$e^+e^- \to 4f$ **(+ photon)**

Markus Roth

Paul Scherrer Institut, ETH Zurich

CERN, June 25,1999

in collaboration with

A. Denner (PSI), S. Dittmaier (Uni Bielefeld)
and D. Wackeroth (PSI)

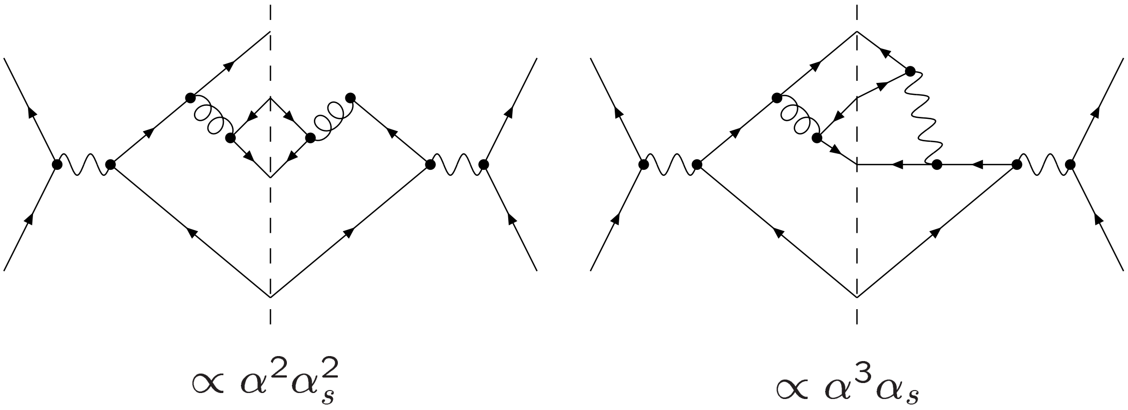
Content

- Processes $e^+e^- \to 4f\gamma$
- Outlook

$e^+e^- \to 4f\gamma$

- Helicity amplitudes for all processes
  $e^+e^- \to 4f$ and $e^+e^- \to 4f\gamma$
  Number of diagrams = 14–1008

- Gluon-exchange implemented
  Example:



- Calculation for massless fermions
  ⇒ Relatively short results

- Possible separation of diagrams including subprocesses:

  $e^+e^- \to WW(\gamma)$
  $e^+e^- \to ZZ(\gamma)$
  $e^+e^- \to ZZ(\gamma), Z\gamma(\gamma), \gamma\gamma(\gamma)$
  $e^+e^- \to WW\gamma, ZZ(\gamma), Z\gamma(\gamma), \gamma\gamma(\gamma)$

- Numerical check of amplitude with *Madgraph* (Stelzer/Long '94) for $\Gamma_{W,Z} = 0$

Monte Carlo programs

- Two independent weighted parton-level event generators

RacoonWW

Radiative corrections on $e^+e^- \to$ WW $\to 4f$

- Multi-channel approach
  ⇒ 14–928 channels

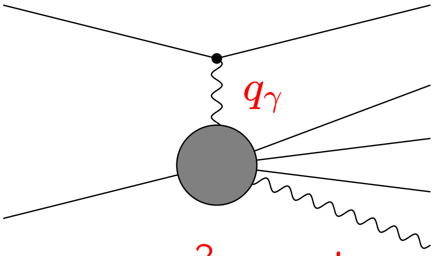
- Adaptive weight optimization

- Channels
  - 10 generic phase-space generators for $e^+e^- \to 4f\gamma$ and 5 for $e^+e^- \to 4f$
  - Each generator responsible for a class of diagrams
  - All propagators are smoothed.

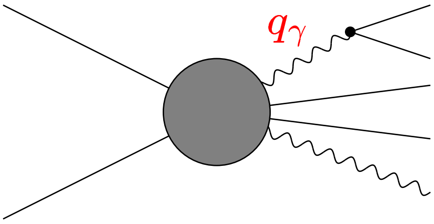
Required cuts

Calculation for vanishing fermion masses!

- Cuts required for $e^+e^- \to 4f\gamma$ as well as for $e^+e^- \to 4f$:



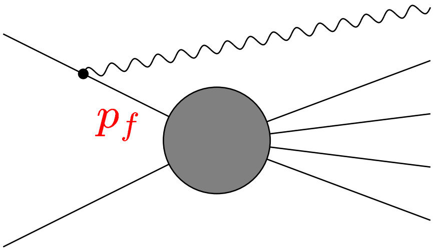
$-q_\gamma^2 > \text{cut}$



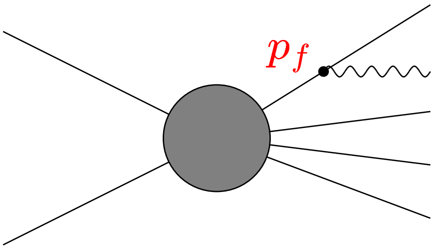
$q_\gamma^2 > \text{cut}$

Infrared and collinear singularities for $q_\gamma^2 \to 0$!
$\Rightarrow$ Invariant mass cut on q_γ^2 required!

- Additional cuts for $e^+e^- \to 4f\gamma$:



$-p_f^2 > \text{cut}$



$p_f^2 > \text{cut}$

Infrared and collinear singularities for $p_f^2 \to 0$!
$\Rightarrow$ Invariant mass cut on p_f^2 required!

Implementation of the width

Fixed-width scheme: $\Gamma_\mathrm{W}(k_\pm^2) = \Gamma_\mathrm{W}$

- Ward identities violated in general
- $\mathrm{e}^+\mathrm{e}^- \to 4\mathrm{f}$: $U(1)$ o.k.; $SU(2)$ violated,
  but $\mathcal{O}(\Gamma_\mathrm{W} M_\mathrm{W}/s)$-effect for $s \to \infty$

Running-width scheme: $\Gamma_\mathrm{W}(k_\pm^2) = \Gamma_\mathrm{W} k_\pm^2/M_\mathrm{W}^2 \theta(k_\pm^2)$

- Ward identities violated in general
- $\mathrm{e}^+\mathrm{e}^- \to 4\mathrm{f}$: $U(1)$ and $SU(2)$ violated,
  wrong results for high energies

Complex-mass scheme

Replace boson masses everywhere with complex mass: $M^2 \to M^2 - \mathrm{i}M\Gamma$,

in particular:

$$\cos^2\theta_W = 1 - \sin^2\theta_W = \frac{M_\mathrm{W}^2 - \mathrm{i}M_\mathrm{W}\Gamma_\mathrm{W}}{M_\mathrm{Z}^2 - \mathrm{i}M_\mathrm{Z}\Gamma_\mathrm{Z}}$$

- Ward identities respected!

$e^+e^- \to 4f\gamma$: Width schemes

$\sigma(e^+e^- \to u\,\bar{d}\,\mu^-\bar{\nu}_\mu)/\,\mathrm{fb}$:

$\sqrt{s}$	189 GeV	500 GeV	2 TeV	10 TeV
constant width	703.5(3)	237.4(1)	13.99(2)	0.624(3)
running width	703.4(3)	238.9(1)	34.39(3)	498.8(1)
complex-mass scheme	703.1(3)	237.3(1)	13.98(2)	0.624(3)

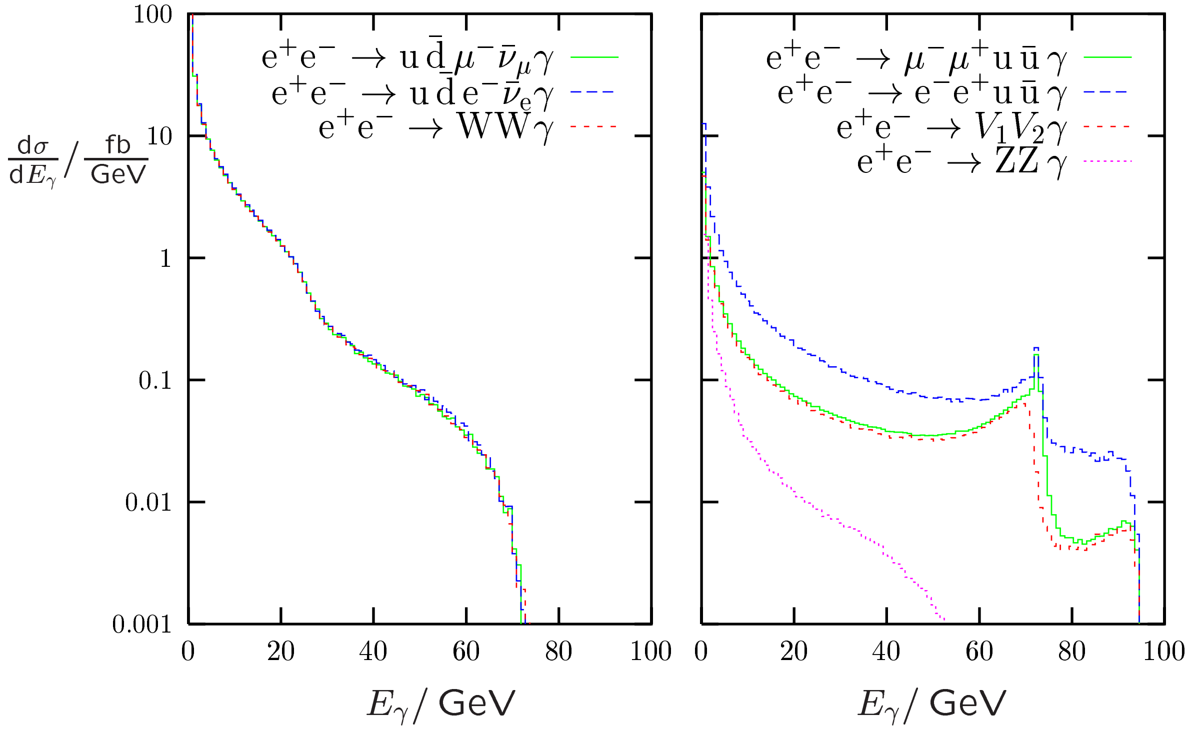
$\sigma(e^+e^- \to u\,\bar{d}\,\mu^-\bar{\nu}_\mu\gamma)/\,\mathrm{fb}$:

$\sqrt{s}$	189 GeV	500 GeV	2 TeV	10 TeV
constant width	224.0(4)	83.4(3)	6.98(5)	0.457(6)
running width	224.6(4)	84.2(3)	19.2(1)	368(6)
complex-mass scheme	223.9(4)	83.3(3)	6.98(5)	0.460(6)

⇒ Treat finite width as carefully as for $e^+e^- \to 4f$!

Results for $e^+e^- \to 4f\gamma$

Energy=189 GeV, ADLO/TH cuts



$e^+e^- \to V_1V_2\gamma$: Diagrams including triple-gauge-boson-production subprocesses $V_1, V_2 = W, Z, \gamma$

Cross sections:

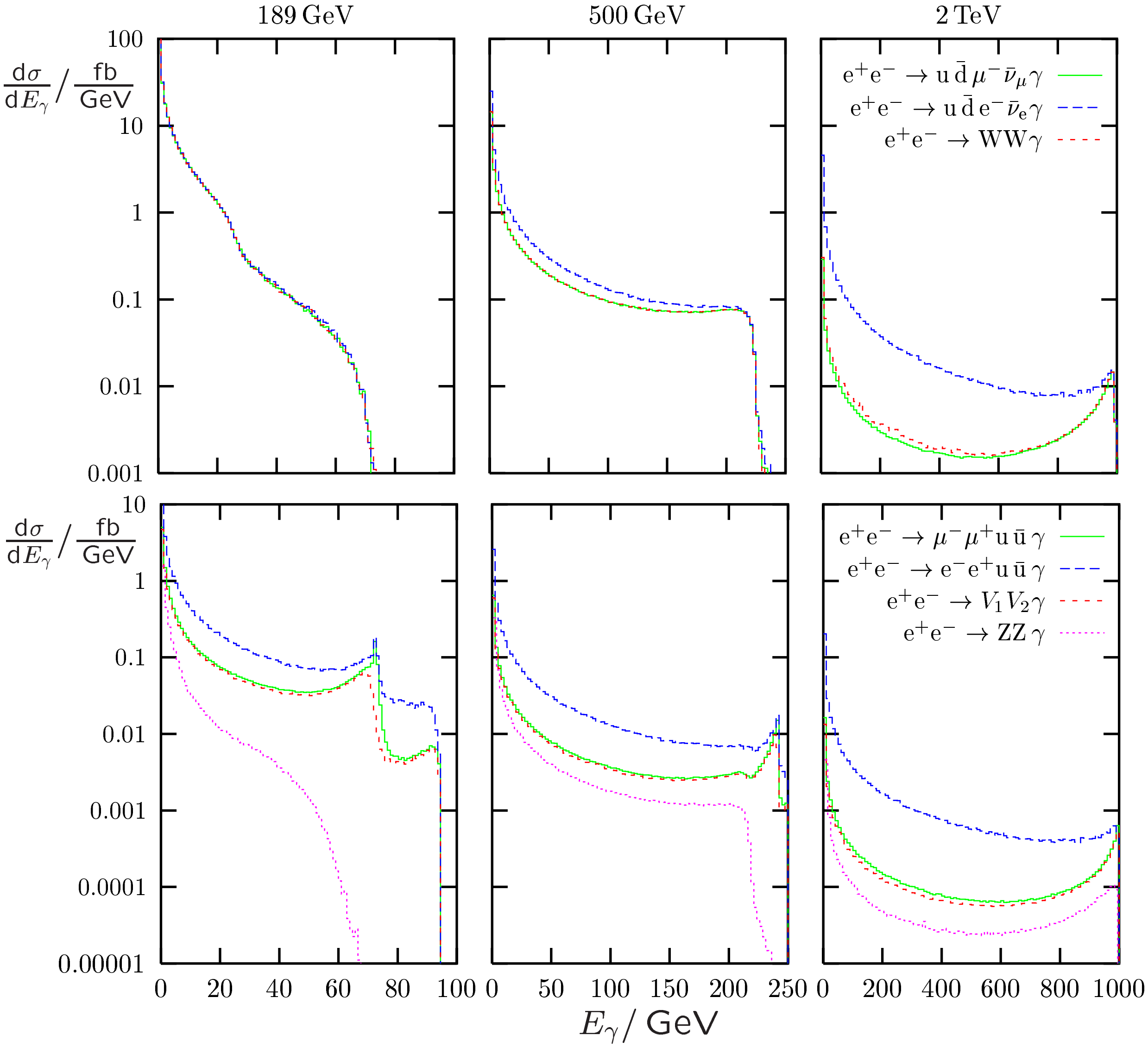
$\sigma(e^+e^- \to u\,\bar{d}\,\mu^-\bar{\nu}_\mu\gamma) = 223.9(4)$ fb

$\sigma(e^+e^- \to u\,\bar{d}\,e^-\bar{\nu}_e\gamma) = 229.9(4)$ fb

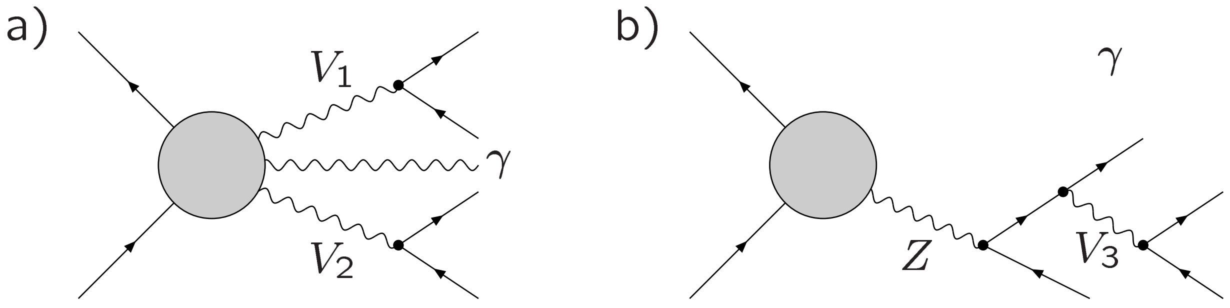
$\sigma(e^+e^- \to \mu^-\mu^+u\,\bar{u}\gamma) = 13.13(4)$ fb

$\sigma(e^+e^- \to e^-e^+u\,\bar{u}\gamma) = 32.3(2)$ fb

Results for $e^+e^- \to 4f\gamma$



$e^+e^- \to 4f\gamma$: Triple-gauge-boson subprocesses



$V_1V_2 =$ WW, ZZ:

Reduction of energy $\sqrt{s'}$ for $e^+e^- \to V_1V_2$ after ISR:

$s' = s - 2\sqrt{s}\,E_\gamma$

Threshold condition: $\sqrt{s'} > M_{V_1} + M_{V_2}$

⇒ Kink in the photon-energy spectra

$V_1V_2 = \gamma\gamma$:

⇒ Large cross section at $E_\gamma \lesssim \sqrt{s}$

$V_1V_2 = Z\gamma$ and $V_3 = \gamma, g$:

Soft virtual photon/gluon kinematically unimportant

Important subprocess is $e^+e^- \to Z\gamma$

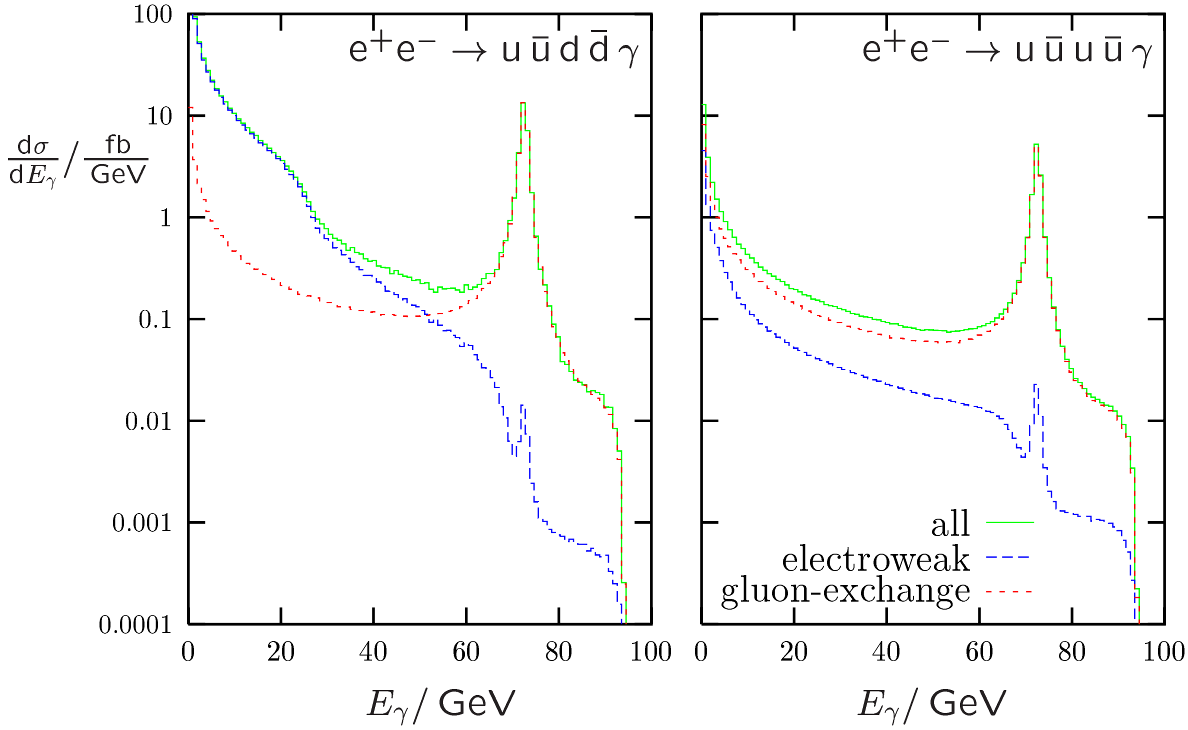
Resonance condition: $\sqrt{s'} \approx M_Z$

⇒ Resonance in the photon energy spectra!

⇒ Enhanced by gluon-exchange contributions!

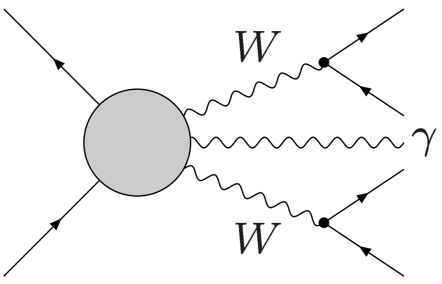
$e^+e^- \to 4f\gamma$: Gluon-exchange diagrams

Energy=189 GeV, ADLO/TH cuts

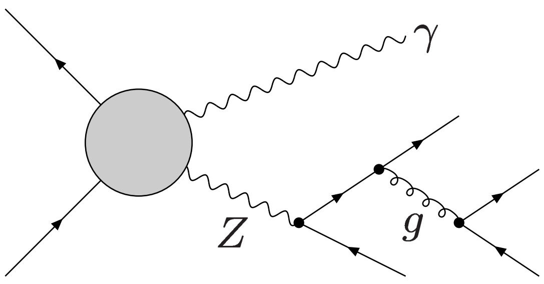


Example:

electroweak



gluon-exchange



Status and further strategy

Aim: Weighted event generator including all $\mathcal{O}(\alpha)$-corrections in double-pole approximation

- Virtual corrections:

  $\Rightarrow$ Double-pole approximation

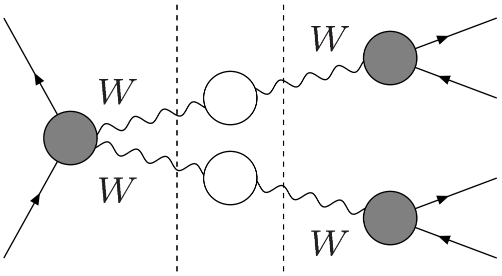
  - Doubly-resonant corrections are gauge-invariant!
  - Program for doubly-resonant virtual corrections exits.
  - Exact off-shell Coulomb singularity included
  - Non-doubly-resonant contributions can be neglected:

    $\sim \mathcal{O}\left(\frac{\Gamma_{\mathrm{W}}}{M_{\mathrm{W}}}\frac{\alpha}{\pi}\log(\cdots)\right) \approx 0.025 \times 0.05 \approx 0.001$
  - Virtual corrections to on-shell W-pair production and on-shell W decay can be used.
  - Non-factorizable corrections calculated

Doubly-resonant virtual corrections

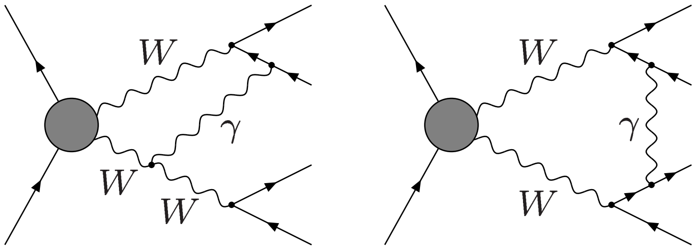
Composed of

- Factorizable corrections:



on-shell production   on-shell decay

- Non-factorizable corrections: Melnikov, Yakovlev
  Beenakker, Berends, Chapovsky
  Denner, Dittmaier, R.



etc.

Doubly-resonant virtual corrections=
factorizable corrections
+non-factorizable corrections

Real corrections

- Real corrections:

  $\Rightarrow$ Complete processes $e^+e^- \to 4f\gamma$

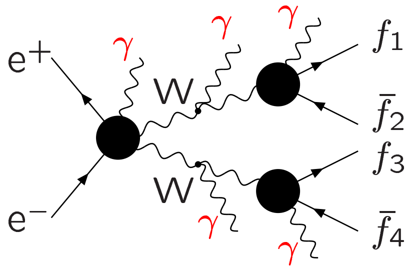
  – Weighted event generator exists.

  – gauge invariant

  – Explicite photons generated with full p_t

  – Involving initial, intermediate and final state radiation

    $\Rightarrow$ In particular important for realistic description of the line shape



$\frac{d\sigma}{dk_\pm^2}$ with $k_\pm = k_{f_a} + k_{\bar{f}_b}$ $(+k_\gamma)$

$\Rightarrow$ Definition of $k_\pm^2$ depends on photon resolution!

  – Numerical stability in the soft and collinear region not yet tested?!

Treatment of singularities and status

Extraction of infrared and collinear singularities:

- Subtraction method
  - Subtraction terms for mass regularization calculated
  - Implementation in progress
  - Numerical stability not yet tested?!
- Second program with phase-space slicing in preparation

Status:

Checking the single routines

Combining all building blocks into a program

First results expected in September '99

Public version of the program later

Not yet included

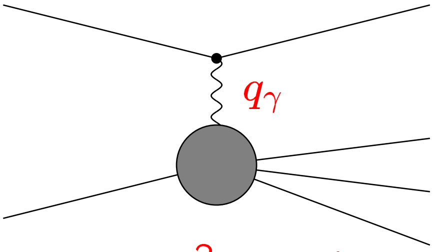
Can be implemented later:

- Initial state radiation of order $\mathcal{O}(\alpha^2)$ LL and higher
- QCD one-loop corrections
- Non-standard couplings
  Triple- and quartic-gauge-boson couplings
- Interface to hadronization packages

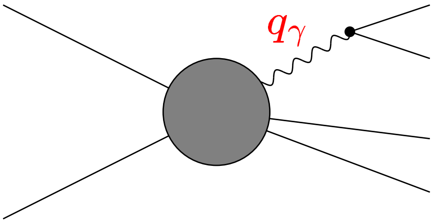
Requirements

Fermion masses neglected whenever possible!

- Required Cuts



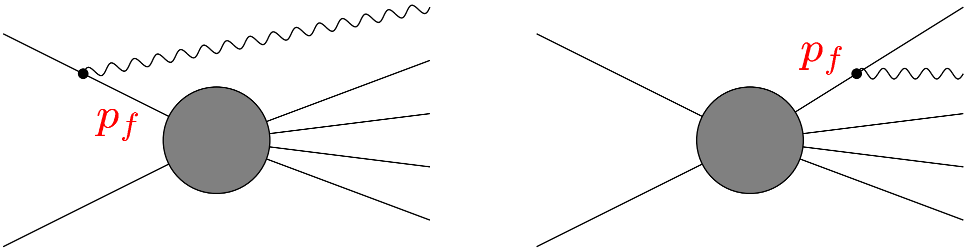
$-q_\gamma^2 >$ cut



$q_\gamma^2 >$ cut

Singularity for vanishing invariant mass of virtual photon γ!
$\Rightarrow$ Invariant mass cut on q_γ^2 required!

- Collinear and soft region at $p_f^2 = 0$ have to be included in the phase space!



$\Rightarrow$ Observables have to be inclusive in the soft and collinear region!