On photon radiation in single-W process

LEP2 Monte Carlo Workshop - CERN, October 12, 1999

G.Montagna[ab], M.Moretti[c], O.Nicrosini[ba], A.Pallavicini[ab], F.Piccinini[ba]

[a]Dipartimento di Fisica Nucleare e Teorica, Università di Pavia
[b]INFN, Sezione di Pavia
[c]Dipartimento di Fisica, Università di Ferrara and INFN, Sezione diFerrara

Contents

Photon radiation via collinear structure functions

- In principle, the best strategy to estimate the uncertainty due to photon radiation corrections to four-fermion production would be to work out the exact $O(\alpha)$ calculation...
  ... *a hard task indeed.*
- Dominant photon corrections must be considered due to their relevance. One of the standard way of including the leading-log contribution is the structure function formalism (hereafter denoted as SF), which consists in convoluting the kernel amplitude with suitable probability densities.

$$d\sigma = \prod_i \int dx_i D(\Lambda^2, x_i)\; d\sigma_0 \qquad (1)$$

- Equation (1) holds since the leading-log contributions of photon radiation factorize with respect to the Born amplitude.
- The SF technique needs, as input parameter, a scale Λ, whose choice is not dictated by the SF method itself.
- A common solution is given by the comparison of equation (1) with an analytical calculation, provided it reproduces the leading-log contribution.

Fixing the photon radiation scale - I

- Consider, as a first step, the QED contributions to Bhabha scattering.
  1. If the dynamics follows the annihilation channel, the SF scale is the centre-of-mass energy of the Born process, i.e. s. On the other hand, photon radiation in the scattering channel is correctly described by setting the scale to $|t|$.
  2. Thus a possible solution for the "scale fixing problem", for this process, is to set the scale either to s or to $|t|$, according to the dynamical channel.
  3. Another relevant solution is given by fixing the scale to the invariant mass combination $\Lambda^2 = st/u$ for all the dynamical channels [1].
- Consider, as a second step, the single-W process, our case of interest.
  1. In the literature the scale of the SF is usually fixed to s.
  2. Recently it has been proposed to fix the scale to s for the SF pertaining to s-channel diagrams and to $|t|$ for the SF pertaining to t-channel diagrams [2].
- The aim of the present approach is to find an invariant mass combination, as done for the QED contributions to Bhabha scattering in [1], for the single-W case too. Further, the possibility of introducing different scales for each SF and of treating both soft radiation and hard radiation collinear to the final-state particles is considered.

---

1. M. Greco and O. Nicrosini, *Phys. Lett. B*, 240:219, 1990.
2. E. E. Boos and M. N. Dubinin, `hep-ph/9909214`.

Fixing the photon radiation scale - II

- Equation (1) can be rewritten stressing the possibility of different scales for each SF

$$d\sigma = \prod_i \int dx_i D(\Lambda_i^2, x_i)\, d\sigma_0 \tag{2}$$

- Then, it is useful to expand equation (2) at $O(\alpha)$, once the integrations over the dx_i are performed, ($l := \log \Delta E/E$ is the infrared log, where ΔE is the energy threshold of the calorimeter to detect a photon, while L is the collinear log)

$$d\sigma = \left(1 + \sum_i \frac{\alpha}{\pi} l L(\Lambda_i)\right) d\sigma_0 \tag{3}$$

- When final-state radiation is considered, L is modified by factors coming from the angular integration over the photon variables, due to the interference between the radiation cones.

Soft photon contribution

- Consider a n-body process and take all the external particles as ingoing ones. Label their momenta by q_i, their energies by E_i, their masses by m_i, and their charges by Q_i. Define $s_{ij\ldots} := (q_i + q_j + \ldots)^2$.
- The soft-photon contribution, provided that $s_{ij} \gg m_i^2, m_j^2$, can be calculated:

$$d\sigma_{\mathrm{soft}} = d\sigma_0 \frac{d\omega}{\omega} \frac{2\alpha}{\pi} \left(\sum_{i>j}^{n} Q_i Q_j \log \frac{s_{ij}}{m_i m_j} + \sum_{i}^{n} \frac{1}{2} Q_i^2 \right) \tag{4}$$

- Otherwise, in the limit $s_{ij} \ll m_i^2, m_j^2$, the following substitution holds:

$$\log \frac{s_{ij}}{m_i m_j} \to \frac{1}{3} \frac{s_{ij}}{m_i m_j} \tag{5}$$

- Since the problem is to fix the SF scale, only the real contribution is explicitly calculated, because the virtual corrections, in order to preserve the infrared cancellations, must share the same leading collinear structure of the real part itself.

Hard-collinear photon contribution

- Consider the case of a calorimetric set-up which does not permit to disentangle the photons collinear to the final-state particles. Name δ the half-opening of the calorimetric cone.
- Re-consider the previous process with n ingoing legs and think of m of them to be changed into outgoing legs at the end of the calculation.
- The contribution of hard photons collinear to the final-state particles, can be cast into a gauge invariant form as follows [3,4]

$$d\sigma_{\rm hard} = d\sigma_0 \frac{d\omega}{\omega}\frac{2\alpha}{\pi}\left(\sum_i^m Q_i^2 \log\frac{E_i\delta}{m_i} - \sum_i^m \frac{1}{2}Q_i^2\right) \tag{6}$$

3. M. Caffo, R. Gatto, and E. Remiddi, *Nucl. Phys. B*, 252:378, 1985.
4. M. Cacciari, G. Montagna, and O. Nicrosini, *Phys. Lett. B*, 274:473, 1992.

The resulting recipe for photon radiation

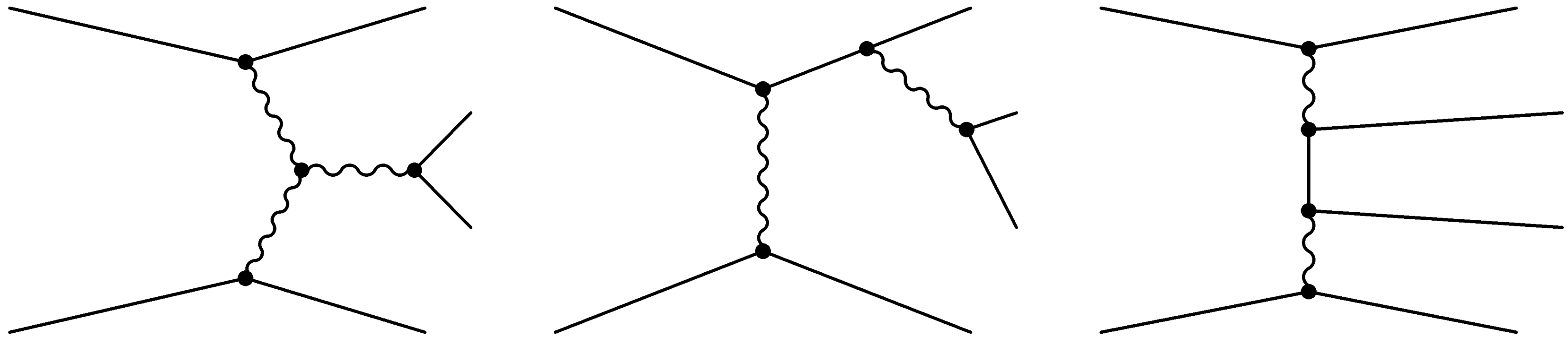
- Equations (4) and (6) give the leading contribution which must be compared with the $O(\alpha)$ perturbative expansion of equation (2) in order to fix the process scales Λ_i.
- Summing up equations (4) and (6) the following formula can be obtained (c_{ij} is the cosine of the angle between particles i and j):

$$
\begin{aligned}
d\sigma_{\mathrm{soft}} + d\sigma_{\mathrm{hard}} \quad = \quad & d\sigma_0 \frac{2\alpha}{\pi} \log \frac{\Delta E}{E} \left\{ \sum_{i=m+1}^{n} Q_i^2 \left(\log \frac{E_i}{m_i} - \frac{1}{2} \right) \; + \right. \\
& \left. - \sum_{i>j}^{n} Q_i Q_j \log 2(1 - c_{ij}) - \sum_{i}^{m} Q_i^2 \log \delta \right\}
\end{aligned} \tag{7}
$$

- Equation (7) does not contain, as expected, any longer the energies and the masses of the final-state particles, as due to KLN theorem.
- If only initial-state SF are considered, as usually done by the MC codes present in the literature, the recipe consists in correcting each initial-state leg by means of the SF $D(\Lambda_i, x_i)$, where the scales Λ_i are given by the inspection of equation (7).

The single-W process

- As an example consider the process $e^+e^- \to e^-\bar{\nu}u\bar{d}$ with the final-state electron lost in the beam pipe (single-W process). Since the electron must be undetected, the leading contributions to this process are the single-resonant and the non-resonant t-channel diagrams:



- Photon radiation will be treated by means of SF with the scale fixed by equation (7). The leading dynamics suggest to introduce one SF for the electron line and one for the positron line, while no SF is introduced for the final-state particles, owing to the calorimetric measure. Further, the smallness of the up-down interference allows a natural sharing of the logarithmic contribution, obtained from the diagrammatic calculation, among the SF.

The scale choice for the single-W case

- For the single-W process the formula (7) translates in the following two scales (Λ_- refers to the electron line, while Λ_+ to the positron line), when compared with equation (2) expanded to $O(\alpha)$

$$\Lambda_-^2 = 4E^2 \frac{(1-c_-)^2}{\delta^2}\,, \quad \Lambda_+^2 = 2^{\frac{14}{9}} E^2 \frac{\left((1-c_{\bar{d}})(1-c_u)^2\right)^{\frac{2}{3}}}{\left((1-c_{u\bar{d}})^2\delta^5\right)^{\frac{2}{9}}} \tag{8}$$

where E is the beam energy, m the electron mass, c_- the cosine of the electron scattering angle, c_u and $c_{\bar{d}}$ the cosine of the quark scattering angle with respect to the initial positron, $c_{u\bar{d}}$ the cosine of the angle between the quarks.

- It is also possible to make a naive ansatz for the process scales without a detailed calculation, by thinking in terms of the Weizsäcker-Williams approximation. This leads to the scale definition:

$$\Lambda^2_{-,\mathrm{naive}} = |t|\,, \quad \Lambda^2_{+,\mathrm{naive}} = M_W^2 \tag{9}$$

• In the numerical implementation, whenever one of the two scales is lesser than a small cut-off, the radiation from the corresponding leg is switched off, in accordance with equation (5).

• Effects due to the finite width of the W-boson, such as radiation decoherence and finite width corrections, are estimated to be small or absent at all.

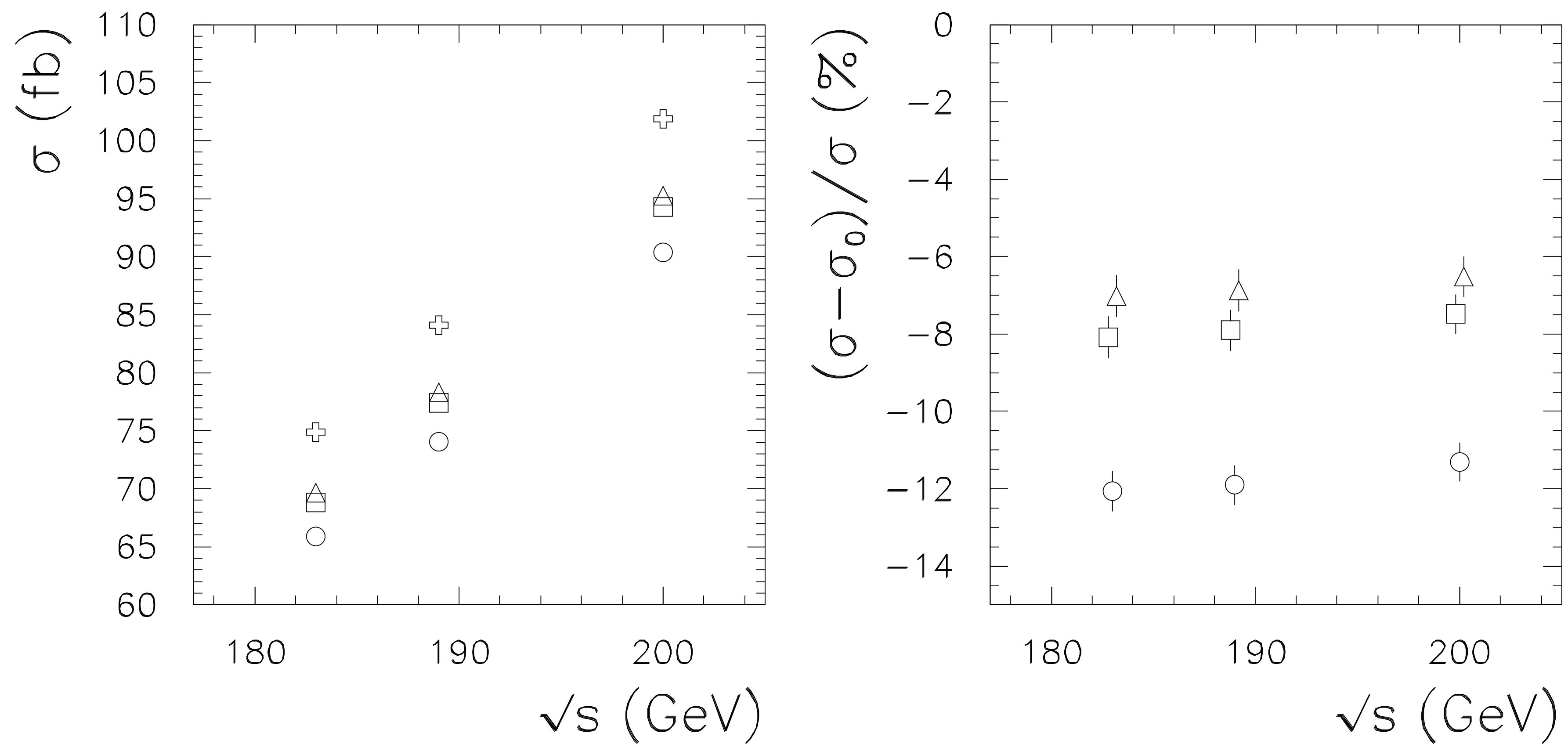
Event selection definition

- The event selection adopted for the single-W signature is derived from the one introduced in the proposal paper for this workshop with slight modifications concerning the angular limits imposed on the quarks.

Beam pipe	$4.44°$
Calo edge for quarks	$9.94°$
Min quark energy	15.0 GeV
Min quark-antiquark mass	45.0 GeV

- The only consequence of introducing the further cut on the quarks is to depress the multiperipheral dynamics. This does not affect the discussion on photon radiation.
- The half-opening angle of the calorimetric cone is set equal to $\delta = 5°$, but numerical results are marginally sensitive to the precise value of δ.

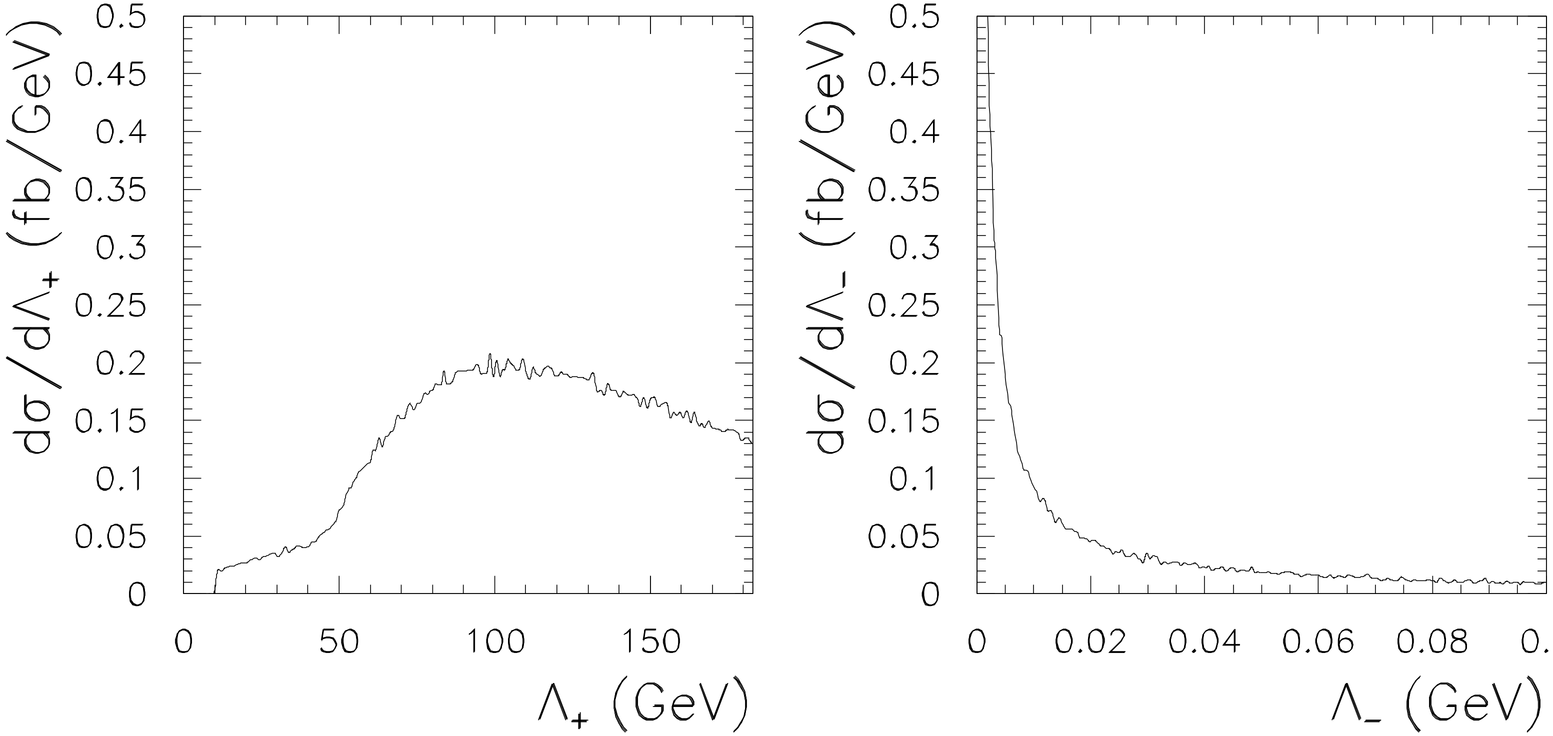
Numerical results - I



- The effects of photon radiation for the single-W process $e^+e^- \to e^-\bar{\nu}u\bar{d}$. On the left side the cross section values are displayed, while on the right side the cross section relative to the Born process. The outlined + represents the Born cross section, ○ represents the correction given by s-channel ISR, □ the correction given by the naive scales of equation (9), △ the correction given by the scales of equation (8). The entries are computed at 183, 189, 200 GeV of center-of-mass energy. The markers are misplaced for better reading.

Numerical results - II

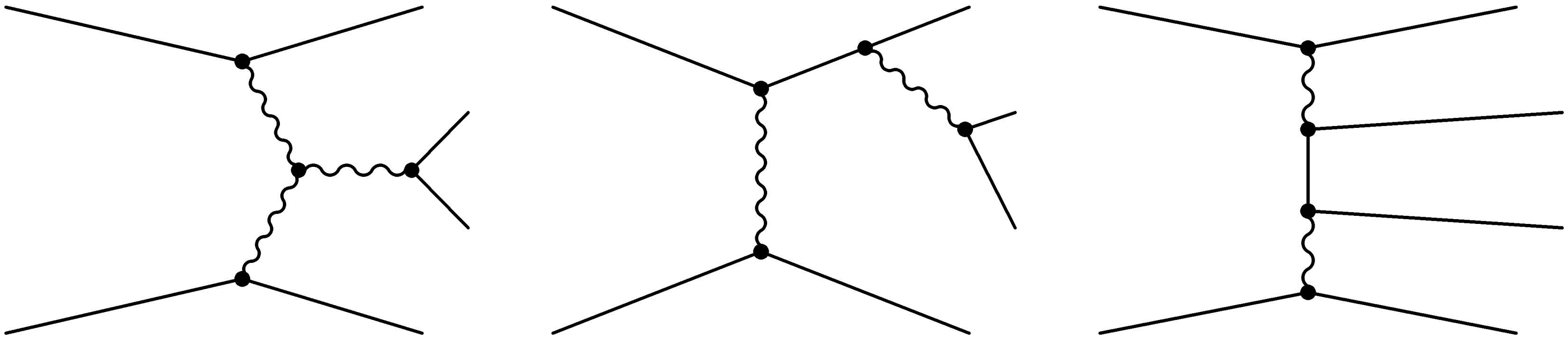
- The s-channel scale common to both legs for initial-state radiation is not a proper choice, because it overestimates by a factor of 2 the effect of radiation.
- The naive scales of equation (9) seem to be a relatively good choice (the discrepancy is at the per cent level) in the inspected range of energies.



- This fact is understood looking at figure above, where, in the left part of the figure, it is shown the differential cross section with respect to the scale Λ_- of equation (8) and it peaks not far from M_W, whereas the other scale peaks as expected at very small t.

Vacuum polarization

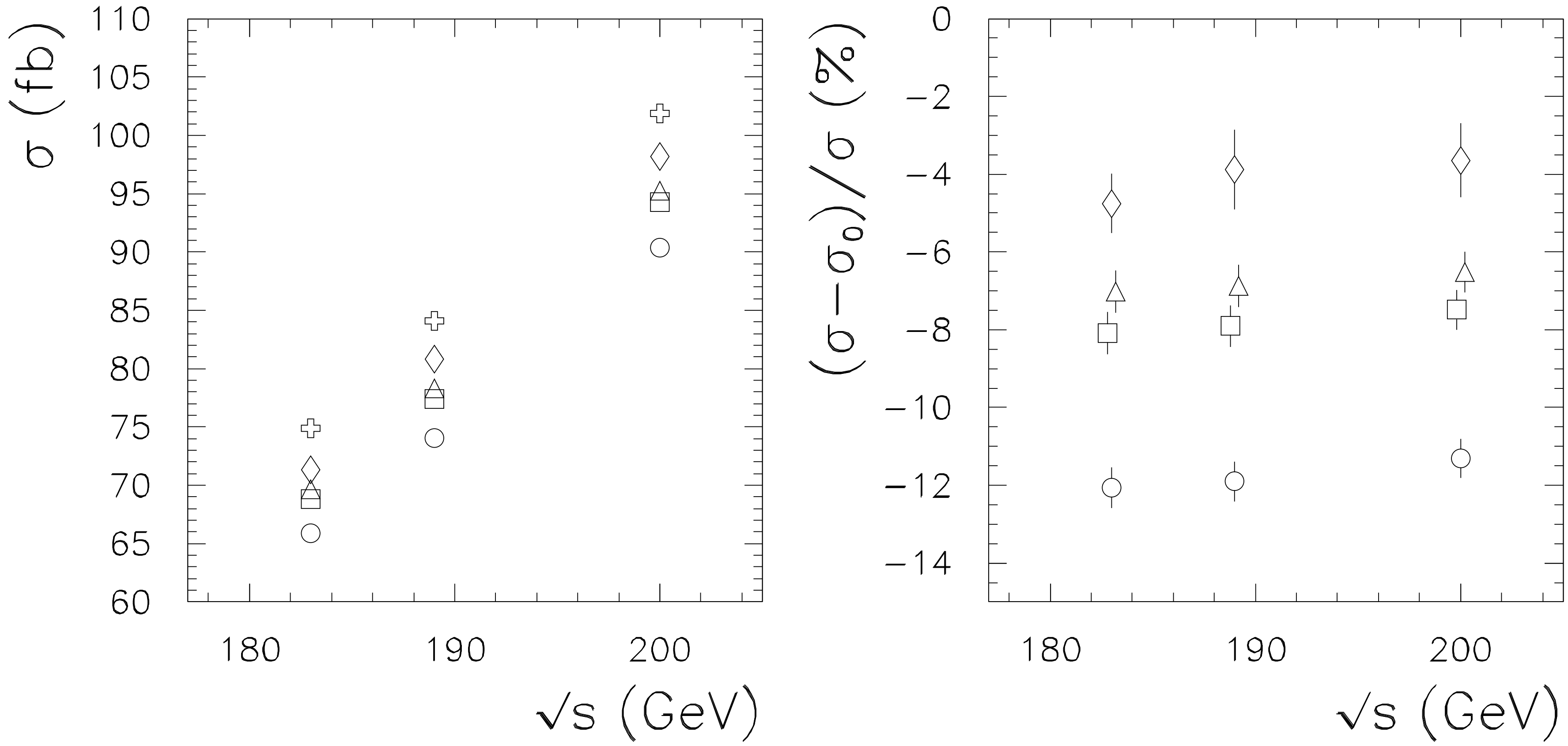
- Beside photon radiation, also the vacuum polarization contribution is to be considered, if radiative corrections must be sized.
- Vacuum polarization is taken into account by means of the running of α_{QED}, computed via dispersion integral techniques [6].



- Since all the above diagrams are characterized by a quasi-real t-channel photon, which is the only photon of the diagram, the vacuum polarization contribution factorizes in front of the amplitude.

6. S. Eidelman and F. Jegerlehner, *Z. Phys. C*, 67:585, 1995.

Numerical results - III



- The effects of photon radiation and vacuum polarization for the single-W process $e^+e^- \to e^-\bar{\nu}u\bar{d}$. On the left side the cross section values are displayed, while on the right side the cross section relative to the Born process. The outlined + represents the Born cross section, $\bigcirc$ represents the correction given by s-channel ISR, $\square$ the correction given by the naive scales of equation (9), $\triangle$ the correction given by the scales of equation (8), $\diamond$ the correction given by the scales of equation (8) plus vacuum polarization. The entries are computed at 183, 189, 200 GeV of center-of-mass energy. The markers are misplaced for better reading.

Conclusions

- The standard SF technique was adopted to valuate the photon radiation contribution.
- The choice of the process scale, a necessary input parameter for the SF, was critically discussed. A detailed procedure to determine the scales was introduced which allows to encompass both soft and hard-collinear photon radiation.
- Particular attention was payed to the single-W process, for which was discussed how to implement both photon radiation and vacuum polarization corrections.
- It was shown for the single-W process that the usual choice of the center-of-mass energy of the reaction, as the scale for process, can lead to overestimate the radiative correction by a factor of two, implying an underestimate of the cross section of about a five per cent.
- Also the choice of fixing the scale to $|t|$ for both the initial-state SF leads to an underestimate of the photon correction of about 5%.
- Vacuum polarization amounts to a correction of about 3%.
- The difference between the "naive" and the "refined" scale choice suggests an estimate of the theory uncertainty due to photon radiation. The error of the present approach is beyond the leading-log approximation.
- The same method can be applied to other four-fermion final states such as single-Z production.

Effects due to the W width

- Owing to the presence of a resonant W, some modifications to the previous results may come from finite-width effects and from radiation decoherence [5].
- Finite-width corrections of the form E_γ/Γ_W arise when the unstable particle propagator goes off its mass-shell, but this is not the present case, since the multi-fermion final state can accommodate a resonant W.
- Radiation decoherence is present whenever a resonance occurs and its effect is to cancel the angular dependence from the scale. As a consequence the scale Λ_+ should be modified by dropping the angular interference factors in equation (8) when the emitted photons have $E_\gamma \gtrsim \Gamma_W$. Yet, in the present case the effect is tiny, since the effects due to angular interference terms are already small by themselves.

---

5. M. Greco *Riv. Nuovo Cim.* **11** (1988) 1 and references therein.

Monte Carlo Features

- A Monte Carlo program was built to compute the cross section for the single-W signature with the following highlights:
  1. Phase space integration performed with the aid of a multi-channel importance sampling with stratification following the guideline of [7].
  2. Realistic event selections can be implemented into the code.
  3. Full massive matrix element, computed by means of the `ALPHA` algorithm [8], inclusive of anomalous gauge coupling.
  4. Photon radiation implemented via SF, as presented in the previous slides.
  5. Vacuum polarization by means of α_{QED} running, as presented in the previous slides.
  6. Weighted and unweighted event generation.

---

7. D. G. Charlton, G. Montagna, O. Nicrosini, F. Piccinini, version 2.0, *Comp. Phys. Comm.* 99:355, 1997. G Montagna, M. Moretti, O. Nicrosini, F. Piccinini *Nucl. Phys. B* 541:31, 1999. G. Montagna, M. Moretti, O. Nicrosini, A. Pallavicini, and F. Piccinini, *Nucl. Phys. B*, 547:39, 1999.
8. F. Caravaglios and M. Moretti, *Phys. Lett. B* 358:332, 1999.