Upgrading APV for Future Electroweak Fits

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Abstract

Atomic Parity Violation revised

Dirty Secrets in APV

courtesy M. Grünewald

Tells you why APV
ain't yet included in EW fits

▷ The most precise experimental results on APV are

$$\begin{aligned} Q_W(\mathrm{Cs}) &= -72.06 \pm 0.28 \pm 0.34 \\ \mathrm{Reference} &: (\mathrm{Boulder}) \mathrm{C.S.Wood et al., Science 275 (1997) 1759-1763}, \\ Q_W(\mathrm{Tl}) &= -114.8 \pm 1.2 \pm 3.4, \\ \mathrm{References} &: (\mathrm{Oxford}) \mathrm{N.H.Edwards et al., Phys.Rev.Lett. 74 (1995) 2654}, \\ & \quad (\mathrm{Seattle}) \mathrm{P.A.Vetter et al., Phys.Rev.Lett. 74 (1995) 2658}. \end{aligned}$$

▷ For a nucleus with Z protons and N neutrons, Q_W is calculated as:

$$Q_W(Z,N) = -2\Big[C_{1u}\,(2\,Z+N) + C_{1d}\,(Z+2\,N)\Big],$$

with $C_{1q} \equiv$ electron axial-vector coupling and quark vector coupling including radiative corrections evaluated at zero momentum transfer scale.

The C_{1q} are now newly calculated in the framework of the MSM by *TOPAZ0* and *ZFITTER*.

Warning

- ▷ The above result for $Q_W(\mathrm{Cs})$ differs from the MSM expectation at the $2.5\,\sigma$ level according.

However

- ▷ in A. Derevianko, **hep-ph/0005274** it is claimed that contributions from the Breit interaction in atomic-structure calculations account for $1.3\,\sigma$ of this difference,
- ▷ as taking these effects into account in the analysis one obtains for the weak charge the new experimental value:

$$Q_W(\mathrm{Cs}) \;=\; -72.65 \pm 0.28 \pm 0.34.$$

- ▷ Including also corrections for the neutron nuclear distribution in **133Cs** shifts $Q_W(\mathrm{Cs})$ by another

$$\Delta Q_W(\mathrm{Cs}) = +0.11,$$

on the SM prediction which reduces the difference to the MSM down to $1.0\,\sigma$.

Question:

▷ by how much must the $Q_W(\mathrm{Tl})$ (and other Q_W) measurement be corrected?

Action:

▷ As soon as these issues are settled, we should include the APV results in electroweak MSM fits!

Ready for Action:

▷ The investigation of APV has been the subject of a number of studies made in the 80's by Marciano and Sirlin.

▷ For *TOPAZ0*, it has been relatively simple to include all recently computed higher-order effects in the old $\overline{MS}$] calculation.

▷ For *ZFITTER* instead, the authors have been able to produce a novel evaluation in the **OMS** scheme.

Upgrading the $\overline{MS}$ calculation

▷ The electron–quark parity-violating Hamiltonian at zero momentum-transfer will be conventionally parametrized as follows:

$$
\begin{aligned}
H_{\rm PV} &= \frac{G_F}{\sqrt{2}}\Big(C_{1u}\bar{e}\gamma_\mu\gamma_5 e\bar{u}\gamma_\mu u + C_{2u}\bar{e}\gamma_\mu e\bar{u}\gamma_\mu\gamma_5 u \\
&+ C_{1d}\bar{e}\gamma_\mu\gamma_5 e\bar{d}\gamma_\mu d + C_{2d}\bar{e}\gamma_\mu e\bar{d}\gamma_\mu\gamma_5 d\Big),
\end{aligned}
$$

▷ In heavy atoms the dominant part of parity-violation is proportional to the so-called weak-charge $Q_{\rm w}$

$$
Q_{\rm w}(Z,A) = 2\Big[(Z+A)\,C_{1u} + (2\,A - Z)\,C_{1d}\Big],
$$

▷ We have taken the calculation by Marciano and Sirlin and have extended it to include all higher-order effects presently known:

1. two-loop leading contribution for the ρ-parameter;
2. exact $\mathcal{O}\left(\alpha\alpha_{_S}\right)$ corrections;
3. $\mathcal{O}\left(\alpha\alpha_{_S}^2\right)$ corrections to ρ;
4. next-to-leading two-loop heavy top corrections.

▷ *TOPAZ0* now returns, among all PO, the two quantities, C_{1u} and C_{1d}. They are defined as follows:

$$
\begin{aligned}
C_{1u} &= -\frac{1}{2}\,\rho'_{\rm PV}\left[1-\frac{8}{3}\,\kappa'_{\rm PV}(0)\sin^2\hat{\theta}(M^2_{_W})\right],\\
C_{1d} &= \frac{1}{2}\,\rho'_{\rm PV}\left[1-\frac{4}{3}\,\kappa'_{\rm PV}(0)\sin^2\hat{\theta}(M^2_{_W})\right],
\end{aligned}
$$

▷ where $\sin^2\hat{\theta}(M^2_{_W})$ is the $\overline{MS}$ weak-mixing angle at the scale $\mu = M_{_W}$. Moreover,

$$
\begin{aligned}
\rho'_{\rm PV} &= \rho - \frac{\alpha}{2\,\pi}\left[1+\frac{1}{\hat{s}^2}+4\,\hat{v}_e\,B_{\rm p(np)}+\frac{9}{16\,\hat{s}^2\hat{c}^2}\left(1-\frac{16}{9}\hat{s}^2\right)\left(1+\hat{v}^2_e\right)\right],\\
\kappa'_{\rm PV}(0) &= \kappa_{\rm PV}(0) - \frac{\alpha}{2\,\pi\,\hat{s}^2}\left[\frac{9-8\hat{s}^2}{8\,\hat{s}^2}-\frac{\hat{v}_e}{6}\left(\ln\frac{M^2_{_Z}}{m^2_e}+\frac{1}{6}\right)\right.\\
&+ \left(\frac{9}{4}-4\hat{s}^2\right)\,\hat{v}_e\,B_{\rm p(np)}\\
&+ \left.\frac{9}{16\,\hat{s}^2\hat{c}^2}\left(\frac{1}{2}\hat{v}_e+\frac{16}{9}\hat{s}^q\right)\left(1+\hat{v}^2_e\right)\right],
\end{aligned}
$$

▷ where $\hat{v}_e = 1-4\,\hat{s}^2$, $\hat{s}^2 = \sin^2\hat{\theta}(M^2_{_W})$ and where we have two different treatments of the Z–γ boxes —

▷ perturbative

$$B_{\rm p} \;=\; \ln\frac{M_Z^2}{m^2}+\frac{3}{2}, \qquad m=m_u=m_d=75\,\text{MeV},$$

▷ and non-perturbative

$$\begin{aligned} B_{\rm np} &= K+\frac{4}{5}\,(\xi_1)^p_B\,, \\ K &= M_Z^2\int_{M^2}^{\infty}\frac{du}{u(u+M_Z^2)}\left[1-\frac{\alpha_S(u)}{\pi}\right], \qquad (\xi_1)^p_B=2.55, \end{aligned}$$

▷ where M is a mass scale representing the onset of the asymptotic behavior, i.e. the regime where α_S becomes small.

▷ We observe a plateau of stability in K for M centered around $0.5\,$GeV and this is the numerical value used.

▷ Furthermore we used the following form for the ρ, κ parameters:

$$\begin{aligned} \rho &= 1+\frac{\alpha}{4\,\pi\,\hat{s}^2}\Bigg[\frac{3}{4\,\hat{s}^2}\,\ln\hat{c}^2-\frac{7}{4}+\frac{3}{4}\,\frac{m_t^2}{M_W^2}\,(1+\delta_{\rm EW}+\delta_{\rm QCD}) \\ &\quad +\frac{3}{4}\,h\left(\frac{\ln(\hat{c}^2/h)}{\hat{c}^2-h}+\frac{1}{\hat{c}^2}\,\frac{\ln h}{1-h}\right)\Bigg], \\ \kappa_{\rm PV}(0) &= 1-\frac{\alpha}{2\,\pi\,\hat{s}^2}\left[\frac{7}{9}-\frac{\hat{s}^2}{3}+\frac{Q_f}{3}\,\sum_f\left(I_e^{(3)}-2\,Q_f\hat{s}^2\right)\,\ln\frac{m_f^2}{M_W^2}\right]. \end{aligned}$$

In the previous equation we have $h=M_H^2/M_Z^2$.

▷ The strange quark mass is effectively chosen to be $m_s = 250\,\mathrm{MeV}$ so that (with effective $m_u = m_d = 75\,\mathrm{MeV}$) we recover the dispersive analysis for the Z–γ transition where $\Pi_{\mathbf{z}\gamma}(0)$ is rewritten in terms of a dispersion relation with the kernel connected to $\sigma(e^+e^- \to \mathrm{hadrons})$.

▷ $\delta_{\mathrm{EW(QCD)}}$ are the LO+NLO electroweak $\Bigl(\mathcal{O}\left(\alpha_S^2 + \alpha_S^3\right)\ \mathbf{QCD}\Bigr)$ correction to ρ.

▷ The evaluation of ρ and $\hat{s}^2$ includes the best available LO+NLO.

▷ *TOPAZ0* default is the perturbative formulation of the *factorized* result. There is the option of using some *additive* formulation where

$$
\begin{aligned}
C_{1u} &= -\frac{1}{2}\,\rho\left[1 - \frac{8}{3}\,\kappa_{\mathrm{PV}}(0)\sin^2\hat{\theta}(M_W^2)\right] + \Delta_u\,, \\
C_{1d} &= \frac{1}{2}\,\rho\left[1 - \frac{4}{3}\,\kappa_{\mathrm{PV}}(0)\sin^2\hat{\theta}(M_W^2)\right] + \Delta_d\,,
\end{aligned}
$$

where $\Delta_{u,d}$ are obtained by expanding and by neglecting terms of $\mathcal{O}\left(\alpha^2\right)$.

Theoretical uncertainty in APV

TOPAZ0 and courtesy **ZFITTER**

m_t [GeV]	170	175	180
Fact/Pert	-72.9712	-72.9632	-72.9551
Fact/Non Pert	-73.1994	-73.1932	-73.1869
Add/Pert	-72.9732	-72.9658	-72.9582
ZFITTER	-72.9762	-72.9698	-72.9637
Add/Non Pert	-73.2026	-73.1969	-73.1912

Table 1: Predictions for $Q_{\mathrm{W}}(\mathrm{Cs})$ from TOPAZ0 and ZFITTER for $M_{_Z} = 91.1875\,\mathrm{GeV}$, $M_{_H} = 150\,\mathrm{GeV}$ and $\alpha_{_S}(M_{_Z}^2) = 0.119$.

m_t/M_H *sensitivity*

- ▷ The m_t-dependence of $Q_{\mathrm{w}}(\mathrm{Cs})$ is shown in Tab.(1) where we register a 0.22(0.17) per-mill increase for m_t between 170 GeV and 180 GeV and for Pert(Non Pert).
- ▷ As for the M_H dependence we have computed a decrease of about 0.7 per-mill for M_H between 150 GeV and 300 GeV.

The associated theoretical uncertainty is

approximately 3.2 *per-mill and*

it is ***largely*** *dominated by QCD effects*

- ▷ In the original calculation of Marciano and Sirlin we have a dependence of the result on light quark masses.
- ▷ In their second paper they suggested how to go beyond the partonic-language.

1. One should distinguish quark masses in the Z–γ transition and in Z–γ **boxes**;

1. light quark masses, m_{uds}, are then fixed to parameterize the dispersive result for the Z–γ transition and are not varied anymore in evaluating the theoretical uncertainty.
2. Furthermore we have Z–γ box diagrams where quark masses show up as the consequence of the zero momentum transfer limit.

▷ Here we split the boxes into a

   1. low-frequency part, approximated with the Born contribution for a physical nucleon,
   2. and an high-frequency part (the K-term) that includes $\mathcal{O}\left(\alpha_S\right)$ corrections where light quark masses disappear.

▷ The mass scale M separating low- from high-frequency parts is, of course, arbitrary and only subjected to the requirement that

   1. $\alpha_S(Q^2)$ starts to become small for $\mid Q^2 \mid > M^2$
   2. and that $M > \Lambda_{\rm QCD}$.

▷ However, we have found a plateau of stability for the result, and we fix $0.5 \leq M \leq 0.6$.

▷ Furthermore, the differences in the factorized versus additive formulation of the coefficients $C_{1u,1d}$ is approximately 0.05 permill signaling that pure electroweak higher orders are relatively under control.

The conclusion is that theoretical uncertainty is completely dominated by QCD effects at zero momentum transfer

Finally, let us define an effective APV weak-mixing angle

$$\sin^2\theta_{\rm APV} = \kappa'_{\rm PV}(0)\sin^2\hat{\theta}(M_W^2).$$

▷ For $M_Z = 91.1875\,$GeV, $M_H = 150\,$GeV and $\alpha_S(M_Z^2) = 0.119$ we obtain

$$\sin^2\theta_{\rm APV} = 0.231601(0.232123),$$

▷ corresponding to perturbative(non-perturbative) treatment.