematic error there etc. References to the observables which can be handled should be given here. Also references to observables the program should not be used for.

Please create individual files for this section. Please refer to observables using labels. Look for the draft with labels on the workshop web-page. Use bibtex file for references so we can combine them all together.

BHWIDE

4.2 Presentation of the program BHWIDE

1) Authors:	**S. Jadach, W. Płaczek and B.F.L. Ward**
2) Program:	**BHWIDE v.1.10, December 1998**
3) Can be obtained from	`http://enigma.phys.utk.edu/pub/BHWIDE/`
4) Reference to main description	[13] bhwide:1997
5) Reference to example	[14] lepybk:1996jn
6) advertisement	

In this subsection, we briefly describe our Monte Carlo (MC) event generator for large angle Bhabha (LABH) scattering called BHWIDE and discuss some important cross-checks of the program.

BHWIDE is based on the YFS exclusive exponentiation procedure [15] yfs:1961, where all the IR singularities are summed-up to infinite order and cancelled out properly in the so-called YFS form factor. The remaining non-IR residuals, $\bar{\beta}_n^{(l)}$, corresponding to the emission of n-real photons, are calculated perturbatively up to a given order l, where $l \geq n$, and $(l-n)$ is a number of loops in the $\bar{\beta}_n^{(l)}$ calculation. In BHWIDE an arbitrary number n of real photons with non-zero p_T are generated according to the YFS MC method of Ref. [16] bhlumi2:1992. The non-IR residuals $\bar{\beta}_n^{(l)}$ are calculated up to $\mathcal{O}(\alpha)$, i.e. $\bar{\beta}_0^{(1)}$ and $\bar{\beta}_1^{(1)}$ corresponding to zero-real (one-loop) and one-real (zero-loop) photons, respectively, are included. In $\bar{\beta}_0^{(1)}$ we implemented two libraries of the $\mathcal{O}(\alpha)$ virtual EW corrections: **(1)** the older one of Refs. [17, 18] Bohm:1988fq, Berends:1988jm, which is not up to date but can be useful for some tests/cross-checks, and **(2)** the more recent one of Ref. [19] Beenakker:1991mb. When the genuine weak corrections are switched off (or numerically negligible) they are equivalent. In $\bar{\beta}_0^{(1)}$ we implemented two independent matrix elements for single-hard-photon radiation: **(1)** our calculation [13] bhwide:1997 in terms of helicity amplitudes, and **(2)** the formula of CALKUL [20] calkul:1982 for the squared matrix element. We have checked that the above two representations agree numerically up to at least 6 digits on an event-by-event basis.

The MC algorithm of BHWIDE is based on the algorithm of the program BHLUMI for small angle Bhabha scattering [16] bhlumi2:1992, however with some important extensions: **(1)** QED interferences between the electron and positron lines ("up-down" interferences) had to be reintroduced as they are important in LABH; **(2)** the full YFS form factor for the $2 \to 2$ process, including all s-, t- and u-channels, was implemented [13] bhwide:1997; **(3)** the exact $\mathcal{O}(\alpha)$ matrix element for the full Bhabha process was included. The multiphoton radiation is generated at the low-level MC stage as for the t-channel process, while the s-channel as well as all interferences are reintroduced through appropriate MC weights. This means that the program is more efficient when the t-channel contribution is dominant, as e.g. at LEP2 energies; however, it proved to work well also near the Z resonance.

The program is written in FORTRAN77 and is particularly suited for use under the Unix operating system[1] for which a special directory structure has been created with useful Makefile's for easy compiling and linking. The program runs in three stages: (1) initialization – where all input parameters are read and transmitted to the program as well as all necessary initializations are performed, (2) event generation – here a single event is generated, and (3) finalization – final bookkeeping for a generated event statistics is done and some useful information is provided (printed-out). There are two main modes of event generation: one can generate either variable weight events (useful for various tests) or constant (=1) weight events (useful for apparatus MC simulations). Various input parameter options, to be set by the user, allow to choose between different contributions/corrections to the cross section, such as weak

[1] Hovewer, it can be used, in principle, on any operating system with a FORTRAN77 compiler.

corrections (two libraries), vacuum polarization (three parametrizations), etc. Other input parameters allow to specify the necessary ingredients for the cross section calculation and the event generation, such as the CMS energy, physical parameters (masses, widths, etc.), phase space cuts, etc. For each generated event, four-momenta of the final state electron, positron and all radiative photons are provided. In the variable-weight-event mode they are supplemented with the main (best) event weight as well a vector of weights corresponding to variuos models/approximations. In the finalization stage, the total cross section corresponding to the generated event sample is calculated and provided (printed-out) together with some other useful information.

So far, several tests/cross-checks of the program have been performed, see e.g. Ref. [21]. First comparisons with other MC programs for LABH were done during the LEP2 Workshop in 1995 [14]. They showed a general agreement of BHWIDE with most of those programs within 2% at LEP2 energies. At that time such a level of precision was expected to be sufficient for LEP2. Discrepancies between variuos calculations can be explained by the fact that most of the programs were designed for LEP1 where the Z s-channel contribution was dominant, while at the LEP2 energy range the t-channel γ exchange dominates. Thus, the physical features of the Bhabha process at LEP1 and LEP2 are very different. Recently, a more detailed study of the theoretical precision of BHWIDE has beed carried out [22]. Comparisons have been made with the MC programs: OLDBIS [16] (a modernized version of the program OLDBAB [23]) and BHLUMI [24], and the semi-analytic code ALIBABA [19]. Tests were done at $\mathcal{O}(\alpha)$ and with higher order corrections for various cuts – by starting from the pure t-channel γ-exchange and switching on gradually other contributions/corrections. This study shows that at $\mathcal{O}(\alpha)$ BHWIDE agrees with OLDBIS within 0.1% for the pure QED process, while ALIBABA differs by up to 0.3%. When higer order corrections are included, BHWIDE is generally within 1% of BHLUMI (0.5% in the forward region: $\cos\theta_e > 0.7$) for the pure t-channel γ-exchange process and within 1.2% of ALIBABA for all kinds of contributions/corrections. From these test we have estimated the overall theoretical precision of BHWIDE at 1.5% for the LEP2 energy range. We expect that by making some improvements of the program (e.g. modifying the "reduction procedures" for the matrix element calculations, including $\mathcal{O}(\alpha^2)$ LL corrections) and performing some new cross-checks (e.g. with the program LABSMC [25]) we can reduce this precision to $\sim 0.5\%$. Futher improvements of the theoretical precision can be made, in our opinion, with the help of the KK MC program [26] after implementing in it the e^+e^- channel.

Barcelona:1998wp
lepybk:1996jn
bhwide:1999ws
bhlumi2:1992
oldbab:1983
bhlumi4:1996
Beenakker:1991mb
Arbuzov:1999db
kkcpc:1999

KORALZ

4.3 Presentation of the program KORALZ

1) Author:	**S. Jadach, B.F.L. Ward and Z. Wąs**
2) Program:	**KORALZ v.4.04,**
3) Can be obtained from	Library of Computer Physics Communication, or from the author (z.was@cern.ch) upon request
4) Reference to main description	[Jadach:1999tr] [27] and references therein.
5) Reference to example, use:	[Jachol,AFBnowa] [28, 29]
6) Status:	this is preliminary version of the description

Initially the KORALZ event generator was written [koralzearly] [30] to simulate τ-pair production and decay for LEP1 physics at the first order of QED bremsstrahlung without any exponentiaition. Only longitudinal τ spin effects were included. Later [31, 32], [koralz:1991,koralz:1994] longitudinal beam polarization was included and higher order QED effects were incorporated using powerful exponentiation techniques [33, 34] [yfs2:1990,yfs3:1992] of initial state bremsstrahlung first, but later of final state bremsstrahlung as well. The interference of initial and final state bremsstrahlung was always neglected, except as parallel mode of operation at single bremsstrahlung level. This assumption was good at the peak of the Z resonance, due to suppression of the correction due to Z life-time, and the estimation of the error based on the single bremsstrahlung calculations were sufficient [29, 35]. [KANCEL,AFBnowa] At that time quite complete system of tests and cross-checks was developped for the effects due to corrections of initial state beremsstrahlung using dedicated methods basedx on comparison of semi-analytical results and Monte Carlo [36] [AFBstara] using importance samling. Tests at technical precision of 10^{-4} and better could be obtained. In general precision of 0.2 % was achievable for quite a range spectrum of observables for $\mu^+\mu^-$ or $\tau^+\tau^-$ final states. The $\nu\bar{\nu}\gamma$ final states were also introduced [37] [rrunu] using assumption that t-channel W-exchange forms contribution which is rather small and in particular $W - W - \gamma$ interaction can be completely neglected.

The use of KORALZ at LEP2 energies impedes substantial decrease of precision. For $\mu^+\mu^-$ or $\tau^+\tau^-$ it is mainly due to lack of interference effects in exponentiation. For $\nu\bar{\nu}\gamma$ final states it is due to approximate treatment of t-channel W-exchange. Even though $W - W - \gamma$ interaction was included, method explained in [28] [Jachol] is not enough for the single photon observables and precision meant to be better than 1-2 %. For double photon observables precision decreases even further as matrix element for two photon configuration is approximate to the pragmatic order α^2 only.

Electroweak corrections are implemented in KORALZ using reduced Born method. This means that the effects of electroweak corrections beyond crude Born are implemented at the leading-log level only. Recently, final version of KORALZ was published and documented [27] [Jadach:1999tr]. That version uses DIZET version 6.05, however, for the sake of tests versions with the up to date DIZET library may be maintained.

Use of the program is expected to be gradually replaced by KK2f Monte Carlo which already at present is superior in all applications (for the time being except $\nu\bar{\nu}\gamma$ final states) and at all energy ranges as far as precision is concerned. Some tools for studying anomalous effects in $\tau\tau\gamma$ final states [38] [Paul:1999tt], $\nu\bar{\nu}\gamma$ [39] [Jacholkowska:1999eleptoki], and leptoquarks [40] are available at present for KORALZ only. A number of flags, to be set by user, allows to switch between different options and perform specific comparisons and investigations, e.g. for calculation of physicap precision.

Program uses following libraries: YFS 3.4 [34] [yfs3:1992] for multiple photon bremsstrahlung, TAUOLA [41] [tauola:1993] for τ-lepton decay and PHOTOS [42] [photos:1994] for radiative corrections in τ-lepton decays.

KK2f

4.4 Presentation of the program KK2f

1) Author:	**S. Jadach, B.F.L. Ward and Z. Wąs**
2) Program:	**KK2f v.4.13,**
3) Can be obtained from	Library of Computer Physics Communication, or from the author (s.jadach@cern.ch) upon request
4) Reference to main description	[26] kkcpc:1999
5) Reference to example, use:	[43] ceex-big
6)Status:	this is preliminary version of the description

KK2f is expected to take over all functionality of KORORALZ event generator. Program includes: (i) complete spin effects in τ-pair production and decay in a manner valid from the threshold up to linear collider Centre-of-Mass sysyem energy. Complete order α^2 matrix element for initial- and final-state bremsstrahlung is included as well as corresponding virtual-corrections. Exponentiation is implemented at the spin amplitude level. Thisw enables systematic treatment of the bremsstrahlung interference. Precision is typically of order of 0.2? % precision for observables where bremsstrahlung photons are not tagged, 1? % for observables with single photon tagged and about 3? % for observables with double photon tagged. **these statements will be verified after tables are filled** Precision estimate consists of two parts: first corresponding to genuine QED effects. The aapropriate schemes of calculation are given in [26, 43]. Second part consits of effects hidden in effective Z and γ couplings, here we rely on results of sudies for electroweak libraries, in particular DIZET library. In general precision due to QED effects depends on the choice of the particular observable. See sections with numerical results for examples. The $\nu\bar{\nu}$ final states are not yet allowed, but are expected to be introduced in the next version.

Electroweak corrections are implemented in KK using effective coupling constants. Present version of KK uses DIZET version 6.21. A number of flags, to be set by user, allows to switch between different options and perform specific comparisons and investigations e.g. in context of the calculation of systematic errors see [43].

Program uses following libraries: TAUOLA [41] for τ-lepton decay and PHOTOS [42] for radiative corrections in τ-lepton decays.

LABSMC

4.5 Presentation of the program LABSMC

1) Author: **A.B. Arbuzov**

2) Program: **LABSMC v.2.02, 10 Febr 2000**

3) Can be obtained from the author (arbuzov@to.infn.it) upon request

4) Reference to main description

5) Reference to example

6) draft

Initially the LABSMC event generator was created [25] to simulate large–angle Bhabha scattering at energies of about a few GeV's at electron positron colliders like VEPP–2M and DAΦNE. The code included the Born level matrix element, the complete set of $\mathcal{O}(\alpha)$ QED RC, and the higher order leading logarithmic RC by means of the electron structure functions. The relevant set of formulae can be found in Ref. [44]. The generation of events is performed using an original algorithm, which combines advantages of semi–analytical programs and Monte Carlo generators.

The structure of our event generator was described in detail in paper [25]. The extension for LEP2 energies is done by introducing electroweak (EW) contributions, such as Z-exchange, into the matrix elements. The third [45] and fourth [46] order leading logarithmic photonic corrections were also included in the new version. The version of the program under consideration is suited for large–angle scattering. The small–angle version, which incorporates some additional second–order corrections [47], will be described elsewhere.

The code contains:

- the tree level electroweak Born cross section;
- the complete set of $\mathcal{O}(\alpha)$ QED radiative corrections (RC);
- vacuum polarization corrections by leptons, hadrons [48], and W-bosons;
- one–loop electroweak RC, either (default) according to Ref. [49] by means of DIZET [50] package, or called directly from the semi-analytical code ALIBABA [19];
- higher order leading log photonic corrections by means of electron structure functions [45,46,51];
- matrix element for radiative Bhabha scattering with both γ- and Z-exchange [17, 18], vacuum polarization RC, and optionally ISR leading log RC (with exponentiation according to Ref. [51]);
- light pair corrections in the $\mathcal{O}(\alpha^2 L^2)$ leading log approximation [52], including the two–photon mechanism.

A number of flags, to be set by user, allows to switch between different options and perform specific comparisons and investigations. In particular one can switch to generation of only radiative events with visible photons. That allows to avoid technical problems due to low statistics in this case.

The inclusion of the third and fourth order LLA photonic corrections allows not to use exponentiation. A simple estimate [46] shows that the difference between the two treatments at LEP2 is negligible, while the exponentiation requires a specific event generation procedure.

LABSMC is a FORTRAN program. It works as follows. First, the code makes initialisation and reads flags and parameters from a list provided by user. Then it performs integration (in semi–analytical branch) and generates events. The 4-momenta of generated particles are to be analysed in a user subroutine to perform a concrete event selection. A certain control of technical precision is provided by comparison of the results from semi–analytical and Monte Carlo branches. Note, that for a case of complicated cuts, which can not be done in the semi–analytical branch, one has to increase the number of generated events to reach the ordered precision.

The accuracy of the code is defined by two main points: technical precision (numerical precision in integrations, errors due to limited statistics, possible bugs *etc.*) and the theoretical uncertainty. Of course, one has than choose the proper, corresponding to his concrete problem, set of flags and parameters.

The technical precision has to be checked and improved, if required, by detailed tests and comparisons with results of other codes. The theoretical uncertainty is defined by: absence of complete set of $\mathcal{O}(\alpha^2 L)$ corrections (for photonic and pair corrections), an uncertainty in definition of vacuum polarization, approximate description of hadronic pair production, and a discrepancy in the treatments of electroweak RC in ZFITTER (recommended option) and ALIBABA. The theoretical uncertainty of the code in description of large–angle Bhabha scattering at LEP2 is estimated now to be of about 0.2%. The corresponding uncertainty for radiative Bhabha scattering with a visible photons is about 1.5%. But for the latter, we have an additional theoretical systematic uncertainty (about 1%), coming from non–standard radiative corrections [53].

4.6 Presentation of the program grc$\nu\nu\gamma$

1. Authors: Y. Kurihara, J. Fujimoto, T. Ishikawa, Y. Shimizu, T. Munehisa
2. Program: grc$\nu\nu\gamma$ v.1.0, 1999.08.20
3. Can be obtained from: http://www-sc.kek.jp/minami/
4. Reference to main description: hep-ph/9908422 to be appeared in CPC.
5. Reference to example discussion of the prediction and its systematic uncertainty: hep-ph/9908422 to be appeared in CPC.
6. Status of grc$\nu\nu\gamma$

grc$\nu\nu\gamma$ is an event generator which combines the exact matrix elements for $e^+e^- \to \nu\bar{\nu}\gamma(\gamma)$, produced by the GRACE system [54], with QEDPS [55] for ISR. The advantages of these packages are:

- The exact matrix elements up to the double-photon emission, including the ν_e process, are used. Double-photon emission is practically sufficient for experimental analysis.
- QEDPS keeps the complete kinematics for the emitted photons and virtual electrons before collisions. It allows a more flexible treatment of the ISR effects in avoiding the double-counting.
- Besides the above-mentioned pure QED corrections, grc$\nu\nu\gamma$ equips with another class of the electro-weak higher order corrections. There is a switch to choose it from the following three schemes; 1) the running coupling constant scheme: the coupling constant of the $fermion$-$fermion$-Z vertex, g_{ffZ}, is determined by the evolution from zero momentum transfer to the mass squared of the $\nu\bar{\nu}$ system, q_Z^2, which differs from one event to another. It varies according to the renormalization group equation (RGE). 2) G_μ scheme [56]: It is such that the weak couplings are determined through the weak-mixing angle, $\sin\theta_W$, which is given by

  $$\sin^2\theta_W = \frac{\pi\alpha(q^2)}{\sqrt{2}G_\mu M_W^2}\frac{1}{1-\Delta r},$$

  where M_W being the W-boson mass and G_μ the muon decay constant. 3) on-shell scheme: the weak couplings are simply fixed by M_W and M_Z though the on-shell relation, $\sin^2\theta_W = 1 - \frac{M_W^2}{M_Z^2}$, where M_Z is the mass of the Z-boson.
- For the ν_μ case the total cross sections and the hard-photon distributions of grc$\nu\nu\gamma$ are compared with those by the $\mathcal{O}(\alpha)$ calculations [57–59], KORALZ [27] and NUNUGPV [60]. The theoretical error uncertainty for the ISR corrections is under control at the 1% level. The systematics of the G_μ scheme, coming from the double energy scales(M_Z,$\sqrt{s}$) involved in the reaction, is estimated to be around 1%. The energy spectrum of the hard-photons is in a reasonable agreement with KORALZ and NUNUGPV up to the double-photon emission.
- Concerning ν_e a similar comparison with NUNUGPV has been done, though in lacking $\mathcal{O}(\alpha)$ calculations.
- In the package the anomalous coupling of the W-W-γ vertex is implemented. The program includes only those terms which conserve C and P invariance, derived from the following

effective Lagrangian [61]:

$$L_{eff} = -ie[(1+\Delta g_{1\gamma})(W^{\dagger}_{\mu\nu}W^{\mu} - W^{\dagger\mu}W_{\mu\nu})A^{\nu} + (1+\Delta\kappa_{\gamma})W^{\dagger}_{\mu}W_{\nu}A^{\mu\nu} + \frac{\lambda_{\gamma}}{M_W^2}W^{\dagger}_{\lambda\mu}W^{\mu}_{\nu}A^{\lambda\nu}],$$

where $W_{\mu\nu} = \partial_{\mu}W_{\nu} - \partial_{\nu}W_{\mu}$, $A_{\mu\nu} = \partial_{\mu}A_{\nu} - \partial_{\nu}A_{\mu}$. Here $\Delta g_{1\gamma}$, $\Delta\kappa_{\gamma}$ and λ_{γ} stand for the anomalous coupling parameters which vanish in the standard model.

4.7 Presentation of the program NUNUGPV

Authors: **G. Montagna, M. Moretti,**
O. Nicrosini and F. Piccinini

Program: **NUNUGPV v.2.0, July 1998**

Can be obtained from:

`http://www.pv.infn.it/~nicrosi/programs/nunugpv/`

The code NUNUGPV [60, 62] has been developed to simulate events for the signatures single- and multi-photon final states plus missing energy in the Standard Model at LEP and beyond.

Matrix elements

In the program the exact matrix elements are implemented for the reactions

$$e^+e^- \to \nu_i\ \bar{\nu}_i\ n\gamma\ ,$$

with $i = e, \mu, \tau$ and $n = 1, 2, 3$.
The matrix element for single-photon production has been computed by means of helicity amplitude techniques [63], while the amplitudes for multi-photon final states are calculated using the numerical algorithm ALPHA [64] for the automatic evaluation of tree-level scattering amplitudes. The contribution of the anomalous couplings Δk_γ and λ_γ to the $WW\gamma$ vertex is included analytically in the matrix element for $e^+e^- \to \nu_e\ \bar{\nu}_e\ \gamma$. The implementation in the ALPHA code of trilinear and quadrilinear anomalous gauge couplings for the processes involving more photons in the final state is currently under development. As an option the program contains also the contribution of a massive neutrino with standard couplings to the Z boson.

Radiative corrections

The phenomenologically relevant Leading Log (LL) QED radiative corrections, due to initial state radiation (ISR), are implemented via the Structure Function (SF) formalism. Due to the presence of a visible photon in the kernel cross section, the inclusion of ISR requires particular care. In order to remove the effects of multiple counting due to the overlap of the phase spaces of pre-emission photons (described by the SF's) and kernel photons (described by the matrix element), the p_t/p_L effects are included in the SF's according to ref. [62]. The generation of the angular variables at the level of the ISR gives the possibility of rejecting in the event sample those pre-emission photons above the minimum detection angle and threshold energy, thus avoiding "overlapping effects". According to such a procedure, the cross section with higher-order QED corrections can be calculated as follows (for the data sample of at least one photon)

$$\sigma^{1\gamma(\gamma)} = \int dx_1 dx_2 dc_\gamma^{(1)} dc_\gamma^{(2)}\ \tilde{D}(x_1, c_\gamma^{(1)}; s)\tilde{D}(x_2, c_\gamma^{(2)}; s)\Theta(cuts) \times \left(d\sigma^{1\gamma} + d\sigma^{2\gamma} + d\sigma^{3\gamma} + ...\right), \quad (2)$$

where $c_\gamma = \cos\vartheta_\gamma$ and $\tilde{D}(x, c_\gamma; s)$ is a proper combination of the collinear SF $D(x, s)$ with an angular factor inspired by the leading behaviour $1/(p \cdot k)$ [62] of the pre-emission photons. According to eq. (2), an "equivalent" photon is generated for each colliding lepton and accepted as a higher-order ISR contribution if:

- the energy of the equivalent photon is below the threshold for the observed photon $E_{\gamma,min}$, for arbitrary angles;
- or the angle of the equivalent photon is outside the angular acceptance for the observed photons, for arbitrary energies.

Within the angular acceptance of the detected photon(s), the cross section is evaluated by summing the exact matrix elements for the processes $e^+e^- \to \nu\bar{\nu}n\gamma$, $n = 1, 2, 3$ ($d\sigma^{1\gamma}, d\sigma^{2\gamma}, d\sigma^{3\gamma}$).

By means of the above sketched formulation, the signatures that can be handled by the program are:

- exactly one(two) visible photon(s) plus undetected radiation;
- at least one(two) visible photon(s) plus undetected radiation;
- exactly three visible photons with QED corrections in the collinear approximation.

Some improvements for Linear Collider energies have been recently introduced. Predictions for the single-photon signature are possible for polarized electron/positron beams. Simulation of beamsstrahlung can be performed by means of the `circe` library [65].
Both integration and unweighted event generation modes are available. More details on technical and theoretical features can be found in ref. [60]. Concerning the LEP2 energy regime, the present theoretical accuracy of `NUNUGPV` is at the per cent level, as due to missing $\mathcal{O}(\alpha)$ electroweak corrections.

ZFITTER

4.8 Presentation of the program ZFITTER with electroweak library DIZET

Authors: **D. Bardin, P. Christova, M. Jack, L. Kalinovskaya, A. Olchevski, S. Riemann, T. Riemann**

Program: **ZFITTER v.6.21 (26 July 1999)**

Can be obtained from: `http://www.ifh.de/~riemann/Zfitter/zf.html`
`/afs/cern.ch/user/b/bardindy/public/ZF6_21`

Reference to main description: [66] Bardin:1999yd-orig

References to examples: [67–75] Bohm:1989pb,Bardin:1989qr,Boudjema:1996qq,Bardin:1997xq,Bardin:1998nm,

Program development: The package is permanently updated, user requests are welcome; last update is v.6.30 (xx March 2000)

ZFITTER is a Fortran program, based on a semi-analytical approach to fermion pair production in e^+e^- annihilation at a wide range of centre-of-mass energies, including LEP1 and LEP2 energies. The main body of the program relies on the analytical results presented in [76–78] Bardin:1991fu,Bardin:1991de,Christova:1 for the QED part and in [79–84] Bardin:1980fe,Bardin:1982sy,Akhundov:1986fc,Bardin:1986fi,Bardin:1989di,Ba Bardin:1999yd-orig for the electroweak physics part. Some of the formulae used may be found only in [66].

The calculation of realistic observables with potential account of complete $\mathcal{O}(\alpha)$ QED and electroweak corrections plus soft photon exponentiation plus some higher order contributions is made possible with several calculational chains:

- Born cross-sections;
- a fast option: cut on s' or combined cuts on acollinearity ξ and minimal energy $E_{\min}$ of the fermions for $\sigma_{T,FB}$;
- cut on s' (or on $\xi, E_{\min}$) for $d\sigma/d\cos\vartheta$; for $\sigma_{T,FB}$ additional cut on the production angle of *anti-fermions* ($\cos\vartheta$).

The scattering angle of *fermions* remains unrestricted if the other cut(s) do not impose an implicit restriction.

Numerical integrations are at most one-dimensional and performed with the Simpson method [85,86] Silin:19xy,Sedykh:19xy. This makes the code so fast and guaranties any practically needed numerical precision.

ZFITTER calculates:

- Δr – the Standard Model corrections to G_μ;
- M_W – the W boson mass from M_Z, M_H, and fermion masses, and Δr;
- $\Gamma_{Z,W} = \sum_f \Gamma_f$ – total and partial Z and W boson decay widths;
- $d\sigma/d\cos\vartheta$ – differential cross-sections;
- σ_T – total cross-sections;
- A_{FB} – forward-backward asymmetries;
- A_{LR} – left-right asymmetries;
- A_{pol}, A_{FB}^{pol} – final state polarization effects for τ leptons;

Various interfaces allow fits to the experimental data to be performed with different sets of free parameters. There are two options to parameterize the Z boson propagator [87] Bardin:1988xt Riemann:1997tj1,Bohm:2000jw (see also [88, 89] and the many references therein).

ZFITTER uses pieces of code from other authors ([85,86,90–94] Silin:19xy,Sedykh:19xy,Matsuura:1987,Jegerlehner:1995Z). We find it important to mention explicitely that the programming of ZFITTER accumulates the efforts of many theoreticians, whose work went into the code either as default programming or as options to be chosen by many flags. A hopefully

complete list (derived from [66]) comprises quite a few references for photonic radiative corrections [45, 51, 68, 95–107] and radiative corrections contributing to the effective Born cross section [108–135]. This is a feature of ZFITTER which makes it very flexible for applications, but also for comparisons with other codes and checks of technical precisions in program development phases. For a systematic presentation of the interplay of the many radiative corrections treated we refer also to [66] and to [136].

ZFITTER is used optionally by other packages, among them are SMATASY [137–139], ZEFIT, [140]. Its electroweak library is used e.g. in KORALZ, [27], KK2f, [141], BHAGENE, [142], and other programs like HECTOR [143] for the study of ep scattering.

ZFITTER v.6.23 differs from v.6.21 by the following features: ...

ZFITTER v.6.30 differs from v.6.23 by the following features: ...

The Fortran package DIZET is part of the ZFITTER distribution.
It can be used in a stand-alone mode and is regularly used by other programs, e.g. [27, 141].

On default, DIZET allows the following calculations:

- by call of subroutine DIZET: W mass and width, Z and W widths;
- by call of subroutine ROKANC: four weak NC form factors, running electromagnetic and strong couplings needed for the composition of effective NC Born cross sections for the production of massless fermions;
- by call of subroutine RHOCC: the corresponding form factors and running strong coupling for the composition of effective CC Born cross sections.

If needed, the form factors may be made to contain the contributions from WW and ZZ box diagrams thus ensuring (over a larger energy range than LEP 1) the correct kinematic behaviour and gauge invariance.

Recent program developments:

ZFITTER v.6.23 differs from v.6.21 by the following features: ...

ZFITTER v.6.30 differs from v.6.23 by the following features: ...

Statement on the precision and the systematic errors:

See the other parts of this report, especially sections 5., section 6., and section 7..

Statement on limitations:

ZFITTER should not be used for precision calculations of Bhabha cross sections. The corresponding QED corrections have to be recalculated. The effective Born approximation for Bhabha scattering is fixed to LEP 1 kinematics. It is relatively easy to improve the latter, but this has to be done yet.

theory_err

5. Theoretical uncertainties of observables

5.1 MCdummy

The program can be used for many purposes it was shown in ref. [1] that in some cases precision is as high as 0.1 % . This analysis is in particular valis for the observables such as (**Obs.** 112) (**Obs.** 113) ..., the analysis can be easily extended to cases such as (**Obs.** 116a) ... and precision of the same order that is not worse than 0.2 % is expected. Numerical values have to be obtained by the users. However in case of observables (**Obs.** 116b) (**Obs.** 117a), for which the program can be still used but it is mandatory for the user to repeat the study of systematic error accordingly to procedure described in [1]. Finally for observables such as (**Obs.** 117b) our program should not be used unless .... is completed.

In the following table we collect some example results of our program for some of the observables from the list. For further numbers we refer the reader to following papers and web-pages ...

5.2 BHWIDE

This chapter which explains rules for calculation of theoretical uncertainties for the predictions from this program is still missing.

5.3 grc$\nu\nu\gamma$

This chapter which explains rules for calculation of theoretical uncertainties for the predictions from this program is still missing.

5.4 KORALZ

This chapter which explains rules for calculation of theoretical uncertainties for the predictions from this program is still missing.

5.5 KK2f

This chapter which explains rules for calculation of theoretical uncertainties for the predictions from this program is still missing.

5.6 LABSMC

This chapter which explains rules for calculation of theoretical uncertainties for the predictions from this program is still missing.

5.7 NUNUGPV

This chapter which explains rules for calculation of theoretical uncertainties for the predictions from this program is still missing.

Concerning the theoretical error, we propose to attribute to our numbers for the time beingan uncertainty which conservatively is given by the maximum between 1codes. To this end we need the predictions by other groups.

5.8 ZFITTER

This chapter which explains rules for calculation of theoretical uncertainties for the predictions from this program is still missing.

5.9 Manual for authors

Here is now the place for main activity. All observables are latex-labelled and should be referred in such a way in the presentations of individual programs and their capabilities In this way any change of observable content will not be a problem at all as indexing will get readjusted

And let us give here an example on how to refer to observables (**Obs.** Nu9) (**Obs.** Nu12)

```
I suggest the following format for program predictions:
Separator                          position    1
observable label  -  character string position    2-14
Separator                          position   15
program name      -  character string position   16-29
Separator                          position   30
Observable type   -   char*1 S or A  position   31
Separator                          position   32
CMS energy                         position   33-39
Separator                          position   40
```

```
prediction        -                    position   41-54
Separator                              position   55
Statistical error -                    position   56-69
Separator                              position   70
Systematic error  -                    position   71-84
Separator                              position   85
Prediction units  -  character string  position   86-94
Separator                              position   95
Error      units  -  character string  position   96-104
Separator                              position  105
Comment                                position  106-131
Separator                              position  132
```

```
        10        20        30        40        50        60        70        80        90       100       110
12345678901234567890123456789012345678901234567890123456789012345678901234567890123456789012345678901234567890123
=             =             = =       =             =             =             =         =         =

=OPAL1        =DummyMC      =S=198.5  =12.3         =0.4          =5.6          =pb       =p-
b       =                       =
=Aleph1       =DummyMC      =A=198.5  =0.123        =0.4          =0.56         =         =         =
=ILT1         =DummyMC      =S=198.5  =12.3         =0.4          =5.6          =pb       =p-
b       =                       =
=Delphi3      =DummyMC      =A=198.5  =0.123        =0.4          =0.56         =         =         =
```

From this I am able to generate all kind of tables automatically, simply combining all input files from you. As separator we can use = symbol.

Here are tables which are authomatically generated from the input file:

In general for numerical input of any calculation we suggest physical parameters from the latest publication of workd averages in PDG. For QCD coupling average $\alpha_S(M_Z)$ should be used as an input. $\Delta\alpha^5_{had}(M_Z) = 0.027782 \pm 0.000254$ is not present in PDG, the suggested value is taken from [144] which contains updates of analysis [48], if other value is used, it should be mentioned.

Jegerlehner:1999hq

Eidelman:1995ny

For the Higggs mass $M_H = 120GeV$, for CMS energies 189, 200 and 206 GeV, and in special cases such as discussion of contribution from $W - W$ boxes also 161 GeV.

Of course authors of programs using different input are welcomed to readjust the numbers to these values, also info about these readjustments/translations should find its place in **Section 3.**.

conventions

compare

6. Architecture of the calculations

At this point, having collected the necessary input from experiments and from authors of the progams used in data analysis, let us review now the basic princliples of the calculations and of the estimations of the systematic errors.

The aim of all the predictions was to obtain numerical predictions form the Standard Model of electroweak and strong interactions. The Standard Theory of interactions provides set of basic rules of calculations which in principle can lead to predcitions of arbitrary high precision. The diffculties which one obtains are however of the double nature. On one side they are technical, on the other they enforce basic re-make of theory ingredients to allow for inclusion of some phenomenologically important effects.

The precision tag which is in most strict cases at the level of 0.04% for LEP1 and 0.14% for LEP 2 is an important guding point. That means that for the terms without any enhancements terms of first order corrections of the full electroweak sector are sufficient. This numerical conclusion originating from the precision demands from experiments is at the core of all of all of phenomenological calculations for LEP[2].

[2]The picture may become by far more complicated if the precision tag of 0.01% is crossed and complete two loop elec-

Before we will go into more detailed discussion let us mention two theoretically most challenging effects which had to be coped, before final attempt to obtain reliable unambiguous predictions for LEP became available.

• The first one is the question of matching of QED photonic vacuum polarization obtained through dispersion relation from data with those calculated from Standard Model. In case of processes at lower than Z mass Center of Mass System energies which could be described by QED and QCD only, the problems were simle and easily solved [PETRA-vacpol]. However at LEP energies comatibility of assumption that $\Pi_{\gamma\gamma}(s)$ for s of order of 100GeV^2 should be taken from data had to lead to change of renormalization scheme of the Standard Model, this subraction rule if used with the other ones defined at M_Z energy scale complicated the renormalization proof of SM. The probem was discussed at the different level of formality an finally solved for our precision tag in [LEP2-L-vacpol].

• Even more diffcult theoretically is the question of the width included in Z propagator. Let us note that the width is intimatelly related with the summation to all orders of some contributions from the loop corrections, but not the others. Let us note that the first calculations of electroweak loop corrections [veltman] were explicitely *not* valid for calculation at the Z peak. Seemingly natural summation of the imaginary part of the Z width into its propagator leads to a lot of problems with the ordering of the perturbation expansion and related difficulties such as gauge invariance breaking were often encountered in LEP phenomenology. Let us list some of the refernces important in this discussion [something], finally in the following papers [znowu] the solution compatible with precision demands of the order of 0.1 % were established.

Typically the work is thus divided int simpler blocks which are later combined into full predictions:

1. pair corrections from QED
2. rest of QED
3. hadronic low energy vacuum polarization
4. QCD corrections for vacuum polarizations
5. QCD effects for final state couplings
6. hadronization
7. leading $\sim M_t^2$ higher order genuine electroweak corrections
8. rest of electroweak corrections taken at order α

The systematic errors originate from every block treated separately as well as from the approximations introduced in the process of their combination. This discussion is thus essential to define frame in which individual quality of the programs which are actually used in the interpretation of the experimental data can be presented.

It is thus essential to discuss these points, and how they affect the quality of programs predictions for our observables, that is what is their systematic error as predictions obtained from Standard Field Theory. Note that in some applications theoretical calculatuions contribute to actual *measurements* of the more idealized quantities such as masses couplings etc.

Let us go now into details:

6.1 hadronic low energy vacuum polarisation

Vacuum polarization makes about 6% correction to the value of α_{QED} at s = m_Z^2. The leptonic part of this contribution is known with excellent precision [VAP-Steinhauser:1998]. The quark part, however, is more difficult since the quark masses are not unambiguously defined and there is no general agreement that the pertubartive QCD can be use for reliable calculations. Several reevaluations of the hadronic contribution to the QED

troweak predictions are needed. Such may be the case of e.g. Z factories, where two loop effects may become available for experimental tests in full.

vacuum polarization have been performed to determine the effective QED coupling $\alpha(m_Z^2)$ [146–158]. They are compared on Fig. 1. The results marked with * are obtained using a dispersion integral of R_{had}

$$R_{had} = \frac{\sigma(e^+e^- \to hadrons)}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

measured experimentally. The uncertainties are dominated by the precision of measurement of $\sigma(e^+e^- \to hadrons)$ at the e^+e^- centre-of-mass energies below 5 GeV. The values not marked with * are obtained by using perturbative QCD in this region.

The values of hadronic contributions presented in Fig. 1 are in good agreement. The precision of calculations using a dispersion integral of R_{had} measured experimentally is sufficient for the precision required at LEP 2. The values of the effective QED coupling $\alpha(s)$ at different center-of-mass are given in Table 37.

$\sqrt{s}$ (GeV)	α^{-1}(s)
80.364	129.08 ± 0.09
91.1871	128.89 ± 0.09
161	128.07 ± 0.09
172	127.98 ± 0.09
183	127.89 ± 0.09
186	127.86 ± 0.09
192	127.82 ± 0.09
196	127.79 ± 0.09
200	127.76 ± 0.09
205	127.72 ± 0.09

Table 37: α^{-1}(s) using results of calculations of Ref. [148].

alpha_table

6.2 QCD corrections for vacuum polarisation at higher energies

6.3 electroweak libraries

question of dyson summation and gauge invariance should be discussed here, question of renormalization scheme (effective subtraction at the point of smooth continuation from perturbative to data driven vacuum polarization) approximations in treatment of hadronic stuff.

6.4 pairs as of M. Kobel but with some addups by M. Skrzypek

This is by far only the second approach to this section based on the approaches developped in OPAL.

In discussing question of pair effect the essential issue is how the related effect compares to expected total experimental error of combined all four experiments at the end of LEP. Let us use as the starting point in our discussion the study performed in OPAL [159]. Simmilar works were performed in other collaborations also ...

Let us evaluate now how important pair effects are for the final studies of 2-fermion final states and whether in practical cases pair effects once estimated can be still dropped out from final predictions or limited to approximations of the leading-log extended type such as in TOPAZ0 [160] or ZFITTER [161] and can be dropped out from the Monte Carlo predictions completely. We have here in mind corrections of the class ISNS$_\gamma$ + FS$_\gamma$ as defined in [160].

In order to setup a general framework for the analysis of pair corrections we follow the approach of [GiampPP]. The key point of the analysis is the separation of pair corections into two components: *signal* and *background*. We begin by dividing real pair contributions into four groups: (1) Multi-Peripheral **MP**, (2) Initial State Singlet **ISS**, (3) Initial State Non-Singlet **ISNS** and (4) Final State **FS**, cf. Fig. 1. of [GiampPP].

The signal part is now conveniently defined as the ISNS + ISS + matching virtual corrections and is provided by the semianalytical programs such as TOPAZ0 [pairTOPAZ0] or ZFITTER [pairZFITTER]. In practice its accuracy is therefore given by the accuracy of the corresponding programs. Appropriate theoretical calculations for ISNS and ISS pair corrections are available in the literature [Arbuzov:1999uq,Berends:1988ab, Jadach] with the precision 0.1% in the LEP2 range [Arbuzov:1999uq], matching the required experimental precision.

The background part contains all the other contributions, including MP and FS pairs. It is simulated by four-fermion Monte Carlo programs and minimized with the help of appropriate cuts.

Let us now go into details.

6.41 Signal

this part should be updated!

Having in mind our definitions of 2-fermion 4-fermion separation as defined in observable section of our repport we believe (see eg. [MichOPAL2] and ...) that the improvement of pair correction modelling in programs such as ZFITTER and TOPAZ0 (especially common exponentialtion of photons and pairs) should be sufficient. common exponentiation of photons and Pairs. This effort has been started but not yet fully completed, **we hope for the updated references here**.

Let us be more specific numerically. On pages 28 , 29 of [MichOPAL2] it is argued that the pair correction problem problem is not an issue at all for large s' All effects one observe there, are below 0.1%. Note, that up to now we averaged these large s' data.

For inclusive studies (e.g. s'/s > 0.01) (see page 28) real pairs add about 1.0% to the cross-section (as verified by independent 4f MC generators) with the signal definition of page 26 (feynman diagrams). The interference of virtual pairs subtracts 0.4% deduced from the (old!) Zfitter5.0 prediction of 0.6% total effect minus the 1.0% real pairs from above. This 0.4% number is uncertain due to the non-optimal ZFITTER5.0 pair treatment for LEP2, but should lie between 0 and -1.0%. (it has to be negative, and is very probably not absolutely bigger than the effect of real pairs.) In any case the *total* real+virtual pair correction from any SA program *must* not exceed the effect of real pairs: (Thus some numbers in 5.2x ZFITTER seem to be wrong) This issue is in my point of view the most important point for clarification, but it interfers in no way with the problem of vhow to define the 2f+4f signal (see below).

In conclusion, the accuracy of signal calculation is ???? for the LEP2 energy range.

6.42 Background

In addition to predictions of semianalytical programs, the experimentalists need a full 2f+4f Monte carlo dataset to determine: (i) signal efficiency, (ii) background. Again, with the OPAl definition (or any other definition close to it) the effect is 0.1% or less for high s'.

For inclusive selections one will have about:

1. 1% of the signal x-section from pairs (i.e. 4f MCs), which must be excluded from background subtraction
2. a (very experiment and channel dependent) correction to the selection efficiency due to these 4f events of up to 0.5% (in OPAL).

The interference with virtual pairs can be inlcuded in these efficiency correction as explained on page 29 of [MichOPAL2]. For that, we again use the interference prediction from semianalytical programs, (e.g.

the -0.4% from ZFITTER5.0.) but in effect it plays no role at all, since it represents a correction to the (small) efficiency correction. Quantitatively: completly ignoring the 0.4% interference would alter the efficiency by only 0.4%*0.5% = 0.002% !

So, the remaining point is the definition of the two-fermion signal and its realisation in 4f Monte carlo event generators. Here, we see four possible ways to go (all close to OPAL). We assume in all these 4 ways that we do not want to include any

1. CC diagrams (WW,Wev)
2. Multiperipheral 2-photon diagrams

The experimentalists have tried hard to optimize their selection to get rid of them. They were rather successful because these corrections are very forward peaked. Including them, would mean to get all this complications, which are either cut or even not observable at all back into the signal via efficiency corrections. We would have to accept then also additional theoretical errors in their prediction. These are the arguments against including 2-photon diagrams in any 2f signal.

Now let us go to four ways of signal definition (we use notation s'/s = z_p):

1. ISNS$_\gamma$ and FS$_\gamma$ diagrams as explained above (and page 26 of [163]) using mass cuts only to avoid overlap in ISNS$_\gamma$ with both primary and secondary pairs from γ^*. The mass cut will be just q^2(SIM)$< Q^2$(HIM)
2. same as (1), but allowing in addition for all other diagrams except MP and ZZ, using the mass cut now for all diagrams, so that the primary pair (*defined* to be the one from e+e- annihilation) is the HIM pair. Practically this makes little difference to (1) for experimentalists, since all "new" diagrams (apart from ISS$_Z$) will be rejected by the definition primary pair= HIM. This option might trigger considerable additional work for theorists, though.
3. Defining signal by a HIM and SIM mass cut, only, thereby again including ISNS$_Z$, FS$_Z$ and both ISS$_\gamma$ and ISS$_Z$ as in (2), but even ZZ, if they pass this mass cuts. The primary pair is here *defined* as the HIM, so that also e.g. the FSR pair in FS$_Z$ can be called "primary". Still, MP two-photon and CC diagrams are excluded! To again exlude the bulk of ZZ, an additional mass cut (z_s) for the SIM pair (e.g. $q^2 < 70$ GeV^2) is necessary. This pattern of mass cut is shown in [163] page 27.
4. same as (3), but exclude not only MP and CC , but also ISS diagrams, for reasons similar to those explained for MP:

Options (1,2) and (3,4) coresspond respectively to [160] choice between diagram based and mass cut based definition

Numerically (1,4) and (2,3) will be very similar, and even the difference between the selected sets of events will be relatively small, and dominated by ISS$_Z$. Actually, what OPAL did, is a (1',4) mixture: Experimentally they use (4) to measure the cross-section (with generating MP, ISS and all other diagrams in three separate MC datasets, ignoring their interference.) The measurement is then compared to (1) w/o the mass cut. (Since ZFITTER/TOPAZ0 does not allow mass cuts) We convinced ourselves that any systematic bias from this procedure is really tiny (I dont have the number at hand, but way below 0.1%)

Taking Giampieros arguments, the pros and cons fro 1-4 are:

1. (pro): least additional theory work to do. (pro): pairing ambigities might be solved by event weights (con): (i) not easy to model exactly this in full 4-vectors suggestion: event weights (also resolving the paring ambiguity) (already existing in OPAL, but not yet used in 2f publications): (ii) each Mc event gets a weight according to its ME for a given diagram - some effective procedure similar to OPALs (4)
2. Same arguments as for (1) except: (pro): always fulfills primary pair =HIM, incontrast to (1) (con): much more difficult for SA programmers
3. (pro): closer to measurement (philospophically) (con): how to solve pairing ambiguity? (con): more complicated definition than (1)

4. (con): same as 3, even more complicated (pro): tiny numerical differnce to the predictable x-section of (1), can be used as an alternative to event weights.

One could consider solution (1) as the preferable one. But the other three (2)-(4), seem to be good as well. Actually, it should be possible to quantitatively calculate differences(or Upper limits on differences) between all these options, which should be very small if it would turn necessary. This would mean that each experiment could pick just one experimental definition in 1-4, and later corrected to a common definition (e.g. 1), which can be predicted by SA programs.

One could have difficulties, on the other hand with including any MP or CC diagram in the signal definition. Excluding thereby (e)νW, it seems somewhat natural to exclude (e)eZ, i.e. ISS. `ZW: I am lost at this moment what is the purpose of this sentence.`

6.5 pairs -subsection to be removed

`Tthis is by far only the first approach to this section based on the approaches developped in OPAL.`

In discussing question of pair effect the essential issue is how the related effect compares to expected total experimental error of combined all four experiments at the end of LEP. Let us use as the starting point in our discussion the study performed in OPAL [MichOPAL]. Simmilar works were performed in other collaborations also ...

Let us evaluate now how important pair effects are for the final studies of 2-fermion final states and whether in practical cases pair effects once estimated can be still dropped out from final predictions or limited to approximations of the leading-log extended type such as in TOPAZ0 [pairTOPAZ0] or ZFITTER [pairZFITTER] and can be dropped out from the Monte Carlo predictions completely. We have here in mind corrections of the class ISNS$_\gamma$ + FS$_\gamma$ as defined in [pairTOPAZ0].

Having in mind our definitions of 2-fermion 4-fermion separation as defined in observable section of our repport we believe (see eg. [MichOPAL2] and ...) that the improvement of pair correction modelling in programs such as ZFITTER and TOPAZ0 (especially common exponentialtion of photons and pairs) should be sufficient. common exponentiation of photons and Pairs. This effort has been started but not yet fully completed, **we hope for the updated references here**.

Let us be more specific numerically. On pages 28 , 29 of [MichOPAL2] it is argued that the pair correction problem problem is not an issue at all for large s' All effects one observe there, are below 0.1%. Note, that up to now we averaged these large s' data.

For inclusive studies (e.g. s'/s > 0.01) (see page 28) real pairs add about 1.0% to the cross-section (as verified by independent 4f MC generators) with the signal definition of page 26 (feynman diagrams). The interference of virtual pairs subtracts 0.4% deduced from the (old!) Zfitter5.0 prediction of 0.6% total effect minus the 1.0% real pairs from above. This 0.4% number is uncertain due to the non-optimal ZFITTER5.0 pair treatment for LEP2, but should lie between 0 and -1.0%. (it has to be negative, and is very probably not absolutely bigger than the effect of real pairs.) In any case the *total* real+virtual pair correction from any SA program *must* not exceed the effect of real pairs: (Thus some numbers in 5.2x ZFITTER seem to be wrong) This issue is in my point of view the most important point for clarification, but it interfers in no way with the problem of vhow to define the 2f+4f signal (see below).

In addition to predictions of semianalytical programs, the experimentalists need a full 2f+4f Monte carlo dataset to determine: (i) signal efficiency, (ii) background. Again, with the OPAl definition (or any other definition close to it) the effect is 0.1% or less for high s'.

For inclusive selections one will have about:

1. 1% of the signal x-section from pairs (i.e. 4f MCs), which must be excluded from background subtraction

2. a (very experiment and channel dependent) correction to the selection efficiency due to these 4f events of up to 0.5% (in OPAL).

The interference with virtual pairs can be inlcuded in these efficiency correction as explained on page 29 of [MichOPAL2 163]. For that, we again use the interference prediction from semianalytical programs, (e.g. the -0.4% from ZFITTER5.0.) but in effect it plays no role at all, since it represents a correction to the (small) efficiency correction. Quantitatively: completly ignoring the 0.4% interference would alter the efficiency by only 0.4%*0.5% = 0.002% !

So, the remaining point is the definition of the two-fermion signal and its realisation in 4f Monte carlo event generators. Here, we see four possible ways to go (all close to OPAL). We assume in all these 4 ways that we do not want to include any

1. CC diagrams (WW,Wev)
2. Multiperipheral 2-photon diagrams

The experimentalists have tried hard to optimize their selection to get rid of them. They were rather successful because these corrections are very forward peaked. Including them, would mean to get all this complications, which are either cut or even not observable at all back into the signal via efficiency corrections. We would have to accept then also additional theoretical errors in their prediction. These are the arguments against including 2-photon diagrams in any 2f signal.

Now let us go to four ways of signal definition (we use notation s'/s = z_p):

1. ISNS$_\gamma$ and FS$_\gamma$ diagrams as explained above (and page 26 of [MichOPAL2 163]) using mass cuts only to avoid overlap in ISNS$_\gamma$ with both primary and secondary pairs from γ^*. The mass cut will be just q^2(SIM)$< Q^2$(HIM)
2. same as (1), but allowing in addition for all other diagrams except MP and ZZ, using the mass cut now for all diagrams, so that the primary pair (*defined* to be the one from e+e- annihilation) is the HIM pair. Practically this makes little difference to (1) for experimentalists, since all "new" diagrams (apart from ISS$_Z$) will be rejected by the definition primary pair= HIM. This option might trigger considerable additional work for theorists, though.
3. Defining signal by a HIM and SIM mass cut, only, thereby again including ISNS$_Z$, FS$_Z$ and both ISS$_\gamma$ and ISS$_Z$ as in (2), but even ZZ, if they pass this mass cuts. The primary pair is here *defined* as the HIM, so that also e.g. the FSR pair in FS$_Z$ can be called "primary". Still, MP two-photon and CC diagrams are excluded! To again exlude the bulk of ZZ, an additional mass cut (z_s) for the SIM pair (e.g. $q^2 < 70$ GeV^2) is necessary. This pattern of mass cut is shown in [MichOPAL2 163] page 27.
4. same as (3), but exclude not only MP and CC , but also ISS diagrams, for reasons similar to those explained for MP:

Options (1,2) and (3,4) coresspond respectively to [pairTOPAZ0 160] choice between diagram based and mass cut based definition

Numerically (1,4) and (2,3) will be very similar, and even the difference between the selected sets of events will be relatively small, and dominated by ISS$_Z$. Actually, what OPAL did, is a (1',4) mixture: Experimentally they use (4) to measure the cross-section (with generating MP, ISS and all other diagrams in three separate MC datasets, ignoring their interference.) The measurement is then compared to (1) w/o the mass cut. (Since ZFITTER/TOPAZ0 does not allow mass cuts) We convinced ourselves that any systematic bias from this procedure is really tiny (I dont have the number at hand, but way below 0.1%)

Taking Giampieros arguments, the pros and cons fro 1-4 are:

1. (pro): least additional theory work to do. (pro): pairing ambigities might be solved by event weights (con): (i) not easy to model exactly this in full 4-vectors suggestion: event weights (also resolving the paring ambiguity) (already existing in OPAL, but not yet used in 2f publications): (ii) each Mc event gets a weight according to its ME for a given diagram - some effective procedure similar to OPALs (4)

2. Same arguments as for (1) except: (pro): always fulfills primary pair =HIM, incontrast to (1) (con): much more difficult for SA programmers
3. (pro): closer to measurement (philospophically) (con): how to solve pairing ambiguity? (con): more complicated definition than (1)
4. (con): same as 3, even more complicated (pro): tiny numerical differnce to the predictable x-section of (1), can be used as an alternative to event weights.

One could consider solution (1) as the preferable one. But the other three (2)-(4), seem to be good as well. Actually, it should be possible to quantitatively calculate differences(or Upper limits on differences) between all these options, which should be very small if it would turn necessary. This would mean that each experiment could pick just one experimental definition in 1-4, and later corrected to a common definition (e.g. 1), which can be predicted by SA programs.

One could have difficulties, on the other hand with including any MP or CC diagram in the signal definition. Excluding thereby (e)νW, it seems somewhat natural to exclude (e)eZ, i.e. ISS. `ZW: I am lost at this moment what is the purpose of this sentence.`

6.6 special problems of hadronic final states

Question of correction to final state γ^5 terms for b-quarks should be discussed here and not only corrections to loops. Gluons from loops and whatever else.

Reference to other chapters of the book should be enough here?

omparisons

7. Tests and comparisons

There are several classes of systematic errors which need to be adressed. First of all, every calculation has its technical precision originating e.g. from integration techniques used in calculation, numerically small programing bugs etc. The second class of systematic precision is originating from the physical assumptions actually used. There is no doubt that the program authors are best suited to give valid numbers for that errors.

This is however not the end. Comparisons of programs constructed by different authors using different calculative schemes are essential[3].

Quite complicated network of relations between different codes was built at this point. Sophisticated cut-off intensive observables[4] are calculated usually with the help of Monte Carlo programs, semi-analytical calculations are missing. Sometimes only single Monte Carlo program or even procedure of combining results from different runs of different programs is available. This is not such a big problem as required precision is usually not very high. The programs are cross checked for other observables. This helps to establish confidence in some but not all ingredients of the programs.

For less cut-off intensive observables such as asymmetries or total cross sections effects of the cuts can be approximated by the simpler idealized ones, and Monte Carlo programs may be used just to estimate size of the small effects beyond these. For idealized observables comparisons between Monte Carlo results and semianalytical calculations can be performed and provide powerful "final check" of the two. Unfortunatelly this is not a final answer at all even for idealized observables, as often both semi-analytical calculations and MC programs use the same buiding blocks – so called electro-weak libraries. The comparisons at this level are essential for establishing correctness of complete semianalytical calculations and MC. On even lower level electroweak libraries use libraries of QCD/(low energy data) calculations for hadronic vacuum polarizations. These need to have some check presented also. The final states of quarks need to be hadronised. Related questions are covered elsewhere, outside 2f group, in QCD chapter of our repport.

[3]The comparisons can help also to fix seemingly trivial problems, related to misuse of input parameters.

[4]Here we have in mind mainly observables of the 1 or 2 hard photons tagged. These are often signatures of searches e.g. ....

Such multi-layer approach is advantageous in cases of problems and provides invaluable help for theoretical code authors in developpment and/or improvement phase of their work. Some of the discussion can be universal, whereas other will strongly depend on particular observable under consideration.

It is essential that comparisons of different programs were performed:

7.1 hadronic low energy vacuum polarisation

7.2 QCD corrections for vacuum polarisation at higher energies

7.3 electroweak libraries

7.4 pairs

7.5 semi-analytical programs and Monte Carlos for idealized observables

approximation in formfactor approach, separation impossible at genuine order α^2

7.6 Monte Carlos for strong cut-offs

7.7 report on final state hadronization

7.8 workshop status

Example how to include a figure, taken from the instruction:

```
At this moment it is difficult to continue work to make this sec-
tion more specific. Our activity is far behind the schedule. Theoret-
ical colleagues still wait for definite form of observables and I must
wait for the results and program definitions.
```

Finally let us note that comparisons with the data can serve only to indicate big problems of the calculations. It can never help in establishing precision at our requested level of $\frac{1}{3}$ of experimental statistical error. Problems at $2-3\sigma$ level can be found in this way at best.

Let me give here suggested time-table for our "future" work which was adopted to delays accumulated until Oct 15:

1. **Oct 20,** Final corrections from experiments on list of observables.
2. **Oct 30,** Feed back from theorists whether these definitions are complete enough in context of required precision tags.
3. **Nov 15,** Preliminary input from MC programs and SAN for the tables.
4. **Dec 3,** Informal (electronic ?) meeting
5. **Dec 15,** Preliminary layout of common sections.
6. **Jan 14,** Informal (electronic ?) meeting
   At this moment we are still finishing work for points 1 and 2, some material for section 3 (thanks to A. Arbuzov) is already included.

conclusion

8. Summary – what will remain to be done by th community later ...

References

example [1] Y. Zxxx, *Journal* **Cxx** (1999).

program [2] X. Yzzz, "example monte carlo program". CERN Program Library Wxxx, CERN, Geneva, 19xx.

ALPUB183 [3] T. A. Collaboration ALEPH Collab., CERN-EP/99-042 (submitted to Euro. Phys. J. C).

DELPUB172 [4] T. D. Collaboration, "P. abreu et al.". EP 99–05 (Accepted by Eur. Phys. J. C).

L3PUB172 [5] L3 Collaboration, M. Acciarri *et al.*, *Phys. Lett.* **B407** (1997) 361–376.

OPAL2F172 [6] OPAL Collaboration, K. Ackerstaff *et al.*, *Eur. Phys. J.* **C2** (1998) 441, hep-ex/9708024.

OPAL2F183 [7] OPAL Collaboration, G. Abbiendi *et al.*, *Eur. Phys. J.* **C6** (1999) 1, hep-ex/9808023.

OPAL2F189 [8] OPAL Collaboration, G. Abbiendi *et al.*, hep-ex/9908008.

PALGAML172 [9] OPAL Collaboration, K. Ackerstaff *et al.*, *Eur. Phys. J.* **C1** (1998) 45–64, hep-ex/9709027.

ALGAMNU172 [10] OPAL Collaboration, K. Ackerstaff *et al.*, *Eur. Phys. J.* **C2** (1998) 607, hep-ex/9801024.

ALGAMNU183 [11] OPAL Collaboration, G. Abbiendi *et al.*, *Eur. Phys. J.* **C8** (1999) 23, hep-ex/9810021.

OPALWW183 [12] OPAL Collaboration, G. Abbiendi *et al.*, *Eur. Phys. J.* **C8** (1999) 191, hep-ex/9811028.

hwide:1997 [13] S. Jadach, W. Płaczek, and B. F. L. Ward, *Phys. Lett.* **B390** (1997) 298.

ybk:1996jn [14] S. Jadach, O. Nicrosini, *et al.*, "Event generators for bhabha scattering", in *Physics at LEP2* (T. S. G. Altarelli and F. Zwirner, eds.), p. 229, CERN, Geneva, Switzerland, 1996.

yfs:1961 [15] D. R. Yennie, S. Frautschi, and H. Suura, *Ann. Phys. (NY)* **13** (1961) 379.

lumi2:1992 [16] S. Jadach, E. Richter-Wąs, B. F. L. Ward, and Z. Wąs, *Comput. Phys. Commun.* **70** (1992) 305.

ohm:1988fg [17] M. Bohm, A. Denner, and W. Hollik, *Nucl. Phys.* **B304** (1988) 687.

nds:1988jm [18] F. A. Berends, R. Kleiss, and W. Hollik, *Nucl. Phys.* **B304** (1988) 712.

ker:1991mb [19] W. Beenakker, F. Berends, and S. C. van der Marck, *Nucl. Phys.* **B349** (1991) 323.

alkul:1982 [20] F. A. B. et al. (CALKUL Collab.), *Nucl. Phys.* **B206** (1982) 61.

ona:1998wp [21] W. Płaczek, S. Jadach, M. Melles, B. Ward, and S. Yost, "Precision calculation of bhabha scattering at lep", in *Radiative Corrections: Applications of Quantum Field Theory to Phenomenology* (J. Sola, ed.), p. 325, World Scientific, Singapore, 1999, Barcelona, Spain, 8-12 September, 1998.

ide:1999ws [22] W. Płaczek, talk given at the meeting of the LEP2MC Workshop CERN, 12 Oct. 1999.

ldbab:1983 [23] F. A. Berends and R. Kleiss, *Nucl. Phys.* **B228** (1983) 537.

lumi4:1996 [24] S. Jadach, W. Płaczek, E. Richter-Wąs, B. F. L. Ward, and Z. Wąs, *Comput. Phys. Commun.* **102** (1997) 229.

zov:1999db [25] A. B. Arbuzov, hep-ph/9907298.

kkcpc:1999 [26] S. Jadach, Z. Wąs, and B. F. L. Ward, preprint DESY-99-106, CERN-TH/99-235,UTHEP-99-08-01.

ach:1999tr [27] S. Jadach, B. F. L. Ward, and Z. Was, *Comput. Phys. Commun.* **124** (2000) 233, hep-ph/9905205.

Jachol [28] A. J. kowska, J. Kalinowski, and Z. W. as, *Eur. J. Phys* **C6** (1999) 485.

AFBnowa [29] S. Jadach, B. Ward, and Z. Wąs, *Phys. Lett.* **B257** (1991) 213.

oralzearly [30] Z. Wąs, *Acta Phys. Polon.* **B18** (1987) 1099.

oralz:1991 [31] S. Jadach, B. F. L. Ward, and Z. Wąs, *Comput. Phys. Commun.* **66** (1991) 276.

oralz:1994 [32] S. Jadach, B. F. L. Ward, and Z. Wąs, *Comput. Phys. Commun.* **79** (1994) 503.

yfs2:1990 [33] S. Jadach and B. F. L. Ward, *Comput. Phys. Commun.* **56** (1990) 351.

yfs3:1992 [34] S. Jadach and B. F. L. Ward, *Phys. Lett.* **B274** (1992) 470.

KANCEL [35] S. J. and Z. WąsPhys Lett. **219** (1989) 103; J.H. Kühn, S. Jadach, R.G. Stuart and Z. Wąs, Z. Phys. **C38** (1988) 609; J.H. Kühn and R. G. Stuart, Phys. Lett. **B200** (1988) 360.

AFBstara [36] S. Jadach and Z. Wąs, *Phys. Rev.* **D41** (1990) 1425.

nunu [37] P. Colas, R. Miquel, and Z. Wąs, *Phys. Lett.* **B246** (1990) 541.

aul:1999tt [38] T. Paul, J. Swain, and Z. Was, hep-ph/9905207.

ska:1999ei [39] A. Jacholkowska, J. Kalinowski, and Z. Was, *Comput. Phys. Commun.* **124** (2000) 238, hep-ph/9905225.

leptoki [40] S. Jadach, B. Ward, and Z. Was, *Phys. Lett.* **B408** (1997) 281.

auola:1993 [41] R. Decker, S. Jadach, J. H. Kühn, and Z. Wąs, *Comput. Phys. Commun.* **76** (1993) 361.

hotos:1994 [42] E. Barberio and Z. Wąs, *Comput. Phys. Commun.* **79** (1994) 291.

ceex-big [43] S. Jadach, Z. Wąs, and B. F. L. Ward, preprint CERN-TH/2000-.., UTHEP-99-09-01.

zov:1997pj [44] A. B. Arbuzov *et al.*, *JHEP* **10** (1997) 001, hep-ph/9702262.

pek:1992vk [45] M. Skrzypek, *Acta Phys. Polon.* **B23** (1992) 135.

zov:1999cq [46] A. B. Arbuzov, hep-ph/9908361.

zov:1997qd [47] A. B. Arbuzov *et al.*, *Nucl. Phys.* **B485** (1997) 457–502, hep-ph/9512344.

man:1995ny [48] S. Eidelman and F. Jegerlehner, *Z. Phys.* **C67** (1995) 585–602, hep-ph/9502298.

din:1999yd [49] D. Bardin *et al.*, hep-ph/9908433.

din:1989tq [50] D. Bardin, M. Bilenky, P. Christova, T. Riemann, M. Sachwitz, and H. Vogt, *Comput. Phys. Commun.* **59** (1990) 303.

aev:1985hb [51] E. Kuraev and V. Fadin, *Sov. J. Nucl. Phys.* **41** (1985) 466–472.

zov:1996vi [52] A. B. Arbuzov, E. A. Kuraev, N. P. Merenkov, and L. Trentadue, *Nucl. Phys.* **B474** (1996) 271–285.

zov:1998ax [53] A. B. Arbuzov, E. A. Kuraev, and B. G. Shaikhatdenov, *J. Exp. Theor. Phys.* **88** (1999) 213, hep-ph/9805308.

GRACE [54] T. Ishikawa, T. Kaneko, K. Kato, S. Kawabata, Y. Shimizu, and H. Tanaka, "GRACE manual". KEK report 92-19, 1993.

QEDPS [55] T. Munehisa, J. Fujimoto, Y. Kurihara, and Y. Shimizu, *Prog. Theor. Phys.* **95** (1996) 375.

gmu [56] W. B. et al., "Ww cross-sections and distributions". in Physics at LEP2, eds. G. Altarelli, T. Sjöstrand, F. Zwirner, CERN 96-01, (1996), Vol.1, p.79.

berends [57] e. a. F.A. Berends, *Nucl. Phys.* **B301** (1988) 583.

1-loop [58] F. Berends, G. Burgers, and W. van Neerven, *Phys. Lett.* **B177** (1986) 191.

IN [59] M. Igarashi and N. Nakazawa, *Nucl. Phys.* **B288** (1987) 301.

mmnp:1999 [60] O. N. G. Montagna, M. Moretti and F. Piccinini, *Nucl. Phys.* **B541** (1999) 31–49, hep-ph/9807465.

iwara:1987 [61] K. Hagiwara, R. D. Peccei, D. Zeppenfeld, and K. Hikasa, *Nucl. Phys.* **B282** (1987) 253.

mnp:1996 [62] O. N. G. Montagna and F. Piccinini, *Comput. Phys. Commun.* **98** (1996) 206–214.

gp:1984 [63] G. Passarino, *Nucl. Phys.* **B237** (1984) 249.

cm:1995 [64] F. Caravaglios and M. Moretti, *Phys. Lett.* **B358** (1995) 332–338, hep-ph/9507237.

to:1997 [65] T. Ohl, *Comput. Phys. Commun.* **101** (1997) 269–288, hep-ph/9607454.

999yd-orig [66] D. Bardin, P. Christova, M. Jack, L. Kalinovskaya, A. Olshevski, S. Riemann, and T. Riemann, "ZFITTER v.6.21: A semi-analytical program for fermion pair production in e^+e^- annihilation", DESY 99–070 (1999), subm. to Comp. Phys. Commun., hep-ph/9908433.

ohm:1989pb [67] D. Bardin *et al.*, "Forward - backward asymmetries", in *Proc. of Workshop on Z Physics at LEP, Geneva, Switzerland, Feb 20-21 and May 8-9, 1989,* report CERN 89–08 (1989) (G. Altarelli, R. Kleiss, and C. Verzegnassi, eds.), vol. 1, pp. 201–234.

din:1989qr [68] D. Bardin *et al.*, "Z line shape", in *Proc. of Workshop on Z Physics at LEP, Geneva, Switzerland, Feb 20-21 and May 8-9, 1989,* report CERN 89–08 (1989) (G. Altarelli, R. Kleiss, and C. Verzegnassi, eds.), vol. 1, pp. 89–128.

ema:1996qg [69] E. Accomando *et al.*, "Standard model processes", in *Physics at LEP2,* report CERN 96–01 (1996) (G. Altarelli, T. Sjöstrand, and F. Zwirner, eds.), pp. 207–248, 1996, hep-ph/9601224.

din:1997xq [70] D. Bardin *et al.*, "Electroweak working group report", in *Reports of the Working Group on Precision Calculations for the Z Resonance,* report CERN 95–03 (1995) (D. Bardin, W. Hollik, and G. Passarino, eds.), pp. 7–162, hep-ph/9709229.

din:1998nm [71] D. Bardin and G. Passarino, "Upgrading of precision calculations for electroweak observables", preprint CERN-TH/98-92 (1998), hep-ph/9803425 v.2.

din:1999gt [72] D. Bardin, M. Grünewald, and G. Passarino, "Precision calculation project report", hep-ph/9902452 (1999).

ova:1999gh [73] P. Christova, M. Jack, S. Riemann, and T. Riemann, "Predictions of ZFITTER v.6 for fermion-pair production with acollinearity cut", DESY preprint 99-037 (1999), hep-ph/9908289.

ack:1999af [74] M. Jack, "QED radiative corrections to $e^+e^- \to \bar{f}f$ with realistic cuts at LEP energies and beyond", talk given at 14th Int. Workshop on High Energy Physics and Quantum Field Theory (QFTHEP 99), Moscow, Russia, 27 May - 2 Jun 1999, preprint DESY 99-166 (1999), hep-ph/9911296.

ova:2000zu [75] P. Christova, M. Jack, S. Riemann, and T. Riemann, "Radiative corrections to $e^+e^- \to \bar{f}f$", ECFA/DESY note LC-TH-2000-008 (2000), contrib. to: *Proc. of 2nd Joint ECFA/DESY Study on Physics and Detectors for a Linear Electron-Positron Collider, held at Orsay, Lund, Frascati, Oxford, and Obernai from April 1998 until Oct 1999*, to appear as DESY report 123F, "Physics Studies for a Future Linear Collider" (R. Heuer, F. Richard, P. Zerwas, eds.), hep-ph/0002054.

din:1991fu [76] D. Bardin, M. Bilenky, A. Chizhov, A. Sazonov, O. Fedorenko, T. Riemann, and M. Sachwitz, *Nucl. Phys.* **B351** (1991) 1–48.

din:1991de [77] D. Bardin, M. Bilenky, A. Sazonov, Y. Sedykh, T. Riemann, and M. Sachwitz, *Phys. Lett.* **B255** (1991) 290–296.

ova:1999cc [78] P. C. Christova, M. Jack, and T. Riemann, *Phys. Lett.* **B456** (1999) 264, hep-ph/9902408.

din:1980fe [79] D. Bardin, P. Christova, and O. Fedorenko, *Nucl. Phys.* **B175** (1980) 435.

din:1982sv [80] D. Bardin, P. Christova, and O. Fedorenko, *Nucl. Phys.* **B197** (1982) 1.

dov:1986fc [81] A. Akhundov, D. Bardin, and T. Riemann, *Nucl. Phys.* **B276** (1986) 1.

din:1986fi [82] D. Bardin, S. Riemann, and T. Riemann, *Z. Phys.* **C32** (1986) 121.

din:1989di [83] D. Bardin, M. S. Bilenky, G. Mitselmakher, T. Riemann, and M. Sachwitz, *Z. Phys.* **C44** (1989) 493.

989tq-orig [84] D. Bardin, M. Bilenky, P. Christova, T. Riemann, M. Sachwitz, and H. Vogt, *Comput. Phys. Commun.* **59** (1989) 303.

Silin:19xy [85] I. N. Silin, Fortran subroutine SIMPS.

edykh:19xy [86] Y. Sedykh, Fortran subroutine FDSIMP.

din:1988xt [87] D. Bardin, A. Leike, T. Riemann, and M. Sachwitz, *Phys. Lett.* **B206** (1988) 539–542.

nn:1997tj1 [88] T. Riemann, "The Z boson resonance parameters", in *Irreversibility and Causality, Proc. of the 21^{st} Int. Colloquium on Group Theoretical Methods in Physics, 15-20 July, 1996, Goslar, Germany, Lecture Notes in Physics, vol. 504* (A. Bohm, H. Doebner, and P. Kielanowski, eds.), pp. 157–177, Springer, Berlin, 1998, hep-ph/9709208.

ohm:2000jw [89] A. R. Bohm and N. L. Harshman, "On the mass and width of the Z-boson and other relativistic quasistable particles", Texas Univ. preprint (2000), hep-ph/0001206.

suura:1987 [90] T. Matsuura, Fortran subroutines TRILOG and S12.

ner:1995ZZ [91] F. Jegerlehner, Fortran function hadr5 (Feb 1995) [Eidelman:1995ny 48].

ssi:1996ZZ [92] G. Degrassi, Fortran package m2tcor (Oct 1996) [Degrassi:1997ps,Degrassi:1999jd 130, 135].

ov:1999uq1 [93] A. Arbuzov, Fortran packages pairho and funang (March 2000) [Arbuzov:1999uq 107].

hl:1990yc1 [94] B. Kniehl, Fortran package bkqcdl5_14.f (1990) [Kniehl:1990yc 164].

eau:1971mk [95] G. Bonneau and F. Martin, *Nucl. Phys.* **B27** (1971) 381–397.

eco:1975rm [96] M. Greco, G. Pancheri-Srivastava, and Y. Srivastava, *Nucl. Phys.* **B101** (1975) 234–262.

eco:1975ke [97] M. Greco, G. Pancheri-Srivastava, and Y. Srivastava, *Phys. Lett.* **56B** (1975) 367.

Greco:1980 [98] M. Greco, G. Pancheri-Srivastava, and Y. Srivastava, *Nucl. Phys.* **B171** (1980) 118–140; E: Nucl. Phys. B197 (1982) 543–546.

ehl:1988id [99] B. A. Kniehl, M. Krawczyk, J. H. Kühn, and R. G. Stuart, *Phys. Lett.* **209B** (1988) 337.

nds:1988ab [100] F. A. Berends, G. Burgers, and W. L. van Neerven, *Nucl. Phys.* **B297** (1988) 429; E: ibid., **B304** (1988) 921.

kii:1989zg [101] M. Bilenky and A. Sazonov, "QED corrections at Z^0 pole with realistic kinematical cuts", Dubna preprint JINR-E2-89-792 (1989).

ker:1989km [102] W. Beenakker, F. Berends, and W. van Neerven, "Applications of renormalization group methods to radiative corrections", in *Proc. of the Int. Workshop on Radiative Corrections for e^+e^- Collisions, Schloß Ringberg, Tegernsee, Germany, April 1989* (J. H. Kühn, ed.), p. 3, Springer, Berlin, 1989.

gna:1989at [103] G. Montagna, O. Nicrosini, and L. Trentadue, *Phys. Lett.* **231** (1989) 492.

gna:1991ku [104] G. Montagna, O. Nicrosini, and L. Trentadue, *Nucl. Phys.* **B357** (1991) 390.

ach:1992aa [105] S. Jadach, M. Skrzypek, and M. Martinez, *Phys. Lett.* **B280** (1992) 129–136.

gna:1997jv [106] G. Montagna, O. Nicrosini, and F. Piccinini, *Phys. Lett.* **B406** (1997) 243–248, hep-ph/9611463 v.2.

zov:1999uq [107] A. Arbuzov, "Light pair corrections to e^+e^- annihilation at LEP/SLC", hep-ph/9907500.

len:1955ks [108] G. Källén and A.Sabry, *K. Dan. Vidensk. Selsk. Mat.-Fys. Medd.* **17** (1955) 29.

erman:1958 [109] S. M. Berman, *Phys. Rev.* **112** (1958) 267.

ita:1959ru [110] T. Kinoshita and A. Sirlin, *Phys. Rev.* **113** (1959) 1652–1660.

allen:1968 [111] G. Källen, *Springer Tracts in Modern Physics* **46** (1968) 67–132.

ine:1979qh [112] M. Dine and J. Sapirstein, *Phys. Rev. Lett.* **43** (1979) 668.

kin:1979bj [113] K. G. Chetyrkin, A. L. Kataev, and F. V. Tkachev, *Phys. Lett.* **85B** (1979) 277.

ter:1980xr [114] W. Celmaster and R. J. Gonsalves, *Phys. Rev. Lett.* **44** (1980) 560.

ann:1984dv [115] G. Mann and T. Riemann, *Annalen Phys.* **40** (1984) 334.

Bij:1984bw [116] J. van der Bij and M. Veltman, *Nucl. Phys.* **B231** (1984) 205.

Bij:1984aj [117] J. J. van der Bij, *Nucl. Phys.* **B248** (1984) 141.

adi:1987gn [118] A. Djouadi and C. Verzegnassi, *Phys. Lett.* **195B** (1987) 265.

Bij:1987hy [119] J. J. van der Bij and F. Hoogeveen, *Nucl. Phys.* **B283** (1987) 477.

adi:1988di [120] A. Djouadi, *Nuovo Cim.* **100A** (1988) 357.

din:1989aa [121] D. Bardin and A. Chizhov, "On the $O(\alpha_{em}\alpha_s)$ corrections to electroweak observables", in *Proc. Int. Topical Seminar on Physics of e^+e^- Interactions at LEP energies, 15-16 Nov 1988, JINR Dubna, USSR,* JINR preprint E2-89-525 (1989) (D. Bardin *et al.*, eds.), pp. 42–48.

nii:1991hw [122] S. Gorishny, A. Kataev, and S. Larin, *Phys. Lett.* **B273** (1991) 141–144.

aev:1992dg [123] A. L. Kataev, *Phys. Lett.* **B287** (1992) 209–212.

zov:1992pr [124] A. B. Arbuzov, D. Bardin, and A. Leike, *Mod. Phys. Lett.* **A7** (1992) 2029–2038, E: ibid. **A9** (1994) 1515.

her:1992fq [125] J. Fleischer, O. V. Tarasov, F. Jegerlehner, and P. Raczka, *Phys. Lett.* **B293** (1992) 437–444.

eri:1993ra [126] R. Barbieri, P. Ciafaloni, and A. Strumia, *Phys. Lett.* **B317** (1993) 381.

eev:1994db [127] L. Avdeev, J. Fleischer, S. Mikhailov, and O. Tarasov, *Phys. Lett.* **B336** (1994) 560–566, E: ibid., **B349** (1995) 597, hep-ph/9406363.

in:1994js3 [128] K. Chetyrkin, J. Kühn, and A. Kwiatkowski, "QCD corrections to the e^+e^- cross-section and the Z boson decay rate", in *Reports of the Working Group on Precision Calculations for the Z Resonance,* report CERN 95–03 (1995) (D. Bardin, W. Hollik, and G. Passarino, eds.), pp. 175–263, hep-ph/9503396.

ssi:1996mg [129] G. Degrassi, P. Gambino, and A. Vicini, *Phys. Lett.* **B383** (1996) 219–226, hep-ph/9603374.

ssi:1997ps [130] G. Degrassi, P. Gambino, and A. Sirlin, *Phys. Lett.* **B394** (1997) 188–194, hep-ph/9611363.

ser:1998rq [131] M. Steinhauser, *Phys. Lett.* **B429** (1998) 158–161, hep-ph/9803313.

gen:1998hn [132] T. van Ritbergen and R. G. Stuart, *Phys. Lett.* **B437** (1998) 201, hep-ph/9802341.

gen:1998yd [133] T. van Ritbergen and R. G. Stuart, *Phys. Rev. Lett.* **82** (1999) 488, hep-ph/9808283.

der:1998zb [134] R. Harlander, T. Seidensticker, and M. Steinhauser, *Phys. Lett.* **B426** (1998) 125–132, hep-ph/9712228.

ssi:1999jd [135] G. Degrassi and P. Gambino, "Two loop heavy top corrections to the Z^0 boson partial widths", Padua Univ. preprint DFPD-99-TH-19 (1999), hep-ph/9905472.

din:1999ak [136] D. Bardin and G. Passarino, "The standard model in the making: Precision study of the electroweak interactions" (Oxford, UK, Clarendon, 1999).

ike:1991pq [137] A. Leike, T. Riemann, and J. Rose, *Phys. Lett.* **B273** (1991) 513–518, hep-ph/9508390.

ann:1992gv [138] T. Riemann, *Phys. Lett.* **B293** (1992) 451–456, hep-ph/9506382.

ch:1995cf1 [139] M. Grünewald, S. Kirsch, and T. Riemann, Fortran program SMATASY v.6.10 (27 May 1999), obtainable from
/afs/cern.ch/user/g/gruenew/public/smatasy/smata6_10.fortran
or from http://l3www.cern.ch/homepages/gruenew/welcome.html.

ann:1997aa [140] S. Riemann, Fortran program ZEFIT v. 5.0 (1997);
obtainable from http://www.ifh.de/~riemanns/uu/zefit5_0.uu.

ach:1999vf [141] S. Jadach, B. F. L. Ward, and Z. Was, "The precision Monte Carlo event generator KK for two-fermion final states in e^+e^- collisions", preprint (1999), hep-ph/9912214.

eld:1996dk [142] J. H. Field and T. Riemann, *Comput. Phys. Commun.* **94** (1996) 53–87, hep-ph/9507401.

zov:1995id [143] A. Arbuzov, D. Bardin, J. Blümlein, L. Kalinovskaya, and T. Riemann, *Comput. Phys. Commun.* **94** (1996) 128, hep-ph/9511434.

ner:1999hg [144] F. Jegerlehner, hep-ph/9901386.

auser:1998 [145] M. Steinhauser, *Phys. Lett.* **B429** (1998) 158.

-BJPV:1989 [146] G. P. H. Burkhardt, F. Jegerlehner and C. Verzegnassi, *Z. Phys.* **C43** (1989) 497.

eJege:1995 [147] S. Eidelman and F. Jegerlehner, *Z. Phys.* **C67** (1995) 585.

Bolek:1995 [148] H. Burkhardt and B. Pietrzyk, *Phys. Lett.* **B356** (1995) 398.

zorov:1994 [149] A. N. R.B. Nevzorov and M.I.Vysotsky, *JETP Lett.* **60** (1994) 399.

nbein:1994 [150] B. Geshkenbein and V. Morgunov HEPPH–9407228.

nfeld:1994 [151] A. Martin and D. Zeppenfeld, *Phys. Lett.* **B345** (1994) 558.

wartz:1995 [152] M. Swartz, *Phys. Rev.* **D53** (1996) 5268.

-AbYn:1995 [153] K. Abel and F. Yndurain FTUAM-95-32, September 1995.

emany:1998 [154] *et al.*. R. Alemany, *Eur. Phys. J.* **C2** (1998) 123.

avier:1997 [155] M. Davier and A. Höcker, *Phys. Lett.* **B419** (1998) 419.

-Kuhn:1998 [156] J. Kühn and M. Steinhanser, *Phys. Lett.* **B437** (1998) 425.

roote:1998 [157] J. K. S. Groote and A. Pivovarov, *Phys. Lett.* **B440** (1998) 375.

avier:1998 [158] M. Davier and A. Höcker, *Phys. Lett.* **B435** (1998) 427.

MichOPAL [159] T. O. Collaboration, see also:
`http://home.cern.ch/ mkobel/lep2mc/index.html`.

pairTOPAZ0 [160] G. P. ..., reference on pair treatment in topaz0.

airZFITTER [161] D. B. ..., reference on pair treatment in dizet.

GiampPP [162] G. Passarino, "The pp-lep2poll". The PP-LEP2Poll, transparencies, December 8, 1999, **www** page at http://www.to.infn.it/ giampier/latex/pp_quest.ps.

MichOPAL2 [163] M. Kobel, `http://home.cern.ch/ mkobel/lep2mc/P026.html` and further pages.

ehl:1990yc [164] B. Kniehl, *Nucl. Phys.* **B347** (1990) 86–104.

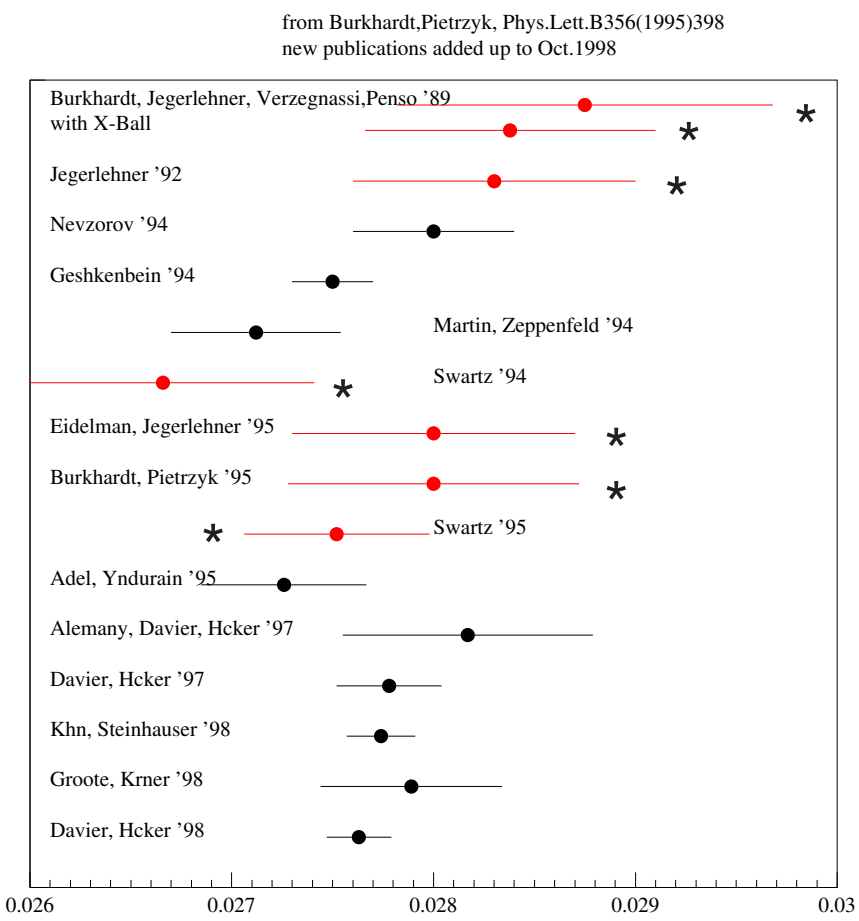

Fig. 1: Results of calculations of the hadronic contribution to the QED vacuum polarization. The results marked with * are obtained using a dispersion integral of R_{had} measured experimentally.

alpha_figure

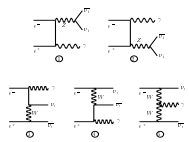

Fig. 2: The CERN $\bar{p}$ complex.