Non-Annihilation Processes Fermion-Loop Scheme and QED Radiation

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Abstract

How to deal with scales.

PRESENT STATUS of RC

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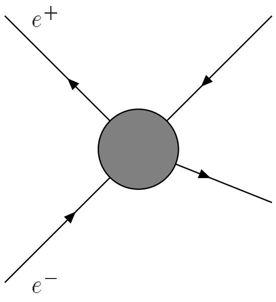


one-loop $\Longrightarrow$ **ALL**

two-loop $\Longrightarrow$ **LO + NLO**

+ re-summations

IS QED, **SF-method** (or **equivalent**)
FS QED, **available**
IS $\otimes$ **FS QED**, **see Wąs**

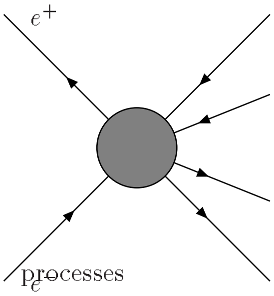
Fine Points in QED for $2 \to 2$

s-channel $\mathcal{O}\left(\alpha^2 L^n\right)$, $n = 0, 1, 2$ (Calculations), $\mathcal{O}\left(\alpha^3 L^3\right)$
important for Z **lineshape**

Differences and uncertainties amount to at most $\pm 0.1\,\mathrm{MeV}$ on M_Z and Γ_Z and $\pm 0.01\%$ on σ^0_{h} (**MIZA, TOPAZ0** and **ZFITTER**) LEP EWWG/LS 2000-01

non-annihilation (Bhabha) both **SF** and **PS** have been analyzed, uncertainty estimated to be **0.061%** (**BHLUMI**)

FULL two-loop EW needed for **GigaZ** ($10^9 Z$ events),
Numerical evaluation of **FD**?



$e^+e^- \to 4\,\mathbf{f}$ **Born** $\Longrightarrow$ **ALL**

$\mathcal{O}(\alpha)$ EW $\Longrightarrow$ **only in DPA**[†] **for** WW **signal**

(†) **see Denner, Dittmaier and Ward**)

$e^+e^- \to 4\mathrm{f} + \gamma \Longrightarrow$ **ALL**

hep-ph/0005309

Fine Points in QED for $2 \to 4$

for $e^+e^- \to WW \to 4\,f \;\to$ **see DPA**

for generic $e^+e^- \to 4\,f \;\to$ **s – channel** SF

$\to$ *i.e.* **LL approximation**

◇ the latter strictly applicable only if **ISR** can be separated (may lead to **excess** of **ISR**)

◇ **preliminar** investigations towards **non–s SF** by **GRACE** and **SWAP** **hep-ph/0005309**

Bulk of the effect but *ad hoc*

There are several processes
(those with t-channel γs)
that are not dominated by annihilation

single-W production, **two-photon** processes, etc

$\frown$ $\boxed{\text{How to **include** the **bulk** of **RC**?}}$

Born life Still requires the **notion** of **IPS**, i.e. the choice of some set of **input parameters** (improperly called **RS**) and of **certain relations** among them, e.g.

$$s_\theta^2 = 1 - \frac{M_W^2}{M_Z^2}, \qquad \alpha \equiv \alpha_{G_F} = 4\sqrt{2}\,\frac{G_F M_W^2 s_\theta^2}{4\,\pi},$$

Roughly speaking the **TU** associated with the choice of the **RS** is **most severe** whenever **low-q^2** photons dominate.

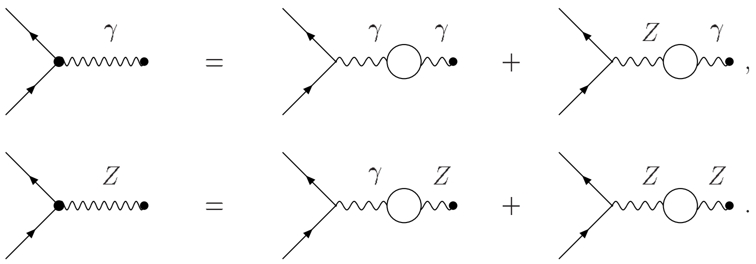
$\boxed{\textbf{Getting the right scales}}$ Complex-Mass **Renormalization** in the **FL**-approximation which, by the end of the day, gives **W. Beenakker et al.**

$$\mathbf{Couplings} \quad \Longrightarrow \quad \mathbf{Running\,Couplings}$$

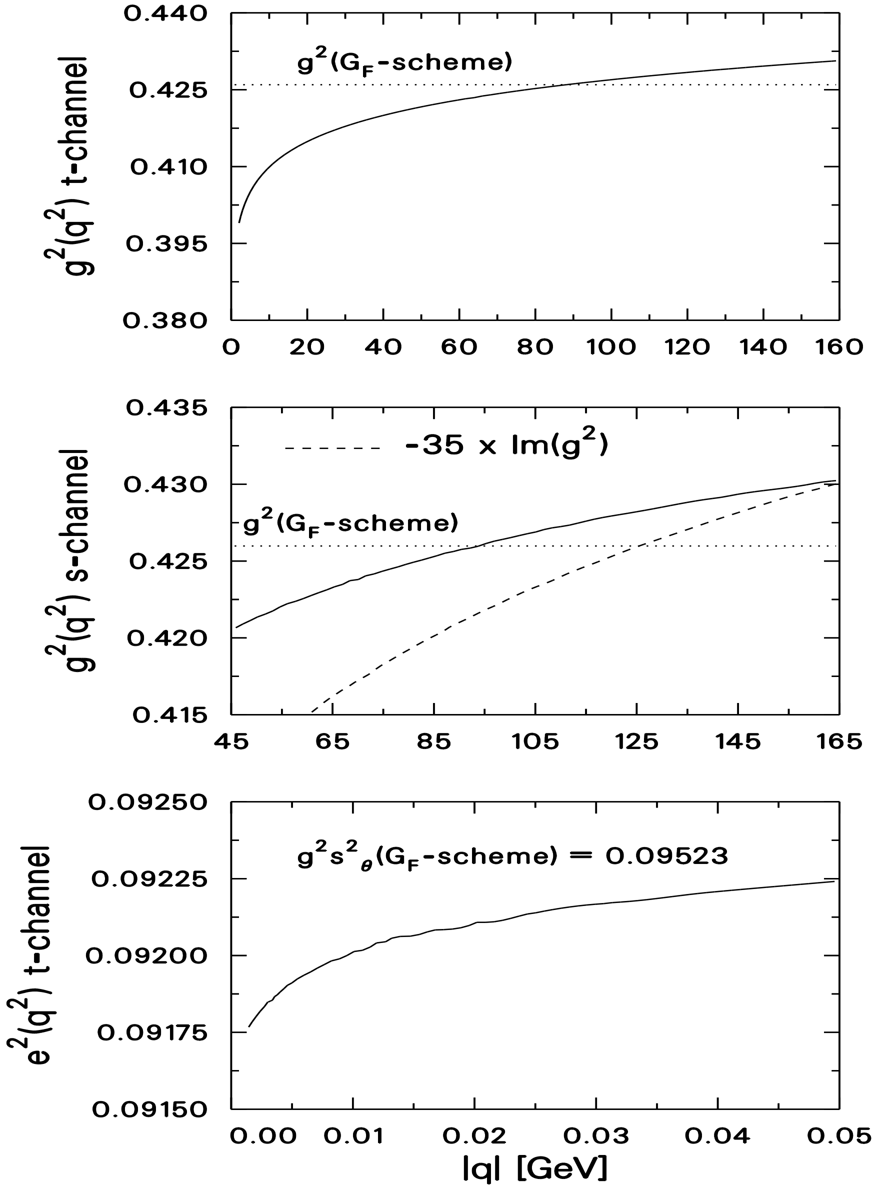
$$\mathbf{Transitions} \quad \Longrightarrow \quad \mathbf{Diagonal\,Propagator - Functions}$$

$$\searrow \quad \mathbf{showing\,a\,pole\,in\,the\,2nd\,sheet}$$

$$\mathbf{Born\,Vertices} \quad \Longrightarrow \quad \mathbf{one\,fermion - loop\,corrected\,Vertices}$$



$\nearrow$ **open circles** denote re-summed propagators and the **dot** a vertex.



Running in Practice: Plots

$\boxed{\text{Running in Practice}}$

▷ In **figure** the running of $e^2(q^2)$ is shown for $q^2 \to 0_+$, compared with the **fixed value** in the G_F-**scheme**.

▷ Furthermore the **evolution** of $g^2(q^2)$ is shown for q^2 **time-like** or **space-like**

▷ The **SIZEABLE** difference that one gets between

1. e^2 **running** in t-**channel**
2. e^2 **fixed** in the G_F-**scheme**

$\swarrow$

▷ is one of the major improvements

$\boxed{\textbf{induced by the FL-scheme}}$

in **Non-Annihilation Born** processes

⌢ The **original FL** works only for **conserved** external currents,

⌣ The extension **exists** and requires one additional **replacement**

$\searrow$

go to the 't H – F gauge
neglect unphysical scalars

$$\delta_{\mu\nu}\,(\textbf{in propagators}) \quad\Longrightarrow\quad \delta_{\mu\nu} + \frac{p_\mu p_\nu}{M^2(p^2)}$$

Connection with Complex-poles p_W, p_Z
(here only for a massless internal world)

$$W \quad\Longrightarrow\quad M^2(p^2) = \frac{g^2(p^2)}{g^2(p_W)}\,p_W,$$

$$Z \quad\Longrightarrow\quad \frac{1}{M_0^2(p^2)} = \frac{g^2(p_Z)}{g^2(p^2)}\,\frac{c^2(p^2)}{c^2(p_Z)}\,\frac{1}{p_Z}$$

$$\triangleright \qquad \frac{1}{M_0^2(p^2)} = \frac{c^2(p^2)}{M^2(p^2)}$$

and **gives** $\Longrightarrow$

M(assive) FL, where

- Gauge Invariance is respected
- Collinear Regions are accessible

Applications to single-W

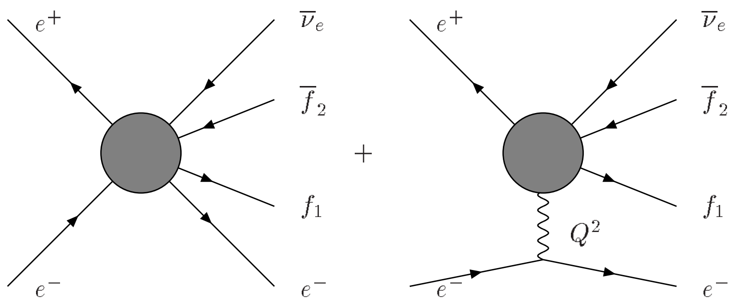


Figure 1: **The CC20 family of diagrams with the explicit component containing a t-channel photon.**

▷ there is a **maximal decrease** of about 7% in the result but,

▷ the effect is rather **sensitive** to the relative weight of multi-peripheral contributions and is **process** and **cut** dependent.

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one scale to go

Is multi-γ **radiation** a **one**-scale or a **multi**-scale **convolution** phenomenon?

$$\sigma\,(p_+p_- \to q_1\dots q_n + \textbf{QED}) \stackrel{?}{=} \int dx_+\,dx_-\,D(x_+,?)D(x_-,?) \times \sigma\,(x_+p_+x_-p_- \to q_1\dots q_n)$$

? to be guessed

How are **standard SFs** related to **exact YFS exponentiation**?

▷ In **standard YFS** treatment,

$$\sigma\left(p_+ + p_- \to \sum_{i=1,2l} q_i + \sum_{j=1,n} k_j\right) \sim \int dPS_q \mid M_0\mid^2 \times E\left(p_+ + p_- - \sum_i q_i\right),$$

$$E(K) = \frac{1}{(2\,\pi)^4}\int d^4x\,\exp(iK\cdot x)\,E(x),$$
$$E(x) = \exp\left\{\frac{\alpha}{2\,\pi^2}\int d^4k e^{ik\cdot x}\delta^+(k^2)\mid j^\mu(k)\mid^2\right\}$$

▷ **do not** separate **soft** from **hard** and compute $(n \neq 4, k^2 = 0)$

$$I = \int d^nk\,e^{ik\cdot x}\,\frac{\delta^+(k^2)}{p_i\cdot k\,p_j\cdot k}$$

▷ In **Dimensional Regularization** one has $\forall x^2$

$$
\begin{aligned}
I(x) &= -\pi\,\rho \int_0^1 \frac{du}{P^2}\left(\frac{1}{\hat{\varepsilon}} + 2\,\ln 2 - \ln x^2 - \xi\,\ln\frac{\xi+1}{\xi-1}\right), \\
\xi &= \frac{|x_0|}{r}, \quad \text{with} \quad x_0 \to x_0 + i\delta(\delta \to 0_+), \\
P &= p_j + (\rho p_i - p_j)\,u,
\end{aligned}
$$

$$
(\rho p_i - p_j)^2 = 0, \quad x_0 = -\frac{P\cdot x}{\sqrt{-P^2}}, \quad r^2 = x_0^2 + x^2.
$$

▷ **Last** integral $\to$ **IR-pole** + collection of Li_2's

▷ $E(K)$ **not** available in close form. **Proposed scheme**

$$\boxed{\longmapsto \textbf{define coplanar approximation}}$$

$$
\begin{aligned}
I^c_{ij} &\stackrel{\text{def}}{=} -\frac{2}{3}\,\pi\,\rho_{ij}\,\mathcal{F}_{\text{cp}}\,\frac{1}{p_j^2 - \rho_{ij}^2\,p_i^2}\,\ln\frac{\rho_{ij}^2 p_i^2}{p_j^2}, \\
I^c_{ii} &\stackrel{\text{def}}{=} -\frac{2}{3}\,\pi\,\rho_{ij}\,\mathcal{F}_{\text{cp}}\,\frac{1}{m_i^2}, \\
\mathcal{F}_{\text{cp}} &= \ln\left\{e^{-\Delta_{\text{IR}}}\,\frac{p_i\cdot x\,p_j\cdot x}{m_i m_j}\right\}, \\
\Delta_{\text{IR}} &= \frac{1}{\hat{\varepsilon}} + \text{constants}.
\end{aligned}
$$

C. Chahine '78

▷ within **coplanar** approximation $\rightsquigarrow$

$$E^{\mathrm{pair}\,<ij>}(K) \;\stackrel{\mathrm{cp}}{\to}\; \frac{1}{(2\,\pi)^2} \left\{ \frac{e^{-\Delta_{\mathrm{IR}}}}{m_i m_j} \right\}^{-\alpha A_{ij}} \frac{1}{\Gamma^2(\alpha A_{ij})}$$

$$\times \int_0^\infty d\sigma d\sigma' \, (\sigma\sigma')^{\alpha A_{ij}-1}\, \delta^4\left(\sigma p_i + \sigma' p_j - K\right).$$

N.B. that's why *coplanar* $\nearrow$

▷ **Note** that $\alpha A \sim \beta$ for **invariant** $\gg$ **mass**^{2} but

▷ the above **expression** is valid for **all regimes** and $\xrightarrow{\text{see next figure}}$

▷ is **easily** generalized to **n emitters** with the **result** that

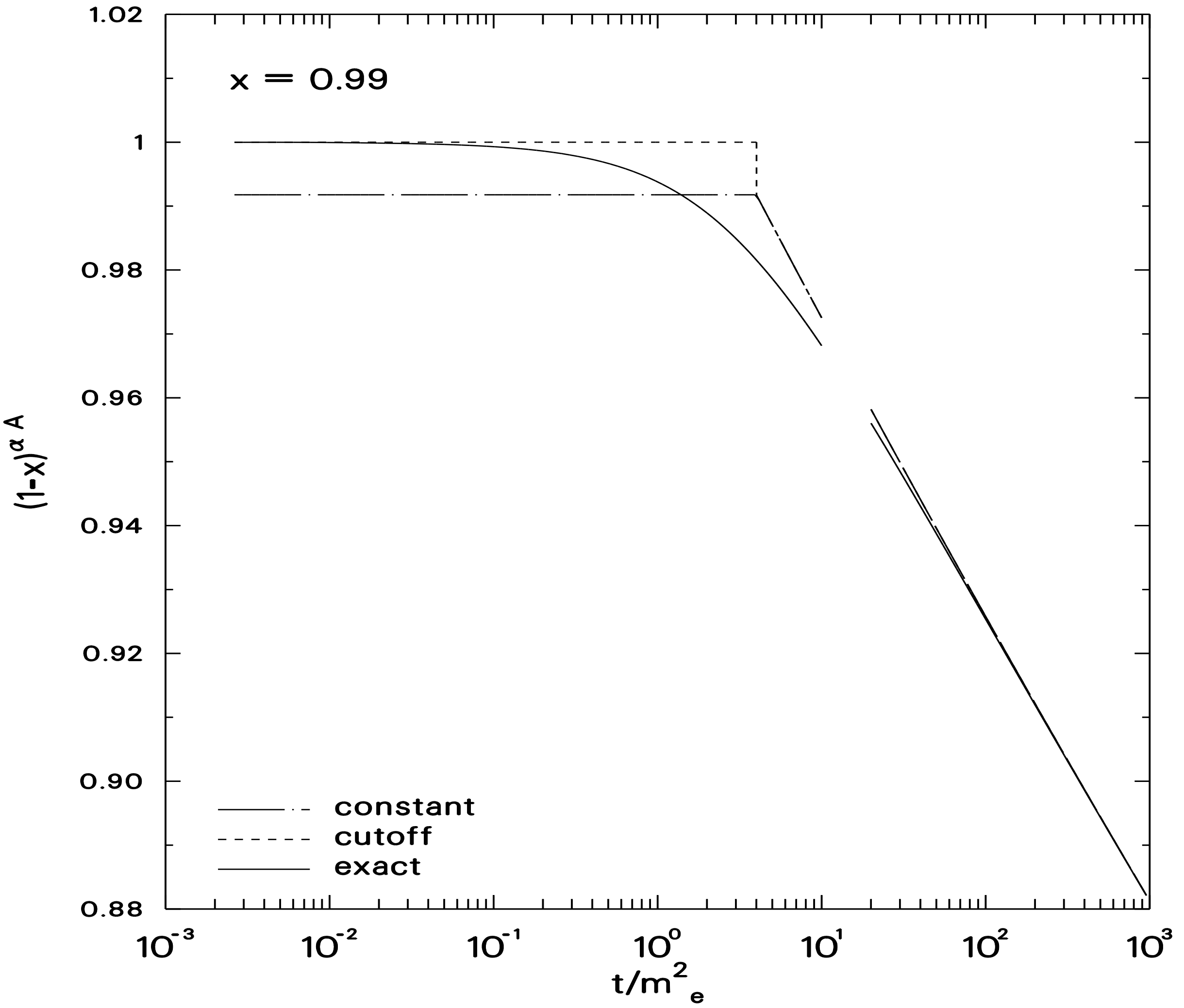
A.Ballestrero,G.P. work in progress

1. In $2 \to n$ **ANY** external charged leg i talks to **ALL** other charged legs,
2. each time with a **known scale** s_{ij},
3. and with a **known TOTAL** weight proportional to

$$x_i^{\alpha\left(A_1^i+\ldots+A_I^i\right)-1} / \, \Gamma\left(\alpha\,\left(A_1^i+\ldots+A_I^i\right)\right), \qquad 0 \le x_i \le 1$$

$\nearrow$ **N.B.** each A has the appropriate sign **in/out, part/antp**

4. where $I(i)$ is the number of **pairs** $< ij >$ with i fixed.



The IR exponent

$$\alpha A \;=\; \frac{2\,\alpha}{\pi}\left\{\frac{1+r^2}{1-r^2}\,\ln\frac{1}{r}-1\right\},\qquad \frac{m_e^2}{|t|}=\frac{r}{(1-r)^2}$$

$$-A(s,m_e)-A(t,m_e)+A(u,m_e)\;\overset{\text{not a guess}}{\sim}\;\frac{2}{\pi}\left[\ln\frac{st}{m_e^2 u}-1\right]$$

CONCLUSION for QED

- **SF**-language still **applicable** but
- **ONE** scale is not **enough** (quite obviously),
- each external **LEG** brings a **Structure Function**,
- since **ALL** charged legs talk to each other, **each SF** is **NOT** function of **ONE** **ad hoc** scale but
- **ALL** $< ij >$ scales enter into $\mathbf{SF}_i$, **exact** spectral-function is a convolution of **SFs**

$$
\begin{aligned}
E^{\mathrm{pair}\,<ij>}(K) &= \int d^4K'\,\Phi(K')E_{\mathrm{cp}}^{\mathrm{pair}\,<ij>}(K-K'),\\
\Phi(K) &= \frac{1}{(2\,\pi)^4}\int d^4x\,\exp\left\{i\,K\cdot x+\alpha\,\left(I-I_{\mathrm{cp}}\right)\right\}\\
&= \delta(K)+\mathcal{O}\left(\alpha\right)
\end{aligned}
$$

- **IR-finite** reminders and **virtual** parts can be added according to standard approach, etc etc

$\leadsto$

$$\left(\mathbf{Multi-Scale\ \ SF}\right)\;\times\;\left(\mathbf{1}+\mathcal{O}\left(\alpha\right)\right)$$