TWOFOLD WORLD

Giampiero PASSARINO[ab],

MOSCOW May 27, 1999

[a] Dipartimento di Fisica Teorica, Università di Torino, Torino, Italy
[b] INFN, Sezione di Torino, Torino, Italy

Summary

- Introduction to 2f.
- What is available and what is not.

PROTOTYPE of comparisons for 2f calculations can be found in hep-ph/9902452, D. Bardin, M. Grünewald and G. P., based on `TOPAZ0` versus `ZFITTER`

At present, the following physicists are active members of the two teams:

`TOPAZ0` : G. Montagna, O. Nicrosini, G. Passarino and F. Piccinini;

`ZFITTER` : D. Bardin, P. Christova, M. Jack, L. Kalinovskaya, A. Olshevski, S. Riemann, T. Riemann.

Prolegomena

Take the LEP-1 measurements of $\sigma_{\mu,\mathrm{had}}$ and A^{l}_{FB} as an example to discuss the data analysis strategy.

- **each LEP experiment extracts a set of**

$$\mathrm{PO} \stackrel{\mathrm{def}}{=} \{M_{Z}, \Gamma_{Z}, R_{e,\mu,\tau}, A^{0,e,\mu,\tau}_{\mathrm{FB}}\}, \qquad (0.1)$$

  from their measured σ and A_{FB} (*realistic observables*).

- **The four sets of PO are combined, taking correlated errors between the LEP experiments into account, in order to obtain a LEP-average set of PO.**

- **The latter is then interpreted, for example within the frame-work of the Minimal Standard Model.**

- **The practical attitude of the experiments is to stay with a *Model-Independent* (MI) fit, i.e.**

$$\mathrm{fromRO} \to \mathrm{PO} \oplus \textbf{a SM remnant} \qquad (0.2)$$

  for each experiment, and these sets of PO are averaged.

- **The extraction of Lagrangian parameters,**

$$M_Z, m_t, M_H, \alpha_S(M_Z^2), \quad \textbf{and} \quad \alpha(M_Z^2), \qquad (0.3)$$

is based on the LEP-averaged PO.

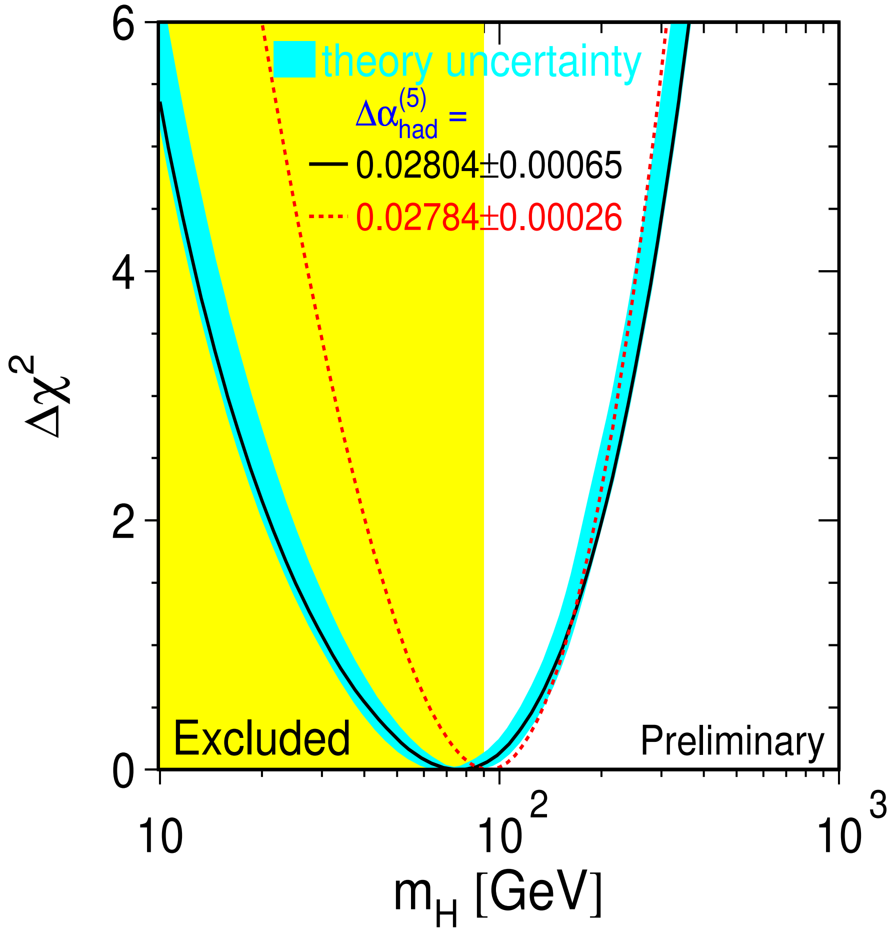
- **Since PO are determined by fitting RO, one has to clarify what is actually taken from the SM, such as**

    1. **imaginary parts,**

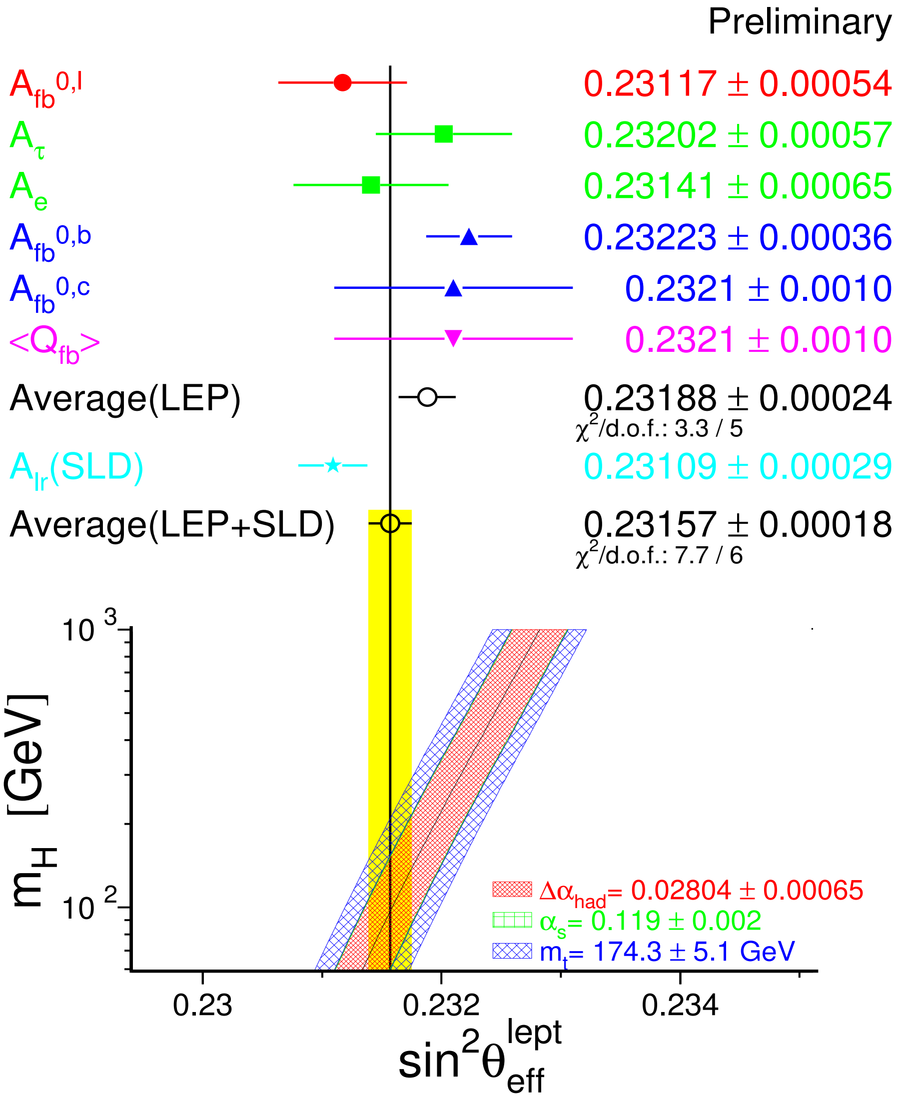
    2. **parts which have been moved to interference terms and photon-exchange terms) making the MI results dependent on the SM.**

- **An alternative way to extract SM parameters from the LEP measurements is to average directly the SM parameters which have been obtained by each experiment through a SM fit to its own RO.**

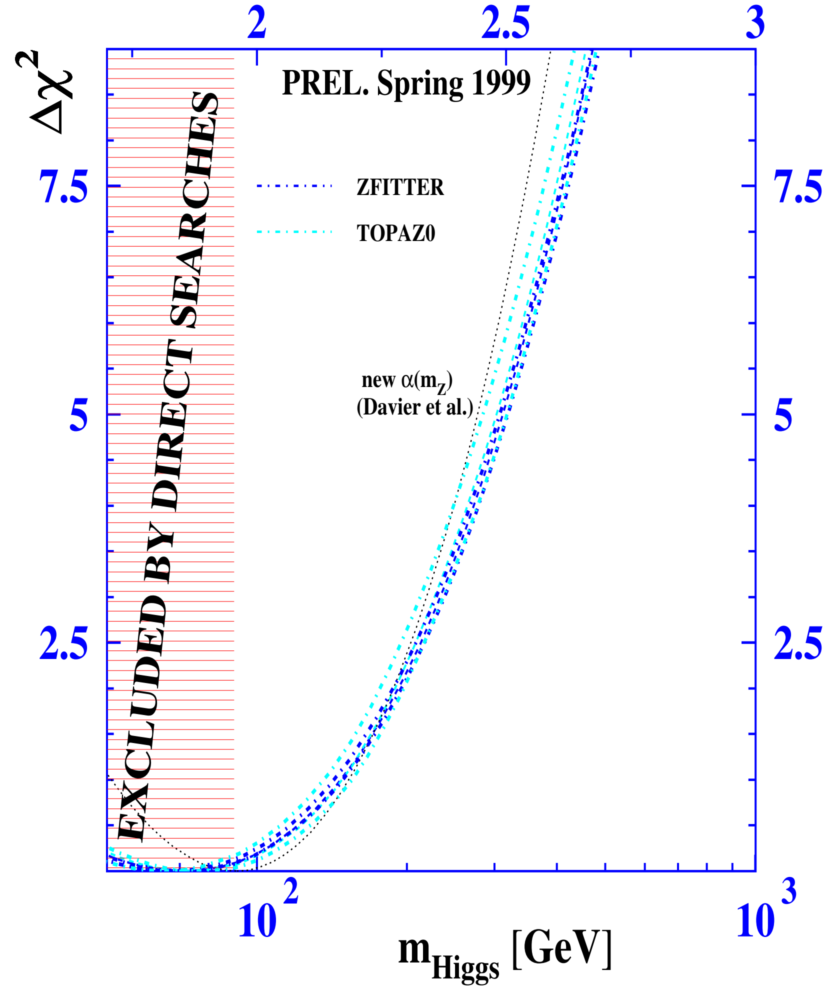
- **This alternative should also be pursued by the experiments.**

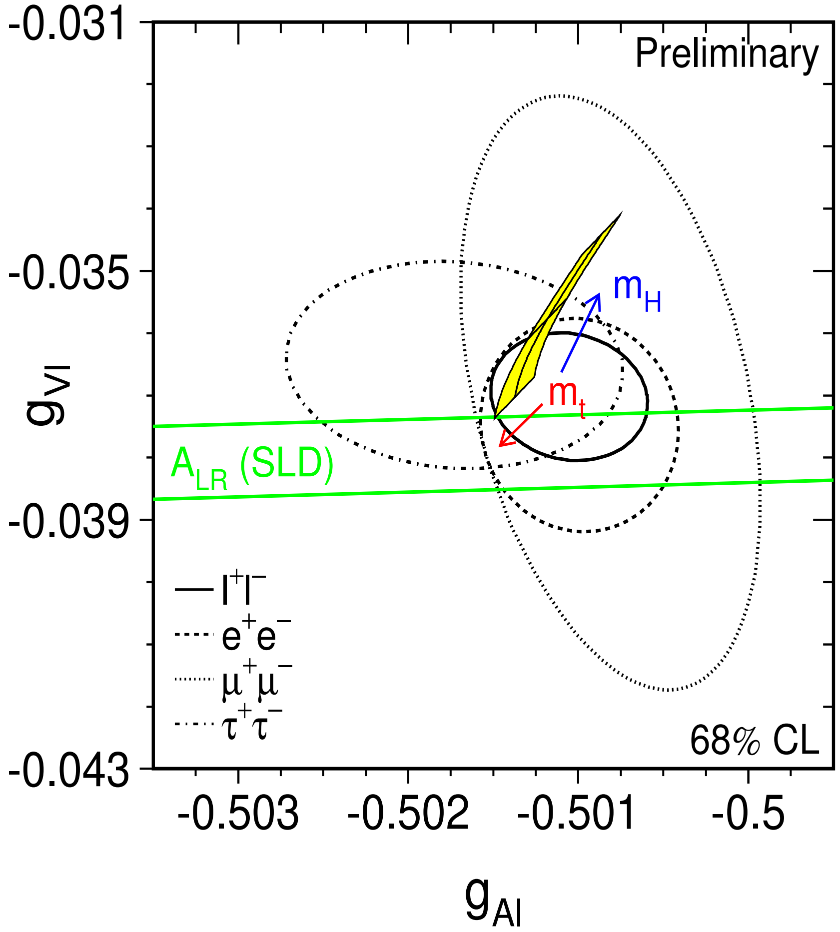


theory uncertainty
$\Delta\alpha^{(5)}_{had}$ =
0.02804±0.00065
0.02784±0.00026
Excluded
Preliminary
$\Delta\chi^2$
m_H [GeV]
0
2
4
6
10
10^2
10^3



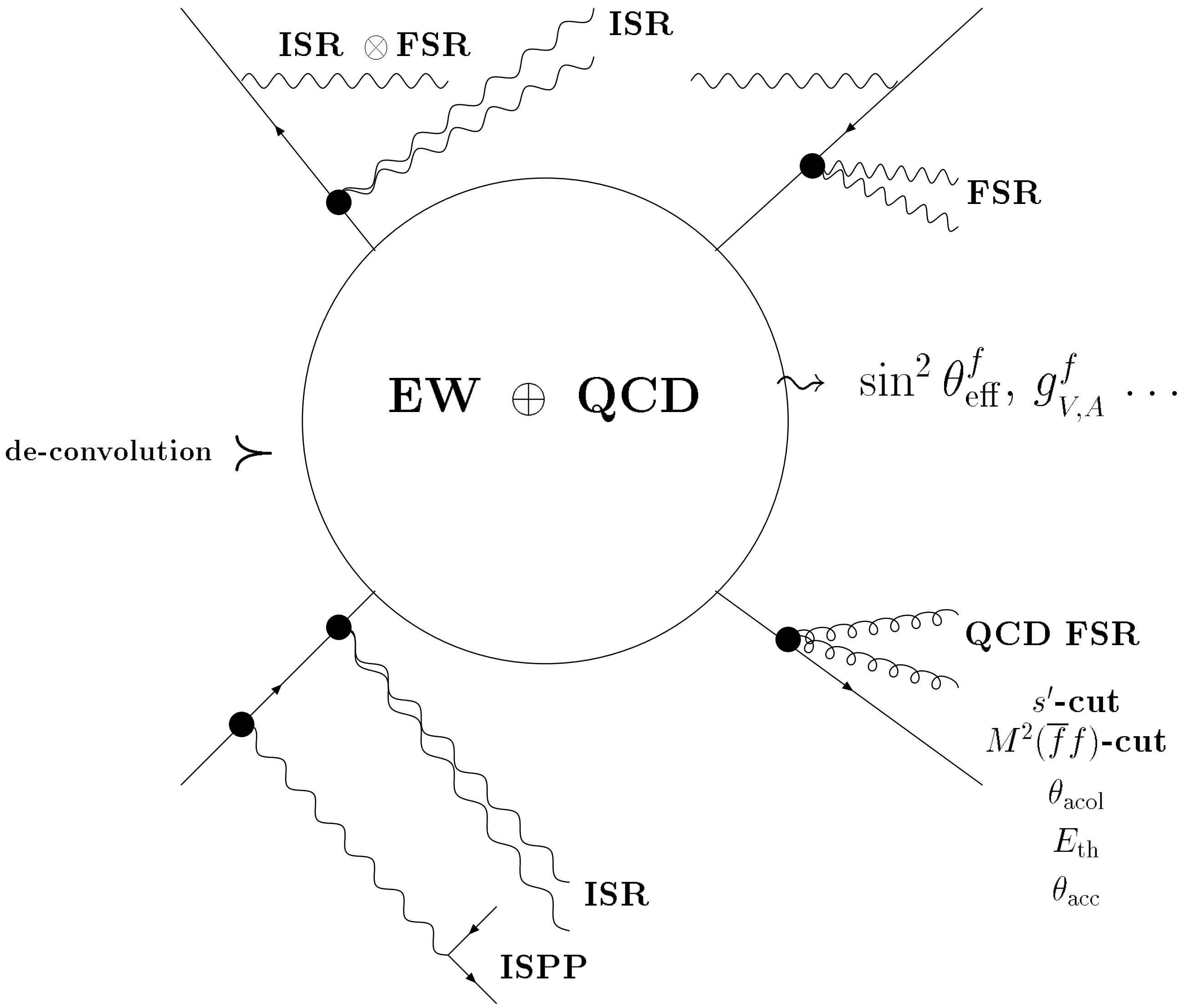
Preliminary
$A_{fb}^{0,l}$ 0.23117 ± 0.00054
A_τ 0.23202 ± 0.00057
A_e 0.23141 ± 0.00065
$A_{fb}^{0,b}$ 0.23223 ± 0.00036
$A_{fb}^{0,c}$ 0.2321 ± 0.0010
<Q_{fb}> 0.2321 ± 0.0010
Average(LEP) 0.23188 ± 0.00024
χ^2/d.o.f.: 3.3 / 5
A_{lr}(SLD) 0.23109 ± 0.00029
Average(LEP+SLD) 0.23157 ± 0.00018
χ^2/d.o.f.: 7.7 / 6
10^3
10^2
m_H [GeV]
$\Delta\alpha_{had}$= 0.02804 ± 0.00065
α_s= 0.119 ± 0.002
m_t= 174.3 ± 5.1 GeV
0.23
0.232
0.234
$\sin^2\theta_{eff}^{lept}$


PREL. Spring 1999
ZFITTER
TOPAZ0
new $\alpha(m_Z)$
(Davier et al.)
EXCLUDED BY DIRECT SEARCHES
$\Delta\chi^2$
m_{Higgs} [GeV]
2
2.5
3
7.5
5
2.5
10^2
10^3



Preliminary
68% CL
m_H
m_t
A_{LR} (SLD)
l^+l^-
e^+e^-
$\mu^+\mu^-$
$\tau^+\tau^-$
-0.031
-0.035
-0.039
-0.043
-0.503
-0.502
-0.501
-0.5
g_{Vl}
g_{Al}

2f kit


ISR ⊗ FSR
ISR
FSR
EW ⊕ QCD
$\rightsquigarrow \sin^2\theta^f_{\rm eff},\ g^f_{V,A}\ \dots$
de-convolution
QCD FSR
s'-cut
$M^2(\overline{f}f)$-cut
$\theta_{\rm acol}$
$E_{\rm th}$
$\theta_{\rm acc}$
ISR
ISPP

Exegesis of comparison

- **Experimental informations are passed through *decoders* like `TOPAZ0` and `ZFITTER`. Therefore, it is of the upmost importance:**

- **to critically upgrade and to compare the complete `TOPAZ0` and `ZFITTER` predictions with a particular emphasis on demanding the following criteria:**

  - **Comparisons, after the upgrading, should be consistently better than what they where in earlier studies.**

  - **At the peak all relative deviations among total cross-sections and absolute deviations among asymmetries should be below 0.1 per-mill.**

  - **At the wings, typically $\sqrt{s} = M_{_Z} \pm 1.8\,\mathrm{GeV}$, they should be below 0.3 per-mill.**

- **(It is important to observe that today's comparisons are at the level of 10^{-4}, never been attempted before).**

PO, RO and all that

- **Within the context of the SM the RO are described in terms of some set of amplitudes**

$$A_{\text{SM}} = A_{\gamma} + A_{Z} + \textbf{non-factorizable}, \qquad (0.4)$$

- **where the last term is due to all those contributions that do not factorize into the Born-like amplitude**

- **Once the matrix element A_{SM} is computed, squared and integrated, a convolution with initial- and final-state QED and final-state QCD radiation follows:**

$$\sigma(s) = \int dz\ H_{\text{in}}(z,s)\, H_{\text{fin}}(z,s)\, \hat{\sigma}(zs), \qquad (0.5)$$

After de-convoluting RO of QED and QCD radiation the set of approximations transform RO into PO.

The inner circle

- **The effective couplings $\in C$, due to the imaginary parts of the diagrams.**

- **In the past this fact had some relevance only for realistic distributions while for PO they were** *conventionally* **taken to be real.**

- **The above convention has changed lately with the introduction of NLO corrections: imaginary parts, although not NLO in a strict sense, are sizeable two-loop effects.**

The explicit formulae for the $Zf\overline{f}$ vertex are always written starting from

- **a Born-like form of a pre-factor $\times$ fermionic current, where**

- **the Born parameters are promoted to effective, scale-dependent parameters,**

$$\rho_Z^f \gamma_\mu \left[\left(I_f^{(3)} + i\, a_L\right)\gamma_+ - 2\, Q_f \kappa_Z^f s^2 + i\, a_Q\right] = \gamma_\mu \left(\mathcal{G}_V^f + \mathcal{G}_A^f \gamma_5\right), \tag{0.6}$$

- **where $\gamma_+ = 1 + \gamma_5$ and $a_{Q,L}$ are the SM imaginary parts.**

- **By definition, the total and partial widths of the Z boson include also QED and QCD corrections. The partial decay width is therefore described by the following expression:**

$$\Gamma_f \equiv \Gamma\left(Z \to f\overline{f}\right) = 4\, c_f\, \Gamma_0 \left(|\mathcal{G}_V^f|^2\, R_V^f + |\mathcal{G}_A^f|^2\, R_A^f\right) + \Delta_{\mathrm{EW/QCD}}\ , \tag{0.7}$$

where $c_f = 1$ or 3 for leptons or quarks ($f = l, q$), and R_V^f and R_A^f describe the final state QED and QCD corrections and take into account the fermion mass m_f.

- **The last term,**

$$\Delta_{\mathrm{EW/QCD}} = \Gamma^{(2)}_{\mathrm{EW/QCD}} - \frac{\alpha_S}{\pi}\, \Gamma^{(1)}_{\mathrm{EW}}\ , \tag{0.8}$$

accounts for the non-factorizable corrections.

- **The standard partial width, Γ_0, is**

$$\Gamma_0 = \frac{G_F M_Z^3}{24\sqrt{2}\,\pi} = 82.945(7)\,\mathrm{MeV}. \tag{0.9}$$

- **The peak hadronic and leptonic cross-sections are defined by**

$$\sigma_h^0 = 12\pi \frac{\Gamma_e \Gamma_h}{M_Z^2 \Gamma_Z^2} \qquad \sigma_\ell^0 = 12\pi \frac{\Gamma_e \Gamma_l}{M_Z^2 \Gamma_Z^2} \,, \tag{0.10}$$

where Γ_Z is the total decay width of the Z boson, i.e, the sum of all partial decay widths.

- **The effective electroweak mixing angles (*effective sinuses*) are always defined by**

$$4\,|Q_f| \sin^2\theta_{\rm eff}^f = 1 - \frac{\mathrm{Re}\,\mathcal{G}_V^f}{\mathrm{Re}\,\mathcal{G}_A^f} = 1 - \frac{g_V^f}{g_A^f} \,, \tag{0.11}$$

where we define

$$g_V^f = \mathrm{Re}\,\mathcal{G}_V^f, \qquad g_A^f = \mathrm{Re}\,\mathcal{G}_A^f. \tag{0.12}$$

- **The forward-backward asymmetry $A_{\rm FB}$ is defined via**

$$A_{\rm FB} = \frac{\sigma_{\rm F} - \sigma_{\rm B}}{\sigma_{\rm F} + \sigma_{\rm B}} \,, \qquad \sigma_{\rm T} = \sigma_{\rm F} + \sigma_{\rm B} \,, \tag{0.13}$$

where $\sigma_{\rm F}$ and $\sigma_{\rm B}$ are the cross sections for forward and backward scattering, respectively.

- **Before analyzing the forward-backward asymmetries we have to describe the inclusion of imaginary parts. $A_{\rm FB}$ is calculated as**

$$A_{\rm FB} = \frac{3}{4}\frac{\sigma_{\rm VA}}{\sigma_{\rm T}} \;, \tag{0.14}$$

where

$$\sigma_{\rm VA} = \frac{G_F M_Z^2}{\sqrt{2}} \sqrt{\rho_e \rho_f}\, Q_e Q_f {\rm Re}[\alpha^*(M_Z^2) \mathcal{G}_V^e \mathcal{G}_A^f \chi(s)] + \frac{G_F^2 M_Z^4}{8\,\pi} \rho_e \rho_f {\rm Re}[\mathcal{G}_V^e \left(\mathcal{G}_A^e\right)^*] {\rm Re}[\mathcal{G}_V^f \left(\mathcal{G}_A^f\right)^*] s\, |\chi(s)|^2. \tag{0.15}$$

- **This result is valid in the realization where ρ_f is a real quantity, i.e., the imaginary parts are not re-summed in ρ_f. In this case**

$$\mathcal{G}_V^f = {\rm Re}\left(\mathcal{G}_V^f\right) + i\,{\rm Im}\left(\mathcal{G}_V^f\right) = g_V^f + i\,{\rm Im}\left(\mathcal{G}_V^f\right), \qquad \mathcal{G}_A^f = I_f^{(3)} + i\,{\rm Im}\left(\mathcal{G}_A^f\right). \tag{0.16}$$

- **Otherwise $\mathcal{G}_A^f = I_f^{(3)}$ is a real quantity but ρ_f is complex valued and Eq.(0.15) has to be changed accordingly, i.e., we introduce**

$$g_V^f = \sqrt{\rho_f}\, v_f \;, \qquad g_A^f = \sqrt{\rho_f}\, I_f^{(3)} \;, \tag{0.17}$$

with

$$v_f = I_f^{(3)} - 2 Q_f \sin^2\theta_{\rm eff}^f \;. \tag{0.18}$$

- **For the peak asymmetry, the presence of ρ's is irrelevant since they will cancel in the ratio. We have**

$$\hat{A}^{0\text{f}}_{\text{FB}} = \frac{3}{4}\,\hat{A}_e\,\hat{A}_f,$$
$$\hat{A}_f = \frac{2\,\text{Re}[\mathcal{G}^f_V\,(\mathcal{G}^f_A)^*]}{(|\mathcal{G}^f_V|^2 + |\mathcal{G}^f_A|^2)}\,. \qquad (0.19)$$

- **The question is what to do with imaginary parts in Eq.(0.19). For partial widths, as they absorb all corrections, the convention is to use**

$$|\mathcal{G}^f_{V,A}|^2 = (\text{Re}\mathcal{G}^f_{V,A})^2 + (\text{Im}\mathcal{G}^f_{V,A})^2. \qquad (0.20)$$

- **On the contrary, the PO peak asymmetry** $A^{0\text{f}}_{\text{FB}}$ **will be defined by an analogy of equation Eq.(0.19) where** *conventionally* **imaginary parts are not included**

$$A^{0\text{f}}_{\text{FB}} = \frac{3}{4}\,\mathcal{A}_e\,\mathcal{A}_f,$$
$$\mathcal{A}_f = \frac{2(g^f_V g^f_A)}{(g^f_V)^2+(g^f_A)^2}\,. \qquad (0.21)$$

- **A definition of the PO heavy quark forward-backward asymmetry parameter which would include mass effects is**

$$\mathcal{A}_b = \frac{2\, g^b_{_V} g^b_{_A}}{\frac{1}{2}\,(3-\beta^2)\,(g^b_{_V})^2 + \beta^2\,(g^b_{_A})^2}\,\beta, \tag{0.22}$$

where β is the b-quark velocity. The difference is very small, due to an accidental cancellation of the mass corrections between the numerator and denominator of Eq.(0.22). This occurs for down quarks where $(g^b_{_V})^2 \approx (g^b_{_A})^2/2$ and where

$$A^{0\mathrm{b}}_{_{\mathrm{FB}}} \approx \frac{3}{4}\,\frac{2\, g^e_{_V} g^e_{_A}}{(g^e_{_V})^2+(g^e_{_A})^2}\,\frac{2\, g^b_{_V} g^b_{_A}}{(g^b_{_V})^2+(g^b_{_A})^2}\,\left(1+\delta_{\mathrm{mass}}\right),$$

$$\delta_{\mathrm{mass}} \approx 4\,\frac{m_q^2}{s}\frac{(g^b_{_A})^2/2-(g^b_{_V})^2}{(g^b_{_V})^2+(g^b_{_A})^2}. \tag{0.23}$$

Therefore, our definition of the PO forward–backward asymmetry and coupling parameter will be as in Eq.(0.21).

Status of art

- **The most important upgradings in the PO calculations consist of the inclusion of**

  1. **higher-order QCD corrections,**

  2. **mixed electroweak-QCD corrections,**

     A. Czarnecki and J.H. Kühn, Phys. Rev. Lett. 77 (1996) 3955;
     R. Harlander, T. Seidensticker and M. Steinhauser, Phys. Lett. B426 (1998) 125.

  3. **NLO two-loop corrections of**

     $\mathcal{O}\left(\alpha^2 m_t^2\right)$

     G. Degrassi and P. Gambino, *in preparation.*

     The only case that is not covered is the one of final b-quarks, because it involves non-universal $\mathcal{O}\left(\alpha^2 m_t^2\right)$ vertex corrections.

- **Another development in the computation of radiative corrections to the hadronic decay of the Z is contained in two papers , which**

  A. Czarnecki and J.H. Kühn, Phys. Rev. Lett. 77 (1996) 3955;

R. Harlander, T. Seidensticker and M. Steinhauser, Phys. Lett. B426 (1998) 125.

together provide complete corrections of $\mathcal{O}(\alpha\alpha_s)$ to $\Gamma(Z \to q\overline{q})$ with $q = u, d, s, c$ and b.

Improved I/O Parameters

- Since all RS use G_F, we refer to

  T. van Ritbergen and R. G. Stuart, Phys. Rev. Lett. 82 (1999) 488

  giving an improved value of $G_F = 1.16637(1) \times 10^{-5}\,\mathrm{GeV}^{-2}$

- An important issue concerns the evaluation of α_{QED} at the mass of the Z. Define

$$\alpha(M_Z) = \frac{\alpha(0)}{1 - \Delta\alpha^{(5)}(M_Z) - \Delta_{\mathrm{top}}(M_Z) - \Delta_{\mathrm{top}}^{\alpha\alpha_S}(M_Z)}\,, \tag{0.24}$$

where one has

$$\Delta\alpha^{(5)}(M_Z) = \Delta\alpha_{\mathrm{lept}} + \Delta\alpha^{(5)}_{\mathrm{had}}. \tag{0.25}$$

- The input parameter is now $\Delta\alpha^{(5)}_{\mathrm{had}}$, as it is the contribution with the largest uncertainty, while

- **the calculation of the top contributions to $\Delta\alpha$ is left for the code.**

- **Next, include for $\Delta\alpha_{\rm lept}$ the $\mathcal{O}\left(\alpha^3\right)$ terms of**

  M. Steinhauser, Phys. Lett. B429 (1998) 158

- **and use as default $\Delta\alpha^{(5)}_{\rm had} = 0.0280398$, taken from**

  S. Eidelman and F. Jegerlehner, Z. Phys. C67 (1995) 585

- **Therefore $1/\alpha^{(5)}(M_z) = 128.877$, to which one must add the $t\bar{t}$ contribution and the $\mathcal{O}(\alpha\alpha_s)$ correction induced by the $t\bar{t}$ loop with gluon exchange,**

  B. Kniehl, Nucl. Phys. B347 (1990) 86

MI Calculus

- **To summarize the MI ansatz,**

  1. **one starts with the SM, which introduces complex-valued couplings, calculated to some order in perturbation theory;**

  2. **next we define**

     * g_V^f, g_A^f **as the real parts of the effective couplings and**

     * Γ_f **as the physical partial width absorbing all radiative corrections including the imaginary parts of the couplings and fermion mass effects.**

  3. **Furthermore,**

  $$R_q = \frac{\Gamma_q}{\Gamma_h}\,, \qquad R_l = \frac{\Gamma_h}{\Gamma_l}\,, \tag{0.26}$$

  for quarks and leptons, respectively.

- **The experimental collaborations report PO for the following sets:**

$$(R_f, A_{\rm FB}^{0,f}), \qquad (g_V^f, g_A^f), \qquad (\sin^2\theta_{\rm eff}^f, \rho_f). \tag{0.27}$$

- **In order to extract g_V^f, g_A^f from Γ_f one has to get the SM-remnant from Eq.(0.7), all else is trivial.**

- **However, the parameter transformation cannot be completely MI, due to the residual SM dependence appearing inside Eq.(0.7).**

- **In conclusion, the flow of the calculation requested by the experimental Collaborations is:**

  1. **pick the Lagrangian parameters m_t, M_H etc. for the explicit calculation of the residual SM-dependent part;**

  2. **perform the SM initialization of everything, such as imaginary parts etc. giving, among other things, the complement $\overline{\mathrm{SM}}$;**

  3. **select g_V^f, g_A^f;**

  4. **perform a SM-like calculation of Γ_f, but using arbitrary values for g_V^f, g_A^f, and only the rest, namely**

$$R_V^f, \quad R_A^f, \qquad \Delta_{\mathrm{EW/QCD}}, \qquad \mathrm{Im}\,\mathcal{G}_V^f, \quad \mathrm{Im}\,\mathcal{G}_A^f, \tag{0.28}$$

     from the SM.

- **An example of the parameter transformations is the following: starting from $M_{_Z}, \Gamma_{_Z}, R_{e,\mu,\tau}$ and $A^{0,e,\mu,\tau}_{\rm FB}$ we first obtain**

$$\Gamma_e = M_{_Z}\Gamma_{_Z}\left[\frac{\sigma^0_h}{12\,\pi R_e}\right]^{1/2},$$
$$\Gamma_{_h} = M_{_Z}\Gamma_{_Z}\left[\frac{\sigma^0_h R_e}{12\,\pi}\right]^{1/2}. \tag{0.29}$$

With

$$\mathcal{A}_e = \frac{2}{\sqrt{3}}\sqrt{A^{0,e}_{\rm FB}}\,, \qquad \text{and} \quad \gamma = \frac{G_{_F}M^3_{_Z}}{6\,\sqrt{2}\,\pi}\,, \tag{0.30}$$

- **we subtract QED radiation,**

$$\Gamma^0_e = \frac{\Gamma_e}{1 + \frac{3}{4}\,\frac{\alpha(M^2_{_Z})}{\pi}}\,, \tag{0.31}$$

- **and get**

$$\sin^2\theta^e_{\rm eff} = \frac{1}{4}\left(1 + \frac{\sqrt{1-\mathcal{A}^2_e}-1}{\mathcal{A}_e}\right),$$
$$\rho_e = \frac{\Gamma^0_e}{\gamma}\left[\left(\frac{1}{2} - 2\,\sin^2\theta^e_{\rm eff}\right)^2 + \frac{1}{4} + \left(\mathrm{Im}\;\mathcal{G}^e_{_V}\right)^2 + \left(\mathrm{Im}\;\mathcal{G}^e_{_A}\right)^2\right]^{-1} \tag{0.32}$$

With

$$\mathcal{A}_f = \frac{4}{3}\,\frac{A^{0,f}_{\rm FB}}{\mathcal{A}_e}\,, \tag{0.33}$$

- **we further obtain**

$$\sin^2\theta^f_{\rm eff} = \frac{1}{4\,|Q_f|}\left(1+\frac{\sqrt{1-\mathcal{A}_f^2}-1}{\mathcal{A}_f}\right),$$
$$\rho_f = \frac{\Gamma_f^0}{\gamma}\left[\left(\frac{1}{2}-2|Q_f|\sin^2\theta^f_{\rm eff}\right)^2+\frac{1}{4}+\left({\rm Im}\,\mathcal{G}^f_{V}\right)^2+\left({\rm Im}\,\mathcal{G}^f_{A}\right)^2\right]^{-1} \quad (0.34)$$

where $f = \mu, \tau$.

- **For quarks one should remember to subtract first non-factorizable terms and then to distinguish between R^f_V and R^f_A.**

Results for PO

- **see tables**

Th Uncertainties for PO

Here we discuss the theoretical uncertainties associated with PO.

Realistic Observables

- The RO are computed in the context of the SM, however one of the goals will be to pin down

  1. the definition of PO;
  2. the calculation of RO in terms of the defined PO for the purpose of MI fits, showing that for PO with values as calculated in the SM, the RO are *by construction* identical to the full SM RO calculation.

- The last point requires expressing ρ's and effective mixing angles in terms of PO, assuming the validity of the SM.

- One should remember that gauge invariance (GI) at the Z pole (on-shell GI) is entirely another story from GI at any arbitrary scale (off-shell GI).

- Some of the re-summations that are allowed at the pole and that heavily influence the definition of effective Z couplings are not trivially extendible to the off-shell case.

- **Therefore, the expression for RO = RO(PO), at arbitrary s, requires a careful examination and should be better understood as**

$$\mathrm{RO} = \mathrm{RO}\left(\mathrm{PO}, \overline{\mathrm{SM}}\right), \tag{0.35}$$

that is, for example:

$$\sigma_{\mathrm{MI}} = \sigma_{\mathrm{SM}}\Big(R_l, A^{0,l}_{\mathrm{FB}}, \ldots \to g^f_{_V}, g^f_{_A} \\ \to \rho_f, \sin^2\theta^f_{\mathrm{eff}}; \textbf{residual SM}\Big). \tag{0.36}$$

- **As long as the procedure does not violate GI and the PO are given SM values, there is nothing wrong with the calculations. It is clear that in this case the SM RO coincide with the ROMI.**

Final-State Radiation

- **One should realize that s' is not equivalent to the invariant mass of the final-state $f\overline{f}$ system, $M^2(f\overline{f})$, due to final state QED and QCD radiation.**

- **Furthermore, in the presence of a s'-cut the correction for final state QED radiation is simply**

$$R_{\rm QED}^{\rm FS} = \frac{3}{4}\,Q_f^2\,\frac{\alpha(s)}{\pi}\,, \qquad (0.37)$$

while for a M^2-cut the correction is more complicated, see

G. Montagna, O. Nicrosini and G. Passarino, Phys. Lett. B309 (1993) 436.

- **For full angular acceptance one derives the following corrections:**

$$\sigma\left(s\right) = \frac{\alpha}{4\pi} Q_f^2 \sigma^0\left(s\right)\Big\{-2x^2 + 4\Big[\left(z + \frac{z^2}{2} + 2\ln x\right)\ln\frac{s}{m_f^2} + z\left(1 + \frac{z}{2}\right)\ln z + 2\zeta(2) - 2{\rm Li}_2\left(x\right) - 2\ln x + \frac{5}{4} - 3z - \frac{z^2}{4}\Big]\Big\}.$$

- **Here we have introduced**

$$x = 1 - z, \qquad z = M^2\left(f\overline{f}\right)/s. \tag{0.38}$$

- **For an s'-cut both QED and QCD final-state radiation are included through an inclusive correction factor.**

- **For M^2-cut the result remains perfectly defined for leptons, however for hadronic final states there is a problem. This has to do with QCD final-state corrections. Indeed we face the following situation:**

1. **for $e^+e^- \to f\overline{f}\gamma$ the exact correction factor is known at $\mathcal{O}(\alpha)$ even in the presence of a $M^2(f\overline{f})$ cut;**

2. **the complete set of final-state QCD corrections are known up to $\mathcal{O}\left(\alpha_s^3\right)$**

   K. G. Chetyrkin et al. in 'Reports of the Working Group on Precision Calculations for the Z Resonance', D. Bardin, W. Hollik and G. Passarino eds., CERN-95-03, p. 175.

   and also

   K. G. Chetyrkin, J. H. Kühn and A. Kwiatkowski, Phys. Rept. 277 (1996) 189.

only for the fully inclusive setup, i.e., no cut on the $f\overline{f}$ invariant mass;

3. **the mixed two-loop QED/QCD final-state corrections are also known only for a fully inclusive setup**

   A. L. Kataev, Phys. Lett. B287 (1992) 209.

4. **at $\mathcal{O}(\alpha_s)$ QCD final-state corrections in presence of a M^2-cut follow from the analogous QED calculation**

   A. Arbuzov, D. Bardin and A. Leike, Modern Phys. Lett. A7 (1992) 2029.

Convoluted Realistic Observables

CA3 Complete RO, with QED initial state radiation implemented through an additive $\mathcal{O}\left(\alpha^3\right)$ radiator

G. Montagna, O. Nicrosini and F. Piccinini, Phys. Lett. B406 (1997) 243.

CF3 Complete RO, with QED initial state radiation implemented through a factorized $\mathcal{O}\left(\alpha^3\right)$ radiator

M. Skrzypek, Acta. Phys. Pol. B23 (1992) 135.

- **The following two equations define cross-sections and forward-backward asymmetry** *convoluted* **with ISR:**

$$\sigma_{\mathrm{T}}\left(s\right) = \int_{z_0}^{1} dz H\left(z;s\right)\hat{\sigma}_{\mathrm{T}}(zs), \qquad (0.39)$$

where $z_0 = s_0/s$ **and**

$$A_{\text{FB}}(s) = \frac{\pi\alpha^2 Q_e^2 Q_f^2}{\sigma_{\text{tot}}} \int_{z_0}^{1} dz \frac{1}{(1+z)^2} H_{\text{FB}}(z;s)\, \hat{\sigma}_{\text{FB}}(zs), \tag{0.40}$$

- **Note that the so-called *radiator* (or *flux function*), $H(z;s)$, is known up to terms of order α^3 while H_{FB} is only known up to terms of order α^2.**

- **The kernel cross-sections $\hat{\sigma}_{\text{T,FB}}$ should be understood as**

  * **the improved Born approximation (IBA),**
  * **corrected with all electroweak and possibly all FSR (QED $\otimes$ QCD) corrections,**
  * **where all coupling constants and effective vector and axial weak couplings are *running*, i.e., they depend on $s' = zs$.**

Uncertainty on QED Convolution

$$\delta^{\mathrm{dec}}\left(O\right) = \frac{O}{O^{\mathrm{SD}}} - 1, \qquad \Delta^{\mathrm{dec}}\left(O\right) = O - O^{\mathrm{SD}}. \qquad (0.41)$$

- **For convenience of the reader we have reproduced in Tab. 0.4 the results for the CA3 and CF3 mode.**

- **From Tab. 0.4 we derive the absolute differences, for $A^{\mu}_{_{\rm FB}}$, and the relative ones, for cross-sections, between additive and factorized versions of the QED radiators. They are shown in Tab. 0.5.**

Initial-Final QED Interference

- If initial-final QED (ISR-FSR) interference (IFI) is included in a calculation we have a conceptual problem with the meaning of the s'-cut.

- In this case the definition of the variable s' is unnatural since one does no longer know the origin of the radiative photon (ISR or FSR).

- Only a cut on the invariant mass of the final-state $f\overline{f}$ system would make sense.

- There is another option: to select events with little initial-state radiation one can use a cut on the acollinearity angle $\theta_{\rm acol}$, between the outgoing fermion and anti-fermion.

- A cut on $\theta_{\rm acol}$ is roughly equivalent to a cut on the invariant mass of the $f\overline{f}$ system, indeed one may write

$$\frac{s'}{s} \approx z_{\rm eff} = \frac{1-\sin(\theta_{\rm acol}/2)}{1+\sin(\theta_{\rm acol}/2)}\,, \tag{0.42}$$

  therefore a cut of $\theta_{\rm acol} < 10^\circ$ is roughly corresponding to the request that $s'/s > 0.84$.

- **The inclusion of initial-final QED interference in** TOPAZ0 **and in** ZFITTER **is done at $\mathcal{O}(\alpha)$, i.e., the $\mathcal{O}(\alpha)$ interference term is added linearly to the cross-sections without entering the convolution with ISR and without cross-talk to FSR.**

- **For loose cuts the induced uncertainty is rather small and the effect of the interference itself is minute.**

- **On the contrary, when we select a tight acollinearity cut the resulting limit on the energy of the emitted photon becomes more stringent and the effect of interference grows.**

- **The corresponding theoretical uncertainty, due to missing higher-order corrections, is therefore expected to be larger.**

- **I/F INT is currently treated dubiously by the experiments:**

  * **The MC used to extrapolate for efficiency and possibly acceptance do not contain I/F interference (eg, KORALZ in multiphoton mode).**

  * **Thus extrapolated and quoted results somehow miss I/F interference.**

- **There is a claim of some serious discrepancy between** KK **/** `TOPAZ0` **/** `ZFITTER` **in** $\sigma_{\rm int}, A_{\rm FB}^{\rm int}$ **for muons.**

  http://home.cern.ch/ jadach

- **It has to do with** *exponentiation* **of** `ISR` $\otimes$ `FSR`

Observable	TOPAZ0	ZFITTER	$10^3 \times \frac{T-Z}{T}$
$1/\alpha^{(5)}(M_Z)$	128.877	128.877	
$1/\alpha(M_Z)$	128.887	128.887	
M_W [GeV]	80.3731	80.3738	-0.009
σ_h^0 [nb]	41.4761	41.4777	-0.04
σ_l^0 [nb]	1.9995	1.9997	-0.12
Γ_h [GeV]	1.74211	1.74223	-0.07
Γ_Z [GeV]	2.49549	2.49573	-0.10
Γ_ν [MeV]	167.207	167.234	-0.16
Γ_e [MeV]	83.983	83.995	-0.14
Γ_μ [MeV]	83.983	83.995	-0.14
Γ_τ [MeV]	83.793	83.805	-0.14
Γ_u [MeV]	300.129	300.154	-0.08
Γ_d [MeV]	382.961	382.996	-0.09
Γ_c [MeV]	300.069	300.092	-0.08
Γ_b [MeV]	375.997	375.993	0.01
$\Gamma_{\rm inv}$ [GeV]	0.50162	0.50170	-0.16
R_l	20.7435	20.7420	0.07
R_b^0	0.215829	0.215811	0.08
R_c^0	0.172245	0.172246	-0.01
$\sin^2\theta_{\rm eff}^{\rm lept}$	0.231596	0.231601	-0.02
$\sin^2\theta_{\rm eff}^{b}$	0.232864	0.232950	-0.37
$\sin^2\theta_{\rm eff}^{c}$	0.231491	0.231495	-0.02
ρ_e	1.00513	1.00528	-0.15
ρ_b	0.99413	0.99424	-0.11
ρ_c	1.00582	1.00598	-0.16
Observable	TOPAZ0	ZFITTER	$10^3 \times (T-Z)$
$A_{\rm FB}^{0,l}$	0.016084	0.016074	0.01
$A_{\rm FB}^{0,b}$	0.102594	0.102617	-0.02
$A_{\rm FB}^{0,c}$	0.073324	0.073300	0.02
$\mathcal{A}_e$	0.146440	0.146396	0.04
$\mathcal{A}_b$	0.934654	0.934607	0.05
$\mathcal{A}_c$	0.667609	0.667595	0.01

Table 0.1 Complete table of PO, from TOPAZ0 and ZFITTER.

Observable	central	minus error	plus error	total exp. error
$1/\alpha^{(5)}(M_z)$	128.877	-	-	
$1/\alpha(M_z)$	128.887	-	-	
M_W [GeV]	80.3731	5.8 MeV	0.3 MeV	90 MeV
σ_h^0 [nb]	41.4761	1.0 pb	1.6 pb	58 pb
Γ_ν [MeV]	167.207	0.017	0.001	
Γ_e [MeV]	83.983	0.010	0.0005	0.10^*
Γ_μ [MeV]	83.983	0.010	0.0005	
Γ_τ [MeV]	83.793	0.010	0.0005	
Γ_u [MeV]	300.129	0.047	0.013	
Γ_d [MeV]	382.961	0.054	0.010	
Γ_c [MeV]	300.069	0.047	0.013	
Γ_b [MeV]	375.997	0.208	0.077	
$\Gamma_{\rm had}$ [GeV]	1.74211	0.26 MeV	0.11 MeV	2.3 MeV*
$\Gamma_{\rm inv}$ [GeV]	0.50162	0.05 MeV	0.002 MeV	1.8 MeV*
Γ_z [GeV]	2.49549	0.34 MeV	0.11 MeV	2.4 MeV

Table 0.2 Theoretical uncertainties for PO from `TOPAZ0`. $*$) assumes lepton universality.

Observable	central	minus error	plus error	total exp. error
R_l	20.7435	0.0020	0.0013	0.026
R_b^0	0.215829	0.000100	0.000031	0.00074
R_c^0	0.172245	0.000005	0.000024	0.0044
$\sin^2\theta_{\rm eff}^{\rm lept}$	0.231596	0.000035	0.000033	0.00018
$\sin^2\theta_{\rm eff}^{b}$	0.232864	0.000002	0.000048	
$\sin^2\theta_{\rm eff}^{c}$	0.231491	0.000029	0.000033	
$A_{\rm FB}^{0,l}$	0.016084	0.000057	0.000060	0.00096
$A_{\rm FB}^{0,b}$	0.102594	0.000184	0.000195	0.0021
$A_{\rm FB}^{0,c}$	0.073324	0.000142	0.000149	0.0044
$\mathcal{A}_e$	0.146440	0.000259	0.000275	0.0051
$\mathcal{A}_b$	0.934654	0.000032	0.000005	0.035
$\mathcal{A}_c$	0.667609	0.000114	0.000103	0.040
ρ_e	1.00513	0.00010	0.000005	0.0012
ρ_b	0.99413	0.00048	0.000001	
ρ_c	1.00582	0.00010	0.000005	

Table 0.3 Theoretical uncertainties for PO from `TOPAZO`.

	LEP 1 energy in GeV				
	$M_z - 3$	$M_z - 1.8$	M_z	$M_z + 1.8$	$M_z + 3$
$\delta^{\rm dec}(\sigma_\mu^{\rm CA3})$ `T`	-23.976	-27.616	-26.257	5.356	30.665
$\delta^{\rm dec}(\sigma_\mu^{\rm CA3})$ `Z`	-24.007	-27.629	-26.261	5.341	30.631
$\delta^{\rm dec}(\sigma_\mu^{\rm CF3})$ `T`	-23.973	-27.611	-26.253	5.364	30.671
$\delta^{\rm dec}(\sigma_\mu^{\rm CF3})$ `Z`	-24.000	-27.624	-26.256	5.348	30.637
$\delta^{\rm dec}(\sigma_{\rm had}^{\rm CA3})$ `T`	-25.867	-28.501	-26.537	4.952	30.564
$\delta^{\rm dec}(\sigma_{\rm had}^{\rm CA3})$ `Z`	-25.873	-28.503	-26.538	4.945	30.550
$\delta^{\rm dec}(\sigma_{\rm had}^{\rm CF3})$ `T`	-25.863	-28.497	-26.533	4.959	30.572
$\delta^{\rm dec}(\sigma_{\rm had}^{\rm CF3})$ `Z`	-25.869	-28.499	-26.533	4.953	30.558
$\Delta^{\rm dec}(A_{\rm FB}^{\mu{\rm CA3}})$ `T`	-2.190	-1.967	-1.804	-6.292	-11.486
$\Delta^{\rm dec}(A_{\rm FB}^{\mu{\rm CA3}})$ `Z`	-2.211	-1.975	-1.803	-6.287	-11.480
$\Delta^{\rm dec}(A_{\rm FB}^{\mu{\rm CF3}})$ `T`	-2.189	-1.966	-1.804	-6.292	-11.487
$\Delta^{\rm dec}(A_{\rm FB}^{\mu{\rm CF3}})$ `Z`	-2.215	-1.977	-1.803	-6.286	-11.479

Table 0.4 RO: the effect in % of initial state QED radiation for CA3 and CF3 modes.

	LEP 1 energy in GeV				
	M_Z-3	M_Z-1	M_Z	M_Z+1	M_Z+3
$10^4\times$ (fact/add-1)					
σ_μ	0.44	0.63	0.61	0.72	0.49
	0.88	0.63	0.68	0.72	0.49
$\sigma_{\rm had}$	0.58	0.58	0.64	0.73	0.59
	0.61	0.62	0.67	0.76	0.62
fact-add [pb]					
σ_μ	0.01	0.03	0.09	0.05	0.02
	0.02	0.03	0.10	0.05	0.02
$\sigma_{\rm had}$	0.26	0.56	1.95	1.04	0.48
	0.27	0.60	2.04	1.08	0.51
$10^5\times$ (fact-add)					
$A^\mu_{\rm FB}$	1.00	1.00	0.00	0.00	-1.00
	-4.00	-2.00	0.00	1.00	1.00

Table 0.5 Absolute and relative differences in `TOPAZ0` and in `ZFITTER` for additive and factorized radiators.