σ_{VV} and A^J_{VV}
as Pseudo-Observables

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There are three regions of interest:

- $\triangleright$ $m_{W,Z} \approx m_H \equiv$ the Higgs is accessible;
- $\triangleright$ $m_{W,Z} \ll m_H \ll \sqrt{s} \equiv$ **Higgs mechanism OK** but partial wave scattering amplitudes stay constant for large s at a value $\propto m_H$. Constraints from perturbative unitarity are then used to derive the critical value of m_H signalling the onset of a **strongly interacting** system.
- $\triangleright$ $m_{W,Z} \ll \sqrt{s} \ll m_H \equiv$ This is supposedly almost the same thing as no Higgs. Amplitudes grow with the energy and **perturbative unitarity** fixes a critical value s_c such that for $s > s_c$ perturbation theory breaks down.

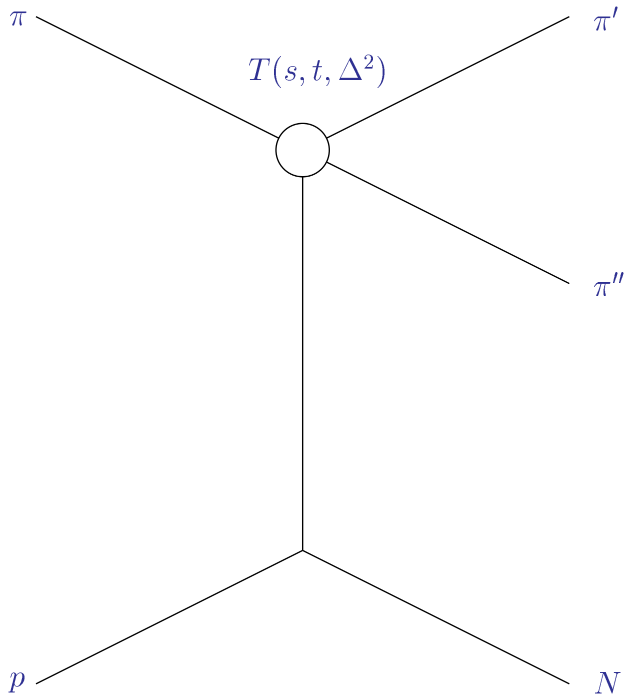
VV *scattering is the main source of information*

We don't have VV-scattering
we have **multi-fermion final states**

How to extract it?
By **DE-CONVOLUTION** and by
EXTRAPOLATION

Old times LEP

OldOld times $\Longrightarrow$


π
$T(s,t,\Delta^2)$
π'
π''
p
N

▷ $\pi\pi$-scattering from $\pi + p \to N + \pi + \pi$

$$
\begin{aligned}
< \pi\pi|T|\pi\pi > \; &= \; T(s,t), \\
< \pi\pi N|T|\pi p > \; &\propto \; T(s,t,\Delta^2),
\end{aligned}
$$

$$
(q_N - q_p)^2 = \Delta^2 m^2,
$$

$$
s = -(k' + k'')^2, \quad \text{and} \quad t = -(k - k'')^2
$$

▷ Procedure

1. Extract

$$
\mid T(s,t,\Delta^2 \mid^2 \quad \text{from} \quad \frac{\partial^3 \sigma}{\partial s \partial t \partial \Delta^2} \tag{1}
$$

2. Compute

$$
\frac{\partial \sigma_{\pi\pi}}{\partial t} = \frac{1}{16\,\pi} \frac{|T(s,t,-1)|^2}{\lambda(s,m^2,m^2)}
$$

- Observe that $\Delta^2 = -1$ while it can only be positive in Eq.(1).

▷ Conclusion: We can investigate $\sigma_{\pi\pi}$ from a

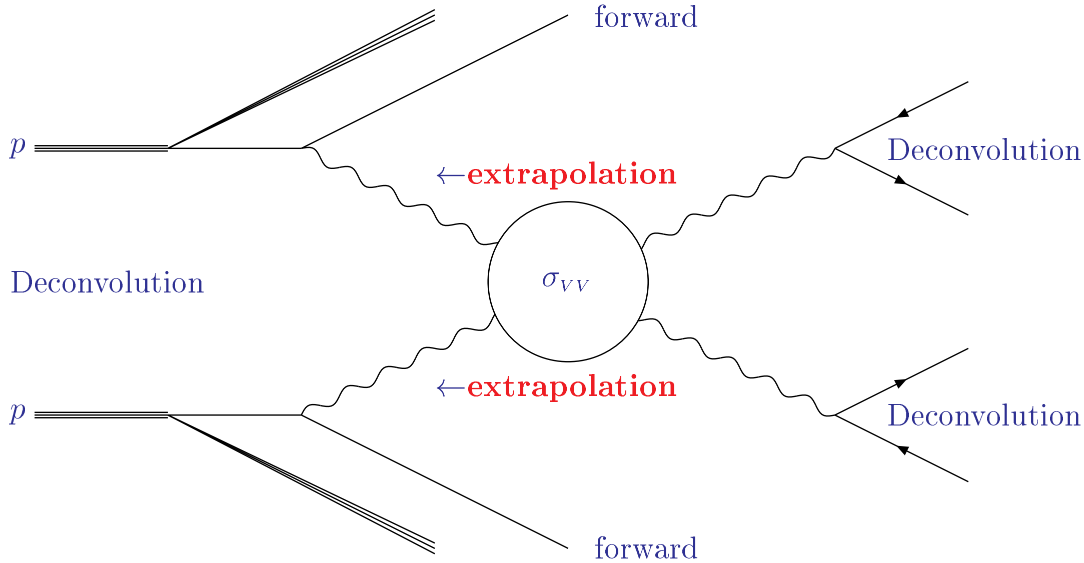
measurement of

▷ the differential σ in $\pi + p \to N + \pi + \pi$

▷ if the **extrapolation procedure** can be reliably performed

▷ see D.D. Carmony and R.T. Van de Walle, Phys. Rev. 127(1962)959.

Back to vectorbosons



▷ The best source of **longitudinally polarized** V's is their bremsstrahlung from fast beams of weak-coupling particles, e.g. $\bar{q}_1 q_2 \to \bar{q}_3 q_4 V V$;

▷ **Deconvolution** $\equiv$ isolating the signal diagram of double V-bremsstrahlung from background;

▷ In computing the **signal cross-section** we replace

$$\delta_{\mu\nu} + \frac{K_\mu K_\nu}{M^2} \;\to\; \sum_\lambda e_\mu(\lambda, K)\, e^\star_{nu}(\lambda, K), \quad K^2 > 0,$$

which is correct only for on-shell momenta $K^2 = -M^2$.

▷ We have $\sigma = \sigma_T + \sigma_L + \sigma_I$.

▷ In the region $M^2 \ll s$, where the cross-section peaks at small angles, σ_L dominates.

This defines the **approximation**:

- $\triangleright$ Only L vectorbosons contribute;
- $\triangleright$ they are taken to be **virtual** when produced by fermion lines and
- $\triangleright$ **real** when re-scattered

At very high energies masses are less important and

- $\triangleright$ in the forward region we may identify virtual with real V, $K^2 \approx 0$.

Let μ^2 be the VV **invariant mass**

Let f be the **luminosity factor**

$$f(x) = (1+x)\ln\frac{1}{x} - 2(1-x),$$

we obtain

$$\mu^2 \frac{d\sigma}{d\mu^2}(\bar{q}_1 q_2 \to \bar{q}_2 q_3 V_L V_L) = (\mathrm{const})\, f(\frac{\mu^2}{s})\, \sigma(V_L V_L \to V_L V_L),$$

Scattered quarks will be in the **forward region**

&

and we measure combinations

$$\mu^2 \frac{d\sigma}{d\mu^2}(\bar{q}_1 q_2 \to X W^+_L W^-_L) = (\mathrm{const})\, f(\frac{\mu^2}{s})$$
$$[h(\theta_W)\, \sigma(Z_L Z_L \to W^+_L W^-_L) + \sigma(W^+_L W^-_L \to W^+_L W^-_L)].$$

Vector boson scattering at $m_{W,Z} \ll m_H \ll \sqrt{s}$

▷ In this region the scattering involving **longitudinally polarized** W**'s** and Z**'s** is computed by using the **effective theory of interacting scalars**.

$$\begin{aligned}\mathcal{L} &= -\frac{1}{2}\partial_\mu H \partial_\mu H - \frac{1}{2} m^2 H^2 - \frac{1}{2}\partial_\mu z \partial_\mu z - \partial_\mu \omega^+ \partial_\mu \omega^- \\ &- \frac{\beta}{(2\,G)^{1/2}} H - \frac{1}{2}\beta\phi^2 - (\frac{G}{2})^{1/2} m^2 H \phi^2 - \frac{1}{4} G m^2 \phi^4, \\ \phi^2 &= H^2 + z^2 + 2\,\omega^+\omega^-.\end{aligned}$$

▷ Given the Lagrangian we compute a two-body matrix element M such that

$$\sigma_{2\to 2} = \frac{1}{(2\,\pi)^{10}\,2s} \int dPS_2 \mid M \mid^2,$$

or

$$\frac{d}{dt}\sigma_{2\to 2} = \frac{\mid T(s,t)\mid^2}{16\,\pi s^2}, \qquad M(s,t) = (2\,\pi)^4 i\,T(s,t),$$

▷ In general $T = T_B + T_{1L} + \ldots$ and we project the $j = 0$ partial-wave for zero total helicity in the center of mass frame. In the Born approximation

$$T^0_B = \frac{1}{16\,\pi s}\int_{-s}^{0} dt\,T_B,$$

▷ Next consider the coupled, neutral channels

$$\omega^+\omega^-, \quad \frac{1}{\sqrt{2}}zz, \quad \frac{1}{\sqrt{2}}HH, \quad Hz,$$

▷ As well known we obtain a 4×4 matrix with eigenvalues

$$-Gm^2/8\,\pi(3,1,1,1).$$

▷ Bounds can be derived and it is important to recall the procedure. **Two-particle unitarity** can be expressed as

$$Im T^0 \;\geq\; \mid T^0 \mid^2$$

▷ T^0 is usually expressed through an **expansion** in some parameter g^2

$$T^0 \;=\; g^2 T^0_B + g^4 T^0_{1L} + O\left(g^6\right),$$

▷ at very high energies $Im T^0_B = 0$ and the **Schwartz inequality** implies

$$Im T^0 \;\leq\; g^4 \mid T^0_{1L} \mid \, + O\left(g^6\right),$$

▷ A **reliable perturbative expansion** requires

$$g^4 \mid T^0_{1L} \mid \; \ll \; g^2 \mid T^0_B \mid,$$

▷ Thus

$$\begin{aligned} g^4 \mid T^0_{1L} \mid \; &\geq \; g^4 \mid T^0_B \mid^2, \\ g^2 \mid T^0_B \mid \; &\gg \; g^4 \mid T^0_B \mid^2, \end{aligned}$$

▷ How small is small becomes clearly a matter of convention; therefore at this order is commonly accepted to conservatively require

$$g^2 \mid T^0_B \mid \; < \; 1$$

If there is **New Physics** we can test it
if we have a model independent determination of
VV -scattering

From **Theory** loop corrections are also needed

▷ In the intermediate region, for

$$\omega^+\omega^-, \quad \frac{1}{\sqrt{2}}zz, \quad \frac{1}{\sqrt{2}}HH, \quad Hz,$$

▷ the eigenvalues of $T^0_B + T^0_{1L}$ (in units of $G_{\exp}\, m^2_H / 8\,\pi$) are

$$\begin{aligned}
e_1 &= -3 + \rho\left(-36\,L + \frac{117}{2} - 6\sqrt{3}\,\pi + 5\,\pi^2 + 18\,i\pi\right)\\
e_2 &= -1 + \rho\left(-12\,L + \frac{47}{2} - 2\sqrt{3}\,\pi + \frac{5}{3}\pi^2 + 2\,i\pi\right)\\
e_3 &= e_4 = -1 + \rho\left(-12\,L + 17 - \sqrt{3}\,\pi + \frac{5}{3}\pi^2 + 2\,i\pi\right).
\end{aligned}$$

▷ with

$$L = \ln\frac{s}{m^2_H}, \quad \rho = \frac{G_{\exp} m^2_H}{16\,\pi^2}$$

$$G_{\exp} = 8\,\pi\,\frac{\Gamma(H \to zz)}{m^3_H}.$$

Scattering at $M^2_{W,Z} \ll s \ll m^2_H$

$$
\begin{aligned}
T^0_{1L}(\omega^+\omega^- \to \frac{1}{\sqrt{2}}zz) &= \frac{G_F^2 s^2}{256\sqrt{2}\,\pi^3}\left(\frac{20}{9}\ln\frac{m_H^2}{s} + 6\sqrt{3}\pi - \frac{905}{27} + 2\,i\pi\right) \\
T^0_B(\omega^+\omega^- \to \frac{1}{\sqrt{2}}zz) &= \frac{G_F\, s}{8\sqrt{2}\,\pi}
\end{aligned}
$$

$$
\begin{aligned}
T^0_{1L}(\frac{1}{\sqrt{2}}zz \to \frac{1}{\sqrt{2}}zz) &= \frac{G_F^2 s^2}{512\,\pi^3}\left(\frac{20}{3}\ln\frac{m_H^2}{s} + 10\sqrt{3}\pi - \frac{512}{9} + 4\,i\pi\right) \\
T^0_B(\frac{1}{\sqrt{2}}zz \to \frac{1}{\sqrt{2}}zz) &= 0
\end{aligned}
$$

- ▷ The coupled $j = 0$ **partial-wave** has two eigen-values
- ▷ In the tree approximation $\lambda_1 < 0$, corresponding to a **repulsive interaction**
- ▷ while λ_2 gives an **attractive interaction**.
- ▷ Including **one-loop corrections** changes the situation. One eigen-channel (corresponding to λ_2) is always **attractive**, the other stays **repulsive** with λ_1 becoming more and more negative till some threshold, after which the **behavior is reversed**.

$$j = 1$$

- ▷ We have

$$T^1_B + T^1_{1L}(\omega^+\omega^- \to \omega^+\omega^-) \quad = \quad \frac{G_F s}{16\,\pi}\left[+\frac{1}{3} + \frac{G_F s}{16\,\pi^2}\left(\frac{143}{27} - \sqrt{3}\pi + \frac{i}{9}\pi\right)\right].$$

- ▷ with vanishing $j = 1$ projection for the other two channels, also at one loop

- ▷ **The vector interaction** is **attractive**

CONCLUSIONS

▷ In all partial-waves, the amplitudes T^j **must be small** and

▷ this protects **radiative effects** from affecting the current-current picture **at low energy**;

▷ Otherwise there exist **strong interactions** of the **vector bosons** with each other but not with **leptons**.