Total Precision of BHWIDE and BHLUMI Monte Carlos for Bhabha Scattering

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OUTLINE

- INTRODUCTION
- BHLUMI4.04
- BHWIDE1.0
- CONCLUSIONS

PAPERS:
1. S. JADACH, M. MELLES, B.F.L. WARD, and S.A. YOST, UTHEP-98-0501, in press at Phys. Lett. B, 1999
2. S. JADACH, W. PLACZEK and B.F.L. WARD, Phys. Lett. B390, 298 (1997).
3. S. JADACH, M. MELLES,W. PLACZEK, S.A. YOST and B.F.L. WARD, UTHEP-98-1101.

INTRODUCTION

- PRECISION SM TESTS AT LEP

$$\Delta\mathcal{O}/\mathcal{O} = \left(\left(\frac{\partial\mathcal{O}}{\partial\sigma_{\mathcal{A}}} \frac{\Delta\sigma_{\mathcal{A}}}{\sigma_{\mathcal{A}}} \frac{\sigma_{\mathcal{A}}}{\mathcal{O}} \right)^2 + \ldots \right)^{\frac{1}{2}}$$

$\Rightarrow \frac{\Delta\sigma_{\mathcal{A}}}{\sigma_{\mathcal{A}}}$ NEEDED

AS SMALL AS POSSIBLE

$\Rightarrow \frac{\Delta\sigma_{\mathcal{A}}}{\sigma_{\mathcal{A}}}|_{TH}$ NEEDED

AS SMALL AS POSSIBLE

$$\mathcal{A} = \begin{cases} \mathcal{L} & \text{Normalization At LEP1(2),} \\ LABH & \Gamma_e, \text{ (SEARCHES),} \cdots, \text{ AT LEP1(2)} \end{cases}$$

- STATUS

PRIOR TO VANCOUVER, 1998

BHLUMI

	LEP1		LEP2
Type of correction/error	Past	06/98	06/98
(a) Missing photonic $\mathcal{O}(\alpha^2 L)$	0.15%	0.10%	0.20%
(b) Missing photonic $\mathcal{O}(\alpha^3 L^3)$	0.015%	0.015%	0.03%
(c) Vacuum polarization	0.04%	0.04%	0.10%
(d) Light pairs	0.03%	0.03%	0.05%
(e) Z-exchange	0.015%	0.015%	0.0%
Total	0.16%	0.11%	0.25%

Table 1: Summary of the total (physical+technical) theoretical uncertainty for a typical calorimetric detector. For LEP1, the above estimate is valid for the angular range within $1^\circ - 3^\circ$, and for LEP2 it covers energies up to 176 GeV, and angular range within $1^\circ - 3^\circ$ and $3^\circ - 6^\circ$.

BHWIDE

Phys. Lett. B390(1997) 298-308:

$$\text{TOTAL PRECISION} = \begin{cases} 0.3\% & \text{LEP1 ON-PEAK,} \\ 0.5\% & \text{LEP1 OFF-PEAK,} \\ 1.5\% & \text{LEP2, to 205GeV} \end{cases}$$

- COMPARISON WITH EXPT

  $\frac{\Delta\sigma_{\mathcal{L}}}{\sigma_{\mathcal{L}}}|_{EXPT} \lesssim 0.05\%$

  $\frac{\Delta\sigma_{LABH}}{\sigma_{LABH}}|_{EXPT} \lesssim 0.5\%$ AT LEP1

  $\frac{\Delta\sigma_{LABH}}{\sigma_{LABH}}|_{EXPT} \lesssim 0.5\%$ AT LEP2(ANTICIPATED)

  $\Rightarrow$ NEED

  $\frac{\Delta\sigma_{\mathcal{L}}}{\sigma_{\mathcal{L}}}|_{TH} \lesssim 0.05\%$

  $\frac{\Delta\sigma_{LABH}}{\sigma_{LABH}}|_{TH} \lesssim 0.2\%$ AT LEP1(ON-PEAK)
  $\frac{\Delta\sigma_{LABH}}{\sigma_{LABH}}|_{TH} \lesssim 0.3\%$ AT LEP1(OFF-PEAK)

  $\frac{\Delta\sigma_{LABH}}{\sigma_{LABH}}|_{TH} \lesssim 0.5\%$ AT LEP2

- WHERE ARE WE?

BHLUMI4.04

★ DOMINANT CONTRIBUTION TO $\frac{\Delta\sigma_{\mathcal{L}}}{\sigma_{\mathcal{L}}}|_{TH}$

(a) MISSING PHOTONIC $\mathcal{O}(\alpha^2 L) \Leftrightarrow 0.1\%$

- PHYSICAL PRECISION

  1. $\mathcal{O}(\alpha)$ CORR. TO 1γ BREMSS. : $\bar{\beta}_1^{(2)}$

  2. $\mathcal{O}(\alpha^2)$ CORR. TO $\bar{\beta}_0$

  (2-LOOP VIR. CORR.)

- TECHNICAL PRECISION

  1. $\mathcal{O}(\alpha^2)$ 2γ BREMSS. : $\bar{\beta}_2$

  (INDEPNDNT REAL'ZN 2γ PHASE SPACE IN 'TEST' MC)

★ EXACT $\mathcal{O}(\alpha)$ CORRECTION TO 1γ BREMSSTRAHLUNG

- JADACH ET AL., PHYS. LETT. **B377** (1996) 168:

  $\Rightarrow \mathcal{O}(\alpha)$ CORRECTION TO $\bar{\beta}_1$, EXACT

- COMPARISON: ARBUZOV ET AL., NUCL.PHYS.**B485**(1997) 457,

  (SEMI-COLLINEAR APPROX.), NLLB

- SIMPLIFIED FORM OF EXACT RESULT:

  SOFT γ ANSATZ, ANSATZ(SOFT)

$\Rightarrow$ RESULTS IN FIGS. 1 AND 2

.

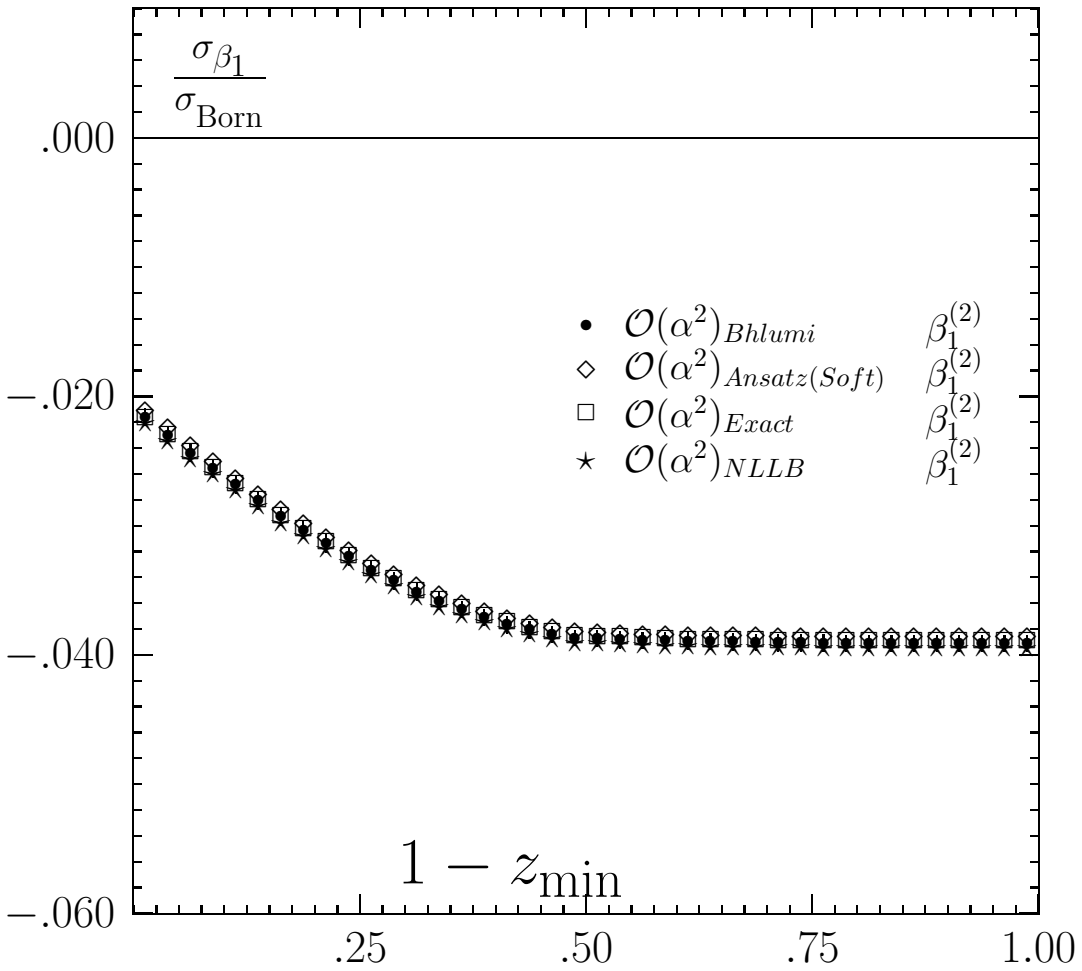
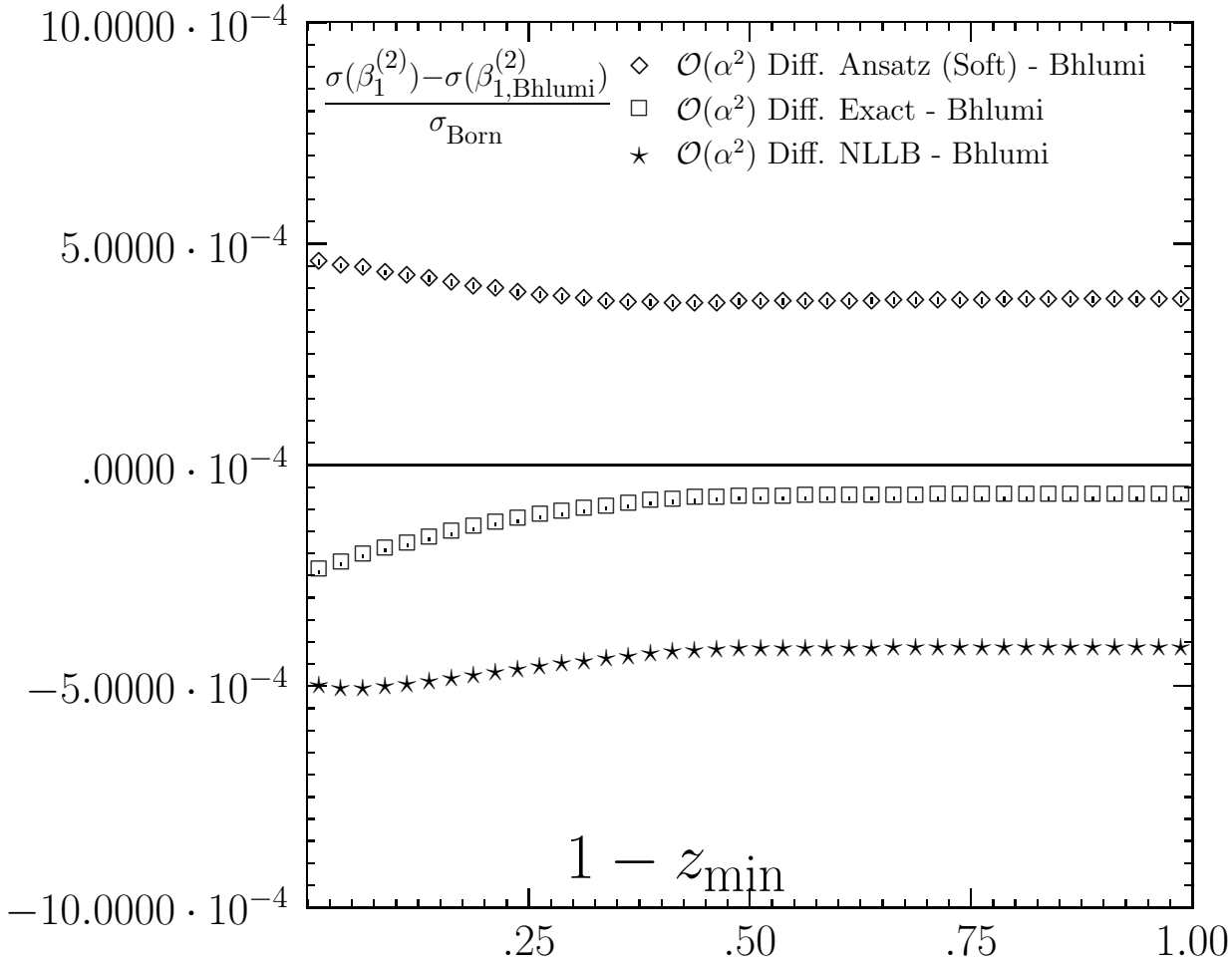


Figure 1: Monte Carlo result (10^6 events) for the entire cross section associated with $\bar{\beta}_1^{(2)}$ for the SICAL Wide-Narrow trigger.The first and second order results are divided by the Narrow-Narrow Born cross section. z_{min} is as it is defined in Fig. 2 of Phys. Lett. **B353**(1995)362.

Figure 2



$$\text{FOR } 0.2 \lesssim 1 - z_{min} \leq 1,\ \frac{\Delta\sigma_{\mathcal{L}}}{\sigma_{\mathcal{L}}}\Big|_{\mathcal{O}(\alpha^2\bar{\beta}_1)} \leq 0.02\%$$

★ EXACT 2γ BREMSSTRAHLUNG: TECHNICAL PRECISION CHECK

- EXACT RESULTS ON 2γ BREMSSTRAHLUNG FOR $\bar{\beta}_2$ (JADACH ET AL., PHYS. REV.**D47**, 2682 (1993); ibid.**D42**, 2977(1990))

- 2γ PHASE SPACE VIA INDEPENDENT 'TEST' MC FOR TECH. PREC. CHK OF $\bar{\beta}_2$

$\Rightarrow$ BY PRODUCT:
CROSS CHK $\bar{\beta}_2$ PHYS. PREC.

$\Rightarrow$ RESULTS IN FIG. 3

$\Rightarrow$

$$\boxed{\frac{\Delta\sigma_{\mathcal{L}}}{\sigma_{\mathcal{L}}}|_{(\mathcal{O}(\alpha^2\bar{\beta}_2);TECH.PREC.)} \leq 0.003\%}$$

$$\boxed{\frac{\Delta\sigma_{\mathcal{L}}}{\sigma_{\mathcal{L}}}|_{(\mathcal{O}(\alpha^2\bar{\beta}_2);PHYS.PREC.)} \leq 0.012\%}$$

★ EXACT 2-LOOP VIRTUAL CORRECTION

- ANALYT. CONT. RESULT OF BERENDS ET AL.(NUCL.PHYS.B297, 429(1988)) TO t-CHANNEL

$\Rightarrow$

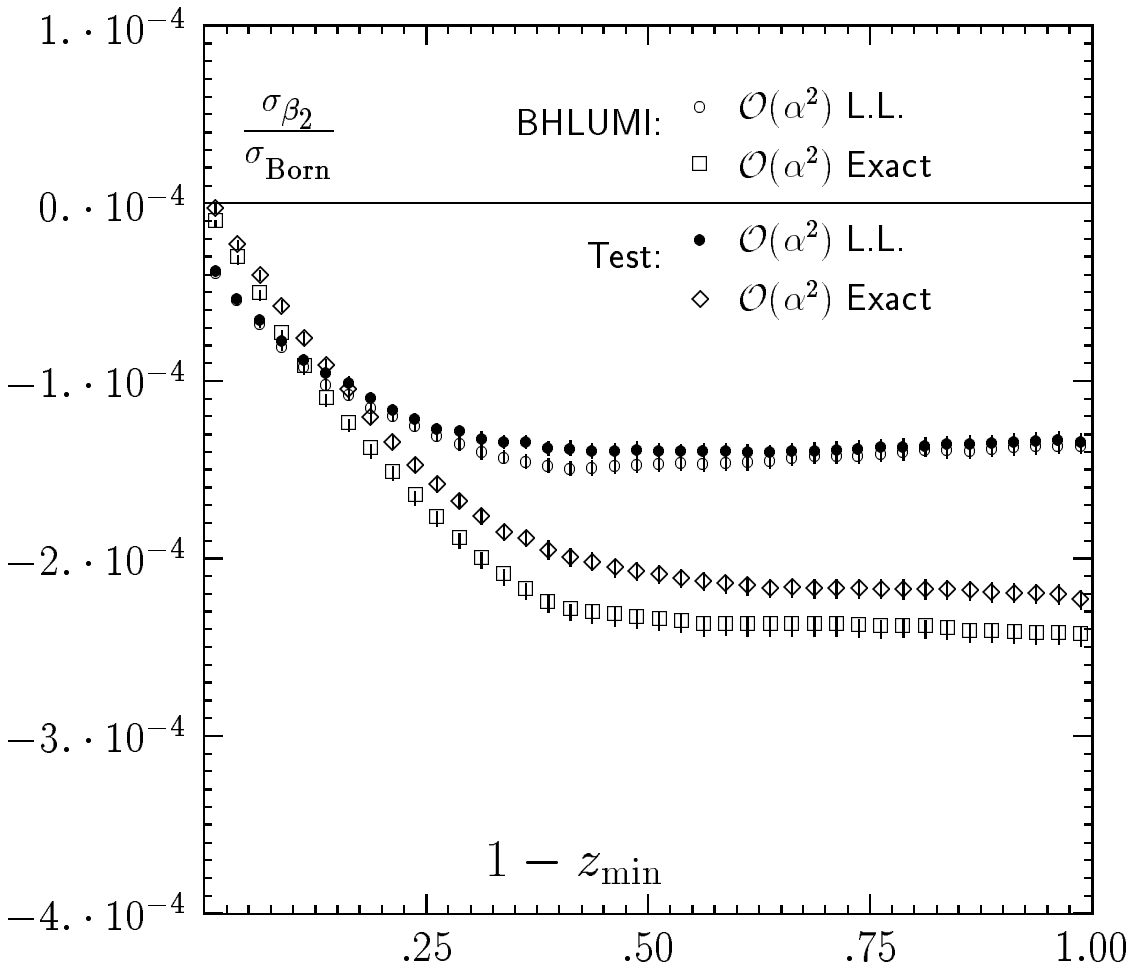

$1. \cdot 10^{-4}$
$0. \cdot 10^{-4}$
$-1. \cdot 10^{-4}$
$-2. \cdot 10^{-4}$
$-3. \cdot 10^{-4}$
$-4. \cdot 10^{-4}$
$\frac{\sigma_{\beta_2}}{\sigma_{\mathrm{Born}}}$
BHLUMI:
$\mathcal{O}(\alpha^2)$ L.L.
$\mathcal{O}(\alpha^2)$ Exact
Test:
$\mathcal{O}(\alpha^2)$ L.L.
$\mathcal{O}(\alpha^2)$ Exact
$1 - z_{\mathrm{min}}$
.25
.50
.75
1.00

Figure 3

$$\Delta\bar{\beta}_0/(\bar{\beta}_0|_{Born}) = (\frac{\alpha}{\pi})^2 L \left(6 + 6\zeta(3) - \frac{45}{8} - \frac{\pi^2}{2}\right) + (\frac{\alpha}{\pi})^2[6 - 9\zeta(3) + (\frac{17}{8} - 2\ln 2)\pi^2 - \frac{8}{45}\pi^4] \tag{1}$$

$$\Rightarrow \boxed{\frac{\Delta\sigma_{\mathcal{L}}}{\sigma_{\mathcal{L}}}|_{\mathcal{O}(\alpha^2\bar{\beta}_0)} \leq 0.014\%}$$

★ NEW ERROR ANALYSIS FOR BHLUMI4.04

- USING THE RESULTS JUST DERIVED WE GET

$$\boxed{\frac{\Delta\sigma_{\mathcal{L}}}{\sigma_{\mathcal{L}}}|_{\mathcal{O}(\alpha^2)-PHOTONIC} = 0.027\%}$$

$\Rightarrow$

	LEP1		LEP2	
Type of correction/error	Past	Present	Past	Present
(a) Missing phot. $\mathcal{O}(\alpha^2)$	0.10%	0.027%	0.20%	0.04%
(b) Missing phot. $\mathcal{O}(\alpha^3 L^3)$	0.015%	0.015%	0.03%	0.03%
(c) Vac. pol.	0.04%	0.04%	0.10%	0.10%
(d) Light pairs	0.03%	0.03%	0.05%	0.05%
(e) Z-exchange	0.015%	0.015%	0.0%	0.0%
Total	0.11%	0.061%	0.25%	0.122%

Table 2: Summary of the total (physical+technical) theoretical uncertainty for a typical calorimetric detector. For LEP1, the above estimate is valid for the angular range within $1^\circ - 3^\circ$, and for LEP2 it covers energies up to 200 GeV, and angular range within $1^\circ - 3^\circ$ and $3^\circ - 6^\circ$.

$\otimes$ OPAL PAIRS CUTS: FIRST LOOK

- BHPAIR: SJ,MS,,BFLW, PHYS. REV. **D47**(1993) 3733

  SEMI-ANALYTIC JW-YFS EXPONENTIATION

  COMPARE TO NICROSINI ET AL.:

  -2.7×10^{-4}, NO ACOL/ACOPL CUT

  ACOL/ACOPL CUT RESULTS IN PROGRESS

$\Rightarrow$ RESULTS IN FIGS. 4, 5 AND TABLE 3

Figure 2: OPAL PAIRS CUTS, BHABHA

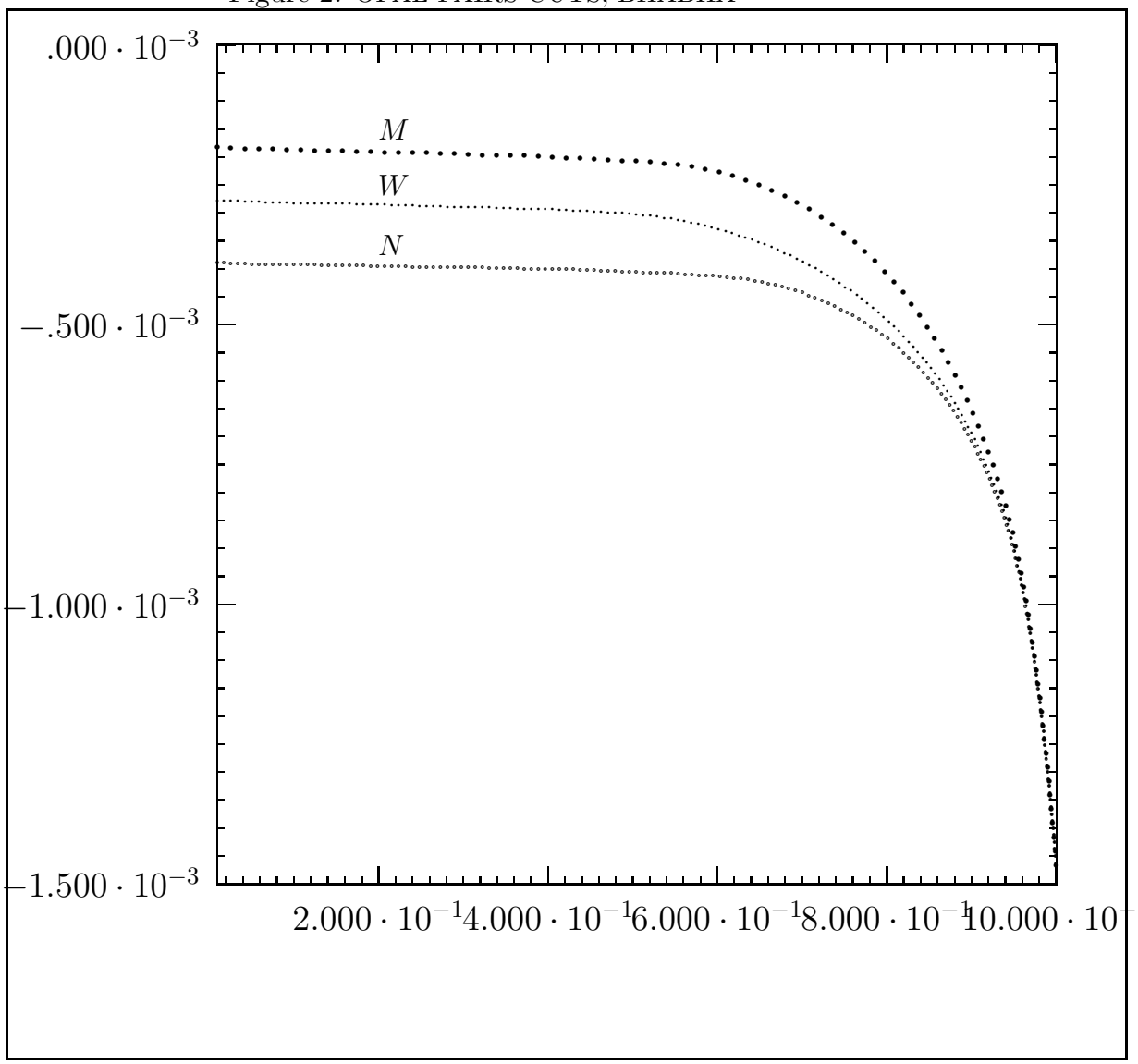


Figure 3: OPAL PAIRS CUTS, BHABHA

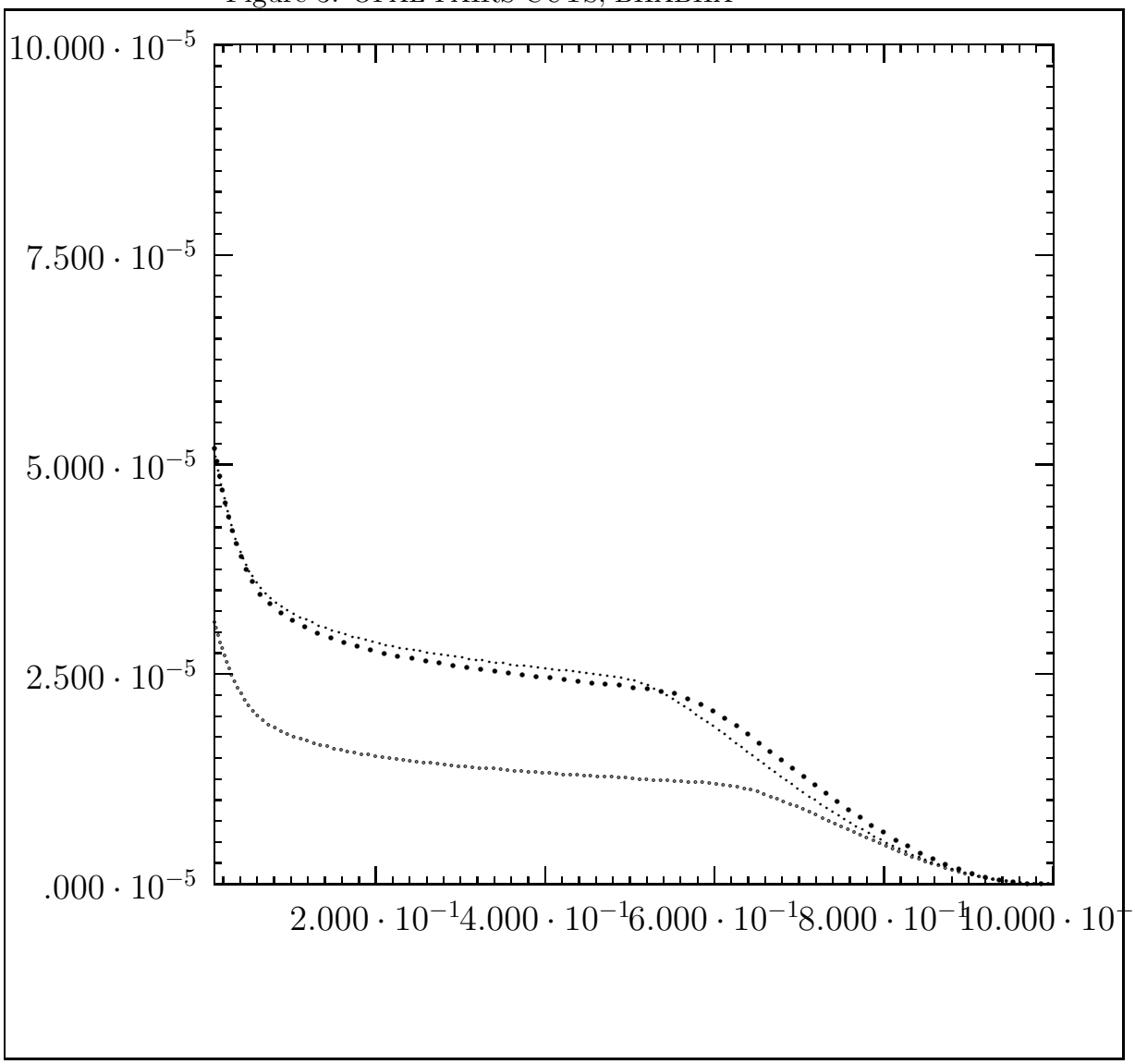


Table 3: $\sigma^S_{(3)}(M_Z \pm \Gamma/2)/\sigma^S_{(3)}(M_Z)$

No.	x_{cut}, W,M,N	$M_Z + \Gamma/2$	$M_Z - \Gamma/2$
1.	.2000000	.9771586	1.0236895
2.	.2000000	.9771867	1.0236597
3.	.2000000	.9772594	1.0235824
4.	.5000000	.9768979	1.0239664
5.	.5000000	.9769224	1.0239404
6.	.5000000	.9769679	1.0238921
7.	.8000000	.9767037	1.0241727
8.	.8000000	.9766981	1.0241786
9.	.8000000	.9767109	1.0241651

BHWIDE1.0

★ RECENT PRECISION STUDIES FOR BHWIDE

- FOCUS ON LEP2
  (RETURN TO Z PEAK LATER)

- APPROACH:

  COMPARE EACH PART OF
  CALCULATION WITH RESULTS OF
  KNOWN PRECISION FROM OTHER SOURCES

  CHECK REDUCTION PROCEDURE
  SENSITIVITY

⇒ FOLLOWING RESULTS IN PROGRESS

Program	Type	Channels	Exchange	Best QED corr.	Vac. Pol.	EW corr.
BHWIDE	MCEG	$s+t$	$\gamma+Z$	$\mathcal{O}(\alpha^1)_{exp}$	Yes	Yes
BHLUMI	MCEG	t	γ	$\mathcal{O}(\alpha^1+\alpha^2 L_t^2)_{exp(t)}$ Models: A and B	Yes	No
OLDBIS	MCEG	$s+t$	γ	$\mathcal{O}(\alpha^1)$	No	No
ALIBABA	SA	$s+t$	$\gamma+Z$	$\mathcal{O}(\alpha^1+\alpha^2 L_s^2)_{exp(s)}$	Yes	Yes

Table 4: Short characteristic of programs used in tests. Notation: MCEG – Monte Carlo Event Generator, SA – Semi-Analytical program, $L_t = \ln(|t|/m_e^2)$, $L_s = \ln(s/m_e^2)$, $exp(t)$ and $exp(s)$ – t-channel- and s-channel-type exponentiation, resp.

Acceptance Regions:

1. $|\cos\theta_-| < 0.7, \quad \theta_{acol} < 10^\circ$
2. $|\cos\theta_-| < 0.9, \quad \theta_{acol} < 10^\circ$
3. $|\cos\theta_\pm| < 0.96, \; \theta_{acol} < 10^\circ$
4. $|\cos\theta_\pm| < 0.9, \quad \theta_{acol} < 170^\circ$

where $\theta_-(\theta_+)$ is the scattering angle of $e^-(e^+)$, and θ_{acol} is the acollinearity angle between the outgoing e^- and e^+.

t-channel γ-exchange $\mathcal{O}(\alpha)$ Up-Down Interference			
No.	Acceptance	$\sigma_{udi}^{\gamma(t)}$ [pb]	$\sigma_{udi}^{\gamma(t)}/\sigma_{Born}^{\gamma(t)}$ [%]
1.	$\lvert\cos\theta_-\rvert < 0.7, \quad \theta_{acol} < 10^\circ$	-0.045 ± 0.002	-0.15
2.	$\lvert\cos\theta_-\rvert < 0.9, \quad \theta_{acol} < 10^\circ$	$+0.059 \pm 0.003$	$+0.05$
3.	$\lvert\cos\theta_\pm\rvert < 0.96, \theta_{acol} < 10^\circ$	$+0.287 \pm 0.004$	$+0.09$
4.	$\lvert\cos\theta_\pm\rvert < 0.9, \quad \theta_{acol} < 170^\circ$	$+0.344 \pm 0.004$	$+0.29$

Table 5: The $\mathcal{O}(\alpha)$ up-down interference correction for the t-channel γ-exchange contribution to the large angle Bhabha (LABH) cross section at the CMS energy $E = 190$ GeV, obtained from OLDBIS and used to correct BHLUMI results.

Weak corrections from BHWIDE			
No.	Acceptance	σ_{weak} [pb]	$\sigma_{weak}/\sigma_{all}$ [%]
1.	$\lvert\cos\theta_-\rvert < 0.7, \quad \theta_{acol} < 10^\circ$	0.110 ± 0.007	0.55
2.	$\lvert\cos\theta_-\rvert < 0.9, \quad \theta_{acol} < 10^\circ$	0.202 ± 0.022	0.20
3.	$\lvert\cos\theta_\pm\rvert < 0.96, \theta_{acol} < 10^\circ$	0.260 ± 0.038	0.09
4.	$\lvert\cos\theta_\pm\rvert < 0.9, \quad \theta_{acol} < 170^\circ$	0.176 ± 0.023	0.16

Table 6: Results for the pure weak corrections to the LABH cross section at the CMS energy $E = 190$ GeV obtained from BHWIDE.

BHWIDE

No.	Acceptance	$Born^{\gamma+Z}_{s+t}$	$Born^{\gamma}_{t}$	$Born^{\gamma}_{s+t}$	$Born^{\gamma+Z}_{s+t}+\delta_{VP}$	$Born^{\gamma+Z}_{s+t}+\delta^{\mathcal{O}(\alpha)}_{QED}$
1.	$\lvert\cos\theta_-\rvert < 0.7,\ \ \theta_{acol} < 10^\circ$	$19.745 \pm .013$	$29.646 \pm .018$	$26.168 \pm .016$	$22.324 \pm .014$	$17.354 \pm .020$
2.	$\lvert\cos\theta_-\rvert < 0.9,\ \ \theta_{acol} < 10^\circ$	$98.207 \pm .034$	$118.762 \pm .039$	$109.307 \pm .037$	$109.921 \pm .038$	$91.477 \pm .050$
3.	$\lvert\cos\theta_\pm\rvert < 0.96,\ \theta_{acol} < 10^\circ$	$299.049 \pm .063$	$328.930 \pm .065$	$313.475 \pm .065$	$331.893 \pm .069$	$271.805 \pm .089$
4.	$\lvert\cos\theta_\pm\rvert < 0.9,\ \ \theta_{acol} < 170^\circ$	$98.207 \pm .034$	$118.762 \pm .039$	$109.307 \pm .037$	$109.921 \pm .038$	$96.618 \pm .053$

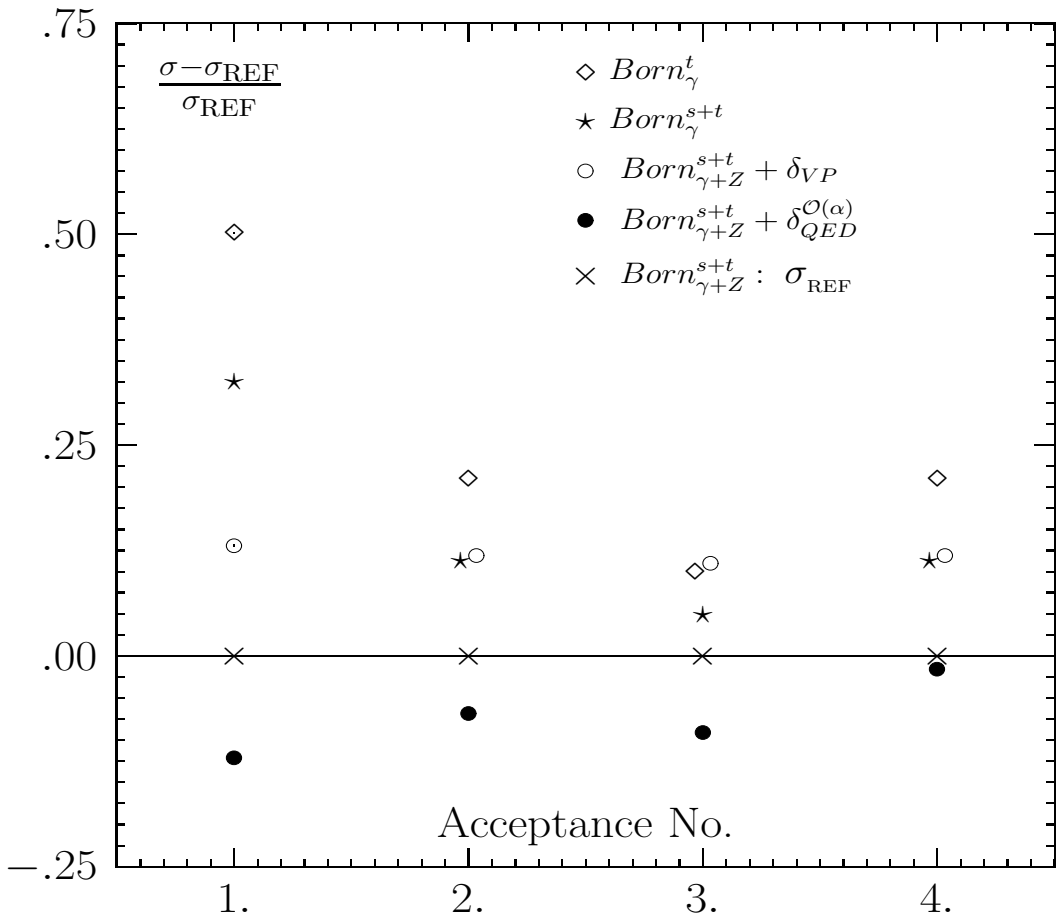


Figure 4: Results for various contributions to LABH at the CMS energy $E = 190$ GeV. Values in the table are the total cross sections [in pb] for four acceptance regions. The value $\sigma_{\rm REF}$ of $Born^{s+t}_{\gamma+Z}$ is used in the figure as a reference cross section.

t-channel γ exchange

No.	Acceptance	BHWIDE	BHLUMI-A	BHLUMI-B	ALIBABA	OLDBIS
	(a) $\mathcal{O}(\alpha)$ QED					
1.	$\lvert\cos\theta_-\rvert < 0.7,\ \ \theta_{acol} < 10^\circ$	$26.091 \pm .028$	$26.164 \pm .062$	$26.173 \pm .061$	$26.170 \pm .009$	$26.093 \pm .026$
2.	$\lvert\cos\theta_-\rvert < 0.9,\ \ \theta_{acol} < 10^\circ$	$110.383 \pm .058$	$110.413 \pm .129$	$110.435 \pm .126$	$110.445 \pm .068$	$110.377 \pm .052$
3.	$\lvert\cos\theta_\pm\rvert < 0.96,\ \theta_{acol} < 10^\circ$	$299.563 \pm .094$	$299.761 \pm .212$	$299.696 \pm .212$	$299.845 \pm .214$	$299.286 \pm .082$
4.	$\lvert\cos\theta_\pm\rvert < 0.9,\ \ \theta_{acol} < 170^\circ$	$116.462 \pm .061$	$116.450 \pm .129$	$116.513 \pm .129$	$116.294 \pm .057$	$116.468 \pm .052$
	(b) Higher-Order QED					
1.	$\lvert\cos\theta_-\rvert < 0.7,\ \ \theta_{acol} < 10^\circ$	$26.292 \pm .006$	$26.411 \pm .006$	$26.472 \pm .006$	$26.539 \pm .010$	—
2.	$\lvert\cos\theta_-\rvert < 0.9,\ \ \theta_{acol} < 10^\circ$	$110.522 \pm .014$	$110.990 \pm .014$	$111.114 \pm .014$	$111.605 \pm .058$	—
3.	$\lvert\cos\theta_\pm\rvert < 0.96,\ \theta_{acol} < 10^\circ$	$301.387 \pm .022$	$302.528 \pm .028$	$302.705 \pm .028$	$304.318 \pm .238$	—
4.	$\lvert\cos\theta_\pm\rvert < 0.9,\ \ \theta_{acol} < 170^\circ$	$116.897 \pm .014$	$117.958 \pm .023$	$118.189 \pm .024$	$118.284 \pm .050$	—

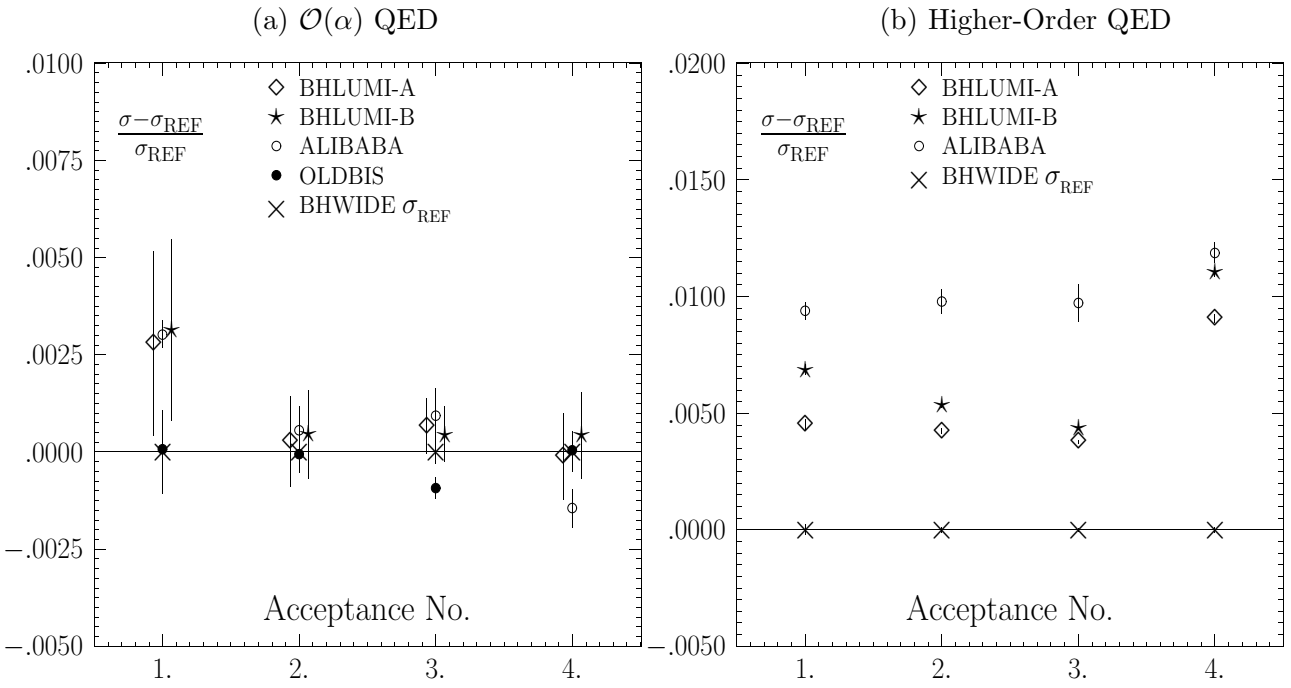


Figure 5: Results for the pure t-channel γ-exchange contribution to LABH at the CMS energy $E = 190$ GeV, including (a) $\mathcal{O}(\alpha)$ QED corrections, and (b) higher order QED corrections. Values in the table are the total cross sections [in pb] for four acceptance regions. The value $\sigma_{\rm REF}$ from BHWIDE is used in the figures as a reference cross section.

t-channel γ exchange

No.	$\cos\theta_-$	BHWIDE	BHLUMI-A	BHLUMI-B	OLDBIS
	(a) $\mathcal{O}(\alpha)$ QED				
1.	$[-0.9, -0,7]$	$1.960 \pm .017$	$2.002 \pm .033$	$2.002 \pm .032$	$1.951 \pm .016$
2.	$[-0.7, -0,5]$	$2.513 \pm .019$	$2.512 \pm .038$	$2.517 \pm .038$	$2.528 \pm .017$
3.	$[-0.5, -0,3]$	$3.449 \pm .022$	$3.429 \pm .046$	$3.435 \pm .045$	$3.401 \pm .020$
4.	$[-0.3, -0,1]$	$4.971 \pm .026$	$4.978 \pm .056$	$4.979 \pm .055$	$4.970 \pm .024$
5.	$[-0.1, \ 0,1]$	$7.769 \pm .033$	$7.824 \pm .072$	$7.824 \pm .071$	$7.746 \pm .030$
6.	$[\ 0.1, \ 0,3]$	$13.411 \pm .045$	$13.460 \pm .098$	$13.514 \pm .096$	$13.491 \pm .041$
7.	$[\ 0.3, \ 0,5]$	$27.048 \pm .064$	$27.160 \pm .142$	$27.144 \pm .139$	$26.949 \pm .059$
8.	$[\ 0.5, \ 0,7]$	$71.291 \pm .105$	$71.455 \pm .235$	$71.449 \pm .229$	$71.381 \pm .096$
9.	$[\ 0.7, \ 0,9]$	$419.520 \pm .251$	$419.147 \pm .562$	$419.306 \pm .548$	$419.469 \pm .227$
	(b) Higher-Order QED				
1.	$[-0.9, -0,7]$	$1.969 \pm .003$	$1.982 \pm .005$	$2.004 \pm .005$	—
2.	$[-0.7, -0,5]$	$2.537 \pm .004$	$2.555 \pm .005$	$2.577 \pm .005$	—
3.	$[-0.5, -0,3]$	$3.465 \pm .004$	$3.481 \pm .006$	$3.505 \pm .006$	—
4.	$[-0.3, -0,1]$	$5.035 \pm .005$	$5.049 \pm .006$	$5.074 \pm .006$	—
5.	$[-0.1, \ 0,1]$	$7.869 \pm .007$	$7.911 \pm .008$	$7.940 \pm .008$	—
6.	$[\ 0.1, \ 0,3]$	$13.585 \pm .010$	$13.668 \pm .009$	$13.709 \pm .009$	—
7.	$[\ 0.3, \ 0,5]$	$27.246 \pm .015$	$27.353 \pm .013$	$27.408 \pm .013$	—
8.	$[\ 0.5, \ 0,7]$	$71.722 \pm .025$	$72.026 \pm .024$	$72.140 \pm .023$	—
9.	$[\ 0.7, \ 0,9]$	$419.180 \pm .063$	$420.910 \pm .065$	$421.204 \pm .064$	—

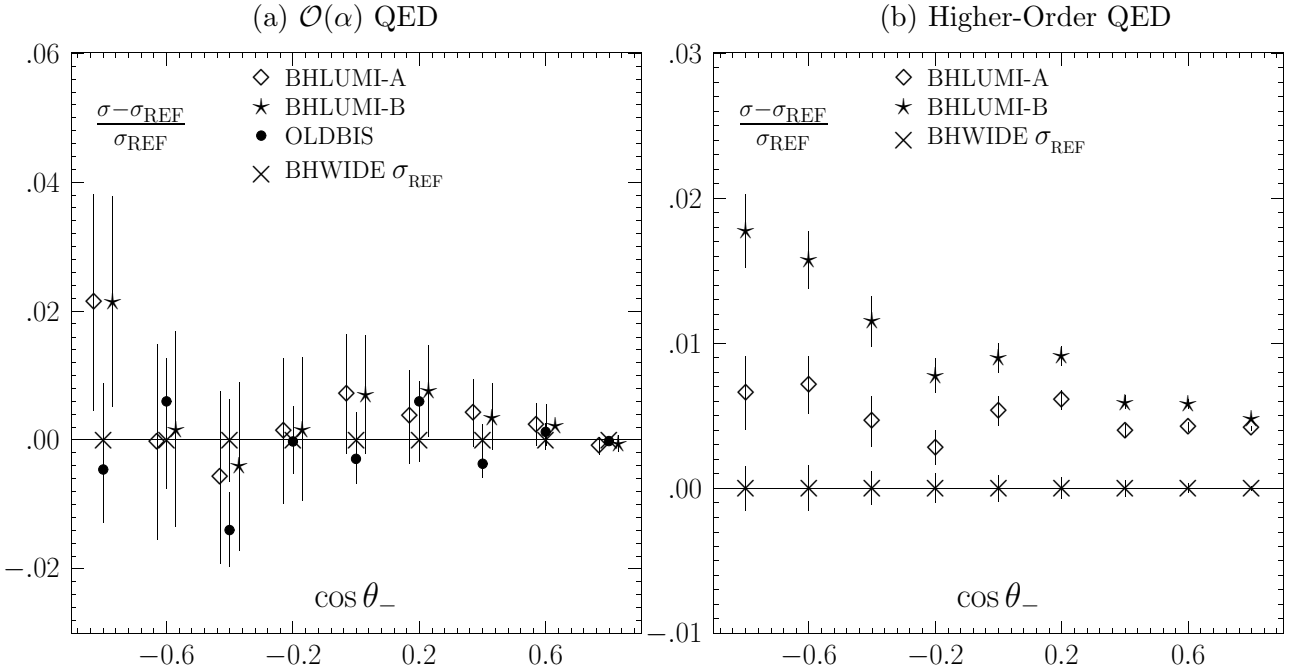


Figure 6: Results for the pure t-channel γ-exchange contribution to LABH at the CMS energy $E = 190$ GeV, including (a) $\mathcal{O}(\alpha)$ QED corrections, and (b) higher order QED corrections. Values in the table correspond to the differential cross section $d\sigma/dcos\theta_-$ [in pb] for the acceptance region no. 2. (i.e. $|\cos\theta_-| < 0.9$, $\theta_{acol} < 10^\circ$). The value $\sigma_{\rm REF}$ from BHWIDE is used in the figures as a reference cross section.

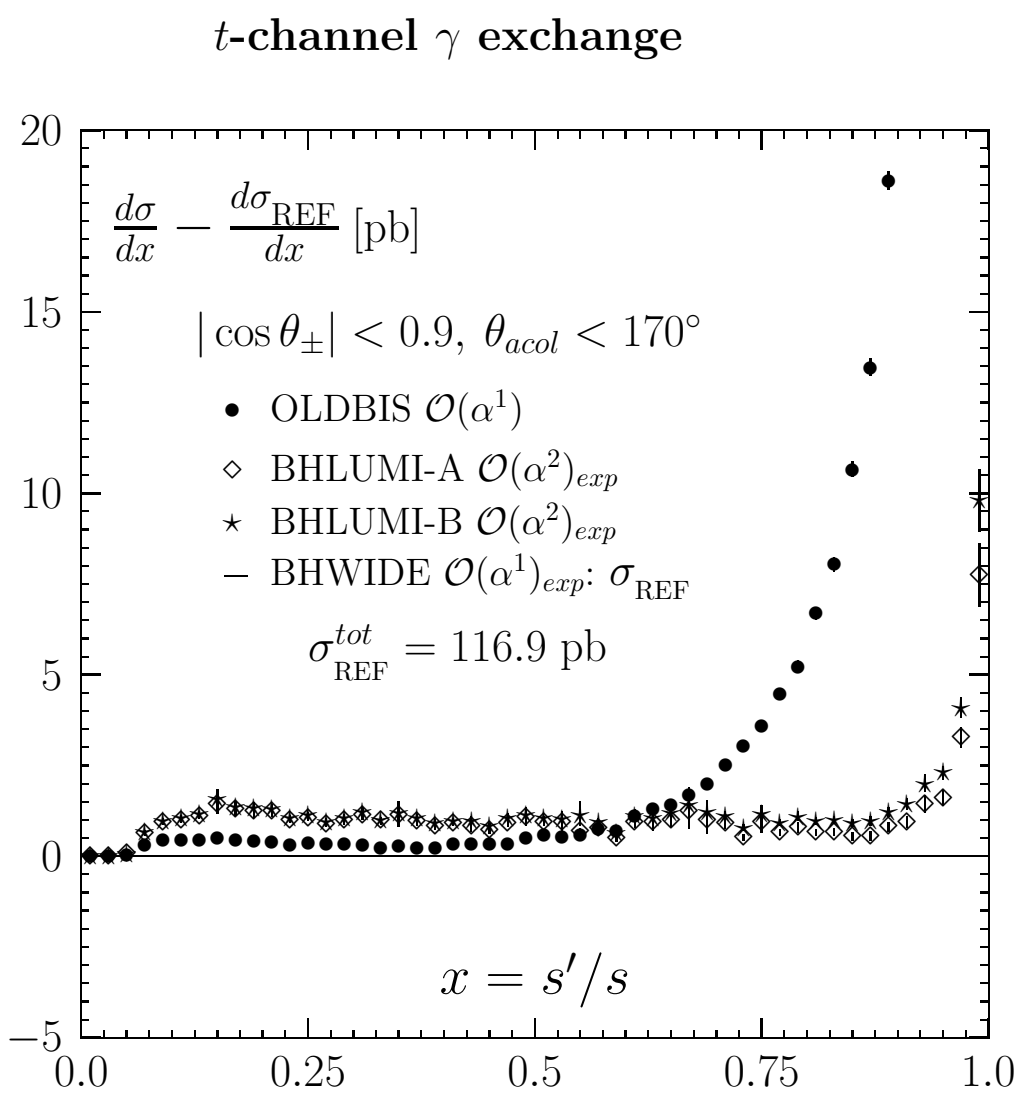


Figure 7: Results for the differential cross section $d\sigma/dx$ [pb], where $x = s'/s$, for the t-channel γ-exchange contribution to LABH at the CMS energy $E = 190$ GeV, for the acceptance region no. 4. (i.e. $|\cos\theta_\pm| < 0.9,\ \theta_{acol} < 170^\circ$).

t-channel γ exchange with vac. pol. correction

No.	Acceptance	BHWIDE	BHLUMI-A	BHLUMI-B
	(a) $\mathcal{O}(\alpha)$ QED			
1.	$\lvert\cos\theta_-\rvert < 0.7,\ \ \theta_{acol} < 10^\circ$	$29.580 \pm .033$	$29.699 \pm .088$	$29.630 \pm .074$
2.	$\lvert\cos\theta_-\rvert < 0.9,\ \ \theta_{acol} < 10^\circ$	$123.798 \pm .067$	$123.837 \pm .180$	$123.813 \pm .151$
3.	$\lvert\cos\theta_\pm\rvert < 0.96,\ \theta_{acol} < 10^\circ$	$333.051 \pm .108$	$333.300 \pm .293$	$333.295 \pm .246$
4.	$\lvert\cos\theta_\pm\rvert < 0.9,\ \ \theta_{acol} < 170^\circ$	$130.679 \pm .071$	$130.662 \pm .180$	$130.673 \pm .151$
	(b) Higher-Order QED			
1.	$\lvert\cos\theta_-\rvert < 0.7,\ \ \theta_{acol} < 10^\circ$	$29.798 \pm .008$	$29.929 \pm .008$	$30.007 \pm .007$
2.	$\lvert\cos\theta_-\rvert < 0.9,\ \ \theta_{acol} < 10^\circ$	$123.952 \pm .016$	$124.451 \pm .020$	$124.603 \pm .016$
3.	$\lvert\cos\theta_\pm\rvert < 0.96,\ \theta_{acol} < 10^\circ$	$335.059 \pm .025$	$336.291 \pm .042$	$336.485 \pm .032$
4.	$\lvert\cos\theta_\pm\rvert < 0.9,\ \ \theta_{acol} < 170^\circ$	$131.130 \pm .016$	$132.275 \pm .033$	$132.530 \pm .026$

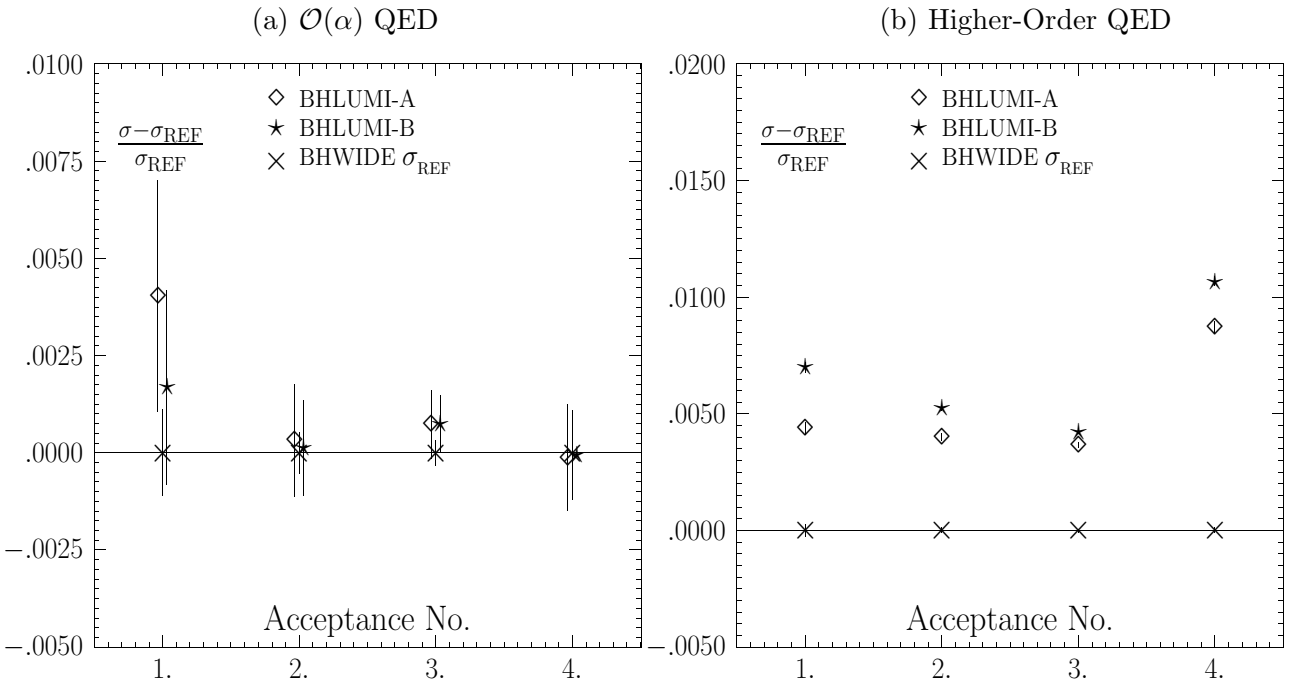


Figure 8: Results for the pure t-channel γ-exchange contribution to LABH including the vacuum polarization correction at the CMS energy $E = 190$ GeV, with (a) $\mathcal{O}(\alpha)$ QED corrections, and (b) higher order QED corrections. Values in the table are the total cross sections [in pb] for four acceptance regions. The value $\sigma_{\rm REF}$ from BHWIDE is used in the figures as a reference cross section.

$(s+t)$-channels γ exchange

No.	Acceptance	BHWIDE	ALIBABA	OLDBIS
	(a) $\mathcal{O}(\alpha)$ QED			
1.	$\lvert\cos\theta_-\rvert < 0.7,\ \ \theta_{acol} < 10^\circ$	$23.008 \pm .026$	$23.066 \pm .008$	$23.003 \pm .023$
2.	$\lvert\cos\theta_-\rvert < 0.9,\ \ \theta_{acol} < 10^\circ$	$101.586 \pm .055$	$101.561 \pm .063$	$101.576 \pm .048$
3.	$\lvert\cos\theta_\pm\rvert < 0.96,\ \theta_{acol} < 10^\circ$	$285.125 \pm .092$	$285.247 \pm .197$	$284.845 \pm .078$
4.	$\lvert\cos\theta_\pm\rvert < 0.9,\ \ \theta_{acol} < 170^\circ$	$106.795 \pm .058$	$106.678 \pm .052$	$106.795 \pm .048$
	(b) Higher-Order QED			
1.	$\lvert\cos\theta_-\rvert < 0.7,\ \ \theta_{acol} < 10^\circ$	$23.180 \pm .006$	$23.400 \pm .009$	—
2.	$\lvert\cos\theta_-\rvert < 0.9,\ \ \theta_{acol} < 10^\circ$	$101.720 \pm .013$	$102.730 \pm .063$	—
3.	$\lvert\cos\theta_\pm\rvert < 0.96,\ \theta_{acol} < 10^\circ$	$286.930 \pm .022$	$289.824 \pm .233$	—
4.	$\lvert\cos\theta_\pm\rvert < 0.9,\ \ \theta_{acol} < 170^\circ$	$107.249 \pm .014$	$108.484 \pm .050$	—

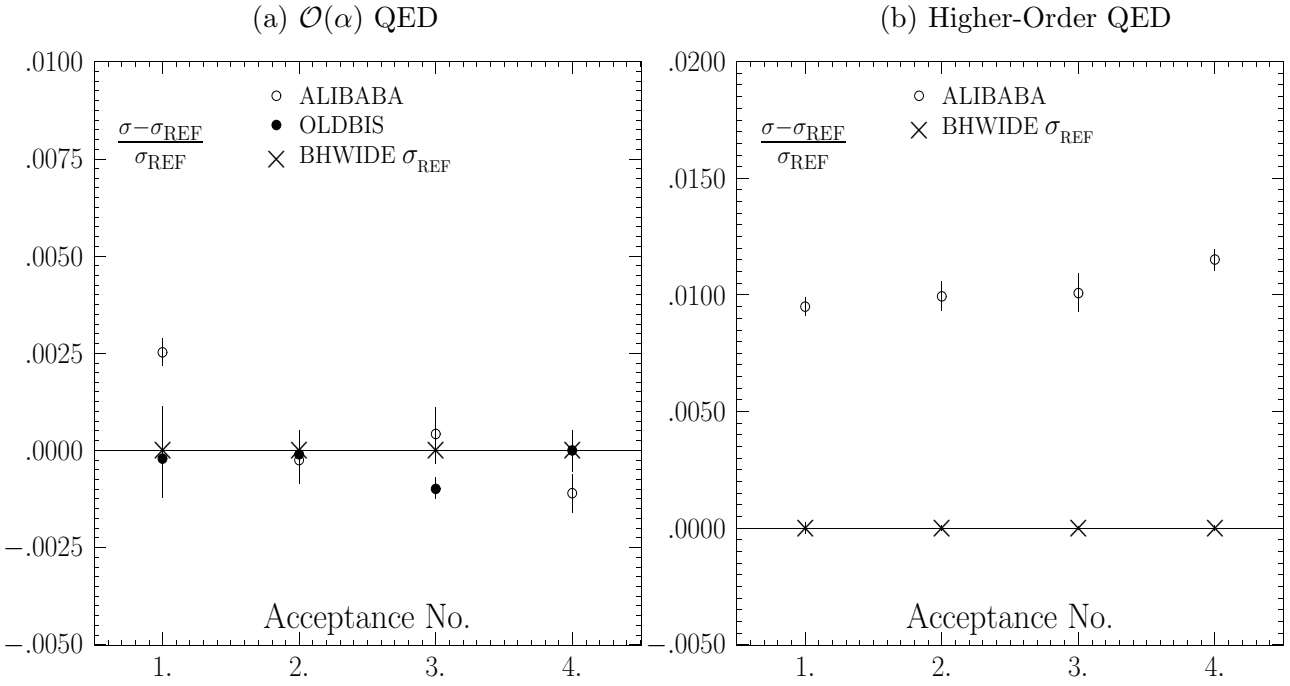


Figure 9: Results for the $(s+t)$-channels γ-exchange contribution to LABH at the CMS energy $E = 190$ GeV, including (a) $\mathcal{O}(\alpha)$ QED corrections, and (b) higher order QED corrections. Values in the table are the total cross sections [in pb] for four acceptance regions. The value $\sigma_{\rm REF}$ from BHWIDE is used in the figures as a reference cross section.

$(s+t)$-channels $(\gamma+Z)$ exchange

No.	Acceptance	BHWIDE	ALIBABA
	(a) $\mathcal{O}(\alpha)$ QED		
1.	$\lvert\cos\theta_-\rvert < 0.7,\ \ \theta_{acol} < 10^\circ$	$17.3540 \pm .020$	$17.4020 \pm .007$
2.	$\lvert\cos\theta_-\rvert < 0.9,\ \ \theta_{acol} < 10^\circ$	$91.4770 \pm .050$	$91.5680 \pm .072$
3.	$\lvert\cos\theta_\pm\rvert < 0.96,\ \theta_{acol} < 10^\circ$	$271.8050 \pm .089$	$272.1190 \pm .202$
4.	$\lvert\cos\theta_\pm\rvert < 0.9,\ \ \theta_{acol} < 170^\circ$	$96.6180 \pm .053$	$96.4120 \pm .049$
	(b) Higher-Order QED		
1.	$\lvert\cos\theta_-\rvert < 0.7,\ \ \theta_{acol} < 10^\circ$	$17.5120 \pm .005$	$17.6860 \pm .007$
2.	$\lvert\cos\theta_-\rvert < 0.9,\ \ \theta_{acol} < 10^\circ$	$91.6100 \pm .012$	$92.5120 \pm .067$
3.	$\lvert\cos\theta_\pm\rvert < 0.96,\ \theta_{acol} < 10^\circ$	$273.6260 \pm .021$	$276.2440 \pm .231$
4.	$\lvert\cos\theta_\pm\rvert < 0.9,\ \ \theta_{acol} < 170^\circ$	$97.1500 \pm .013$	$98.2410 \pm .049$

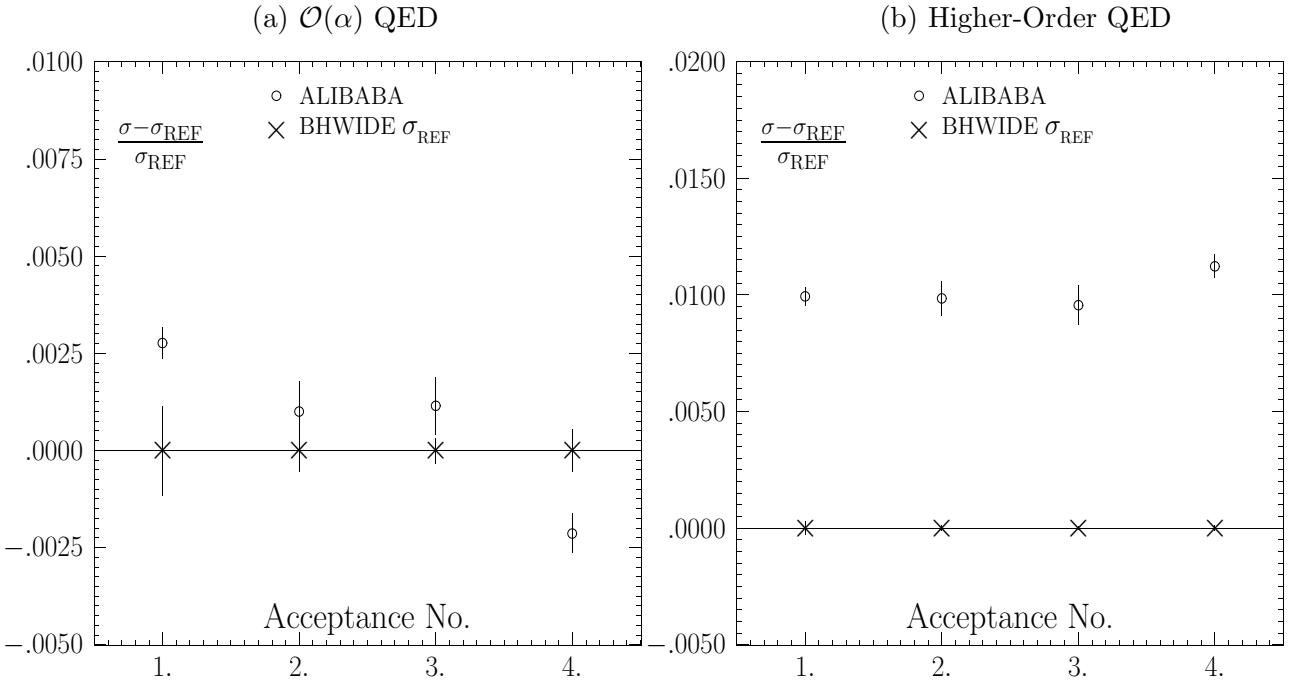


Figure 10: Results for the $(s+t)$-channels $(\gamma+Z)$-exchange contribution to LABH at the CMS energy $E = 190$ GeV, including (a) $\mathcal{O}(\alpha)$ QED corrections, and (b) higher order QED corrections. Values in the table are the total cross sections [in pb] for four acceptance regions. The value $\sigma_{\rm REF}$ from BHWIDE is used in the figures as a reference cross section.

All contributions + VP & EW corrections

No.	Acceptance	BHWIDE	ALIBABA
	(a) $\mathcal{O}(\alpha)$ QED		
1.	$\lvert\cos\theta_-\rvert < 0.7,\ \ \theta_{acol} < 10^\circ$	$19.7240 \pm .025$	$19.7870 \pm .007$
2.	$\lvert\cos\theta_-\rvert < 0.9,\ \ \theta_{acol} < 10^\circ$	$102.4910 \pm .062$	$102.4660 \pm .072$
3.	$\lvert\cos\theta_\pm\rvert < 0.96,\ \theta_{acol} < 10^\circ$	$301.8240 \pm .109$	$302.3520 \pm .203$
4.	$\lvert\cos\theta_\pm\rvert < 0.9,\ \ \theta_{acol} < 170^\circ$	$108.3110 \pm .066$	$108.0890 \pm .050$
	(b) Higher-Order QED		
1.	$\lvert\cos\theta_-\rvert < 0.7,\ \ \theta_{acol} < 10^\circ$	$19.9030 \pm .006$	$20.0560 \pm .008$
2.	$\lvert\cos\theta_-\rvert < 0.9,\ \ \theta_{acol} < 10^\circ$	$102.6590 \pm .015$	$103.3430 \pm .067$
3.	$\lvert\cos\theta_\pm\rvert < 0.96,\ \theta_{acol} < 10^\circ$	$303.8340 \pm .026$	$306.4480 \pm .231$
4.	$\lvert\cos\theta_\pm\rvert < 0.9,\ \ \theta_{acol} < 170^\circ$	$108.8670 \pm .016$	$109.9230 \pm .049$

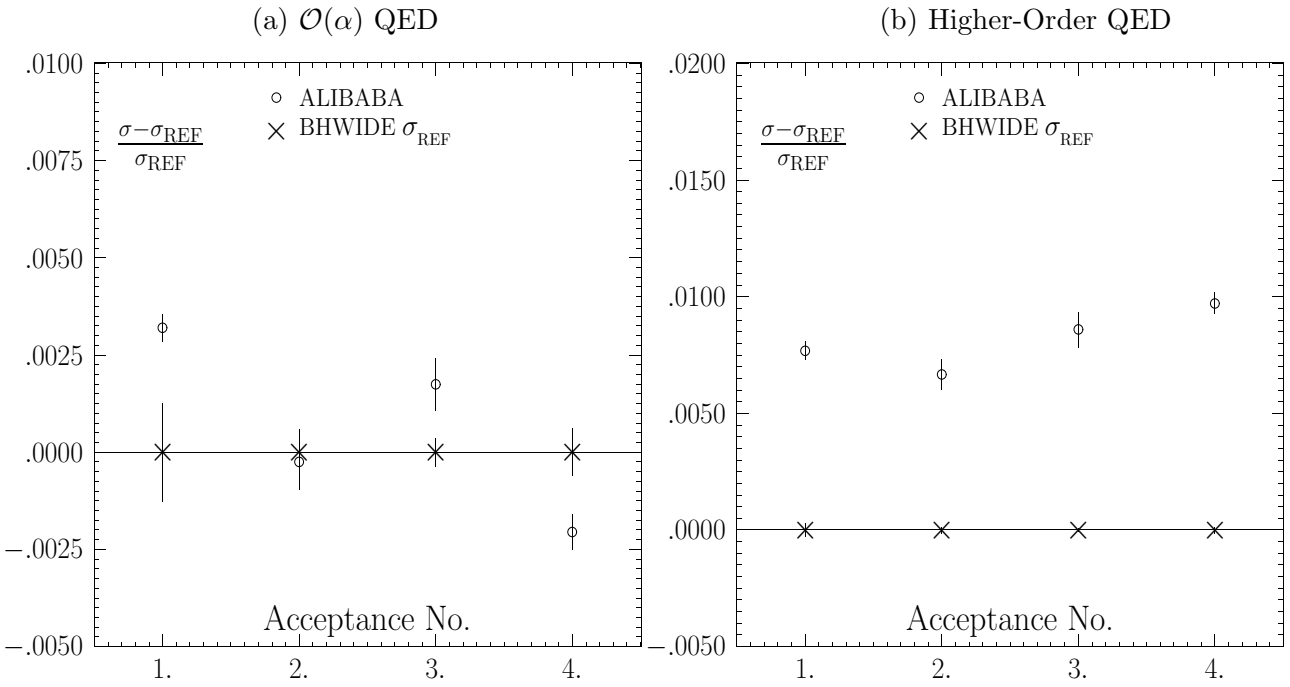


Figure 11: Results for all contributions to LABH, including VP and EW corrections, at the CMS energy $E = 190$ GeV, with (a) $\mathcal{O}(\alpha)$ QED corrections, and (b) higher order QED corrections. Values in the table are the total cross sections [in pb] for four acceptance regions. The value $\sigma_{\rm REF}$ from BHWIDE is used in the figures as a reference cross section.

t-channel γ exchange
(BHWIDE with New Reduction Procedure)

No.	Acceptance	BHWIDE-NR	BHLUMI-A	BHLUMI-B	ALIBABA	OLDBIS
	(a) $\mathcal{O}(\alpha)$ QED					
1.	$\lvert\cos\theta_-\rvert < 0.7,\ \ \theta_{acol} < 10^\circ$	$26.116 \pm .033$	$26.164 \pm .062$	$26.173 \pm .061$	$26.170 \pm .009$	$26.093 \pm .026$
2.	$\lvert\cos\theta_-\rvert < 0.9,\ \ \theta_{acol} < 10^\circ$	$110.375 \pm .068$	$110.413 \pm .129$	$110.435 \pm .126$	$110.445 \pm .068$	$110.377 \pm .052$
3.	$\lvert\cos\theta_\pm\rvert < 0.96, \theta_{acol} < 10^\circ$	$299.557 \pm .110$	$299.761 \pm .212$	$299.696 \pm .212$	$299.845 \pm .214$	$299.286 \pm .082$
4.	$\lvert\cos\theta_\pm\rvert < 0.9,\ \ \theta_{acol} < 170^\circ$	$116.453 \pm .073$	$116.450 \pm .129$	$116.513 \pm .129$	$116.294 \pm .057$	$116.468 \pm .052$
	(b) Higher-Order QED					
1.	$\lvert\cos\theta_-\rvert < 0.7,\ \ \theta_{acol} < 10^\circ$	$26.366 \pm .008$	$26.411 \pm .006$	$26.472 \pm .006$	$26.539 \pm .010$	—
2.	$\lvert\cos\theta_-\rvert < 0.9,\ \ \theta_{acol} < 10^\circ$	$110.853 \pm .016$	$110.990 \pm .014$	$111.114 \pm .014$	$111.605 \pm .058$	—
3.	$\lvert\cos\theta_\pm\rvert < 0.96, \theta_{acol} < 10^\circ$	$302.268 \pm .026$	$302.528 \pm .028$	$302.705 \pm .028$	$304.318 \pm .238$	—
4.	$\lvert\cos\theta_\pm\rvert < 0.9,\ \ \theta_{acol} < 170^\circ$	$117.385 \pm .017$	$117.958 \pm .023$	$118.189 \pm .024$	$118.284 \pm .050$	—

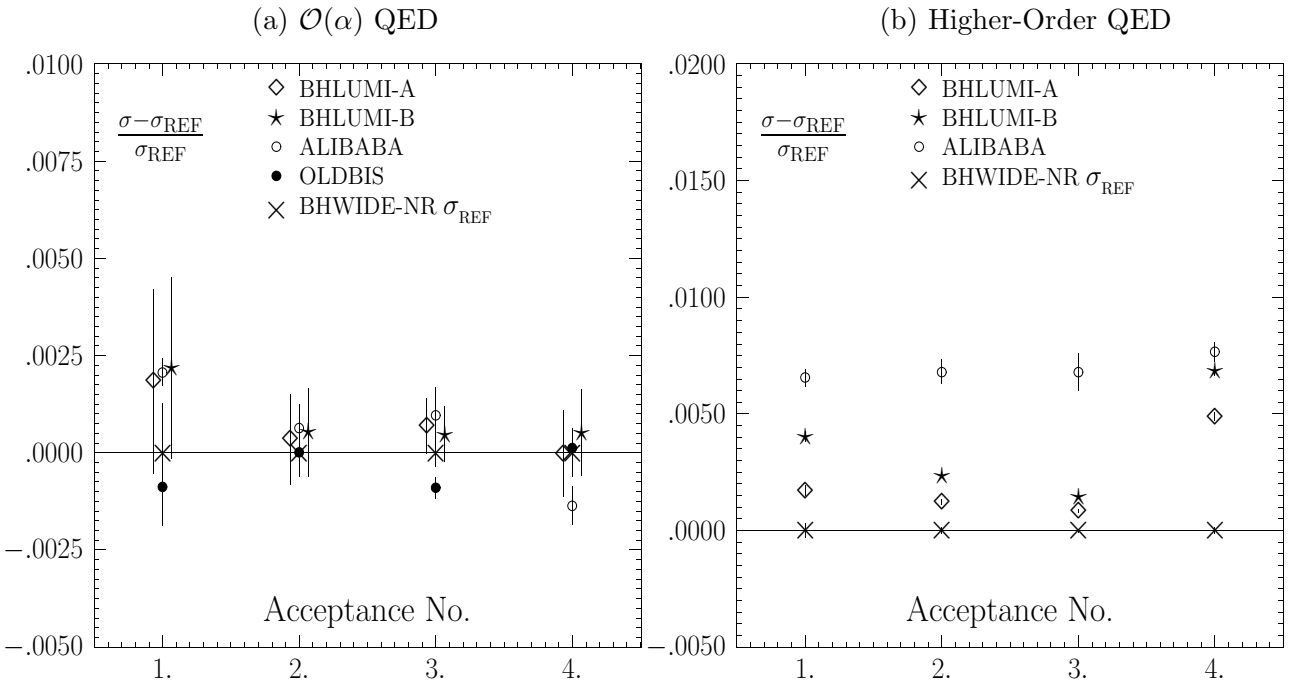


Figure 12: Results for the pure t-channel γ-exchange contribution to LABH at the CMS energy $E = 190$ GeV, including (a) $\mathcal{O}(\alpha)$ QED corrections, and (b) higher order QED corrections. Values in the table are the total cross sections [in pb] for four acceptance regions. The value $\sigma_{\rm REF}$ from BHWIDE is used in the figures as a reference cross section.

CONCLUSIONS

- NEW ANALYSIS OF ERRORS FOR BHLUMI4.04

  $\mathcal{O}(\alpha^2)$ PHOTONIC ERROR REDUCED TO 0.027% AT LEP1

  $\Rightarrow \frac{\Delta\sigma_{\mathcal{L}}}{\sigma_{\mathcal{L}}}|_{TH} = 0.061\%$, LEP1

- FOR LEP2, UP TO 200GeV,

  $\frac{\Delta\sigma_{\mathcal{L}}}{\sigma_{\mathcal{L}}}|_{TH} = 0.122\%$

- PREDICTIONS OF BHLUMI4.04 UNCHANGED

- IF PAIRS EFFECT IS SIMULATED
  (VIA MONTAGNA ET AL.,hep-ph/9811436,
  BHPAIR/BHLUMI2.30, PHYS.REV.**D55**,1206(1997),
  MERENKOV ET AL., ACTA PHYS. POL.**B28**,
  491 (1997), ETC.)

  $\frac{\Delta\sigma_{\mathcal{L}}}{\sigma_{\mathcal{L}}}|_{TH-PAIRS} \rightarrow 0.01\%(0.015\%)$ FOR LEP1(LEP2)

  $\Rightarrow \frac{\Delta\sigma_{\mathcal{L}}}{\sigma_{\mathcal{L}}}|_{TH} \rightarrow 0.054\%(0.113\%)$ FOR LEP1(LEP2)

- NEW ERROR ANALYSIS FOR BHWIDE

  RESULTS IN PROGRESS: WAY TO 0.5%
  PREC. AT LEP2 OPEN