CERN / LEP2 MC workshop, October '99

RacoonWW:
Radiative corrections on $e^+e^- \to WW \to 4f$

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1 Strategy for $\mathcal{O}(\alpha)$ corrections in RacoonWW

1.1 General considerations

Real corrections — $\mathrm{e}^+\mathrm{e}^- \to 4f + \gamma$ at tree level

Exact calculations for all final states exist.

Virtual corrections — $\mathrm{e}^+\mathrm{e}^- \to 4f$ at one loop

Problems:

- Enormous complexity of full one-loop calculation:
  - number of diagrams $\sim \mathcal{O}(10^3\text{–}10^4)$ per final state,
  - many non-equivalent final states,
  - numerical problems with 5- and 6-point functions.
- No convincing procedure for gauge-invariant introduction of finite widths at one loop.

Viable solution: double-pole approximation.

$\Rightarrow$ Take only diagrams for $\mathrm{e}^+\mathrm{e}^- \to \mathrm{WW} \to 4f$
with two nearly resonant W bosons !

- Considerable reduction in number of Feynman graphs.
- On-shell production and decay of W's as building block.
- Generic calculation for all final states possible.
- Expected uncertainty not too close to threshold:

$$\sim \mathcal{O}\left(\frac{\Gamma_\mathrm{W}}{M_\mathrm{W}}\frac{\alpha}{\pi}\ln(\cdots)\right) \sim 0.1\%.$$

1.2 The RacoonWW approach

Denner, S.D., Roth,
Wackeroth ’99

$e^+e^- \to 4f$ at tree level

Full matrix-element calculation with massless fermions.

$\mathcal{O}(\alpha)$ radiative corrections (RCs) to $e^+e^- \to WW \to 4f$

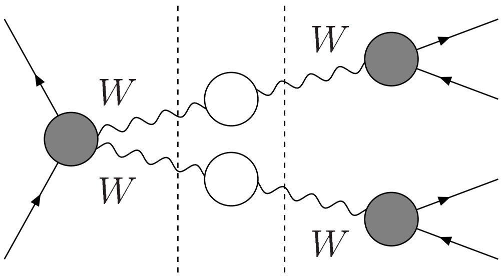
real

$e^+e^- \to 4f + \gamma$ at tree level:
Full matrix-element calculation
→ Talk of M. Roth 6/99

virtual

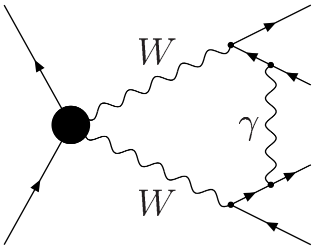
$e^+e^- \to WW \to 4f$ at one loop:
Double-pole approximation

Factorizable corrections



Building blocks:
– RCs to on-shell production
– RCs to on-shell decay
But: respect spin correlations!

Non-factorizable corrections

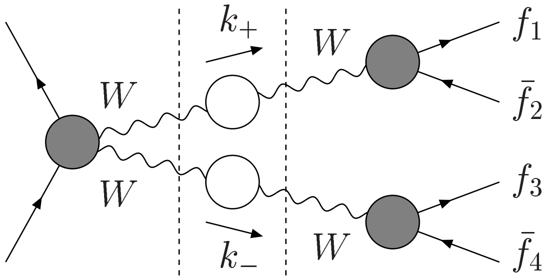


etc.

Photon exchange between
production and decay
subprocesses ($E_\gamma \lesssim \Gamma_W$)

Note: full off-shell phase space everywhere !

1.3 Virtual factorizable corrections



One-loop contribution to matrix element in DPA:

$$\mathcal{M}^{\mathrm{e^+e^-\to WW\to 4}f}_{\mathrm{virt,fact,DPA}} = \sum_{\lambda_+,\lambda_-} \frac{1}{(k_+^2 - M_\mathrm{W}^2 + \mathrm{i}M_\mathrm{W}\Gamma_\mathrm{W})(k_-^2 - M_\mathrm{W}^2 + \mathrm{i}M_\mathrm{W}\Gamma_\mathrm{W})}$$
$$\times \Big(\delta\mathcal{M}^{\mathrm{e^+e^-\to W^+W^-}} \mathcal{M}^{\mathrm{W^+}\to f_1\bar{f}_2}_{\mathrm{Born}} \mathcal{M}^{\mathrm{W^-}\to f_3\bar{f}_4}_{\mathrm{Born}}$$
$$+ \mathcal{M}^{\mathrm{e^+e^-\to W^+W^-}}_{\mathrm{Born}} \delta\mathcal{M}^{\mathrm{W^+}\to f_1\bar{f}_2} \mathcal{M}^{\mathrm{W^-}\to f_3\bar{f}_4}_{\mathrm{Born}}$$
$$+ \mathcal{M}^{\mathrm{e^+e^-\to W^+W^-}}_{\mathrm{Born}} \mathcal{M}^{\mathrm{W^+}\to f_1\bar{f}_2}_{\mathrm{Born}} \delta\mathcal{M}^{\mathrm{W^-}\to f_3\bar{f}_4} \Big).$$

One-loop corrections to subprocesses known:

$\delta\mathcal{M}^{\mathrm{e^+e^-\to W^+W^-}}$ Böhm et al. ’88; Fleischer, Jegerlehner, Zralek ’89

$\delta\mathcal{M}^{\mathrm{W}\to f\bar{f}'}$ Bardin, S. Riemann, T. Riemann ’86; Jegerlehner ’86; Denner, Sack ’90

Full result for $\mathcal{M}^{\mathrm{e^+e^-\to WW\to 4}f}_{\mathrm{virt,fact,DPA}}$ in two ways: Denner, S.D., Roth, Wackeroth ’99

- combination of known results (Böhm et al., Denner/Sack),
- completely new and independent calculation of all loops.

⇒ Perfect numerical agreement!

On-shell projection of momenta:

$\delta\mathcal{M}^{\mathrm{e^+e^-\to W^+W^-}}$ and $\delta\mathcal{M}^{\mathrm{W}\to f\bar{f}'}$ require on-shell kinematics.

$\Rightarrow$ Define mapping of fermion momenta: $k_i \;\to\; \hat{k}_i$

with: $\hat{k}_+^2 = (\hat{k}_1+\hat{k}_2)^2 = M_\mathrm{W}^2, \quad \hat{k}_-^2 = (\hat{k}_3+\hat{k}_4)^2 = M_\mathrm{W}^2.$

Note: actual choice of mapping involves freedom,
but ambiguity is of neglected order $\mathcal{O}\left(\frac{\alpha}{\pi}\frac{\Gamma_\mathrm{W}}{M_\mathrm{W}}\right)$.

Possible choice: fix directions of W^+, f_1, and f_3.

$$\hat{k}_{+0} = \frac{1}{2}\sqrt{s}, \qquad \hat{\mathbf{k}}_+ = \frac{\mathbf{k}_+}{|\mathbf{k}_+|}\beta\frac{\sqrt{s}}{2}, \qquad \hat{k}_-^\mu = p_+^\mu + p_-^\mu - \hat{k}_+^\mu,$$

$$\hat{k}_1^\mu = k_1^\mu \frac{M_\mathrm{W}^2}{2\hat{k}_+ k_1}, \qquad \hat{k}_2^\mu = \hat{k}_+^\mu - \hat{k}_1^\mu,$$

$$\hat{k}_3^\mu = k_3^\mu \frac{M_\mathrm{W}^2}{2\hat{k}_- k_3}, \qquad \hat{k}_4^\mu = \hat{k}_-^\mu - \hat{k}_3^\mu.$$

Some details of the new calculation

$$\mathcal{M}^{\mathrm{e^+e^-\to WW\to 4}f,\sigma}_{\mathrm{virt,fact,DPA}} \quad = \quad \sum_n F_n^\sigma(s,\hat t)\; \mathcal{M}_n^\sigma(p_\pm,\hat k_i,k_+^2,k_-^2)$$

Standard matrix elements $\mathcal{M}_n^\sigma(p_\pm,\hat k_i,k_+^2,k_-^2)$

contain spinor chains and W polarizations:

$$\mathcal{M}_1^\pm = \bar v(p_+)\not{\varepsilon}_+^*(\hat{\not{k}}_+ - \not{p}_+)\not{\varepsilon}_-^*\tfrac{1}{2}(1\pm\gamma_5)u(p_-), \quad \text{etc.}$$

with "effective W polarization vectors"

$$\varepsilon_+^{*,\mu} = \frac{e}{\sqrt{2}s_\mathrm{W}}\,\frac{\bar u(\hat k_1)\gamma^\mu\omega_- v(\hat k_2)}{k_+^2 - M_\mathrm{W}^2 + \mathrm{i}M_\mathrm{W}\Gamma_\mathrm{W}}, \qquad \varepsilon_-^{*,\mu} = \dots$$

⇒ W-spin correlations respected!

Invariant functions $F_n^\sigma(s,\hat t)$

contain coupling structures and loop integrals.

Economic numerical evaluation:

Note: $\mathcal{M}_n^\sigma$ simple, F_n^σ complicated functions!
But: $F_n^\sigma(s,\hat t)$ depend only on single phase-space variable
$\hat t = (p_+ - \hat k_+)^2 = f(\cos\theta)$ with $\theta = \angle(\mathbf{p}_\pm,\hat{\mathbf{k}}_\pm)$.

⇒ Evaluate integrals before phase-space integration:

$$c_{n,l}^\sigma(s) \quad = \quad \tfrac{2l+1}{2}\int_{-1}^{+1}\mathrm{d}\cos\theta\; F_n^\sigma(s,\hat t)\; P_l(\cos\theta), \qquad l = 0,1,\dots$$

Reconstruct F_n^σ algebraically during phase-space integration:

$$F_n^\sigma(s,\hat t) \quad = \quad \sum_{l=0}^{\infty}\; c_{n,l}^\sigma(s)\; P_l(\cos\theta),$$

⇒ Gain in CPU time and stability for $\cos\theta \to \pm 1$!

1.4 Virtual non-factorizable corrections

One-loop contribution to matrix element in DPA:

$$\begin{aligned}\mathcal{M}^{\mathrm{e^+e^-\to WW\to 4}f}_{\mathrm{virt,nonfact,DPA}} &= \delta\mathcal{M}^{\mathrm{e^+e^-\to 4}f}\Big|_{\text{doubly-resonant part}} - \mathcal{M}^{\mathrm{e^+e^-\to WW\to 4}f}_{\mathrm{virt,fact,DPA}} \\ &= \mathcal{M}^{\mathrm{e^+e^-\to WW\to 4}f}_{\mathrm{Born,DPA}}\ \delta_{\mathrm{virt,nonfact,DPA}}.\end{aligned}$$

Features of the correction factor $\delta_{\mathrm{virt,nonfact,DPA}}$:

- gauge invariance by definition,
- non-trivial dependence on off-shellness $k_\pm^2$ of W bosons
  ⇒ no factorization of Breit–Wigner-type resonances
  (=non-factorizability),
- free of mass singularities,
- compensates IR singularities of W bosons in $\mathcal{M}^{\mathrm{e^+e^-\to WW\to 4}f}_{\mathrm{virt,fact,DPA}}$.

Calculation of the correction factor $\delta_{\mathrm{virt,nonfact,DPA}}$:

- most complicated part was known from calculation of the sum $\delta_{\mathrm{virt+real,nonfact,DPA}}$,

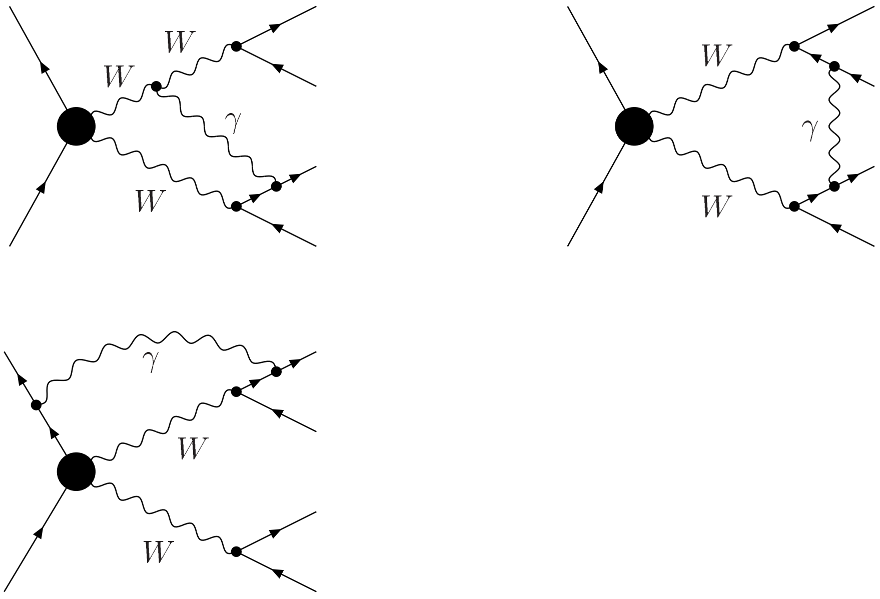
  Melnikov,Yakovlev ’96
  Beenakker,Berends,Chapovsky ’97
  Denner, S.D., Roth ’97

- remainder is calculated.

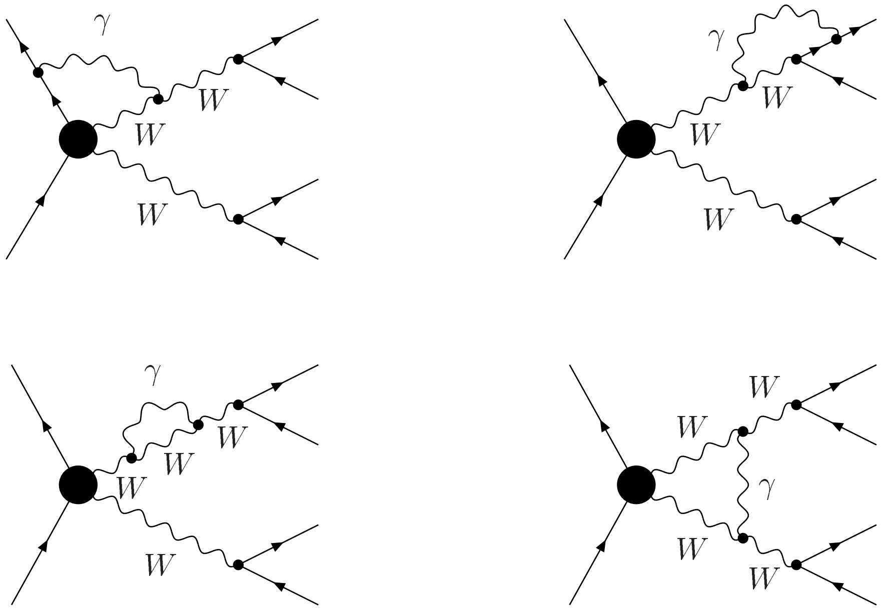
  Denner, S.D., Roth,
  Wackeroth ’99

A representative set of diagrams for non-factorizable RCs:

Genuinely non-factorizable diagrams:



Diagrams contributing to factorizable and non-factorizable RCs:



1.5 Combination of virtual and real corrections

Singularity structure of virtual and real corrections:

- IR singularities due to exchange or emission of soft photons by external charged fermions:
  corrections like $\alpha \ln(m_\gamma)|\mathcal{M}_{\text{Born}}|^2$.
  $\Rightarrow$ Complete cancellation between virtual and real parts!
- Mass singularities due to exchange or emission of collinear photons by light external charged fermions:
  corrections like $\alpha \ln(m_f)|\mathcal{M}_{\text{Born}}|^2$.
  $\Rightarrow$ Partial cancellation between virtual and real parts!

Note:

- Virtual corrections calculated in DPA.
  $\Rightarrow$ Singularities related to $\mathcal{M}^{\mathrm{e^+e^-\to WW\to 4}f}_{\mathrm{Born,DPA}}$.
- Real corrections calculated from full matrix elements.
  $\Rightarrow$ Singularities related to $\mathcal{M}^{\mathrm{e^+e^-\to 4}f}_{\mathrm{Born}}$.

$\Rightarrow$ Avoid mismatch in singularity cancellation!

Strategy for treatment of singularities

1. Determine singular behaviour of $|\mathcal{M}_{\mathrm{Born}}^{\mathrm{e^+e^-}\to 4f\gamma}|^2$:

$$\int \mathrm{d}\Phi_{4f\gamma}\, |\mathcal{M}_{\mathrm{Born}}^{\mathrm{e^+e^-}\to 4f\gamma}|^2 \quad \sim \quad \int \mathrm{d}\Phi_{4f\gamma}\, |\mathcal{M}_{\mathrm{Born}}^{\mathrm{e^+e^-}\to 4f}|^2\, \delta_{\mathrm{sub}}^{4f\gamma}.$$

2. Reintroduce masses m_γ and m_f in singular factor $\delta_{\mathrm{sub}}^{4f\gamma}$.

3. Perform integration over photon phase space analytically:
   (at least over singular region)

$$\int \mathrm{d}\Phi_{4f\gamma}\, |\mathcal{M}_{\mathrm{Born}}^{\mathrm{e^+e^-}\to 4f}|^2\, \delta_{\mathrm{sub}}^{4f\gamma} \quad = \quad \int \mathrm{d}\Phi_{4f}\, |\mathcal{M}_{\mathrm{Born}}^{\mathrm{e^+e^-}\to 4f}|^2 \otimes \delta_{\mathrm{sub}}^{4f}.$$

   Remarks:

   - "$\otimes$" indicates a convolution over an auxiliary parameter.
   - $\delta_{\mathrm{sub}}^{4f}$ contains singular terms like $\alpha \ln(m_\gamma)$ and $\alpha \ln(m_f)$.

4. Split $\delta_{\mathrm{sub}}^{4f}$ into two parts:

$$|\mathcal{M}_{\mathrm{Born}}^{\mathrm{e^+e^-}\to 4f}|^2 \otimes \delta_{\mathrm{sub}}^{4f} = |\mathcal{M}_{\mathrm{Born}}^{\mathrm{e^+e^-}\to 4f}|^2\, \underbrace{\delta_{\mathrm{sub},1}^{4f}}_{\text{related to virtual corr's}} + |\mathcal{M}_{\mathrm{Born}}^{\mathrm{e^+e^-}\to 4f}|^2 \otimes \delta_{\mathrm{sub},2}^{4f}$$

5. Add following contribution to phase-space integral:

$$\begin{array}{ll}
-\int \mathrm{d}\Phi_{4f\gamma}\, |\mathcal{M}_{\mathrm{Born}}^{\mathrm{e^+e^-}\to 4f}|^2\, \delta_{\mathrm{sub}}^{4f\gamma} & \text{cancels singularities of real corr's} \\
+\int \mathrm{d}\Phi_{4f}\, |\mathcal{M}_{\mathrm{Born,DPA}}^{\mathrm{e^+e^-}\to 4f}|^2\, \delta_{\mathrm{sub},1}^{4f} & \text{cancels singularities of virtual corr's} \\
+\int \mathrm{d}\Phi_{4f}\, |\mathcal{M}_{\mathrm{Born}}^{\mathrm{e^+e^-}\to 4f}|^2 \otimes \delta_{\mathrm{sub},2}^{4f} & \text{remaining (mass) singularities} \\
\sim \quad 0 & \text{i.e. non-doubly-resonant !}
\end{array}$$

Two explicit procedures:

Phase-space slicing

$\delta^{4f\gamma}_{\text{sub}} \neq 0$ only for $E_\gamma < \Delta E$ or $\angle(\gamma, f) < \Delta\theta$.

$\Rightarrow$ Cutoff dependences:

- Numerical cutoff dependence of $\int \mathrm{d}\Phi_{4f\gamma}$.
- Explicit cutoff dependence of $\delta^{4f}_{\text{sub},2}$.

$\Rightarrow$ Numerical cancellation for $\Delta E \to 0$, $\Delta\theta \to 0$.

Subtraction procedure

$\delta^{4f\gamma}_{\text{sub}} \neq 0$ over complete phase space.

Features:

- No cutoff-dependent numerical integrations.
  $\Rightarrow$ Better convergence expected!
- Non-trivial construction of δ^{4f}_{sub}.

Convenient general approach:

dipole subtraction formalism, formulated for

- QCD with massless, unpolarized partons, Catani, Seymour '96,'97
- photon radiation off light S.D., Roth '99
  and massive, polarized fermions. S.D. '99

Final master formula:

$$
\begin{aligned}
\int \mathrm{d}\sigma \;=\; \frac{1}{2s}\biggl\{ \int \mathrm{d}\Phi_{4f} \biggl[\, & |\mathcal{M}^{\mathrm{e^+e^-}\to 4f}_{\mathrm{Born}}|^2 \\
& + 2\,\mathrm{Re}\biggl((\mathcal{M}^{\mathrm{e^+e^-}\to \mathrm{WW}\to 4f}_{\mathrm{Born,DPA}})^*\, \delta\mathcal{M}^{\mathrm{e^+e^-}\to \mathrm{WW}\to 4f}_{\mathrm{virt,fact,DPA}} \\
& \qquad + |\mathcal{M}^{\mathrm{e^+e^-}\to \mathrm{WW}\to 4f}_{\mathrm{Born,DPA}}|^2\, \delta_{\mathrm{virt,nonfact,DPA}} \biggr) \\
& \biggr] \\
+ \int \mathrm{d}\Phi_{4f\gamma} \quad & |\mathcal{M}^{\mathrm{e^+e^-}\to 4f\gamma}_{\mathrm{Born}}|^2 \biggr\}.
\end{aligned}
$$

$$\text{non-singular}\left\{\begin{array}{l} \\ \\ \\ + \, |\mathcal{M}_{\mathrm{Born,DPA}}^{\mathrm{e^+e^-\to WW\to 4}f}|^2 \, \delta^{4f}_{\mathrm{sub,1}} \end{array}\right.$$

$$\text{explicit mass singularities}\,\{ \quad + \, |\mathcal{M}_{\mathrm{Born}}^{\mathrm{e^+e^-\to 4}f}|^2 \otimes \delta^{4f}_{\mathrm{sub,2}}$$

$$\underbrace{\Big[\qquad\qquad\qquad - \, |\mathcal{M}_{\mathrm{Born}}^{\mathrm{e^+e^-\to 4}f}|^2 \, \delta^{4f\gamma}_{\mathrm{sub}} \Big]}_{\text{non-singular}}$$

Note:

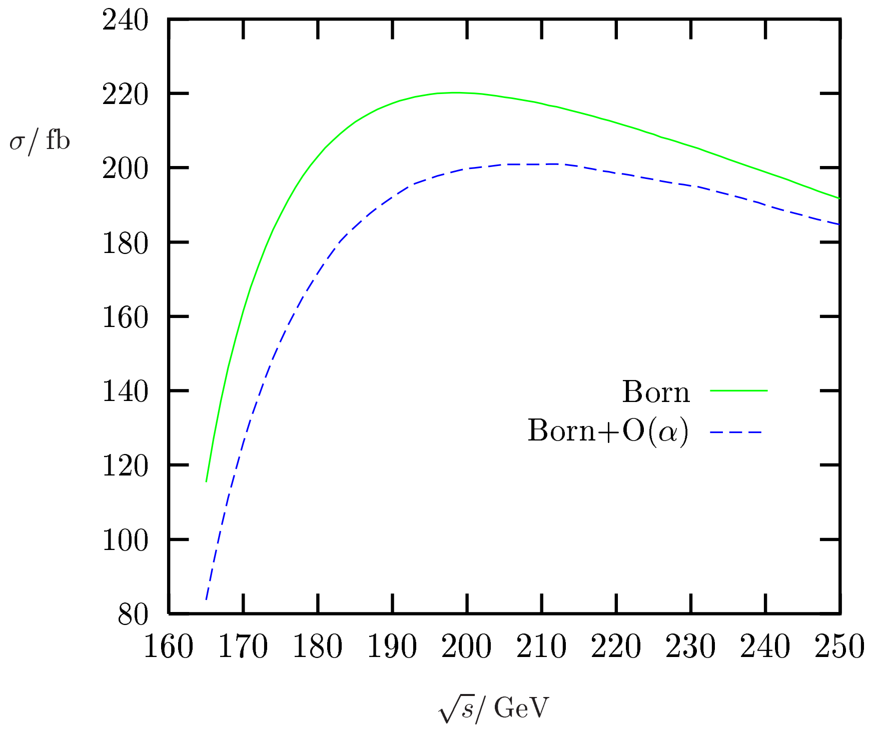
Potentially large ISR logs $\alpha \ln(m_{\mathrm{e}}/E)$ contained in $|\mathcal{M}_{\mathrm{Born}}^{\mathrm{e^+e^-\to 4}f}|^2 \delta^{4f}_{\mathrm{sub,2}}$.

⇒ Leading ISR corrections to background diagrams included!

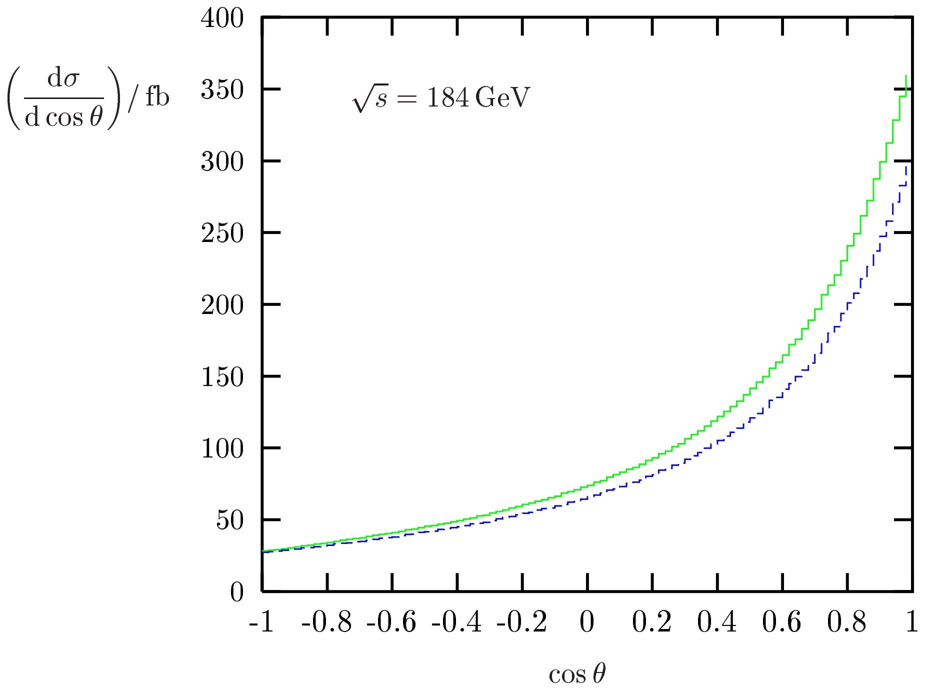
(at present for CC11 class)

2 Numerical results of RacoonWW

Total cross section for $e^+e^- \to WW \to \tau^+\nu_\tau\mu^-\bar{\nu}_\mu$:



W-production angle distribution for $e^+e^- \to WW \to \tau^+\nu_\tau\mu^-\bar{\nu}_\mu$:

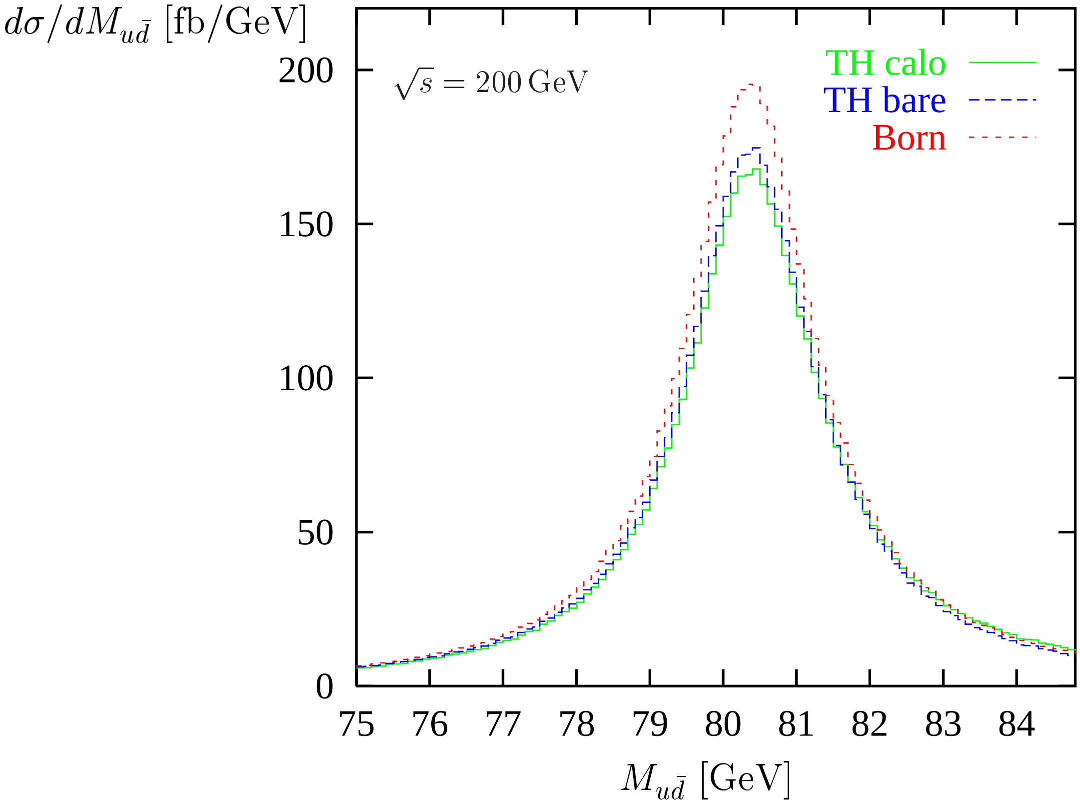


W invariant-mass distributions

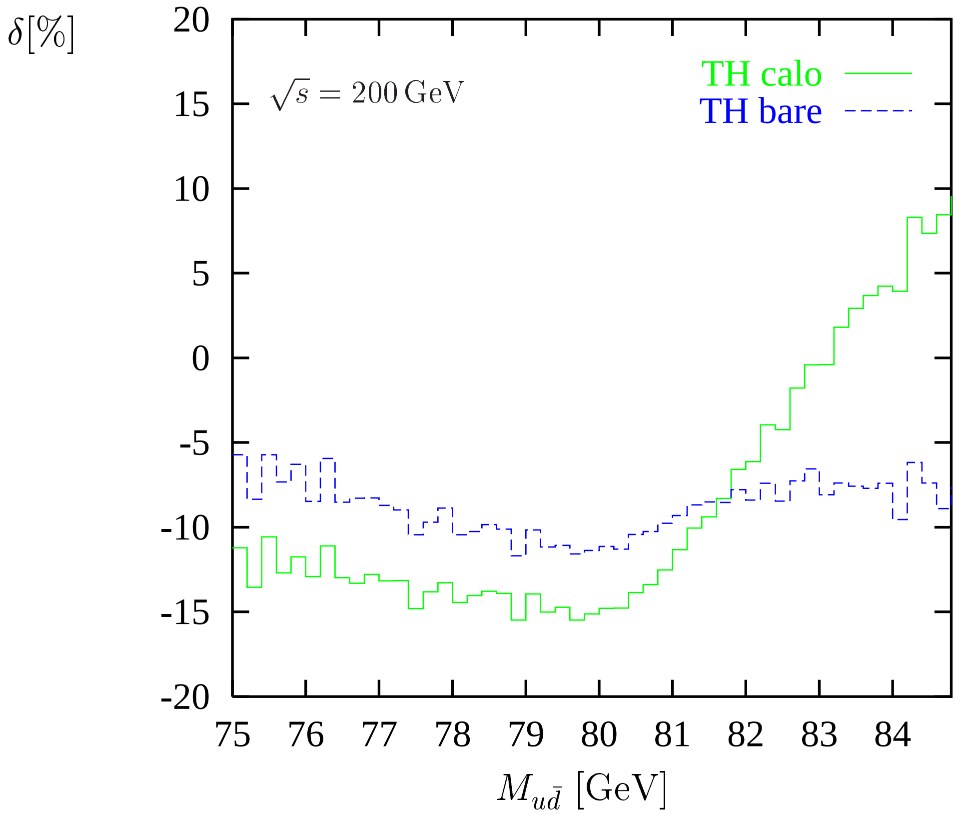
Photon recombination (modified TH prescription of $4f$ proposal):

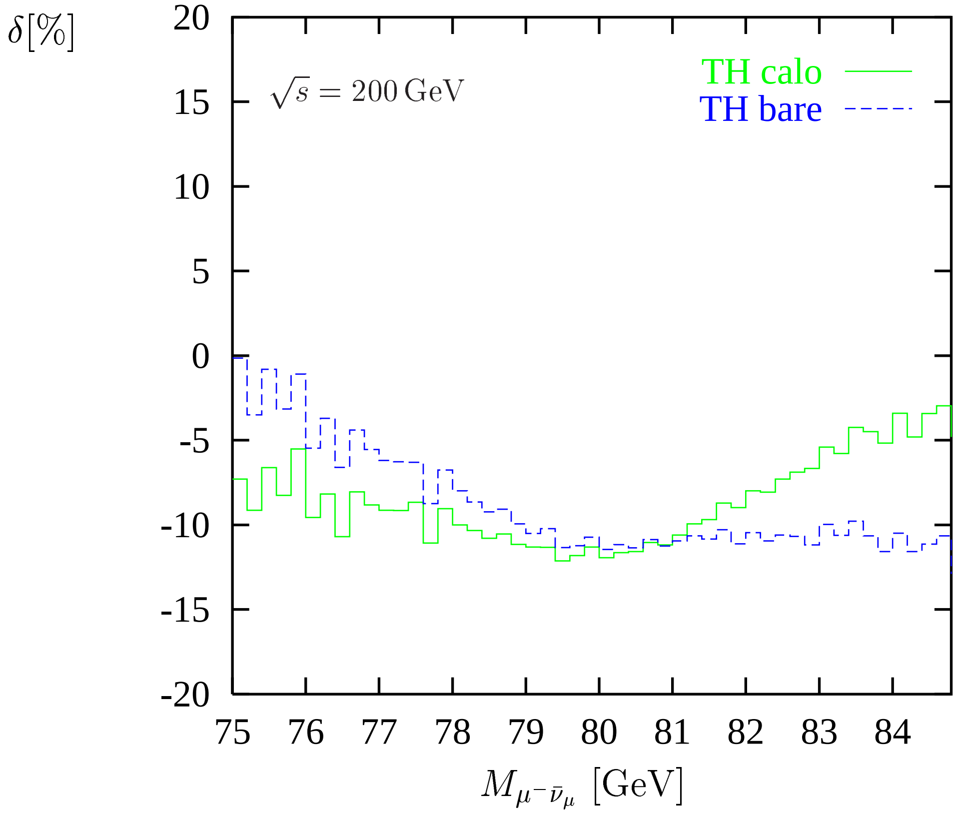
1. Discard photons (not the events!) close to beam
   if $\angle(\gamma, \mathrm{beam}) < 5°$.
2. Recombination of photons with nearest final-state fermion
   - if $M(f+\gamma) < \ \ 5\,\mathrm{GeV}$ (TH bare), or
   - if $M(f+\gamma) < 25\,\mathrm{GeV}$ (TH calo).
3. Omit events with $\angle(f, \mathrm{beam}) < 10°$ for a charged fermions.

W^+ invariant-mass distribution for $\mathrm{e^+e^-} \to \mathrm{u\bar{d}}\mu^-\bar{\nu}_\mu$:

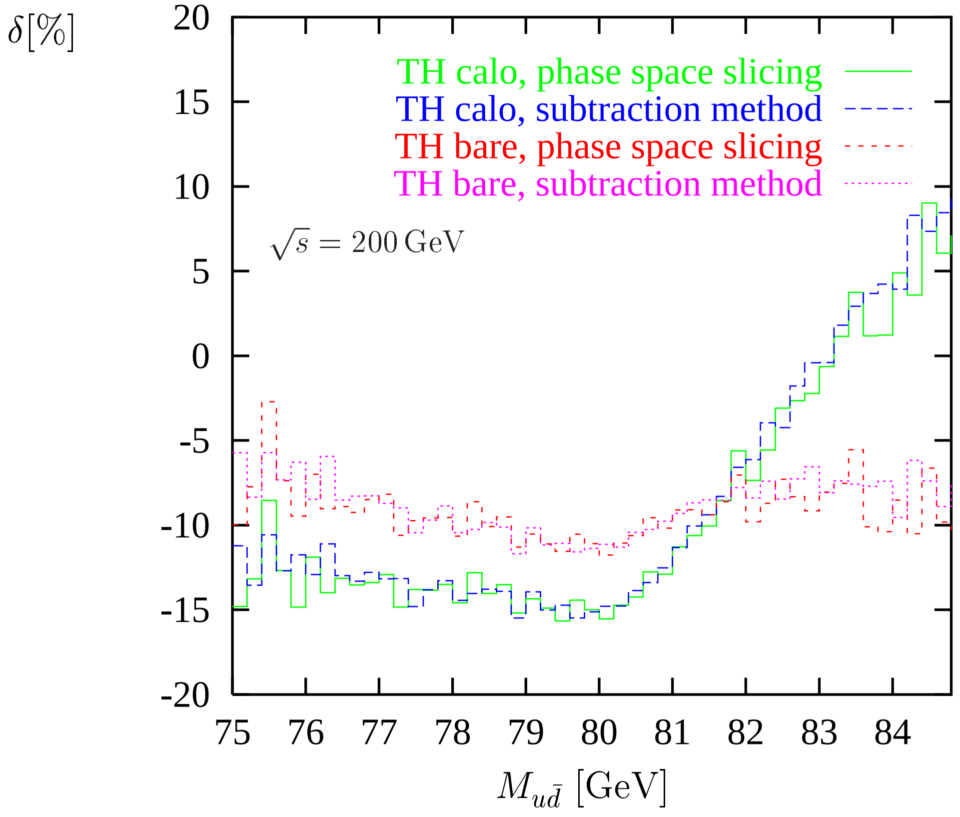


Relative corrections to $\mathrm{d}\sigma/\mathrm{d}M$ for $\mathrm{e}^+\mathrm{e}^- \to \mathrm{u}\bar{\mathrm{d}}\mu^-\bar{\nu}_\mu$:





Internal check: subtraction method vs. phase-space slicing



2.1 Theoretical uncertainties

Estimate quite difficult !

General comments on the DPA:

- Threshold region: DPA not reliable.
  (Background diagrams important!)

  $\Rightarrow$ RacoonWW yields (improved) Born-level accuracy.

- Above threshold, $\sqrt{s} > 2M_{\mathrm{W}} + n\Gamma_{\mathrm{W}}$ with $n = \mathcal{O}(3–5)$:

  corrections to background diagrams

  $$\sim \frac{\alpha}{\pi} \frac{\Gamma_{\mathrm{W}}}{\sqrt{s} - 2M_{\mathrm{W}}} \ln(\cdots) \sim 0.1\%.$$

  (if background diagrams are not enhanced!)

Error estimates in practice: (Work in progress!)

- Estimate corrections to background diagrams

  $$\sim \frac{\alpha}{\pi} (\text{full Born c.s.} - \text{DPA Born c.s.})$$

- Internal ambiguities in DPA:
  Compare results for different on-shell projections.

- Influence of higher-order effects:
  Compare different input or resummation schemes.

2.2 Comparison to results of other authors

Jadach et al. ’99
$\mathcal{O}(\alpha)$ corr’s to $\mathrm{e^+e^-}\to\mathrm{WW}\to 4f$
with YFSWW

Montagna et al. ’99
$\mathrm{e^+e^-}\to 4f+\gamma$ with $m_f \neq 0$
for some final states with ALPHA

others ?

$\Big\}$ → Talk of G. Passarino (?)

Jegerlehner, Kolodziej ’99

$\mathrm{e^+e^-}\to \mathrm{u\bar{d}}\mu^-\bar{\nu}_\mu\gamma$ with $m_f \neq 0$

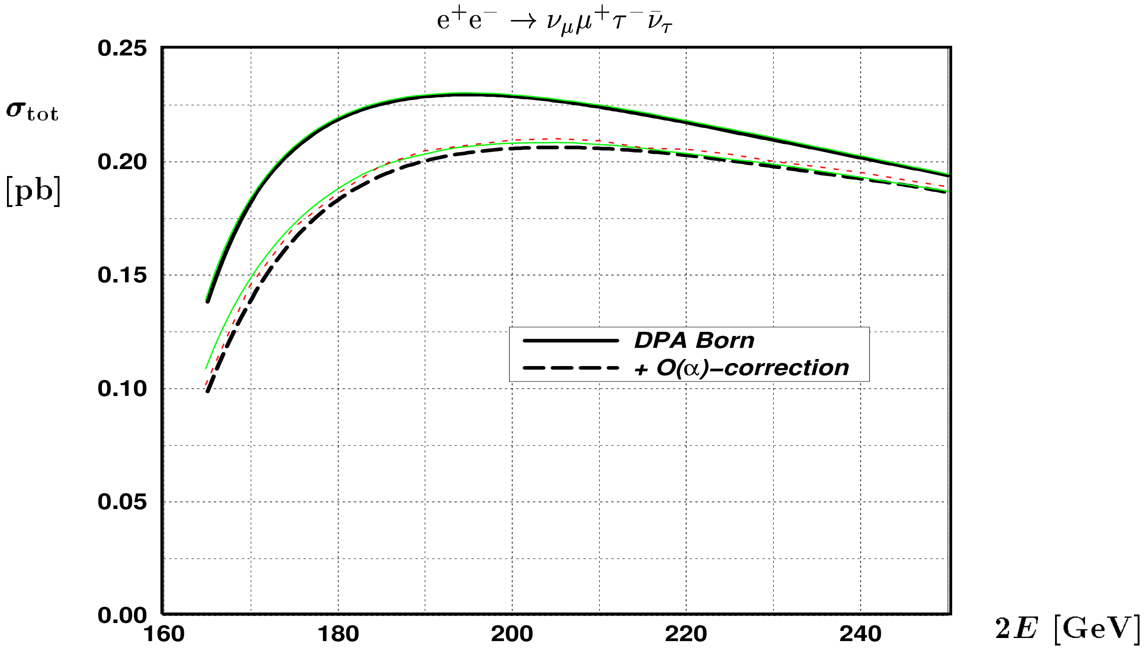
Full real correction to total cross section $\sigma_{\mathrm{real}}/\,\mathrm{fb}$:
(mass- and IR-singular, $m_\gamma = 10^{-6}\,\mathrm{GeV}$)

$\sqrt{s}/\,\mathrm{GeV}$	189	500	2000	10000	$\Delta E_\gamma/\,\mathrm{GeV}$
J/K ’99	1285(1)	571(1)			0.001
	1285.0(8)	570.0(6)	82.5(3)	6.15(6)	0.1
	1286.3(8)	570.1(5)	83.1(3)	6.10(6)	1.0
RacoonWW	1284.6(6)	568.5(4)	82.2(1)	6.31(2)	—

⇒ Reasonable agreement,
but more precise comparisons desirable.

Beenakker, Berends, Chapovsky ’98

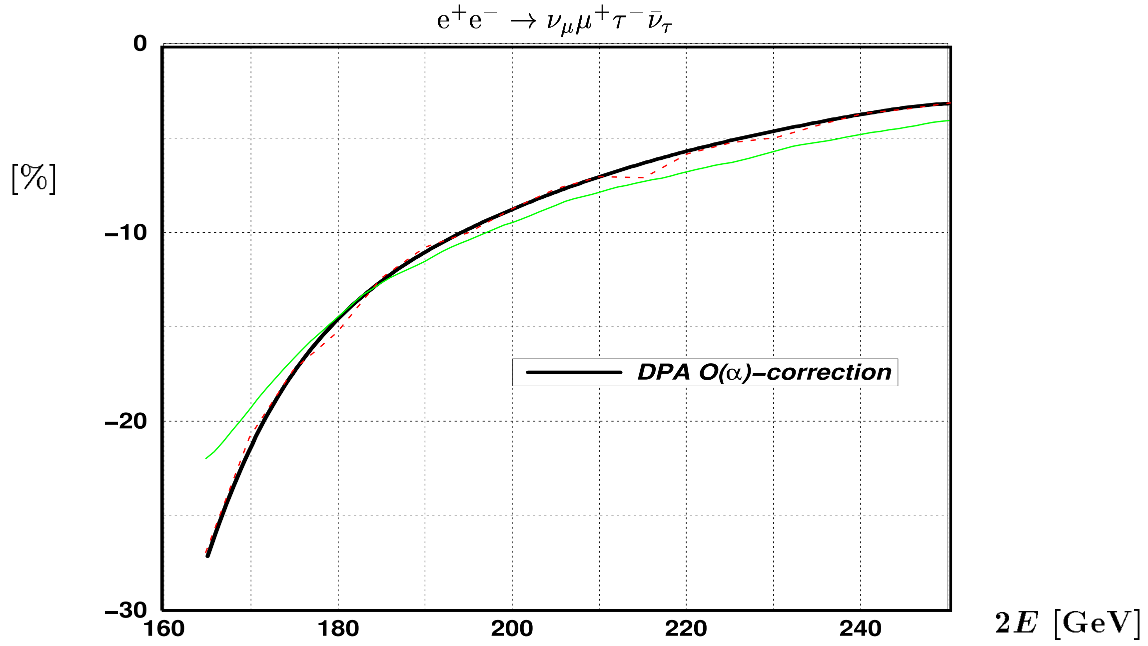
semi-analytical results for $e^+e^- \to WW \to$ 4 leptons in DPA



━━ BBC ’98

- - - BBC ’99

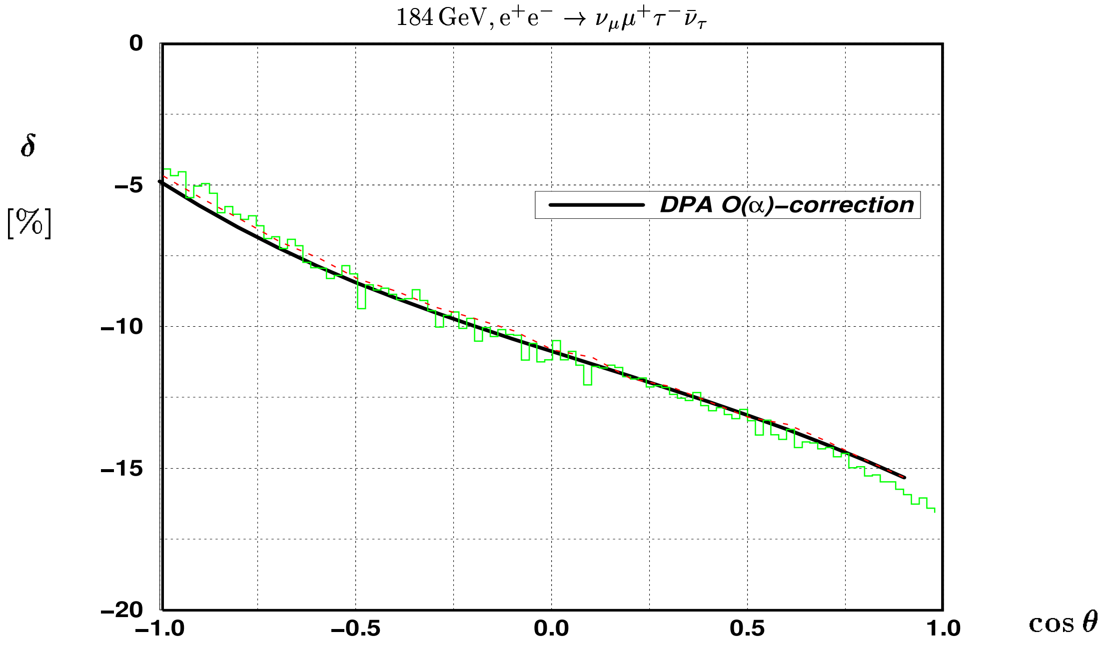
━━ RacoonWW



Origin of differences at $\sqrt{s} \gtrsim 190\,\mathrm{GeV}$ (DPA, phase space) ?

Relative corrections to angular distributions

W production angle θ:

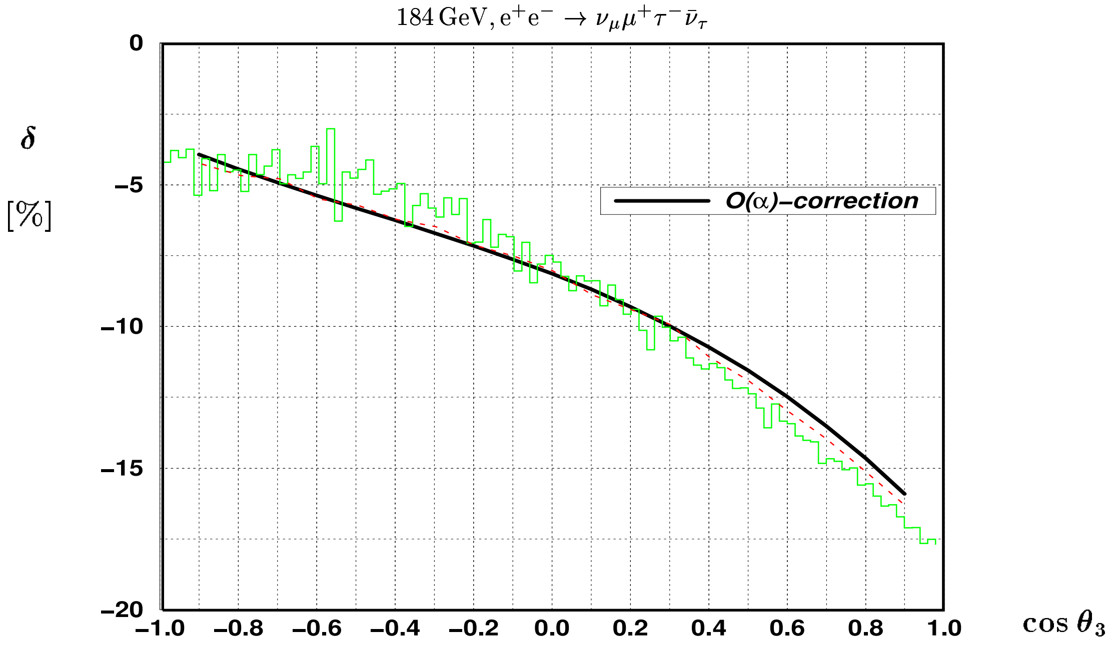


BBC ’98

BBC ’99

RacoonWW

W decay angle $\theta_3 = \angle(\mu^+, W^+)$:



3 Summary and outlook

Event generator RACOONWW:

- Lowest order: full amplitude for $\mathrm{e}^+\mathrm{e}^- \to 4f$
- Real corr's: full amplitude for $\mathrm{e}^+\mathrm{e}^- \to 4f + \gamma$
  (CC11 for invisible photons)
- Virtual corr's: consistent DPA for $\mathrm{e}^+\mathrm{e}^- \to \mathrm{WW} \to 4f$
  - factorizable corr's (on-shell WW production / W decay)
  - non-factorizable corr's

⇒ First event generator with complete DPA for $\mathcal{O}(\alpha)$ corr's

Work in progress:

- Further checks of the program(s)
- Detailed discussion of numerical results
  (cross sections, angular, energy and invariant-mass distributions)
- Comparison to work of other authors

Future extensions:

- QCD corrections
- Leading $\mathcal{O}(\alpha^2)$ corrections
- Anomalous quartic gauge couplings in $\mathrm{e}^+\mathrm{e}^- \to 4f\gamma$