Shadows of the Renormalization Tree

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LoopFest VI

Based on work done in collaboration with Stefano Actis, Christian Sturm and Sandro Uccirati

Outlines

(1 , 2)

1. *The standard model at two loop level*

   A probable decision about its truth is possible inductively by studying its success (verifiable consequences)

2. *Complex poles*

   A prospective case study, per aspera ad astra

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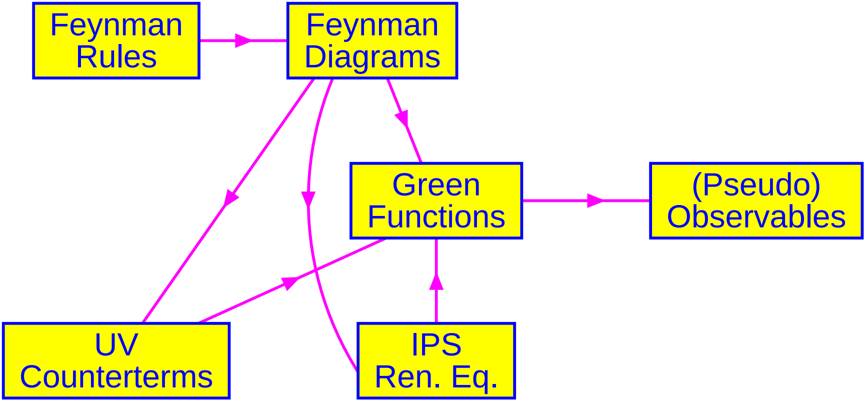
Part I

Renormalization Tree: embedded case study

flow-chart


Feynman Rules
Feynman Diagrams
Green Functions
(Pseudo) Observables
UV Counterterms
IPS Ren. Eq.

Universal UV decomposition: I

One-loop integrals

Any one-loop integral f^1 can be decomposed as

$$f^1(\{l\}) = \sum_{k=-1}^{1} f^1(\{l\};k)\, F^1_k(x),$$

where $\{l\} \equiv$ external kinematical variables, masses of internal particles. x is some kinematical variable and the dependence on the dimensional regulator ϵ is entirely transferred to the universal UV factors,

$$F^1_{-1}(x) = \frac{1}{\epsilon} - \frac{1}{2}\Delta_{UV}(x) + \frac{1}{8}\Delta^2_{UV}(x)\,\epsilon,$$

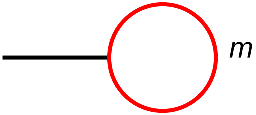
$$F^1_0(x) = 1 - \frac{1}{2}\Delta_{UV}(x)\,\epsilon, \qquad F^1_1(x) = \epsilon.$$

Universal UV decomposition: II

One-loop integrals

Because of overlapping divergencies we include $\mathcal{O}(\epsilon)$ terms in all one-loop results.

$$\Delta_{UV} = \gamma + \ln\pi + \ln\frac{M^2}{\mu^2}, \qquad \Delta_{UV}(x) = \Delta_{UV} - \ln\frac{M^2}{x},$$



$$= -2\,F^{1}_{-1}(M^2) + \left(\ln\frac{m^2}{M^2} - 1\right) F^{1}_{0}(M^2) + \cdots$$

Universal UV decomposition: III

Two-loop integrals

A generic two-loop integral f^2 can be written as

$$f^2(\{l\}) = \sum_{k=-2}^{0} f^2(\{l\}; k)\, F^2_k(x).$$

Here the two-loop UV factors read as follows:

$$F^2_{-2}(x) = \frac{1}{\epsilon^2} - \frac{\Delta_{UV}(x)}{\epsilon} + \frac{1}{2}\Delta^2_{UV}(x),$$

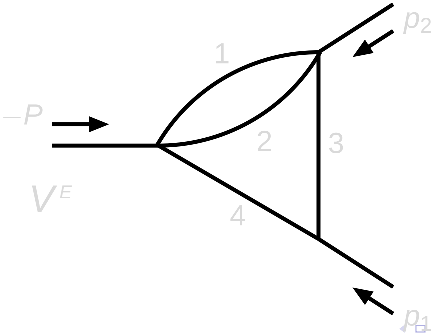
$$F^2_{-1}(x) = \frac{1}{\epsilon} - \Delta_{UV}(x), \qquad F^2_0(x) = 1.$$

Note that the product of two one-loop integrals can be written through the same UV decomposition of a two-loop integral.

Universal UV decomposition: IV

Example

$$v_0^E(\cdots;-2) = -2,$$
$$v_0^E(p_2,P,\{m\}_{1234};-1) = -2\,b_0(1,1,p_1,\{m\}_{34};0)-1.$$



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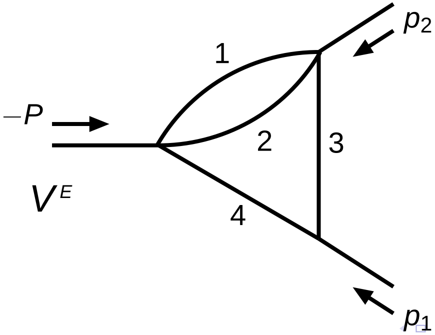


Multiplicative renormalization

c.t. not needed but useful

$$Z_i = 1 + \sum_{n=1}^{\infty} \left(\frac{g_R^2}{16\,\pi^2} \right)^n \delta Z_i^{(n)},$$

Example

masses, parameters

$$\begin{aligned} m &= Z_m^{1/2}\, m_R, \\ p &= Z_p\, p_R, \quad p = g, c_\theta, s_\theta \end{aligned}$$

Example

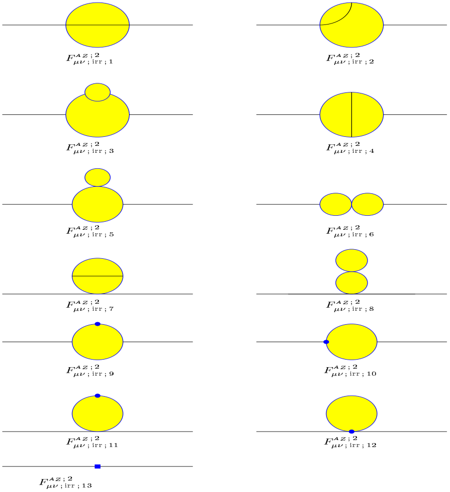
Fields, gauge parameters

$$\begin{aligned} Z_{\xi_{AZ}} &= \sum_{n=1}^{\infty} \left(\frac{g_R^2}{16\,\pi^2} \right)^n \delta Z_{\xi_{AZ}}^{(n)} \\ \phi &= Z_\phi^{1/2}\, \phi_R \quad \psi^{L,R} = Z_{\psi_{L,R}}^{1/2}\, \psi_R^{L,R} \\ A^\mu &= Z_{AA}^{1/2}\, A_R^\mu + Z_{AZ}^{1/2}\, Z_R^\mu \\ Z_{AZ}^{1/2} &= \sum_{n=1}^{\infty} \left(\frac{g_R^2}{16\,\pi^2} \right)^n \delta Z_{AZ}^{(n)} \end{aligned}$$

FP ghost fields are not renormalized

Renormalization constants

$G^{AZ;2}_{\mu\nu;\mathrm{irr}} =$



$\overline{MS}$ and beyond I

$\overline{NMS}$ scheme: advancing renormalization theory

In the spirit of the UV decomposition we define a non-minimal ($\overline{NMS}$) subtraction scheme where

Definition

$$\texttt{1 loop} \quad \to \quad \delta Z_i^{(1)} = \Delta Z_i^{(1)} F_{-1}^{1}(M_R^2),$$

$$\texttt{2 loops} \quad \to \quad \delta Z_i^{(2)} = \sum_{k=-2}^{-1} \Delta Z_{i\,;\,k}^{(2)} F_k^2(M_R^2),$$

$\overline{MS}$ and beyond II

counterterms

c.t. are fixed in order to remove order-by-order the poles at $\epsilon = 0$ for any Green function

Property of $\overline{NMS}$

The product of a one-loop c.t. with a one-loop diagram (i.e., a one-loop c.t. insertion) has the same UV decomposition of a two-loop function thus simplifying two-loop renormalized Green functions

The $\overline{NMS}$ scheme has the virtue of respecting the universal UV decomposition

The two facets of renormalization

Step 1

promote bare quantities p to renormalized ones p_R

Step 2

- fix the c.t. at 1 L $\equiv$ to remove the UV poles from all 1 L GF;
- check that 2 L GF develop local UV residues;
- fix the 2 L c.t. to remove 2 L local UV poles.

Finite renomalization

the absorption of UV poles into local c.t. does not exhaust the procedure; we have to connect p_R to POs, thus making the theory predictive.

The two facets of renormalization

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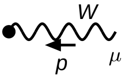
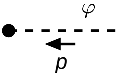
WST identities



$-(2\pi)^4 i \; i p_\mu Z_W^{1/2} Z_{\xi_W}^{-1}$



$(2\pi)^4 i \; M Z_\varphi^{1/2} Z_M^{1/2} Z_{\xi_\varphi}$

Figure: Sources related to the gauge-fixing functions $\mathcal{C}^\pm$

WST identities

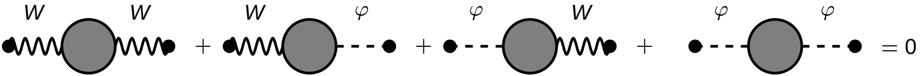


Figure: Doubly-contracted WST identity with two external $\mathcal{C}^{\pm}$ sources

Summary

New

$SU(3) \otimes SU(2) \otimes U(1)$

We have been able to verify that the SM can be made (two-loop) UV finite by adding *local* c.t.

Generalization of $1\,L$

The well-known one-loop result that self-energies suffice in performing renormalization can be extended up to two loops.

Overlapping divergencies

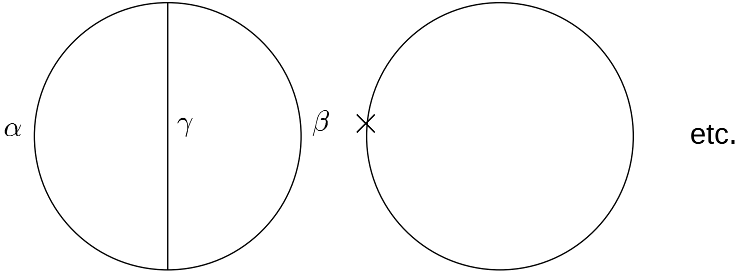


Figure: The arbitrary two-loop diagram $G_L^{\alpha\beta\gamma}$ and one of the associated subtraction sub-diagrams. Only in the sum we have cancellation of non-local residues

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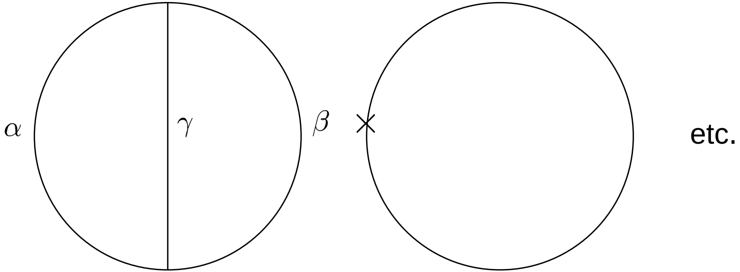

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Example: α

Renormalizing $g\, s_\theta$

In extracting α from Thomson scattering at zero momentum transfer we find four classes of two-loop diagrams:

(1, 2, 3, 4)

1. *irreducible two-loop vertices and wave-function factors, product of one-loop corrected vertices with one-loop wave-function factors;*
2. *one-loop vacuum polarization $\otimes$ one-loop vertices or one-loop wave-function factors;*
3. *irreducible two-loop AA, AZ, $A\phi^0$ transitions;*
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Consistency

A matter of life $\cdots$

You can't prove something by assuming it's true

We have verified that the non-vanishing contribution originates from III and IV only and, within these terms, only the reducible and irreducible AA transition survives.

Bluntly:

We have proven that the SM it's only slightly more complex than QED.

Example: G_F

Renormalizing g, M

In extracting the Fermi coupling constant from the muon lifetime all corrections to

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8\,M^2}\,(1+\Delta g)$$

which do not originate from the W self-energy and that are UV (and IR) finite at one loop remain finite at two loops after one-loop renormalization (i.e. two-loop counterterms are not needed)

A sample of $\overline{MS}$ counterterms: I

an $\overline{MS}$ example

$$\delta Z_i^{(2)} = \left(\frac{1}{\epsilon} - \Delta_{UV}\right)\left(\frac{\Delta Z_{i;1}^{(2)}}{\epsilon} + \Delta Z_{i;2}^{(2)}\right) + \Delta_{UV}^2\,\Delta Z_{i;3}^{(2)}.$$

Higgs field renormalization

$$\Delta Z_{H\,;\,1}^{(2)} = 4 + \frac{43}{4}\frac{1}{c_\theta^4} + \frac{5}{4}\frac{x_t}{c_\theta^2} - \frac{37}{2}\frac{1}{c_\theta^2} - \frac{7}{2}x_t - \frac{9}{4}x_t^2 + 24x_t\frac{g_S^2}{g^2},$$

A sample of $\overline{MS}$ counterterms: II

Higgs field renormalization

$$\Delta Z^{(2)}_{H\,;\,2} = \frac{7}{6} - \frac{431}{96}\frac{1}{c_\theta^4} - \frac{85}{48}\frac{x_t}{c_\theta^2} + \frac{101}{12}\frac{1}{c_\theta^2} - \frac{25}{24}x_t + \frac{27}{16}x_t^2 - \frac{3}{32}x_H^2 - 10x_t\frac{g_s^2}{g^2},$$

$$\Delta Z^{(2)}_{H\,;\,3} = -2 + \frac{43}{16}\frac{1}{c_\theta^4} + \frac{5}{16}\frac{x_t}{c_\theta^2} - \frac{37}{8}\frac{1}{c_\theta^2} - \frac{7}{8}x_t - \frac{9}{16}x_t^2 + 6x_t\frac{g_s^2}{g^2}.$$

γ^5 in a nutshell

HVBM scheme

The HVBM scheme breaks all WST identities (so-called spurious or avoidable violations) which can be restored afterwards by introducing suitable ultraviolet finite counterterms. The procedure, however is lengthy and cumbersome.

pseudo-regularization of chiral theories

After considerable wrangling one is lead to the conclusion that the only sensible solution is the one proposed by Jegerlehner: n-dimensional γ-algebra with strictly anti-commuting γ^5 together with 4-dimensional treatment of the hard anomalies.

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About gauge independence I

Problem

$$\Sigma_{VV}(s,\xi) \quad = \quad \sum_{n=1}^{\infty} \Sigma_{VV}^{(n)}(s,\xi)\, g^{2n}$$

On mass shell

$$\Sigma_{VV}^{(1)}(s,\xi) = \Sigma_{VV\,;\,I}^{(1)}(s) + \quad (s - M_V^2)\,\Phi_{VV}(s,\xi)$$

Nielsen identity

$$\frac{\partial}{\partial\xi}\,\Sigma_{VV}(s_P,\xi) = 0$$

$$s_P - M_V^2 + \Sigma_{VV}(s_P) = 0$$

Decomposition

$$\Sigma_{VV}^{(n)}(s,\xi) = \Sigma_{VV\,;\,I}^{(n)}(s) + \quad \Sigma_{VV\,;\,\xi}^{(n)}(s,\xi)$$

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About gauge independence II

Solution

$$\Sigma^{(n)}_{VV\,;\,\xi}(s_P,\xi) \;=\; \Sigma^{(n-1)}_{VV\,;\,I}(s_P)\,\Phi_{VV}(s_P,\xi),$$

Theorem

As a result we can prove that all ξ-dependent parts cancel,

$$\Sigma_{VV}(s_P) \;=\; \sum_{n=1}^{\infty}\Sigma^{(n)}_{VV\,;\,I}(s_P)\,g^{2n}.$$

However, this example shows how an all-order relation should be carefully interpreted while working at some fixed order.

Renormalization equations

From ren. parameters to an IPS

$$G\left[M^2 - \frac{g^2}{16\,\pi^2}\,F_W(0)\right] = \frac{g^2}{8}$$

$$4\,\pi\,\alpha\left[1 - \frac{g^2}{16\,\pi^2}\,\Pi_{QQ}(0)\right] = g^2 s_\theta^2$$

Warning

One of our renormalization equations will always be of the type

$$s_V = M_V^2 - \frac{g^2}{16\,\pi^2}\,\Sigma_{VV}(s_V, M_V^2).$$

Options

(1, 2, 3,)

1. *At* $\mathcal{O}\left(g^4\right)$ *in* Σ_{VV} *we keep* $p^2 = -s_V$ *with* $\mathcal{O}\left(g^6\right)$ *violation;*
2. *we replace* Σ_{VV} *with* $\Sigma_{VV\,;\,I}$

$$\Sigma^{(2)}_{VV\,;\,I}(s_V) \;=\; \Sigma^{(2)}_{VV}(s_V,1) - \Sigma^{(1)}_{VV\,;\,I}(s_V)\,\Phi_{VV}(s_V,1).$$

3. *we expand,*

$$\begin{aligned} s_V \;=\;& M_V^2 - \frac{g^2}{16\,\pi^2}\,\Sigma^{(1)}_{VV}(M_V^2,M_V^2) \\ &- \left(\frac{g^2}{16\,\pi^2}\right)^2 \Big[\Sigma^{(2)}_{VV}(M_V^2,M_V^2) \\ &- \Sigma^{(1)}_{VV}(M_V^2,M_V^2)\,\Sigma^{(1)}_{VV\,;\,p}(M_V^2,M_V^2)\Big] \end{aligned}$$

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Finite renormalization in running couplings

(intermediate) renormalized theory

$$\frac{1}{g^2(s)} = \frac{1}{g^2} - \frac{1}{16\,\pi^2}\,\Pi_{3Q}^{(1)}(s) - \frac{g^2}{(16\,\pi^2)^2}\,\Pi_{3Q}^{(2)}(s).$$

theory in terms of POs

$$\frac{1}{g^2(s)} = \frac{1}{8\,G\,\mu_W^2} - \frac{1}{16\,\pi^2\,\mu_W^2}\,\delta g^{(1)} - \frac{G}{32\,\pi^4}\,\delta g^{(2)},$$

$$\delta g^{(n)} = \mu_W^2\,\Pi_{3Q}^{(n)}(s) + \tilde{F}_W^{(n)}(s_W).$$

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Dressed propagators

From finite order

$$\Delta^{(n)}(p^2) \;=\; \Delta^{(0)}(p^2)\left[1-\Delta^{(0)}(p^2)\,\Sigma^{(n)}\left(p^2\,,\,\Delta^{(n-1)}(p^2)\right)\right]^{-1},$$

To all orders

$$\overline{\Delta}(p^2) \;=\; \lim_{n\to\infty}\Delta^{(n)}(p^2),$$

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$$\overline{\Delta}(p^2) \;=\; \Delta^{(0)}(p^2)\left[1-\Delta^{(0)}(p^2)\,\Sigma\left(p^2\,,\,\overline{\Delta}(p^2)\right)\right]^{-1},$$

Properties: I

all orders

We assume that a dressed propagator obeys Källen - Lehmann representation; using the relation

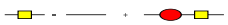
$$\begin{aligned} \mathrm{Im}\,\overline{\Delta}(p^2) &= \mathrm{Im}\,\Sigma \left[\left(p^2 + m^2 - \mathrm{Re}\,\Sigma \right)^2 + (\mathrm{Im}\,\Sigma)^2 \right]^{-1} \\ &= \pi\,\rho(-p^2), \end{aligned}$$

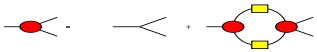
Källen - Lehmann

the Källen - Lehmann representation follows:

$$\overline{\Delta}(p^2) = \int_0^\infty ds\, \frac{\rho(s)}{p^2 + s - i\,\delta}.$$

Properties: II





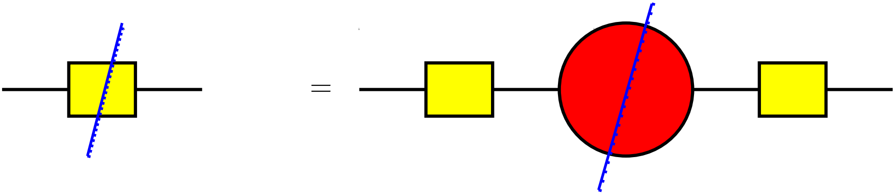


Figure: Cutting equation for a dressed propagator; the red circle is the (all-orders) cut self-energy.

Figure 1: Schwinger-Dyson equation for any permissible (standard model) dressed propagator (yellow box); the red oval is the SD self-energy; solid lines represent (without further distinction) any of the permissible fields of the standard model.

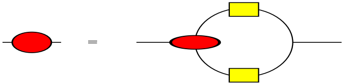


Figure 1: Schwinger - Dyson equation for the self-energy (l.h.s). In the r.h.s. the red oval is the SD vertex and the yellow box is the dressed propagator; solid lines represent (without further distinction) any of the permissible fields of the standard model.

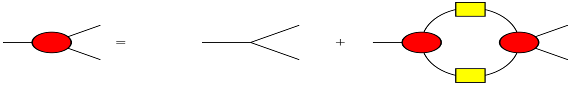


Figure 1: Schwinger-Dyson equation for a dressed vertex (l.h.s); in the r.h.s. we have SD three- and four-point vertices (red ovals) and dressed propagators (yellow boxes); solid lines represent (without further distinction) any of the fields of the standard model and vertices must be understood as any of the permissible standard model vertices.

Unitarity

all orders

- Unitarity follows if we add all possible ways in which a diagram with given topology can be cut in two.
- The shaded line separates S from $S^{\dagger}$.
- For a stable particle the cut line, proportional to $\overline{\Delta}^{+}$, contains a pole term $2\,i\,\pi\,\theta(p_0)\,\delta(p^2+m^2)$,
- whereas there is no such contribution for an unstable particle.

Theorem

- *We express $\mathrm{Im}\,\Sigma$ in terms of cut self-energy diagrams and repeat the procedure ad libidum, therefore proving that unstable particles contribute to the unitarity of the $S-$matrix only via their stable decay products.*

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WSTI with dressed propagators/vertices

- We assume that WST identities hold at any fixed order in perturbation theory for diagrams that contain bare propagators and vertices;
- they again form dressed propagators and vertices when summed.

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Any arbitrary truncation that preferentially resums specific topologies will lead to violations of WST identities.

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Finite renormalization with unstable particles

Give up unitarity (a quizzical approach)

Use

$$m^2 = s_m + \Sigma(s_m);$$

At one loop

$m^2 \to s_m$ everywhere;

At two loops

- $2L$ no Σ insertions: $m^2 = s_m$;
- $1L$: $m^2 = s_m + \Sigma(s_m)$ and the factor

$$\frac{\Sigma(s) - \Sigma(s_m)}{s - s_m},$$

expanded to first order;

- vertices: $m^2 = s_m$ in $2L$, $m^2 = s_m + \Sigma(s_m)$ in $1L$

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Part II

Higgs decay: exploratory case study

$H \to \gamma\gamma$ and WSTI

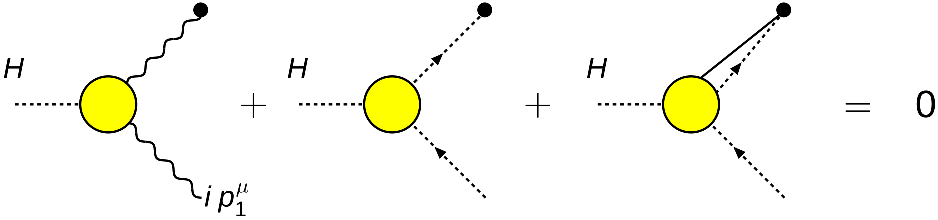
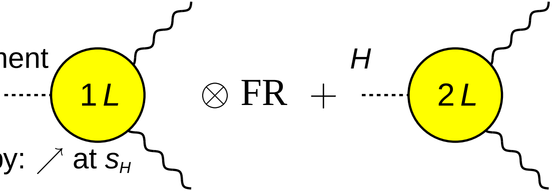


Figure: Simply-contracted off-shell Ward identity for $H \to \gamma\gamma$. The thick dot represents an unphysical, off-shell photon source, the dotted-arrow-line is a FP ghost.

$H \longrightarrow \gamma\gamma$: a practical application for nonsense

LSZ treatment

$\hookrightarrow$



alternative therapy: $\nearrow$ at s_H

Interlude: QCD corrections

M_H	Our Result	Harlander(anal.)	Degrassi(τ^3)	Tarasov(τ^6)
100	$-3.8682(7)$	-3.8687	-3.8688	-3.8687
110	$-4.5947(6)$	-4.5953	-4.5955	-4.5953
120	$-5.3535(6)$	-5.3540	-5.3543	-5.3540
130	$-6.1323(6)$	-6.1329	-6.1336	-6.1329
140	$-6.9175(6)$	-6.9180	-6.9196	-6.9180
150	$-7.6928(6)$	-7.6932	-7.6964	-7.6932

with $\tau = M_H^2/(4\, m_t^2)$

Calibration: $H \to gg$ light quarks

M_H	Our (numerical)	Degrassi et al
115	5.31(5)	5.28
120	5.55(4)	5.62
125	5.97(4)	5.98
130	6.44(6)	6.36
135	6.74(3)	6.76
140	7.12(4)	7.20
145	7.73(4)	7.69
150	8.25(4)	8.26

Numerical - mass regulated vs. harmonic polylogarithms - dimensionally regulated for one generation.

Problems: WST

(Unstable) external H

- Based on validity of WSTI one form factor suffices in describing the amplitude;
- but at two-loops the WSTI is:

$$\begin{aligned} \mathrm{WSTI} &= \mathrm{Re}\, F\,(M_H, M_W\,;\, 1L\ \mathrm{LSZ}, \mathrm{OS}, \mathrm{FR}) - F\,(M_H, M_W\,;\, 2L) \\ &= 0, \quad \text{below threshold}, \\ &\neq 0, \quad \text{above threshold} \end{aligned}$$

Remove the Re tag, F known in analytical form

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(Unstable) internal W

- From both H LSZ - wave function renormalization and pure two-loop diagrams it emerges an unphysical singularity:

$$M(H \to \gamma\gamma) \;=\; \frac{M_s}{\beta_W} + M_f, \quad \beta_W^2 = 1 - 4\,\frac{M_W^2}{M_H^2}$$

Strategy

- Remove the Re tag as before,
- prove WSTI for M_s and M_f separately (analytical extraction of M_s from pure two-loops),
- replace M_W^2 with s_W everywhere in M_s/β_W

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Around the WW threshold

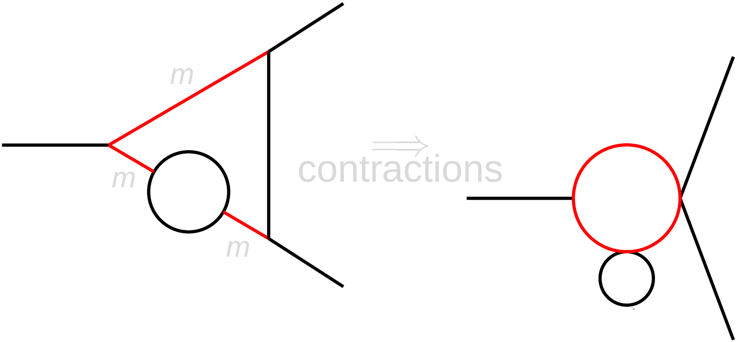
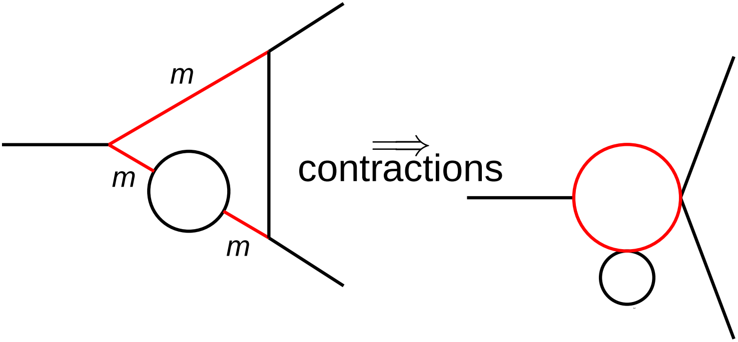


Figure: Contraction of a V^M configuration leading to a β^{-1} behavior at the normal m threshold.

Around the WW threshold



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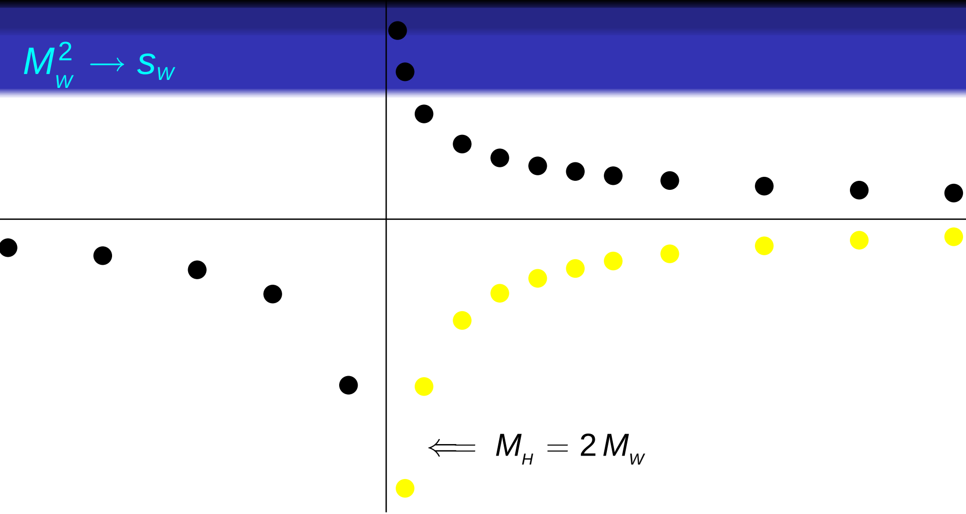

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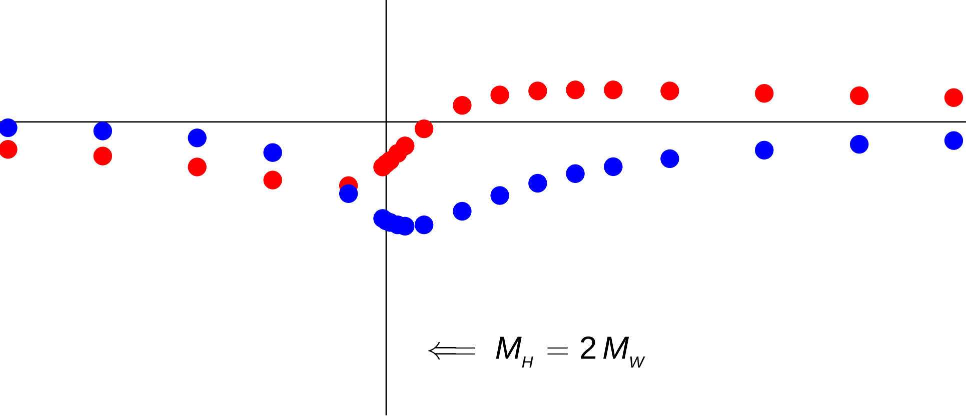

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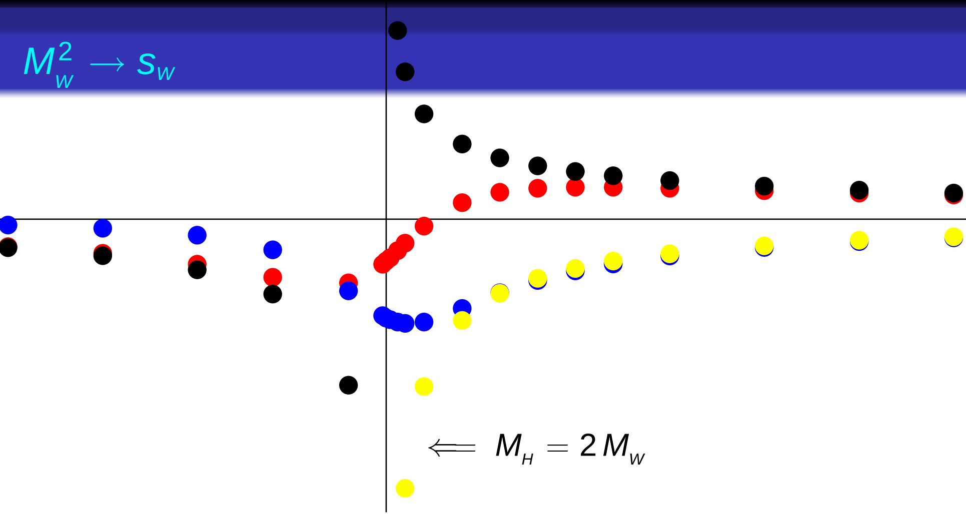

Figure: V_0^M around the $M_H = 2\,M_W$ threshold; black(red) dots give the real part with real (complex) masses. Yellow(blue) dots give the imaginary part with real (complex) masses.

$H \longrightarrow \gamma\gamma$: technical details

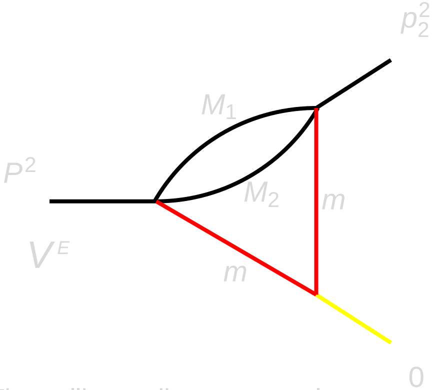

Figure: The collinear-divergent two-loop vertex diagrams V^E.

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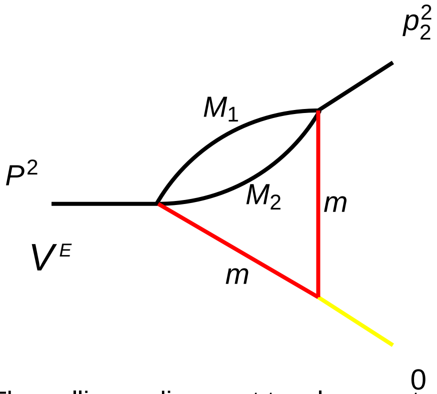


Figure: The collinear-divergent two-loop vertex diagrams V^E.

$H \to \gamma\gamma$: collinear configurations

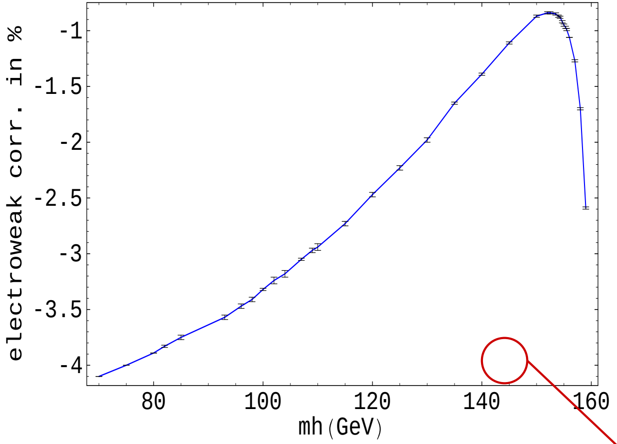
integrals of one-loop functions

$$
\begin{aligned}
IB^i_n\left(P^2, p_i^2, \{m\}\right) &= \int_0^1 dx\, B_n(X_i, \{m\}), \\
X_i &= (1-x)\, P^2 + x\, p_i^2
\end{aligned}
$$

Analytical extraction of coll. logs

$$
\begin{aligned}
V_0^E(0, p_2^2, P^2, \{M\}, \{m\}) &= -\frac{L_m^2}{2} + IB_0^2(P^2, p_2^2, \{M\})\, L_m \\
&+ \text{c.f.}
\end{aligned}
$$

$H \longrightarrow \gamma\gamma$: EW corrections



terra incognita, ready for numerical explorations

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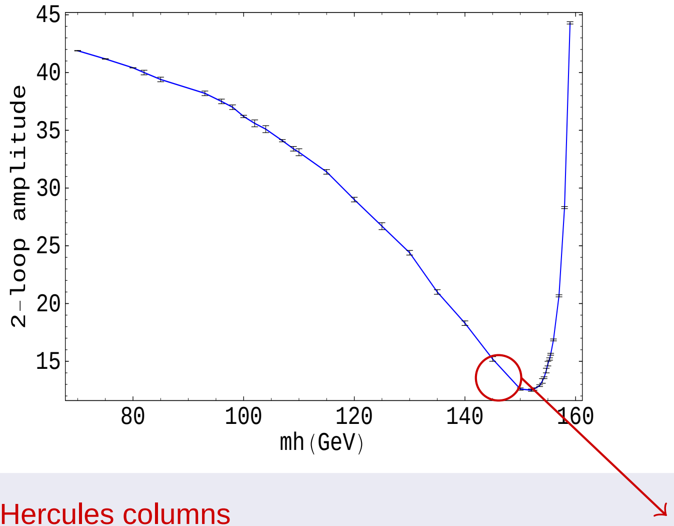

Hercules columns

$H \to \gamma\gamma$: complex-masses

M_H [GeV]	with M_W^2 [%]	with s_W (%)
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155	$-0.95(1)$	$-0.71(2)$
160	$-5.23(1)$	$+0.35(1)$
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C&C

Citations

won't win me many friends . . .

Alwall . . . Willenbrock in a stroke

Conclusions

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