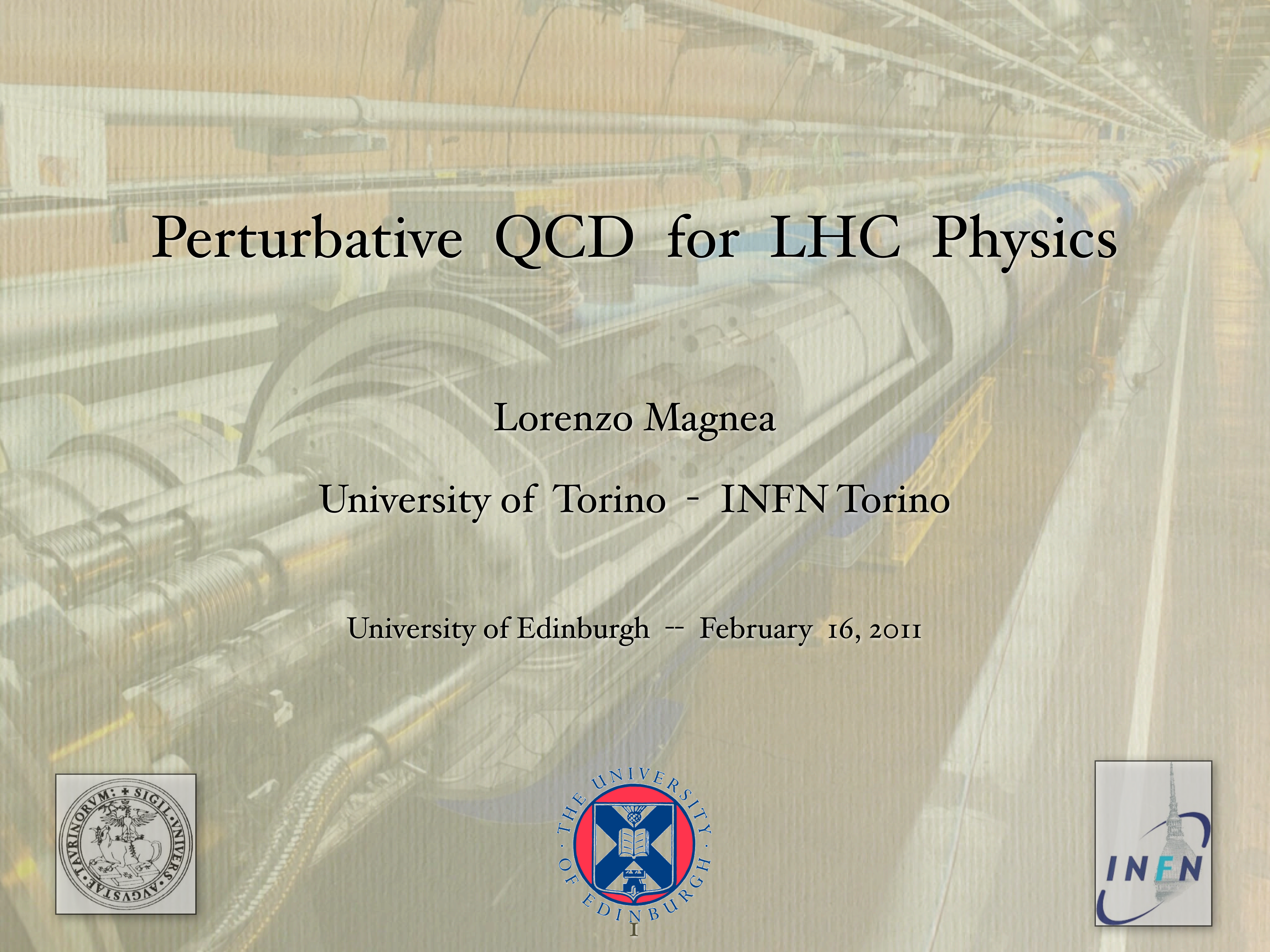


Perturbative QCD for LHC Physics

Lorenzo Magnea

University of Torino - INFN Torino

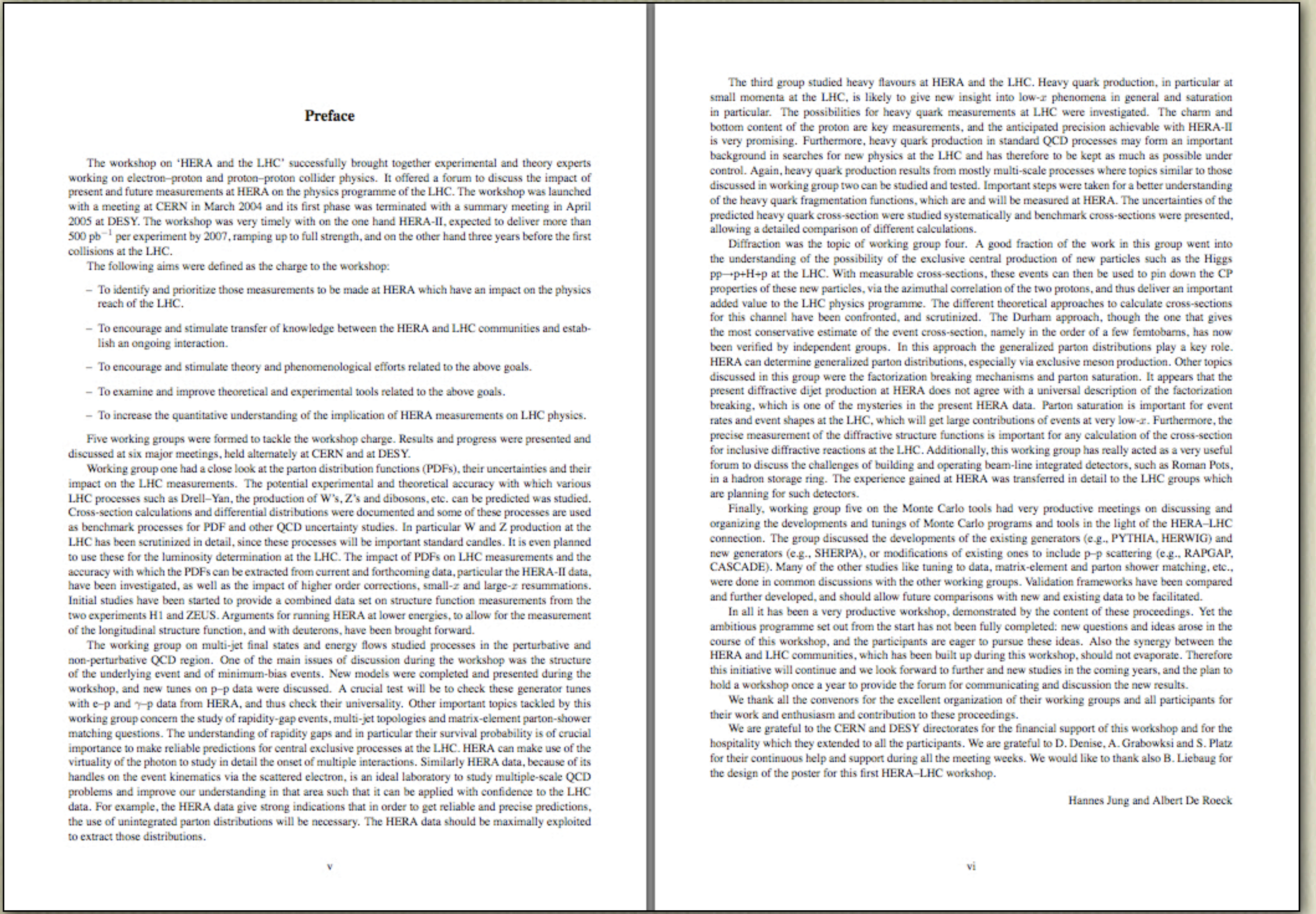
University of Edinburgh -- February 16, 2011

Outline

- Motivation and introduction
    - The why and how of PQCD at LHC
- Parton distributions
    - Picking quarks and gluons inside protons
- QCD Jets
    - The long road to the calorimeter
- Hard scattering cross sections
    - N^kLO and beyond: where we stand
- A Perspective

Motivation (I)

Motivation (I)



Preface

The workshop on 'HERA and the LHC' successfully brought together experimental and theory experts working on electron–proton and proton–proton collider physics. It offered a forum to discuss the impact of present and future measurements at HERA on the physics programme of the LHC. The workshop was launched with a meeting at CERN in March 2004 and its first phase was terminated with a summary meeting in April 2005 at DESY. The workshop was very timely with on the one hand HERA-II, expected to deliver more than 500 pb^{-1} per experiment by 2007, ramping up to full strength, and on the other hand three years before the first collisions at the LHC.

The following aims were defined as the charge to the workshop:

- To identify and prioritize those measurements to be made at HERA which have an impact on the physics reach of the LHC.
- To encourage and stimulate transfer of knowledge between the HERA and LHC communities and establish an ongoing interaction.
- To encourage and stimulate theory and phenomenological efforts related to the above goals.
- To examine and improve theoretical and experimental tools related to the above goals.
- To increase the quantitative understanding of the implication of HERA measurements on LHC physics.

Five working groups were formed to tackle the workshop charge. Results and progress were presented and discussed at six major meetings, held alternately at CERN and at DESY.

Working group one had a close look at the parton distribution functions (PDFs), their uncertainties and their impact on the LHC measurements. The potential experimental and theoretical accuracy with which various LHC processes such as Drell–Yan, the production of W's, Z's and dibosons, etc. can be predicted was studied. Cross-section calculations and differential distributions were documented and some of these processes are used as benchmark processes for PDF and other QCD uncertainty studies. In particular W and Z production at the LHC has been scrutinized in detail, since these processes will be important standard candles. It is even planned to use these for the luminosity determination at the LHC. The impact of PDFs on LHC measurements and the accuracy with which the PDFs can be extracted from current and forthcoming data, particular the HERA-II data, have been investigated, as well as the impact of higher order corrections, small-x and large-x resummations. Initial studies have been started to provide a combined data set on structure function measurements from the two experiments H1 and ZEUS. Arguments for running HERA at lower energies, to allow for the measurement of the longitudinal structure function, and with deuterons, have been brought forward.

The working group on multi-jet final states and energy flows studied processes in the perturbative and non-perturbative QCD region. One of the main issues of discussion during the workshop was the structure of the underlying event and of minimum-bias events. New models were completed and presented during the workshop, and new tunes on p–p data were discussed. A crucial test will be to check these generator tunes with e–p and γ–p data from HERA, and thus check their universality. Other important topics tackled by this working group concern the study of rapidity-gap events, multi-jet topologies and matrix-element parton-shower matching questions. The understanding of rapidity gaps and in particular their survival probability is of crucial importance to make reliable predictions for central exclusive processes at the LHC. HERA can make use of the virtuality of the photon to study in detail the onset of multiple interactions. Similarly HERA data, because of its handles on the event kinematics via the scattered electron, is an ideal laboratory to study multiple-scale QCD problems and improve our understanding in that area such that it can be applied with confidence to the LHC data. For example, the HERA data give strong indications that in order to get reliable and precise predictions, the use of unintegrated parton distributions will be necessary. The HERA data should be maximally exploited to extract those distributions.

The third group studied heavy flavours at HERA and the LHC. Heavy quark production, in particular at small momenta at the LHC, is likely to give new insight into low-x phenomena in general and saturation in particular. The possibilities for heavy quark measurements at LHC were investigated. The charm and bottom content of the proton are key measurements, and the anticipated precision achievable with HERA-II is very promising. Furthermore, heavy quark production in standard QCD processes may form an important background in searches for new physics at the LHC and has therefore to be kept as much as possible under control. Again, heavy quark production results from mostly multi-scale processes where topics similar to those discussed in working group two can be studied and tested. Important steps were taken for a better understanding of the heavy quark fragmentation functions, which are and will be measured at HERA. The uncertainties of the predicted heavy quark cross-section were studied systematically and benchmark cross-sections were presented, allowing a detailed comparison of different calculations.

Diffraction was the topic of working group four. A good fraction of the work in this group went into the understanding of the possibility of the exclusive central production of new particles such as the Higgs $pp\rightarrow p+H+p$ at the LHC. With measurable cross-sections, these events can then be used to pin down the CP properties of these new particles, via the azimuthal correlation of the two protons, and thus deliver an important added value to the LHC physics programme. The different theoretical approaches to calculate cross-sections for this channel have been confronted, and scrutinized. The Durham approach, though the one that gives the most conservative estimate of the event cross-section, namely in the order of a few femtobarns, has now been verified by independent groups. In this approach the generalized parton distributions play a key role. HERA can determine generalized parton distributions, especially via exclusive meson production. Other topics discussed in this group were the factorization breaking mechanisms and parton saturation. It appears that the present diffractive dijet production at HERA does not agree with a universal description of the factorization breaking, which is one of the mysteries in the present HERA data. Parton saturation is important for event rates and event shapes at the LHC, which will get large contributions of events at very low-x. Furthermore, the precise measurement of the diffractive structure functions is important for any calculation of the cross-section for inclusive diffractive reactions at the LHC. Additionally, this working group has really acted as a very useful forum to discuss the challenges of building and operating beam-line integrated detectors, such as Roman Pots, in a hadron storage ring. The experience gained at HERA was transferred in detail to the LHC groups which are planning for such detectors.

Finally, working group five on the Monte Carlo tools had very productive meetings on discussing and organizing the developments and tunings of Monte Carlo programs and tools in the light of the HERA–LHC connection. The group discussed the developments of the existing generators (e.g., PYTHIA, HERWIG) and new generators (e.g., SHERPA), or modifications of existing ones to include p–p scattering (e.g., RAPGAP, CASCADE). Many of the other studies like tuning to data, matrix-element and parton shower matching, etc., were done in common discussions with the other working groups. Validation frameworks have been compared and further developed, and should allow future comparisons with new and existing data to be facilitated.

In all it has been a very productive workshop, demonstrated by the content of these proceedings. Yet the ambitious programme set out from the start has not been fully completed: new questions and ideas arose in the course of this workshop, and the participants are eager to pursue these ideas. Also the synergy between the HERA and LHC communities, which has been built up during this workshop, should not evaporate. Therefore this initiative will continue and we look forward to further and new studies in the coming years, and the plan to hold a workshop once a year to provide the forum for communicating and discussion the new results.

We thank all the convenors for the excellent organization of their working groups and all participants for their work and enthusiasm and contribution to these proceedings.

We are grateful to the CERN and DESY directorates for the financial support of this workshop and for the hospitality which they extended to all the participants. We are grateful to D. Denise, A. Grabowksi and S. Platz for their continuous help and support during all the meeting weeks. We would like to thank also B. Liebaug for the design of the poster for this first HERA–LHC workshop.

Hannes Jung and Albert De Roeck

Motivation (II)

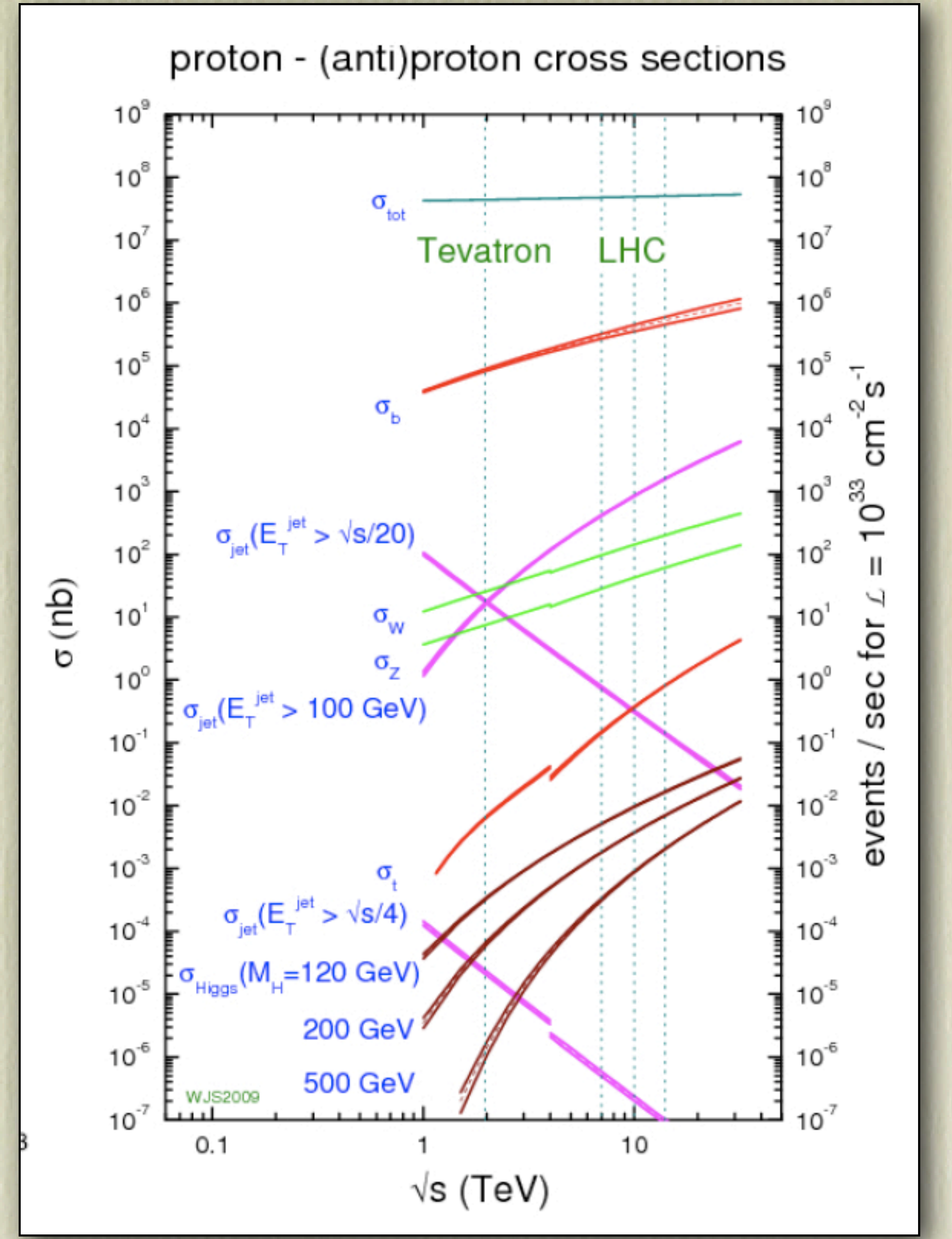
Motivation (II)

LHC is a

large

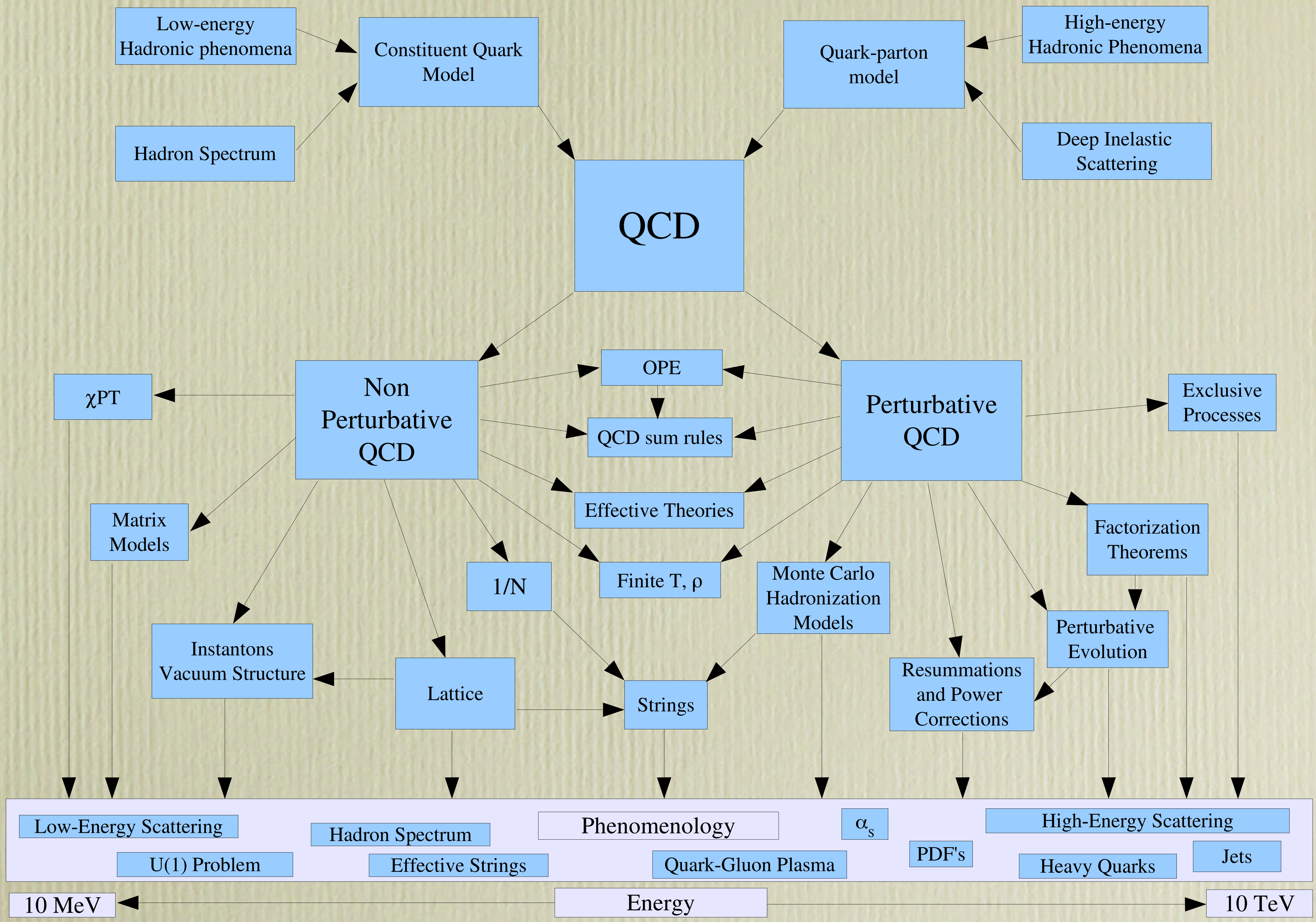
HADRON

collider



Motivation (III) and Disclaimer

Motivation (III) and Disclaimer



Low-energy Hadronic phenomena

Constituent Quark Model

Quark-parton model

High-energy Hadronic Phenomena

Hadron Spectrum

Deep Inelastic Scattering

QCD

χPT

Non Perturbative QCD

OPE

QCD sum rules

Perturbative QCD

Exclusive Processes

Effective Theories

Matrix Models

Factorization Theorems

1/N

Finite T, ρ

Monte Carlo Hadronization Models

Perturbative Evolution

Instantons Vacuum Structure

Lattice

Strings

Resummations and Power Corrections

Low-Energy Scattering

Hadron Spectrum

Phenomenology

α_s

High-Energy Scattering

U(1) Problem

Effective Strings

Quark-Gluon Plasma

PDF's

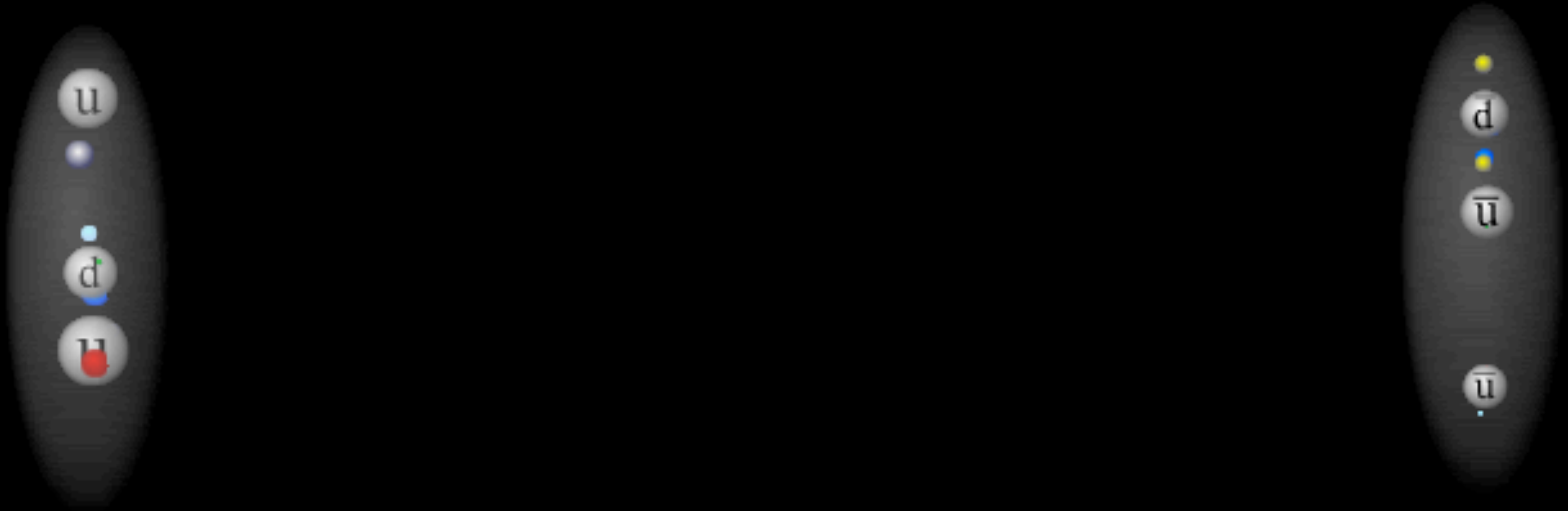
Heavy Quarks

Jets

10 MeV

Energy

10 TeV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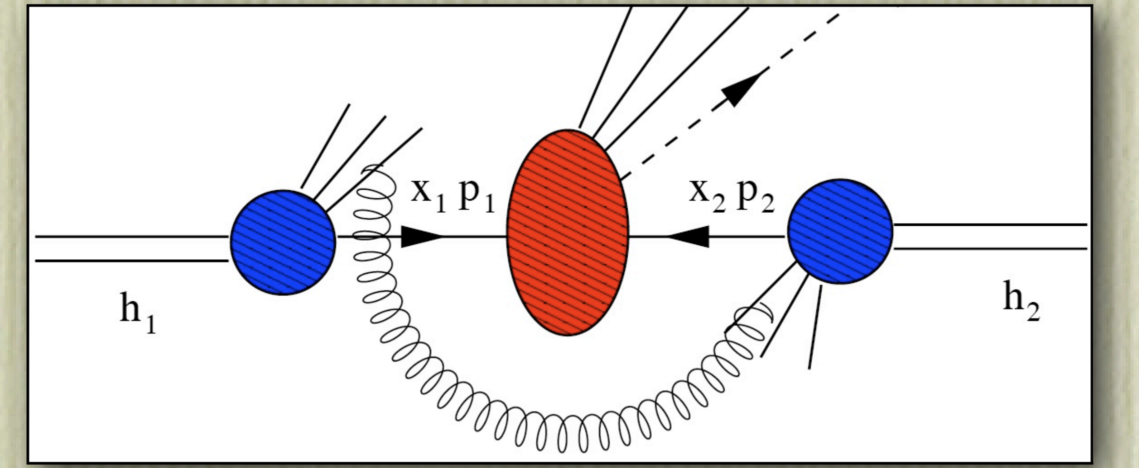
PQCD Master Formulas

$$\sigma_H^{h_1 h_2}(S, Q^2) = \sum_{a,b} \int_0^1 dx_1\, dx_2\ f_{a/h_1}(x_1, \mu_f)\ f_{b/h_2}(x_2, \mu_f)\ \widehat{\sigma}_P^{ab}\left(x_1 x_2 S, Q^2, \mu_f\right)$$

$$\mu_f \frac{\partial}{\partial \mu_f} f_{a/h}(x, \mu_f) = \sum_b \int_x^1 \frac{dy}{y}\, P_{ab}\left(\frac{x}{y}, \alpha_s(\mu^2)\right)\ f_{b/h}(y, \mu_f)$$



- Factorization proofs are highly non-trivial.
- Soft gluons rearrange partons before collision.
- Correlations are suppressed by powers of Q.

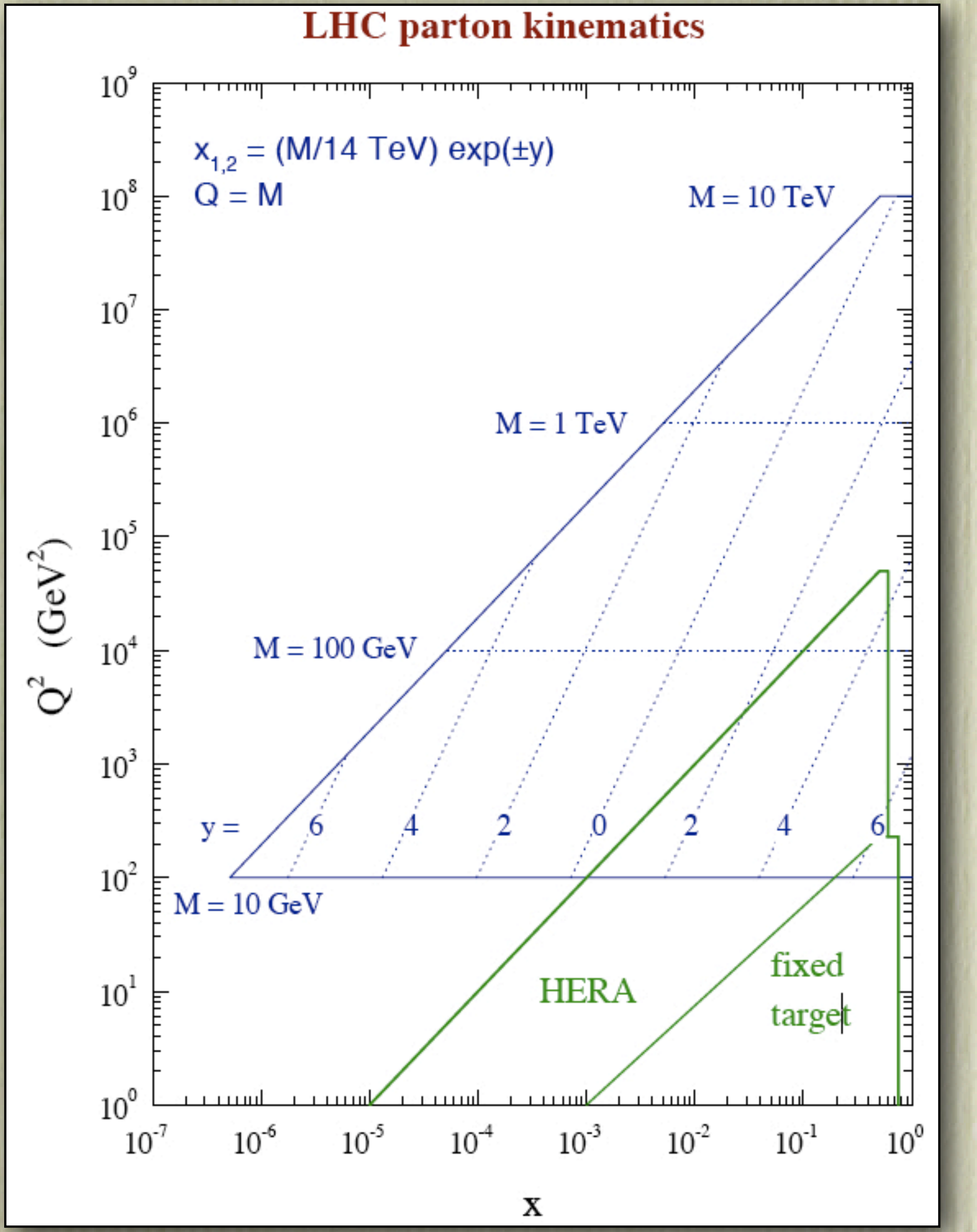
Strategy

- Choose a factorization scheme.
- Compute $\widehat{\sigma}_P^{ab}(\mu_0)$ for process A.
- Measure $\sigma_H(Q \sim \mu_0)$ for process A.
- Determine $f_{a/h}(\mu_0)$.
- Evolve $f_{a/h}(\mu_0)$ to the scale μ_1.
- Compute $\widehat{\sigma}_P^{ab}(\mu_1)$ for process B.
- Predict $\sigma_H(Q \sim \mu_1)$ for process B.

QCD to understand the initial state

Parton Distributions

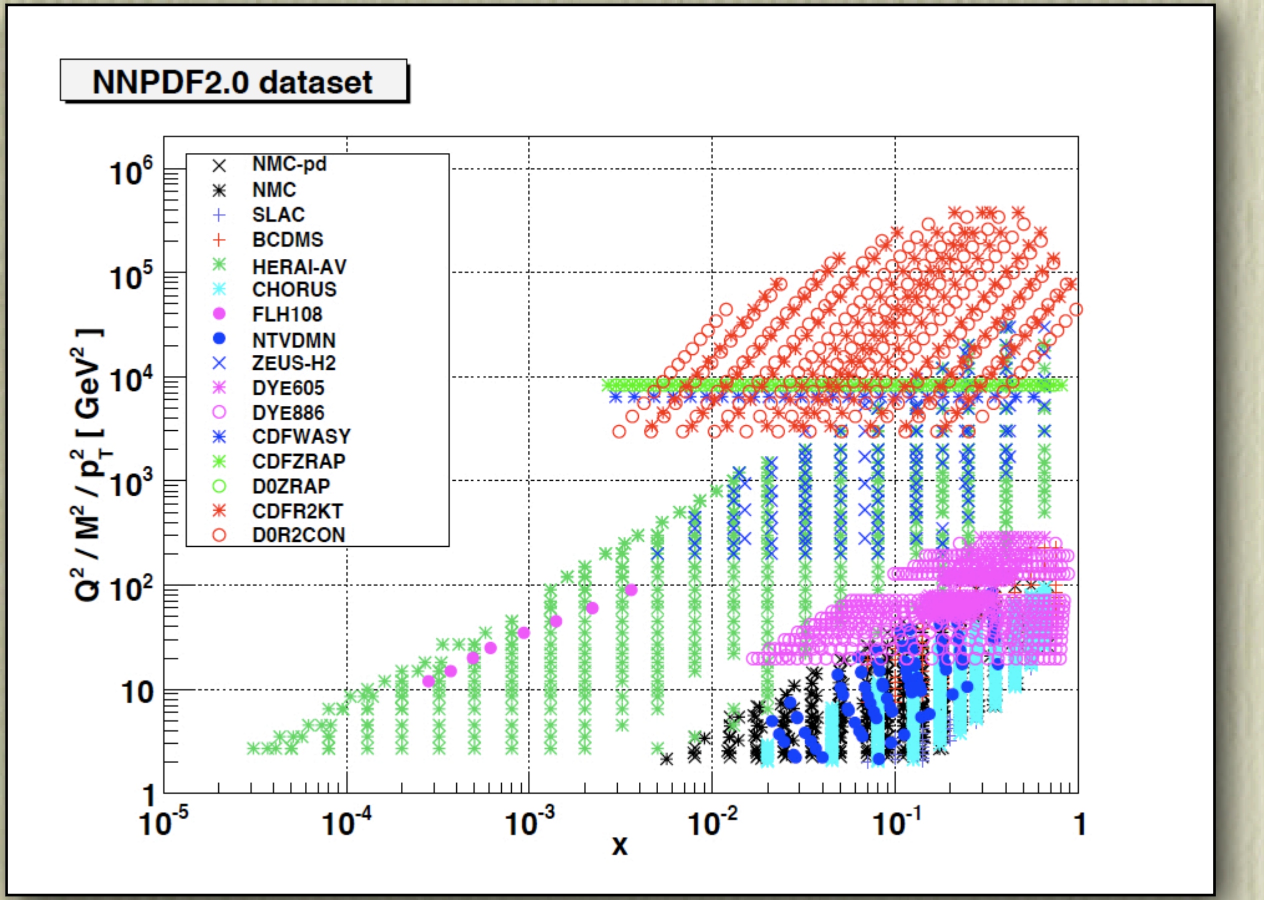
Parton kinematics



- Large mass states are made at large x and central rapidities.
  - ✦ ... though not a light Higgs!
- Small x means limited Q^2.
  - ✦ ... which implies big uncertainties!
- Altarelli-Parisi evolution is up, feeding from the right.
- Precise evolution codes are needed and available
  - ✦ ... splitting functions are known at three loops!
- LHC will measure parton distributions on its own

Partons from data

- Different data sets determine different combinations of parton distributions.
- Global constraints imposed by sum rules (momentum, charge, ...).
- Strategy: “global fit”. Players: CTEQ, MSTW, NNPDF, ABKM, ...
- A highly nontrivial statistical problem!



Data set for NNPDF 2.0 parton fit.

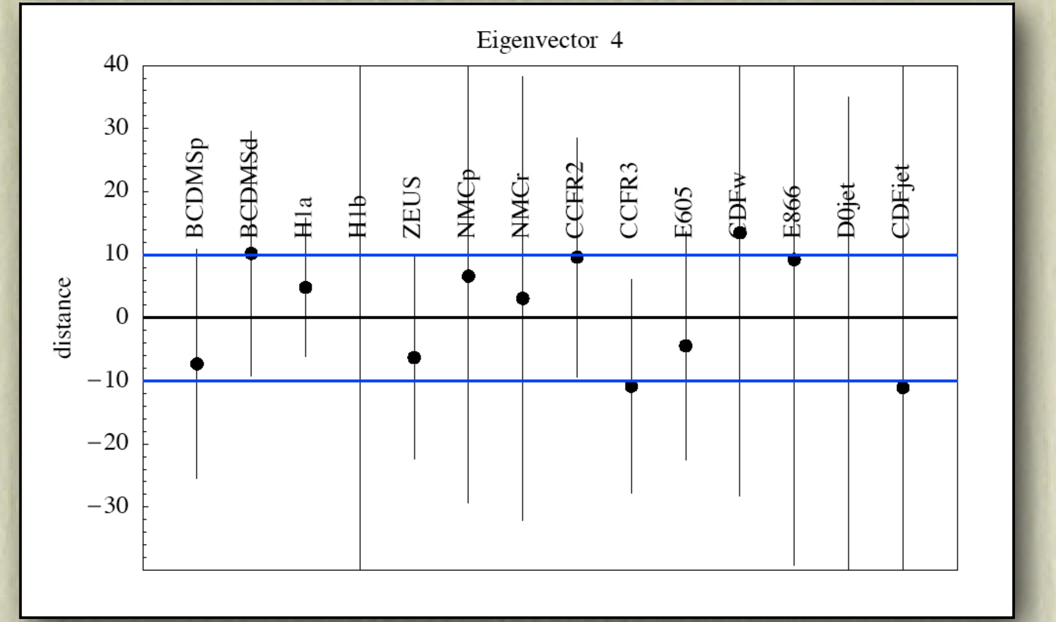
- Photon DIS determines quark + antiquark combinations.
- W DIS determines flavor decomposition.
- Scaling violations determine small-x gluons.
- High p_T jets determine large-x gluons.
- Drell-Yan p-n asymmetry determines antiquark asymmetry.
- W asymmetry determines quark asymmetry.
- Heavy quark production and evolution determine heavy quark pdfs.

Parton parametrizations: standard approach

- **Select** a functional form for each distribution.

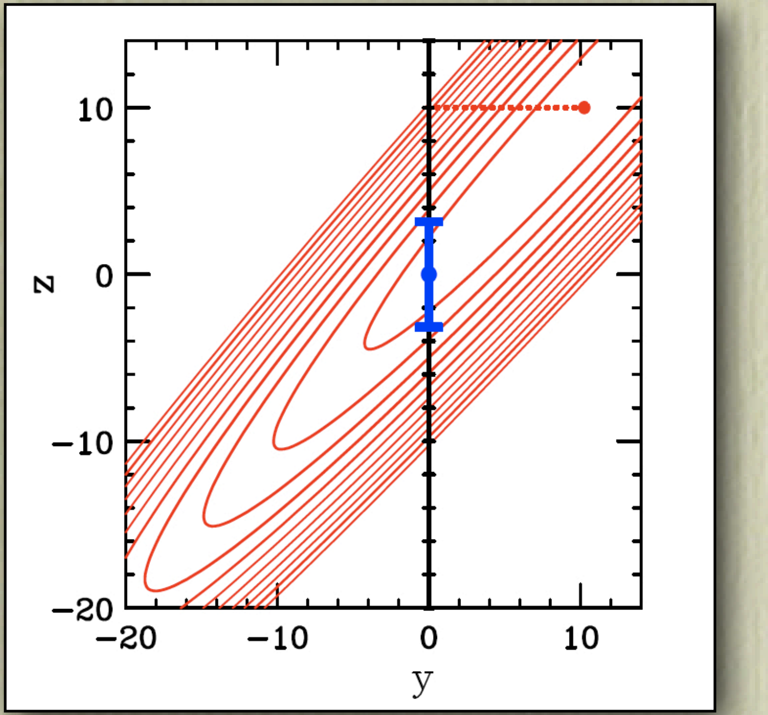
$$f_{a/h}\left(x,\mu_0^2\right) = x^{\alpha}\,(1-x)^{\beta}\,P(x,\gamma_i)$$

- **Fit** the parameters to experimental data.
- **Typically**: 7 functions, 20-30 parameters, ~3000 experimental points.
- **Characterize** fit based on experimental errors and correlations.



CTEQ tolerance criterion for hessian eigenvalues

- **Perform** standard χ^2 analysis with Hessian matrix.
- **Adjust** $T^2 = \Delta\chi^2$ so that PDF's agree with all experiments to 90% confidence level
  - Warning: requires $\Delta\chi^2 >> 1$!
- Problems:
  - χ^2 analysis requires **gaussian** statistics
  - different experiments are not always **compatible**
  - function space **not covered** by parametrization

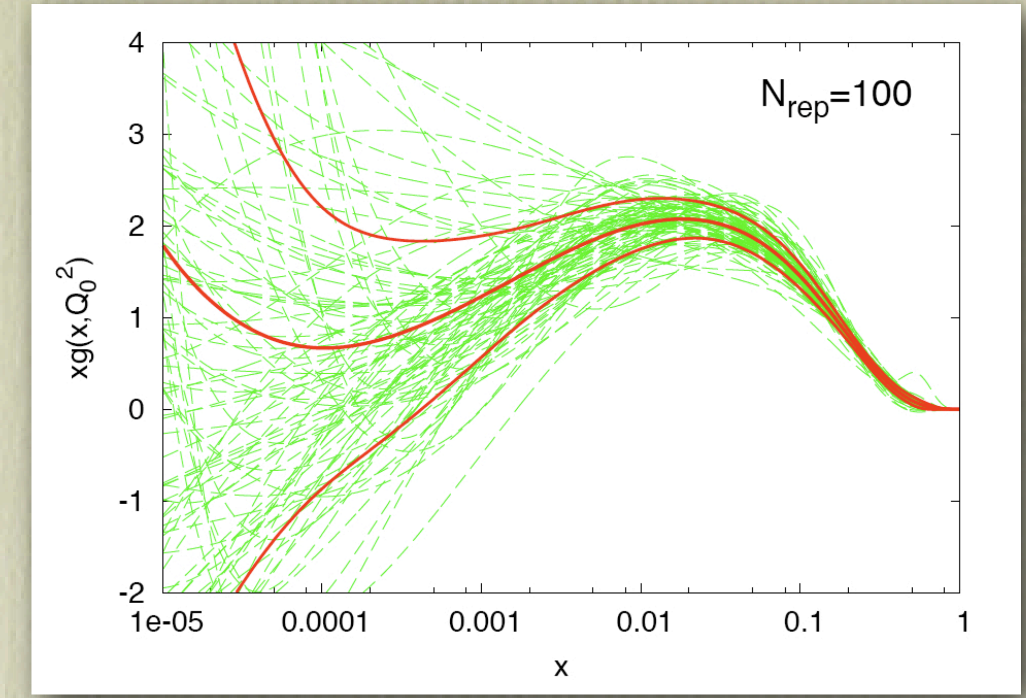


Parametrization rigidity (Pumplin 09)

A novel approach: NNPDF

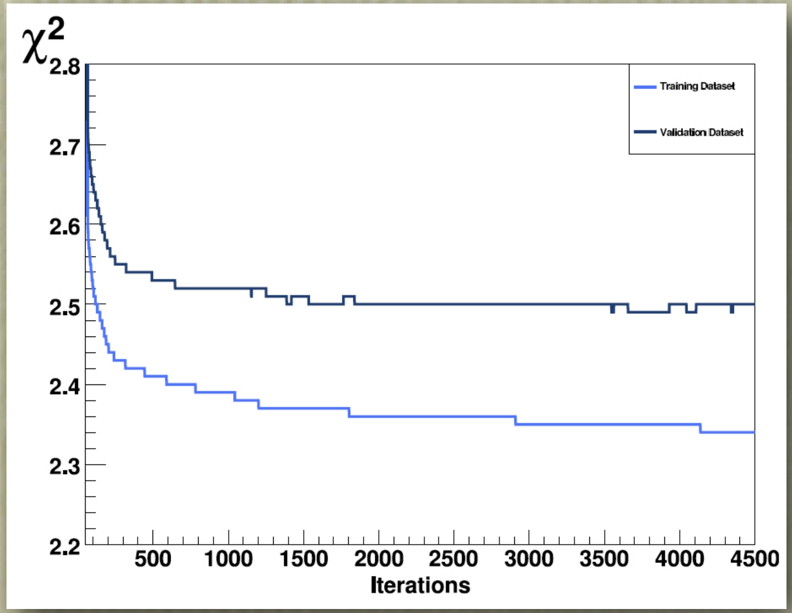
GOAL: Provide a faithful representation of the probability distribution in the functional space of parton distributions.

- Generate N_{rep} Monte Carlo copies of experimental data reproducing central values, errors and correlations.
- Train N_{rep} neural networks, one on each copy of the data.
- Compute any function of PDF’s with its error.



$$\langle \mathcal{F}\left[f_a(x)\right]\rangle = \int \left[\mathcal{D}f_a\right] \mathcal{F}\left[f_a(x)\right] \mathcal{P}\left[f_a(x)\right] \quad \Rightarrow \quad \frac{1}{N_{\rm rep}} \sum_{k=1}^{N_{\rm rep}} \mathcal{F}\left[f_a^{(k,\,{\rm net})}(x)\right]$$

$$\sigma_{\mathcal{F}} = \sqrt{\langle \mathcal{F}\left[f_a(x)\right]^2\rangle - \langle \mathcal{F}\left[f_a(x)\right]\rangle^2}$$



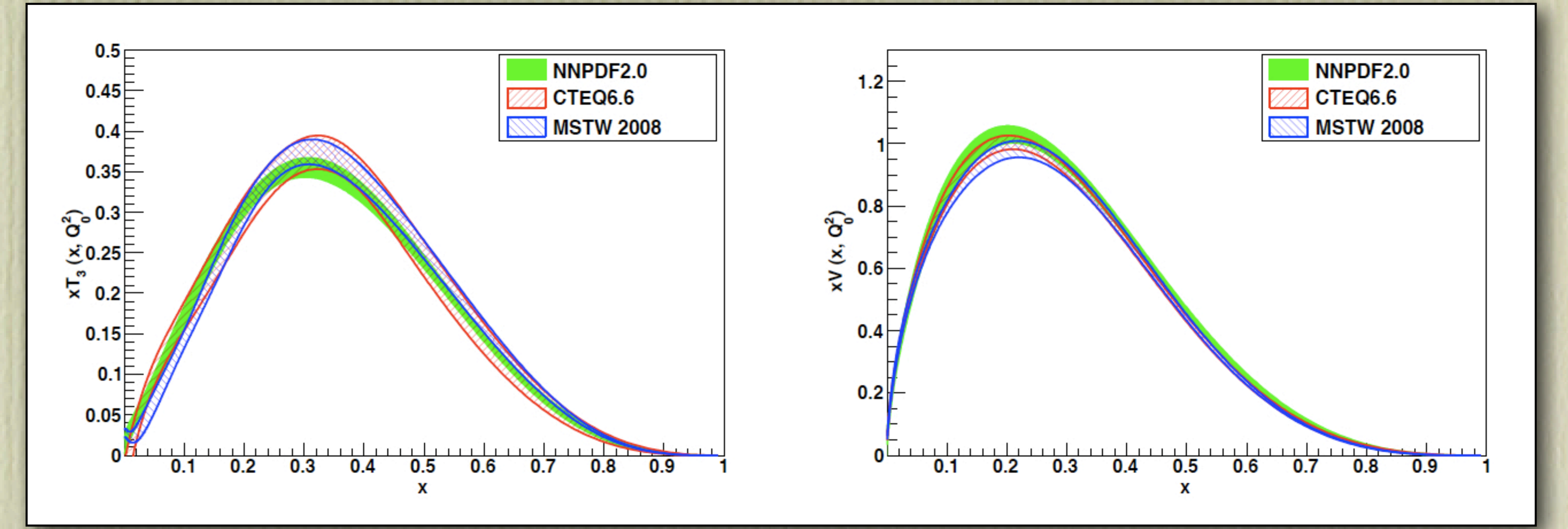
FEATURES:

Unbiased sampling of functional space.

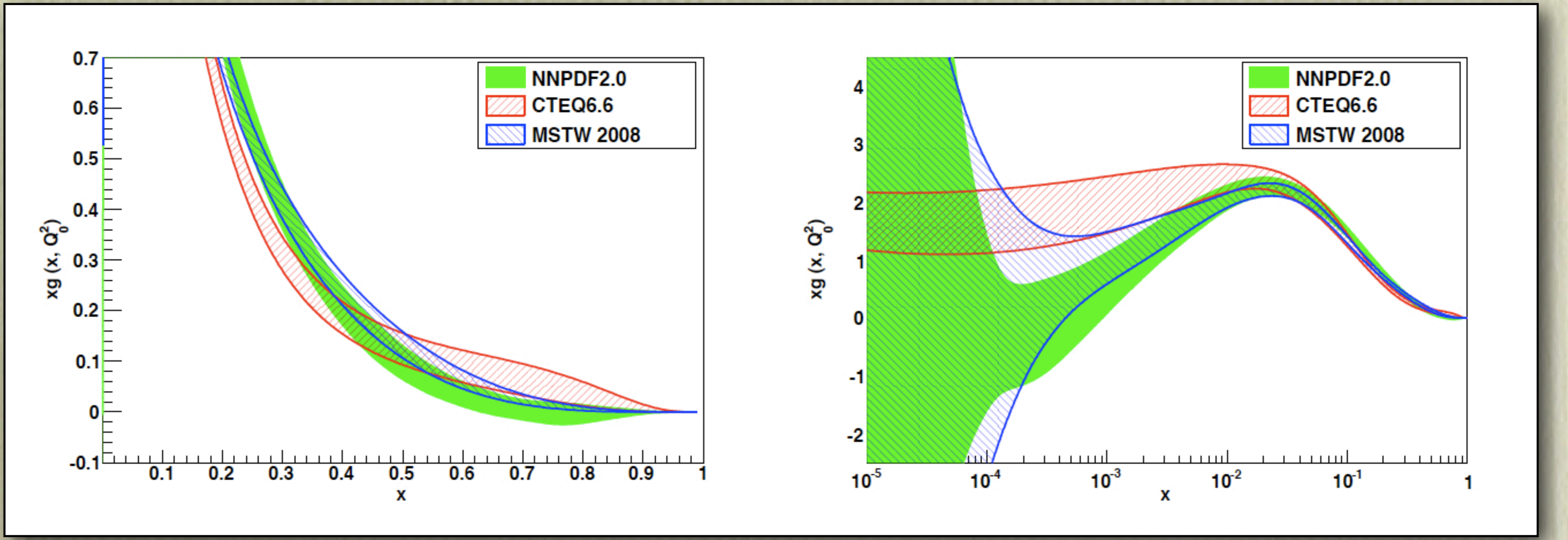
Very many parameters (250 ~ ∞), allow fitting “any function”

Cross-validation prevents overlearning

Sample distributions with error bands



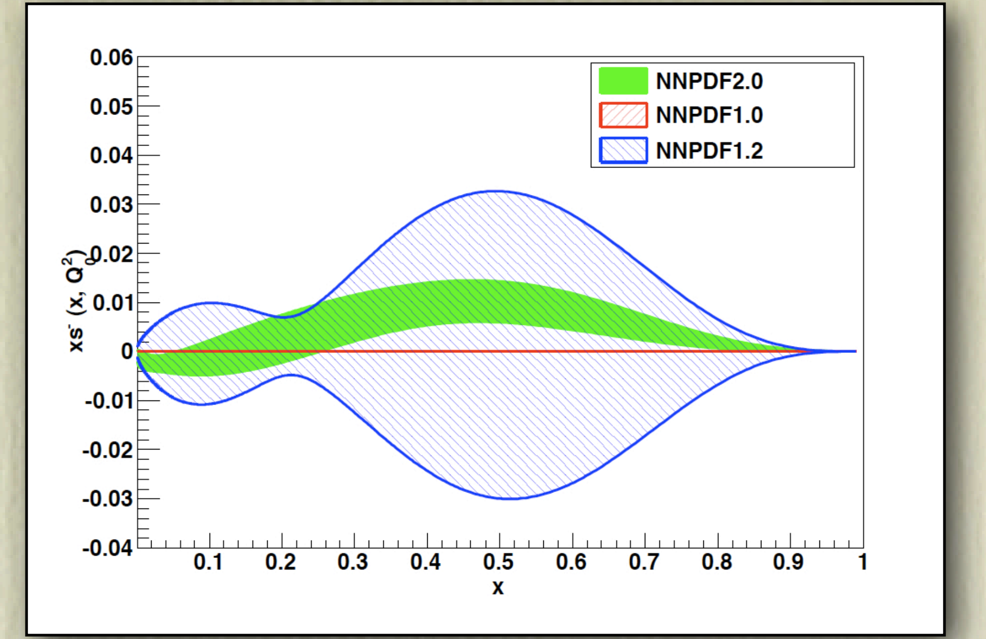
Triplet and valence quark distributions with uncertainties, linear scale (NNPDF).



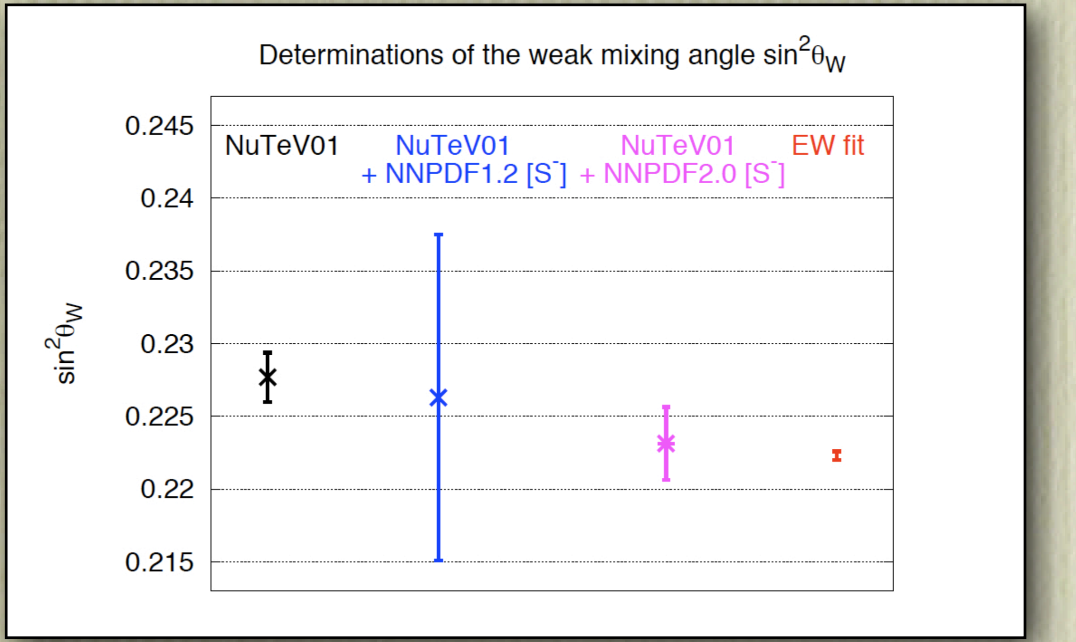
Gluon distributions with uncertainties, log and linear scale (NNPDF).

Strange distributions

- The strange quark distribution is important.
  - It drives the uncertainty on σ_W/σ_Z.
  - It influences the determination of CKM parameters V_{cs} and V_{cd}.
  - It affects the NuTeV anomaly.
- Previously assumed proportional to light antiquarks, with vanishing asymmetry.
- Now determined using neutrino DIS (NuTeV) and fixed-target Drell-Yan data.



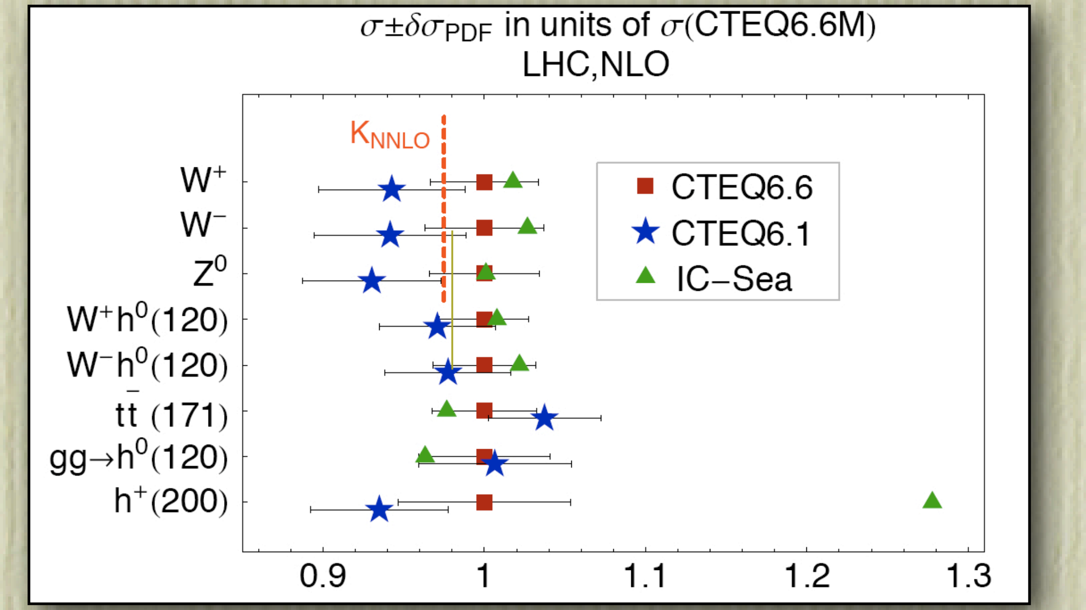
NNPDF determinations of the strange asymmetry



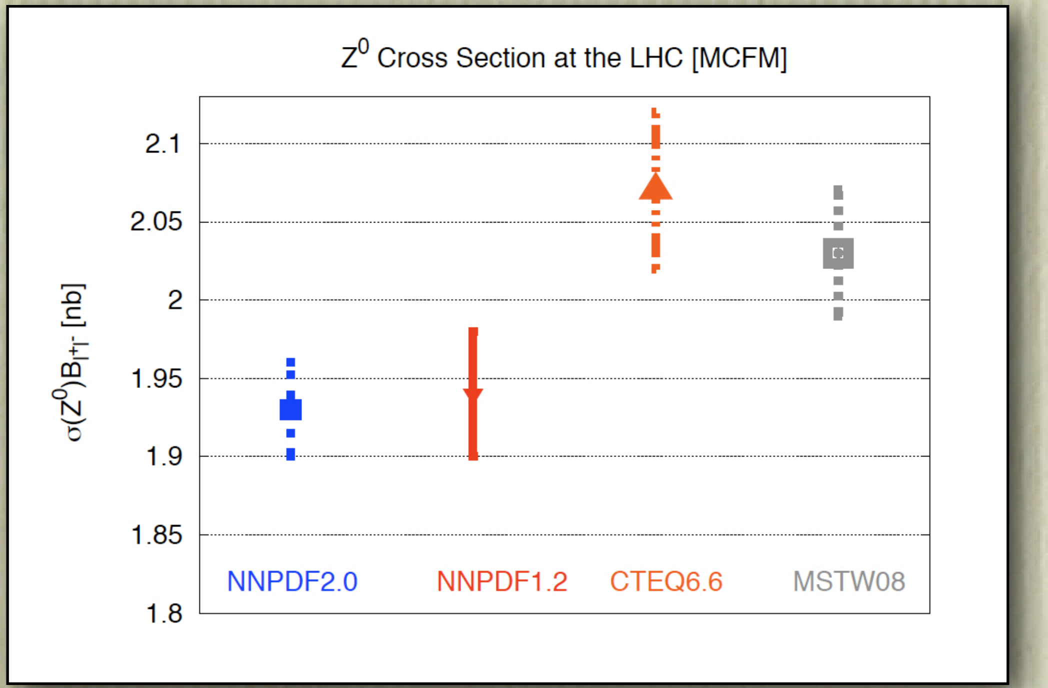
The determination of the strange asymmetry solves the NuTev anomaly.

- There is now 2σ evidence for a non-vanishing strange asymmetry
- Large uncertainty before inclusion of DY data nullified the NuTeV anomaly
- Improved determination with DY data brings NuTev determination of Weak angle in line with EW fit.

Caveat Emptor



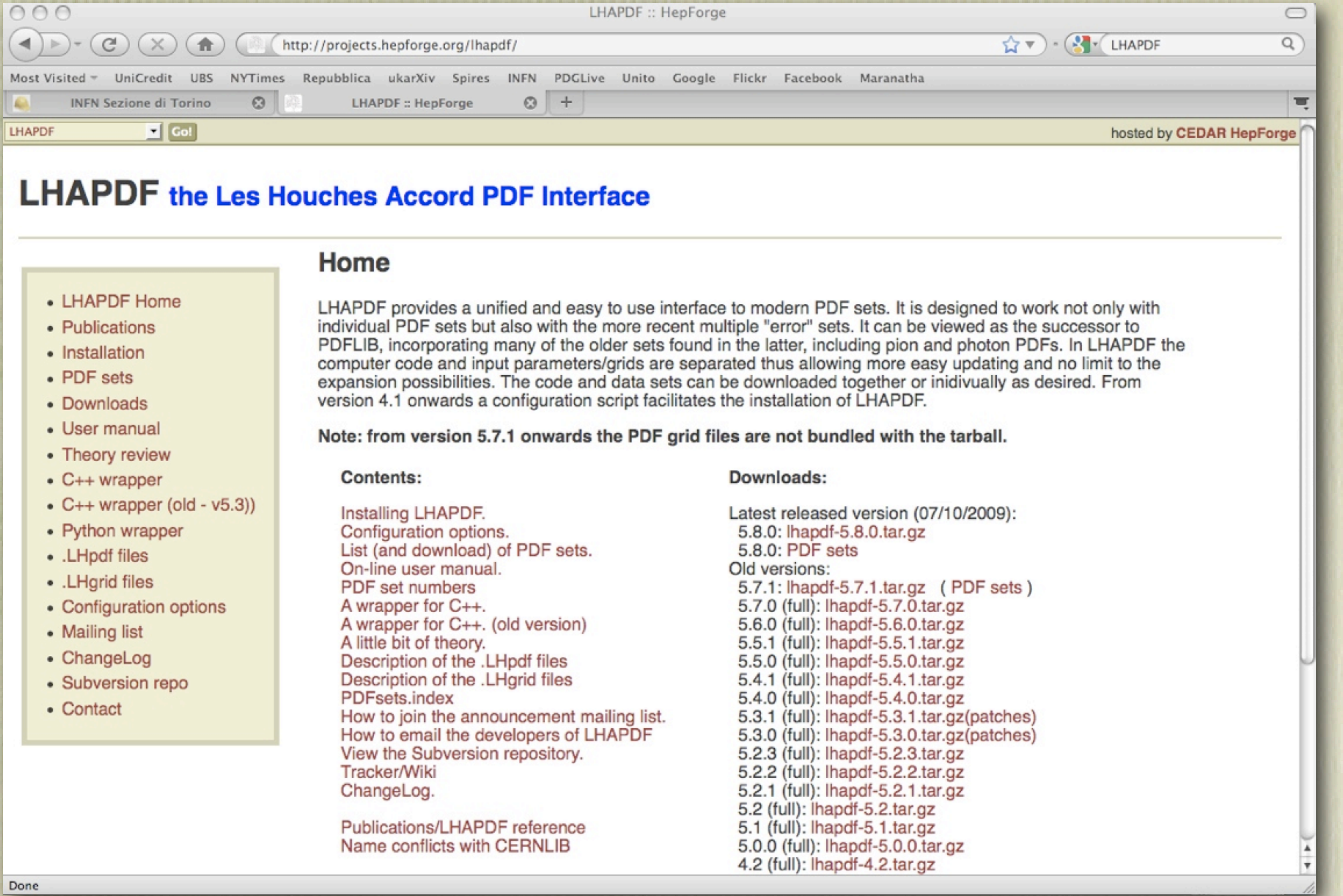
Comparison of different CTEQ pdfs for LHC processes



Different predictions for Z production at LHC

- Parton distributions are the dominant uncertainty for “standard candle” processes such as W or Z production at LHC.
- The expected uncertainties at LHC are a few percent.
- A technical change by CTEQ in the treatment of quark mass thresholds (“ZM-VFN” to “GM-ACOT”) moved the cross section by 2−3 σ.
- Smaller heavy quark PDFs by sum rules imply larger light quark PDFs: these make W’s.
- MSTW reported a similar increase for related (though not identical) reasons.
- This illustrates the complexity and impact of PDF uncertainties: they must be well understood!

A parton distribution interface



LHAPDF the Les Houches Accord PDF Interface

- LHAPDF Home
- Publications
- Installation
- PDF sets
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- User manual
- Theory review
- C++ wrapper
- C++ wrapper (old - v5.3))
- Python wrapper
- .LHpdf files
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- Configuration options
- Mailing list
- ChangeLog
- Subversion repo
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Home

LHAPDF provides a unified and easy to use interface to modern PDF sets. It is designed to work not only with individual PDF sets but also with the more recent multiple "error" sets. It can be viewed as the successor to PDFLIB, incorporating many of the older sets found in the latter, including pion and photon PDFs. In LHAPDF the computer code and input parameters/grids are separated thus allowing more easy updating and no limit to the expansion possibilities. The code and data sets can be downloaded together or inidivually as desired. From version 4.1 onwards a configuration script facilitates the installation of LHAPDF.

Note: from version 5.7.1 onwards the PDF grid files are not bundled with the tarball.

Contents:

Installing LHAPDF.
Configuration options.
List (and download) of PDF sets.
On-line user manual.
PDF set numbers
A wrapper for C++.
A wrapper for C++. (old version)
A little bit of theory.
Description of the .LHpdf files
Description of the .LHgrid files
PDFsets.index
How to join the announcement mailing list.
How to email the developers of LHAPDF
View the Subversion repository.
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Publications/LHAPDF reference
Name conflicts with CERNLIB

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5.8.0: lhapdf-5.8.0.tar.gz
5.8.0: PDF sets
Old versions:
5.7.1: lhapdf-5.7.1.tar.gz (PDF sets)
5.7.0 (full): lhapdf-5.7.0.tar.gz
5.6.0 (full): lhapdf-5.6.0.tar.gz
5.5.1 (full): lhapdf-5.5.1.tar.gz
5.5.0 (full): lhapdf-5.5.0.tar.gz
5.4.1 (full): lhapdf-5.4.1.tar.gz
5.4.0 (full): lhapdf-5.4.0.tar.gz
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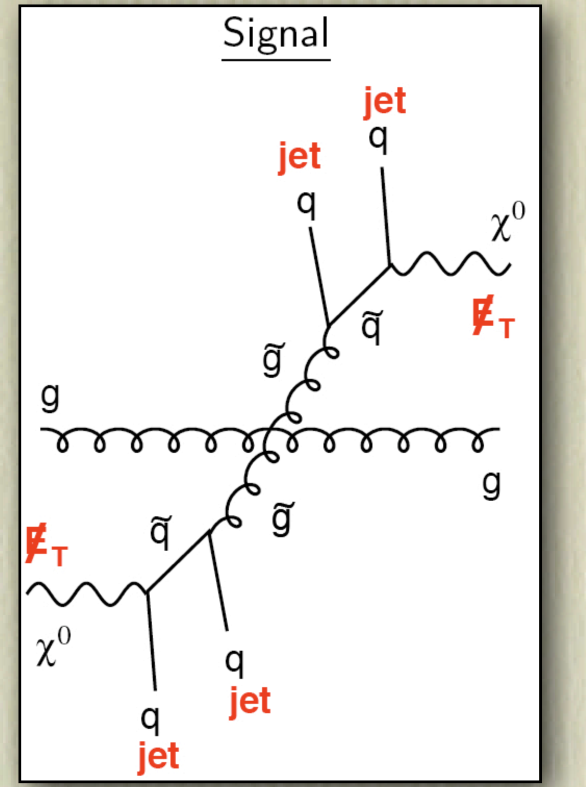
4.2 (full): lhapdf-4.2.tar.gz

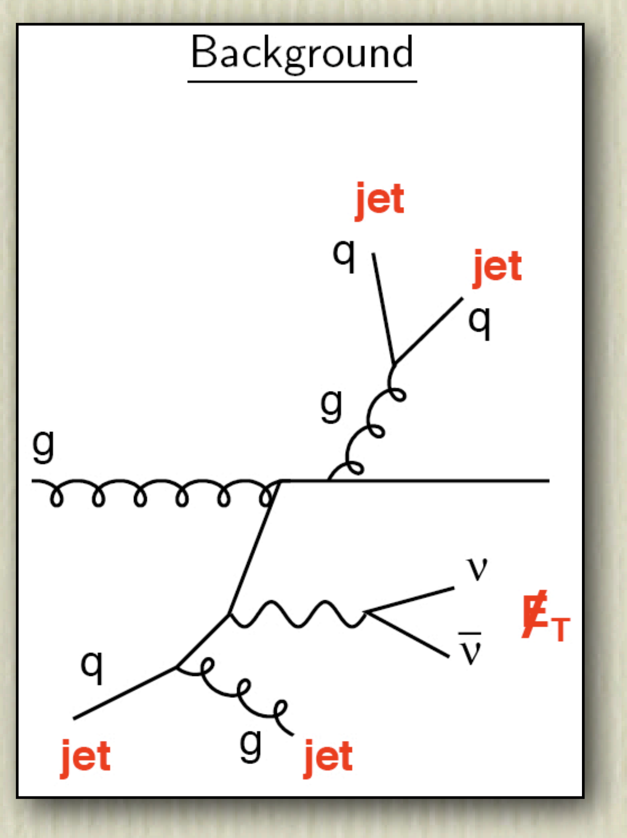
QCD for the observables in the final state

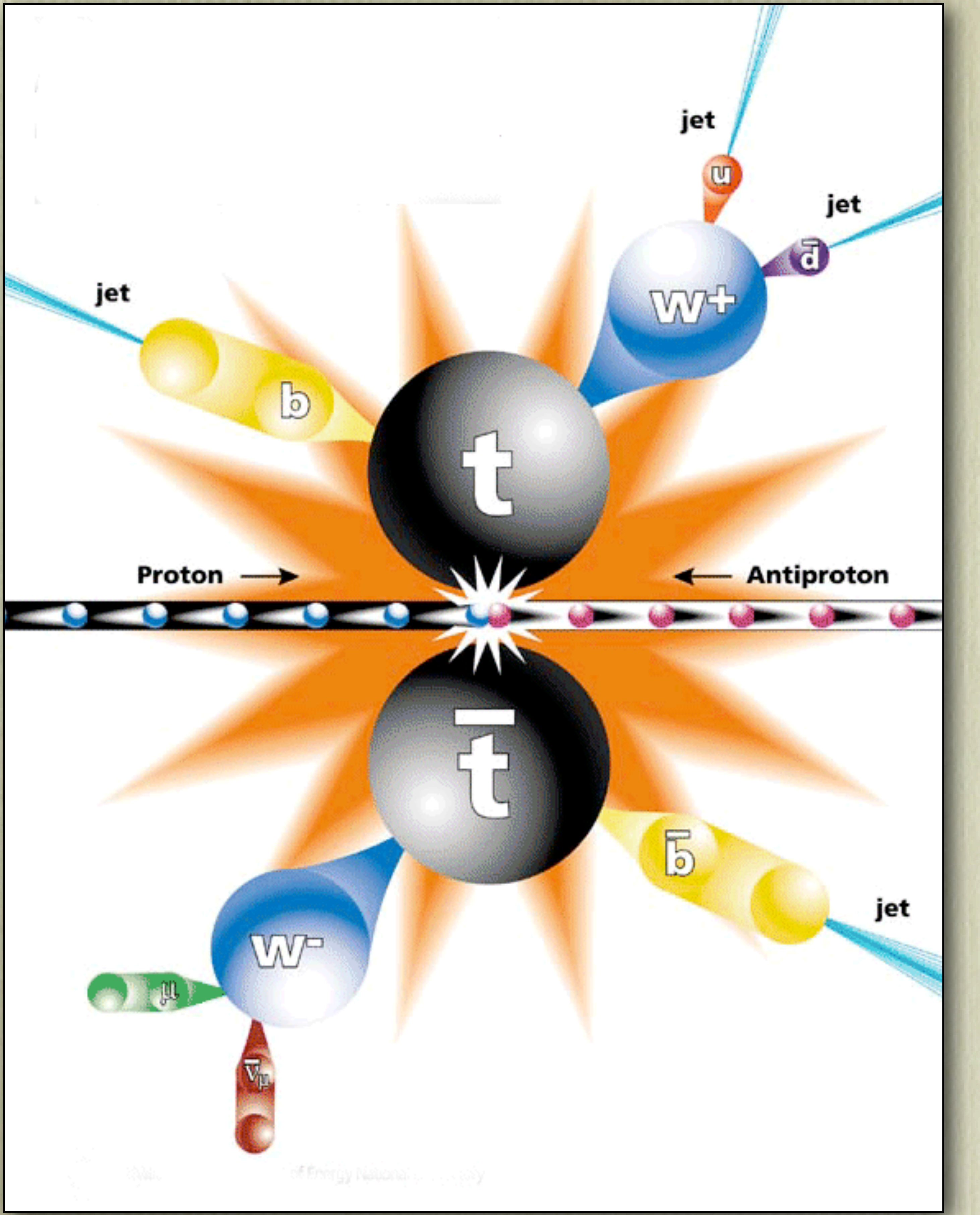
Jets

Jets at Tevatron and LHC

- Jets are ubiquitous at hadron colliders
  - the most common high-p_T final state
- Jets need to be understood in detail
  - top mass, Higgs searches, QCD studies, new particle cascades
- Jets at LHC are numerous and complicated
  - top-antitop-Higgs to 8 jet final state ... , underlying event, pileup ...
- Jets are inherently ambiguous in QCD
  - no unique link between hard parton and jet
- Jets are theoretically interesting
  - Infrared and collinear safety, resummations, hadronization ...

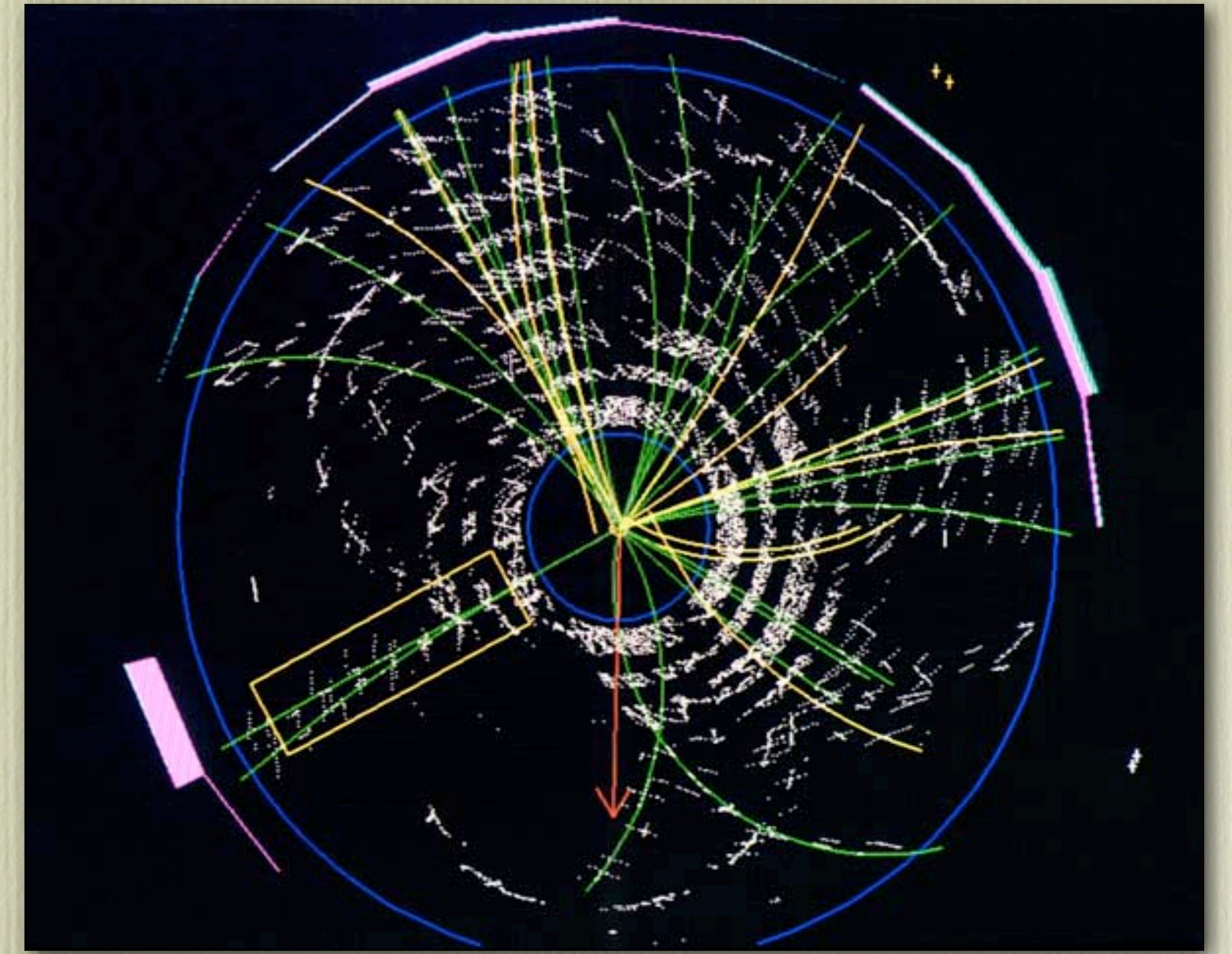






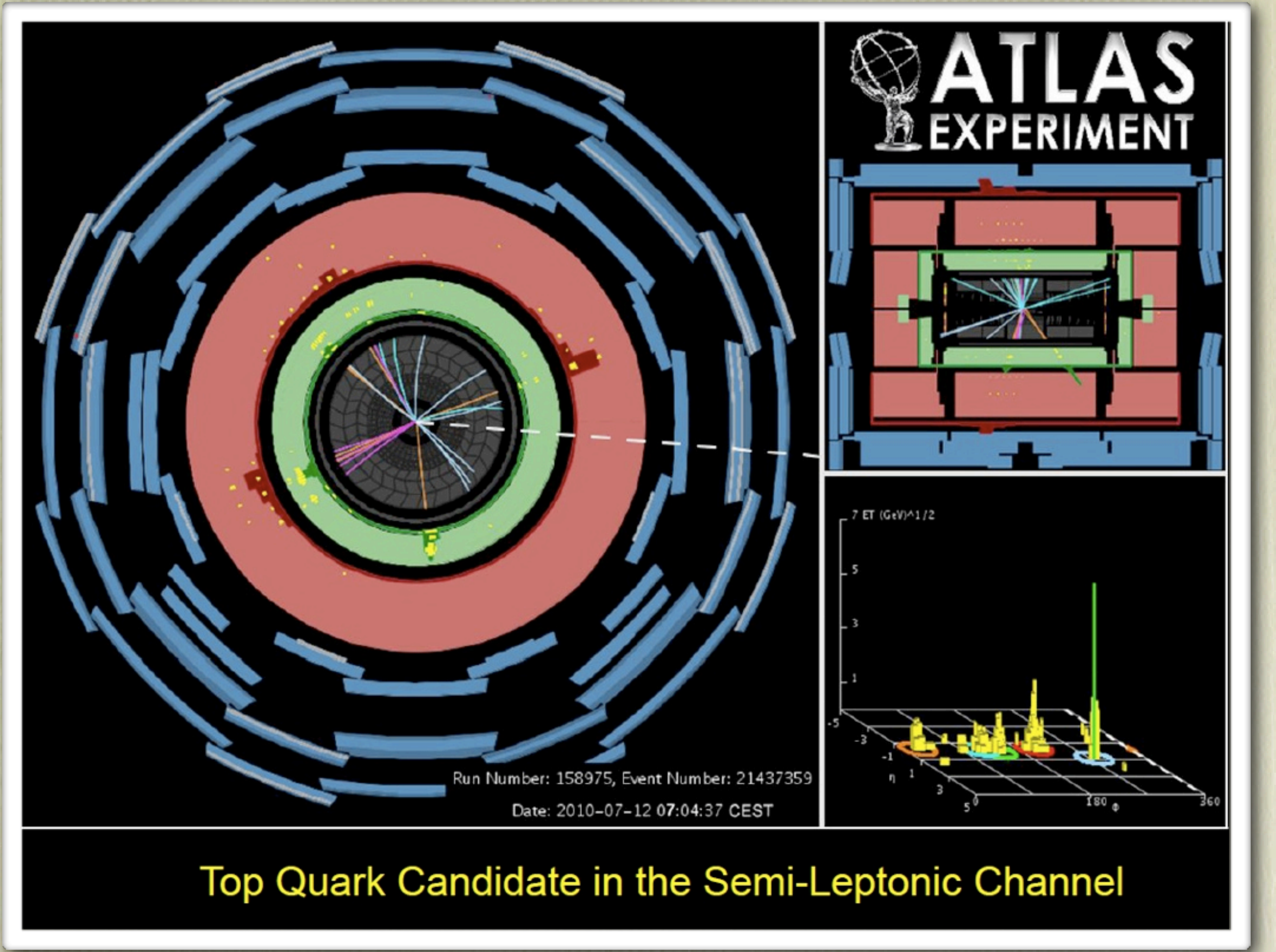
Collisions at Tevatron: a cartoon

- A proton-antiproton collision produces a top-antitop pair.
- Each top quark decays into a bottom quark and a W boson.
- One of the W bosons decays hadronically, into a pair of quarks.
- The other W boson decays into leptons, yielding a muon with its antineutrino.
- All quarks hadronize into jets of colorless particles (pions, kaons, protons ...).
- The observed final state consists of four jets, one lepton, and missing energy.



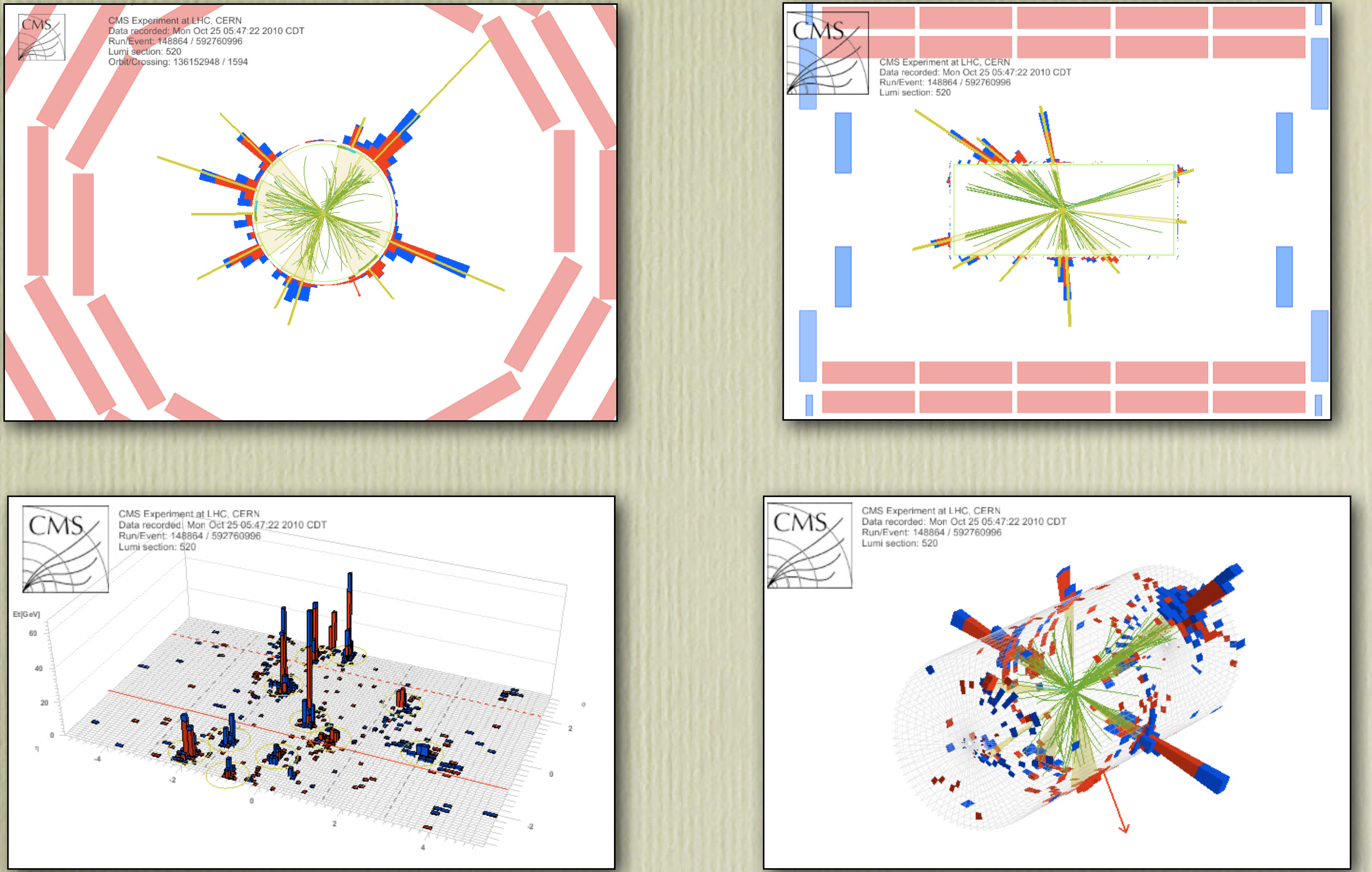
Collisions at Tevatron: real life

Real life collisions at LHC ...



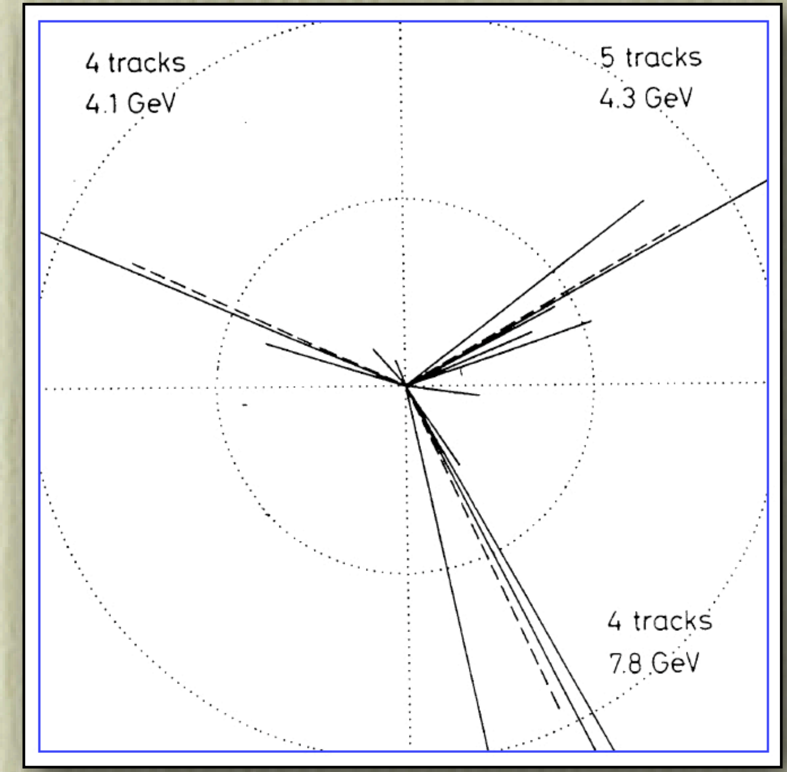
Top Quark Candidate in the Semi-Leptonic Channel

… involve many, many jets.



Different visualizations of a candidate ten jet event recorded at CMS

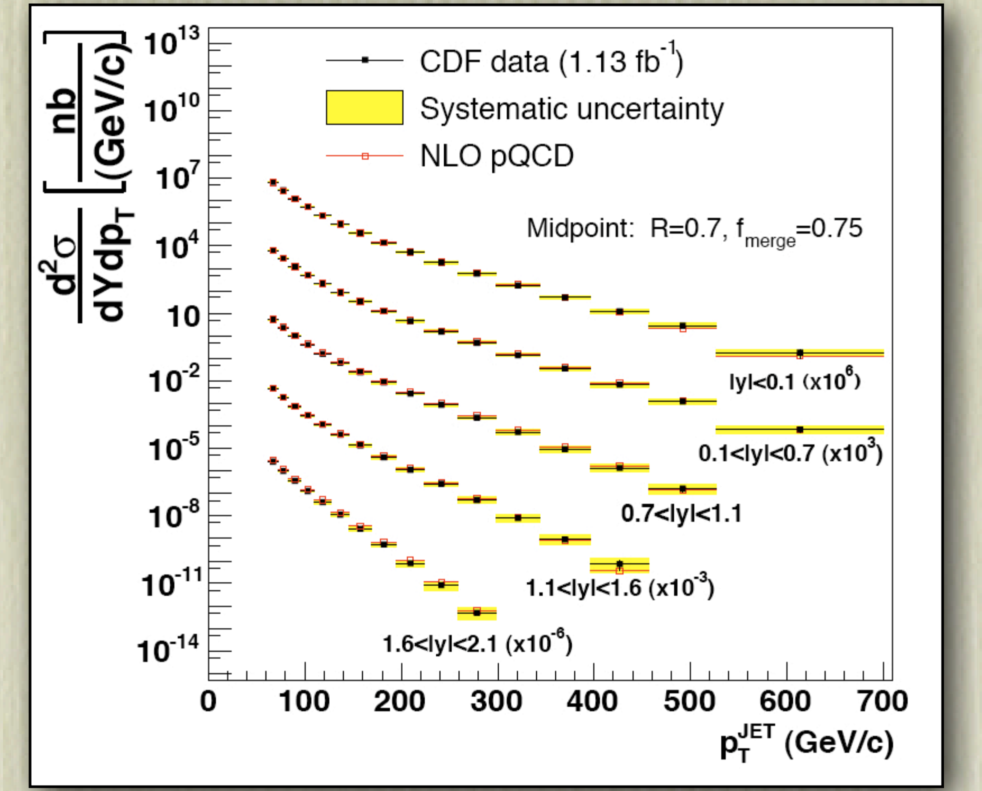
The making of QCD jets



The first QCD jets seen by TASSO at PETRA demonstrate gluon radiation

- Hard partons produced in the collision may emit further perturbative hard radiation.
  - Need higher order perturbative calculations.
- All hard partons are dressed by soft/collinear radiation.
  - Need parton shower Monte-Carlo.
- Parton coalesce forming color singlet hadrons.
  - Need tuned hadronization models.

- A jet algorithm clusters measured hadrons into jets.
  - It must be stable against soft/collinear emissions
- Background radiation not associated with the hard event must be subtracted.
  - Hadronization
  - Underlying event
  - Pileup



Single inclusive jet distribution measured at Tevatron by CDF

Jet Algorithms

- **Requirements**.
  - Infrared and collinear safety, for theoretical (and experimental!) stability.
  - Speed, for implementation in simulations and real life.
  - Limited hadronization corrections.

- **Algorithm structures**.
  - Cone algorithms: top-down, intuitive, Sterman-Weinberg inspired.
    Warning! Infrared/collinear safety issues. Solved by SISCone.
  - Sequential recombination: bottom-up, clustering, adapted from e^+e^- collisions.
  - Define a distance between partons (hadrons)

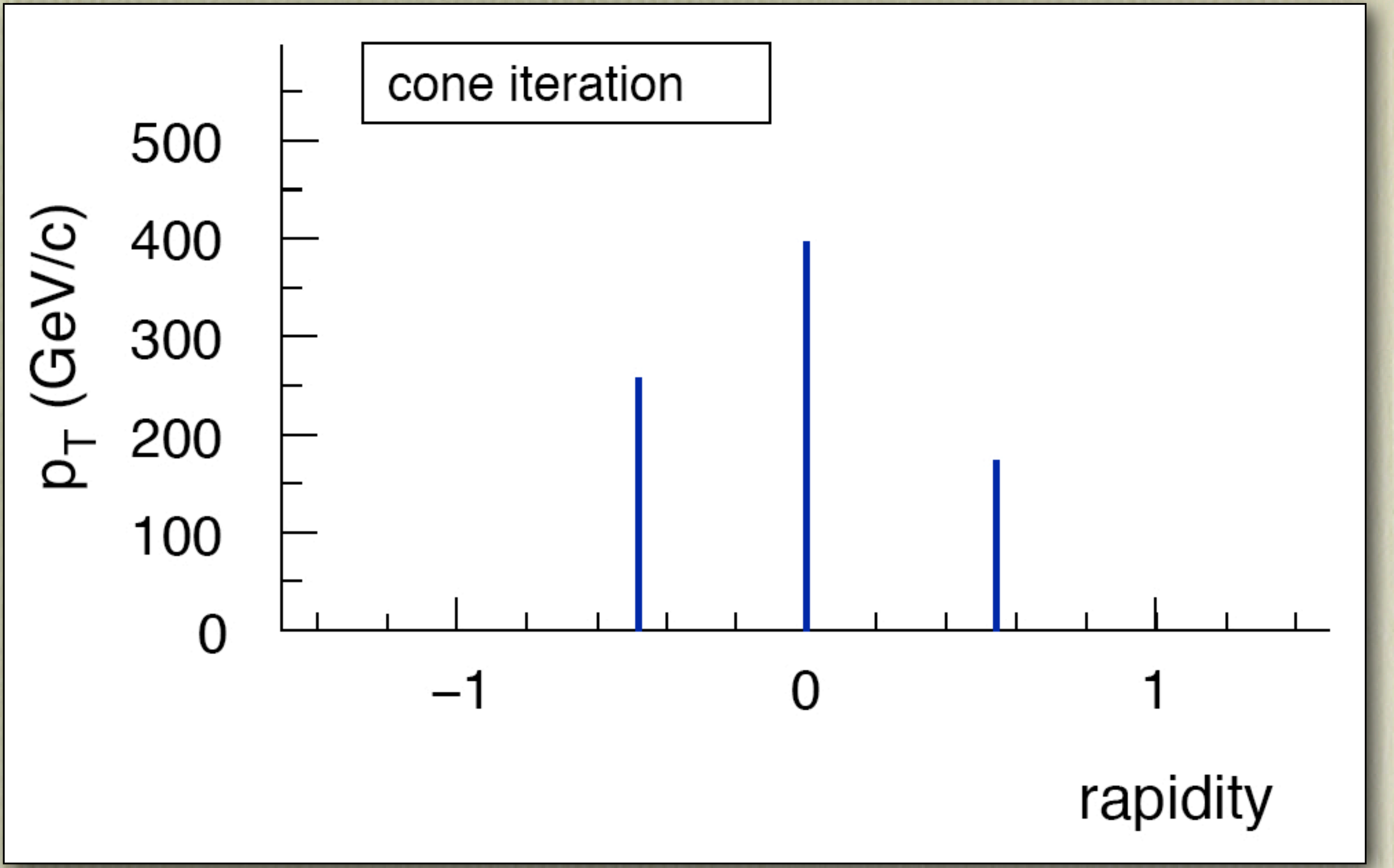
$$d_{ij}^{(p)} \equiv \min\left(k_{T,i}^{2p}, k_{T,j}^{2p}\right) \frac{\Delta y_{ij}^2 + \Delta\phi_{ij}^2}{R^2} , \qquad d_{iB}^{(p)} \equiv k_{T,i}^{2p} .$$

  - Choices: $p = 1$ (k_T) ; $p = 0$ (Cambridge); $p = -1$ (Anti-k_T)

- **A lot of recent progress!**
  - Gavin Salam *et al.*: FastJet, SISCone, Anti-kT,
    Jet Area, Jet Flavor, Analytic Hadronization models.
  - Steve Ellis *et al.*: SpartyJet.

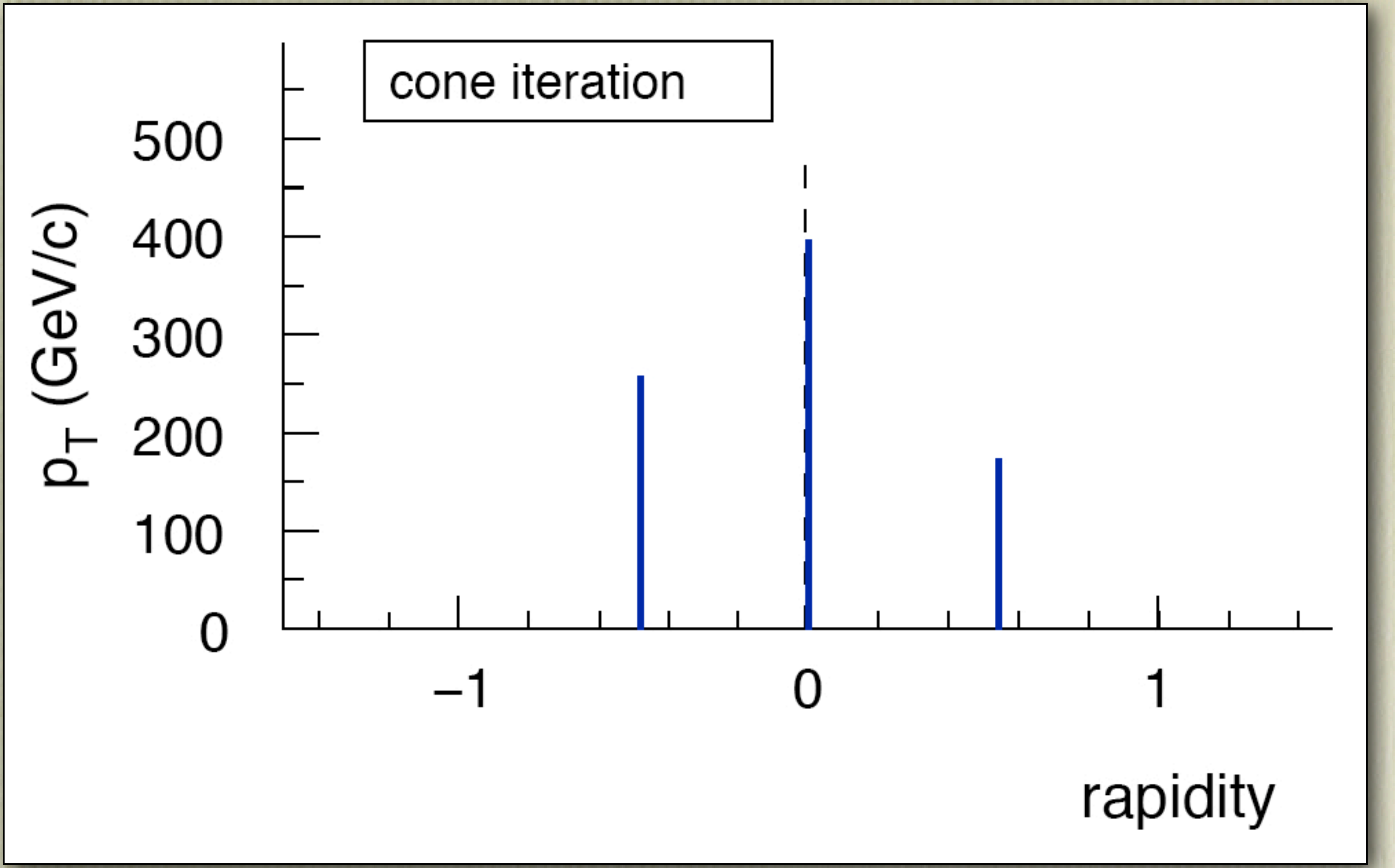
Safety of Jet Algorithms: a cartoon

Safety of Jet Algorithms: a cartoon



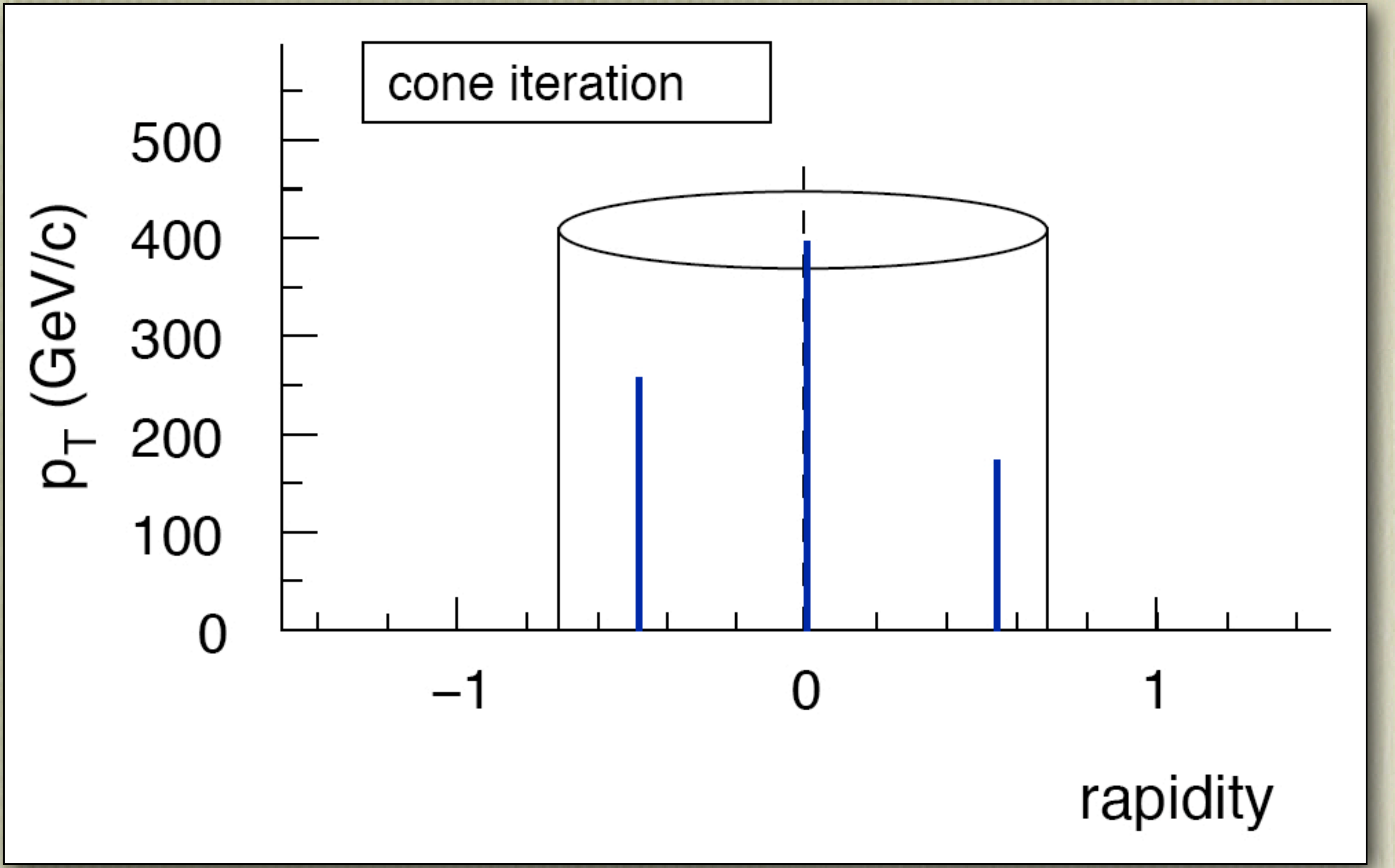
Three Hard Partons ...

Safety of Jet Algorithms: a cartoon



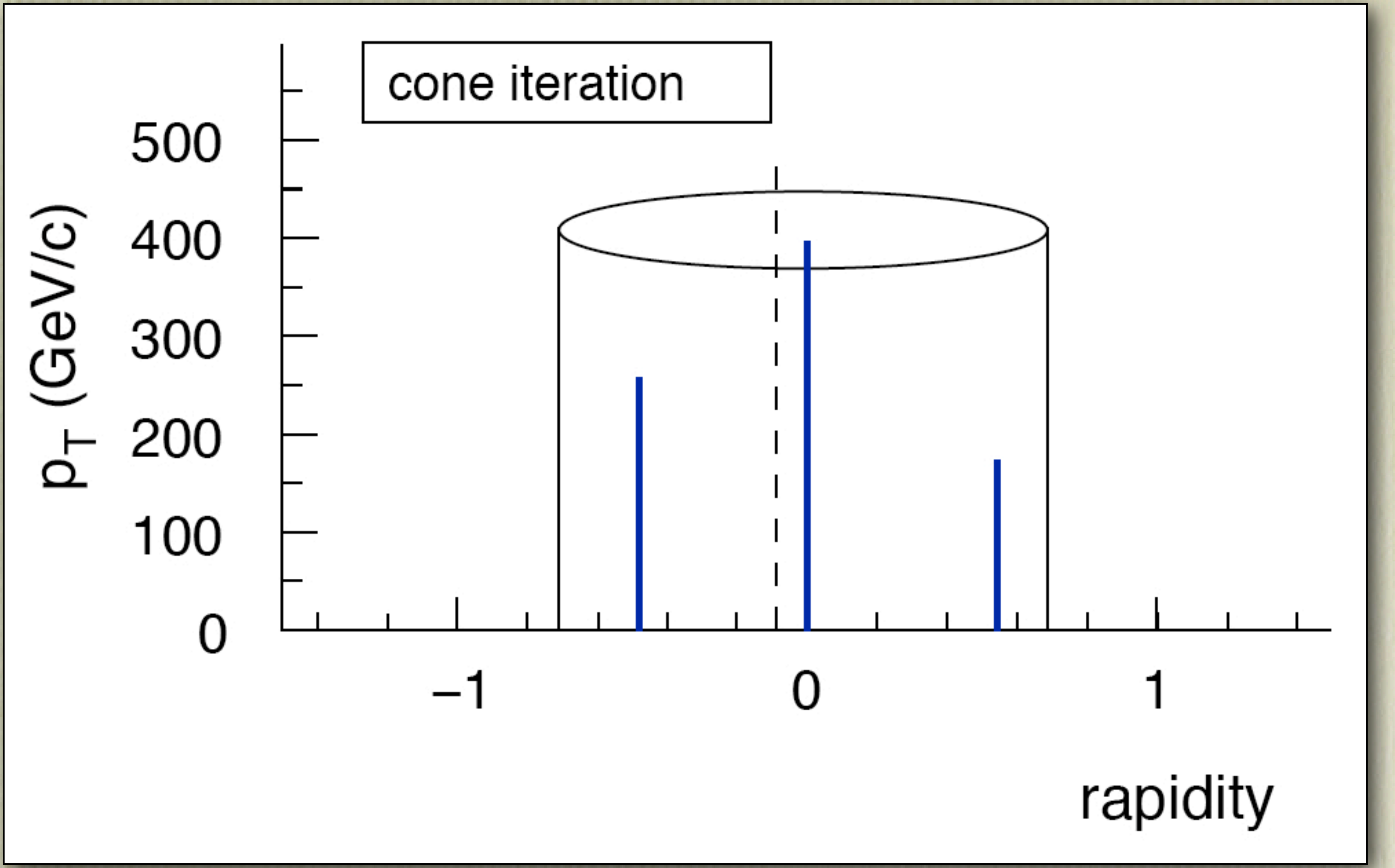
Pick the hardest parton as seed ...

Safety of Jet Algorithms: a cartoon



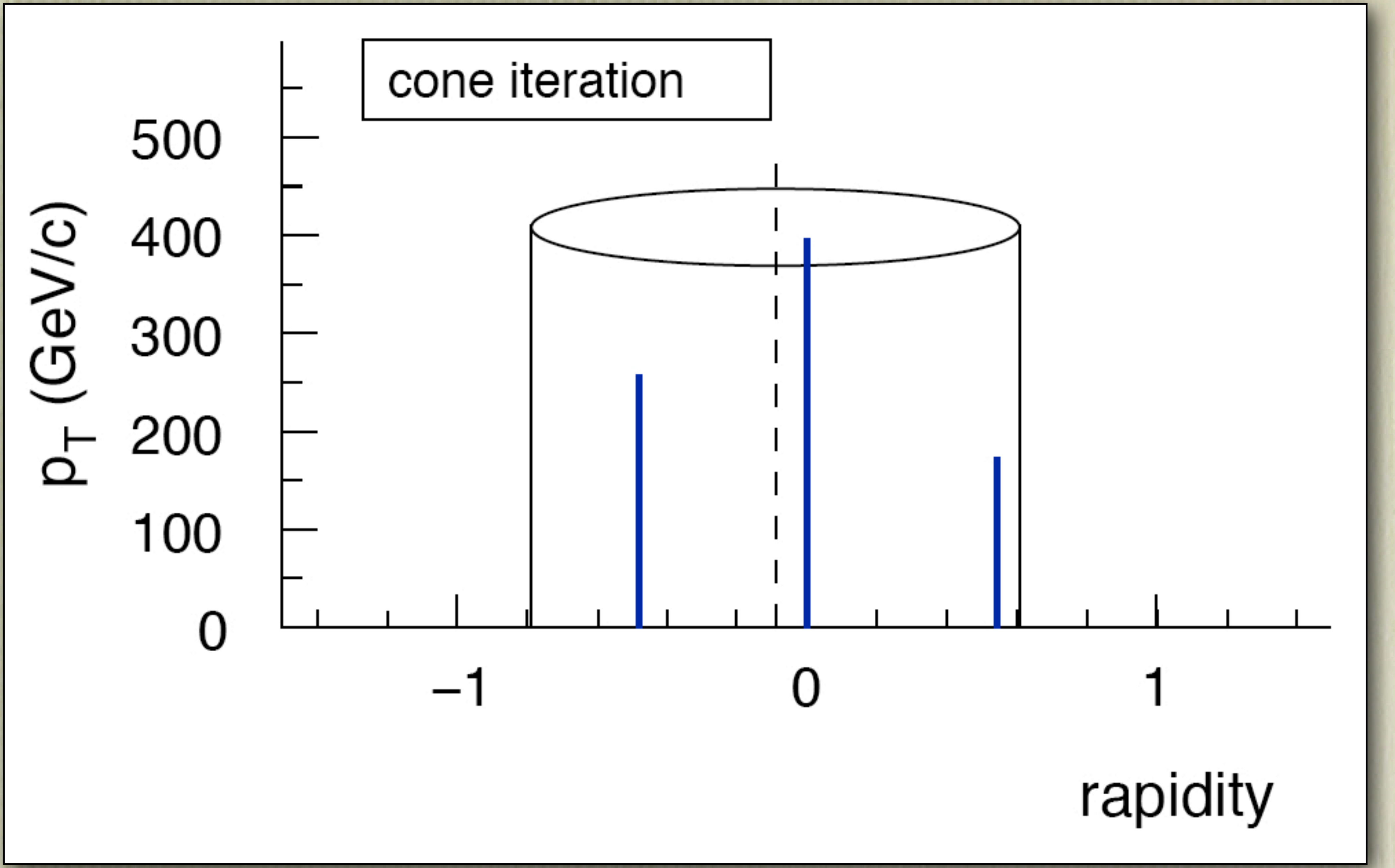
Draw a cone ...

Safety of Jet Algorithms: a cartoon



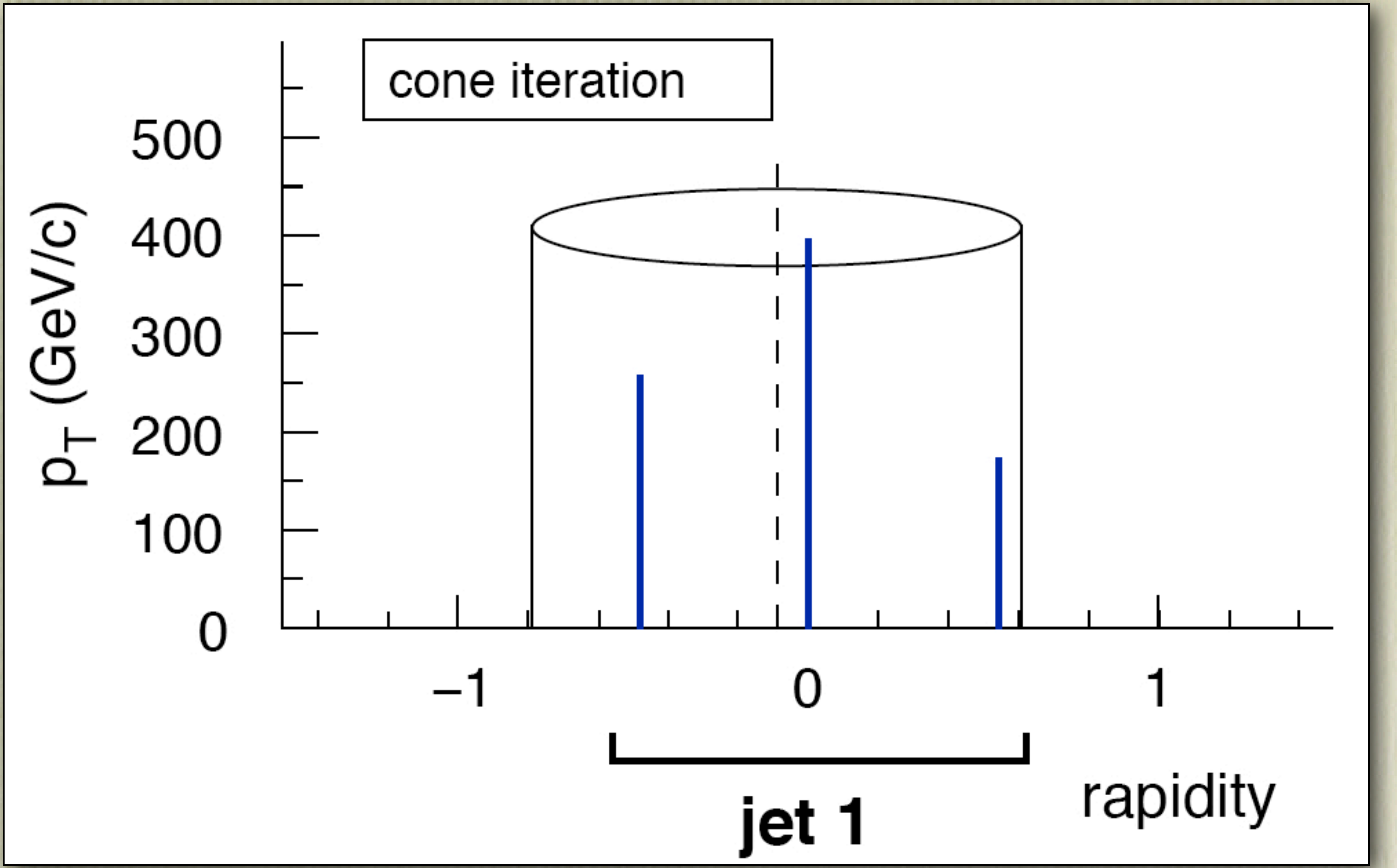
Sum the momenta to get a new seed ...

Safety of Jet Algorithms: a cartoon



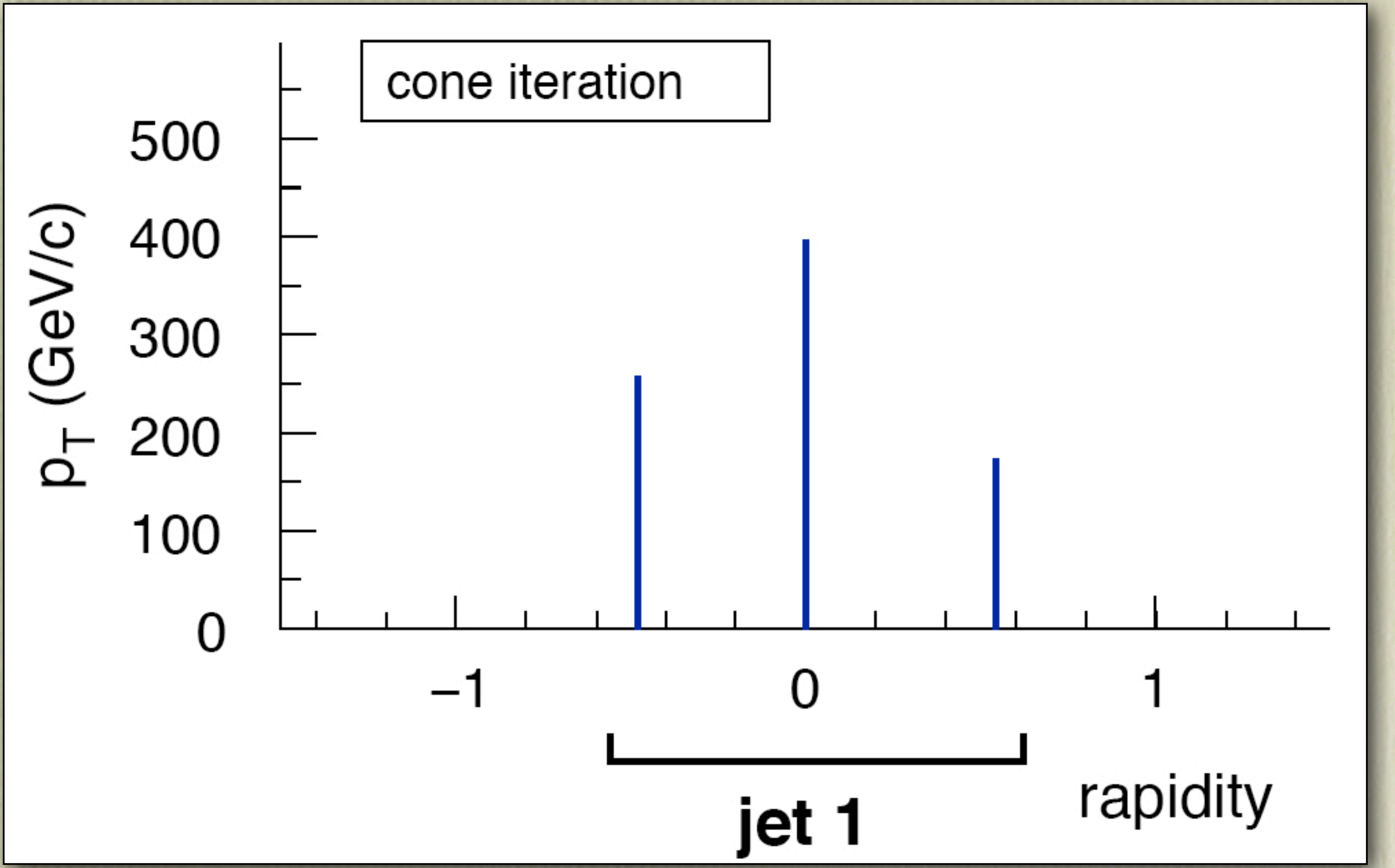
Draw a new cone ...

Safety of Jet Algorithms: a cartoon



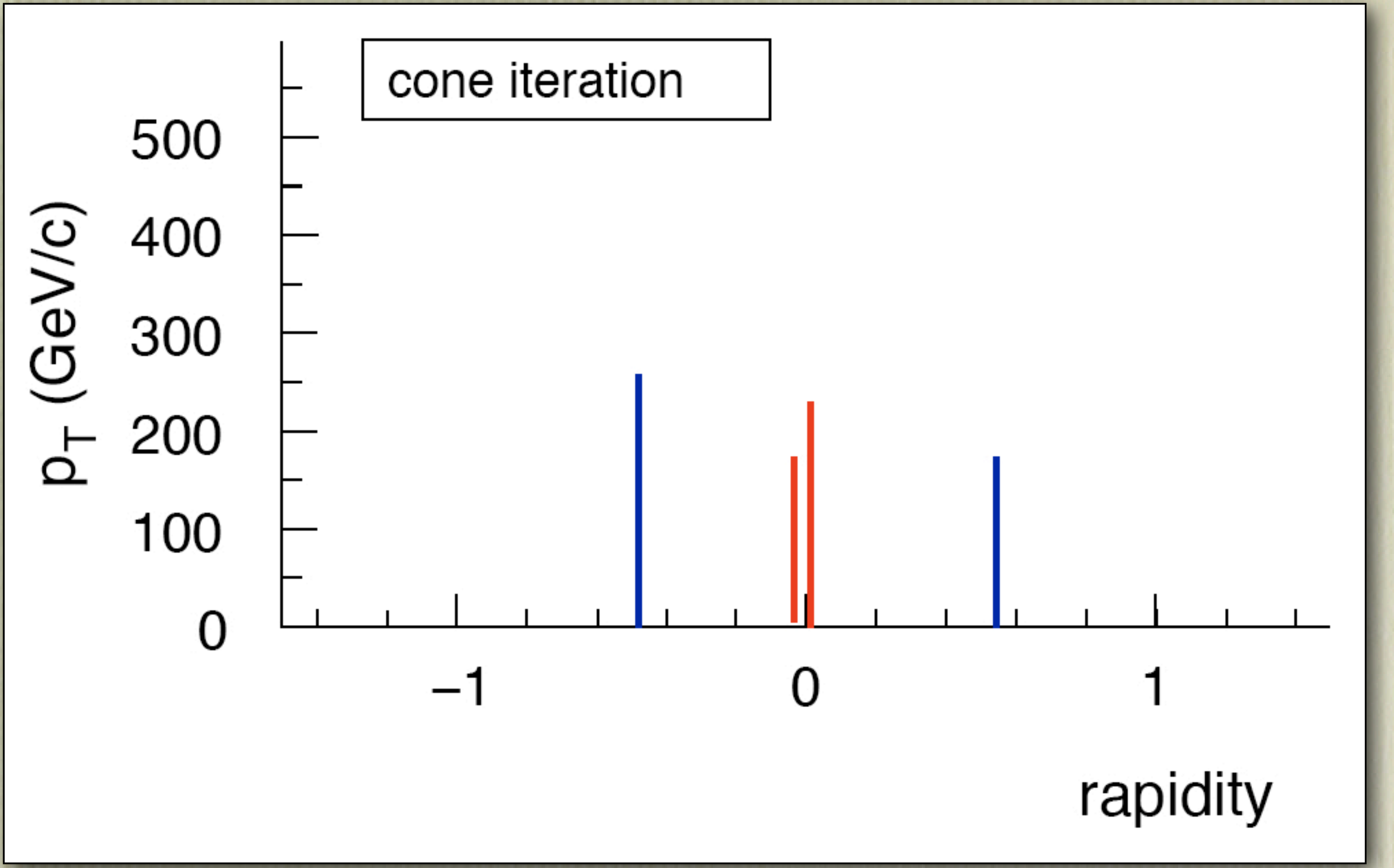
It is stable: call it a jet ...

Safety of Jet Algorithms: a cartoon



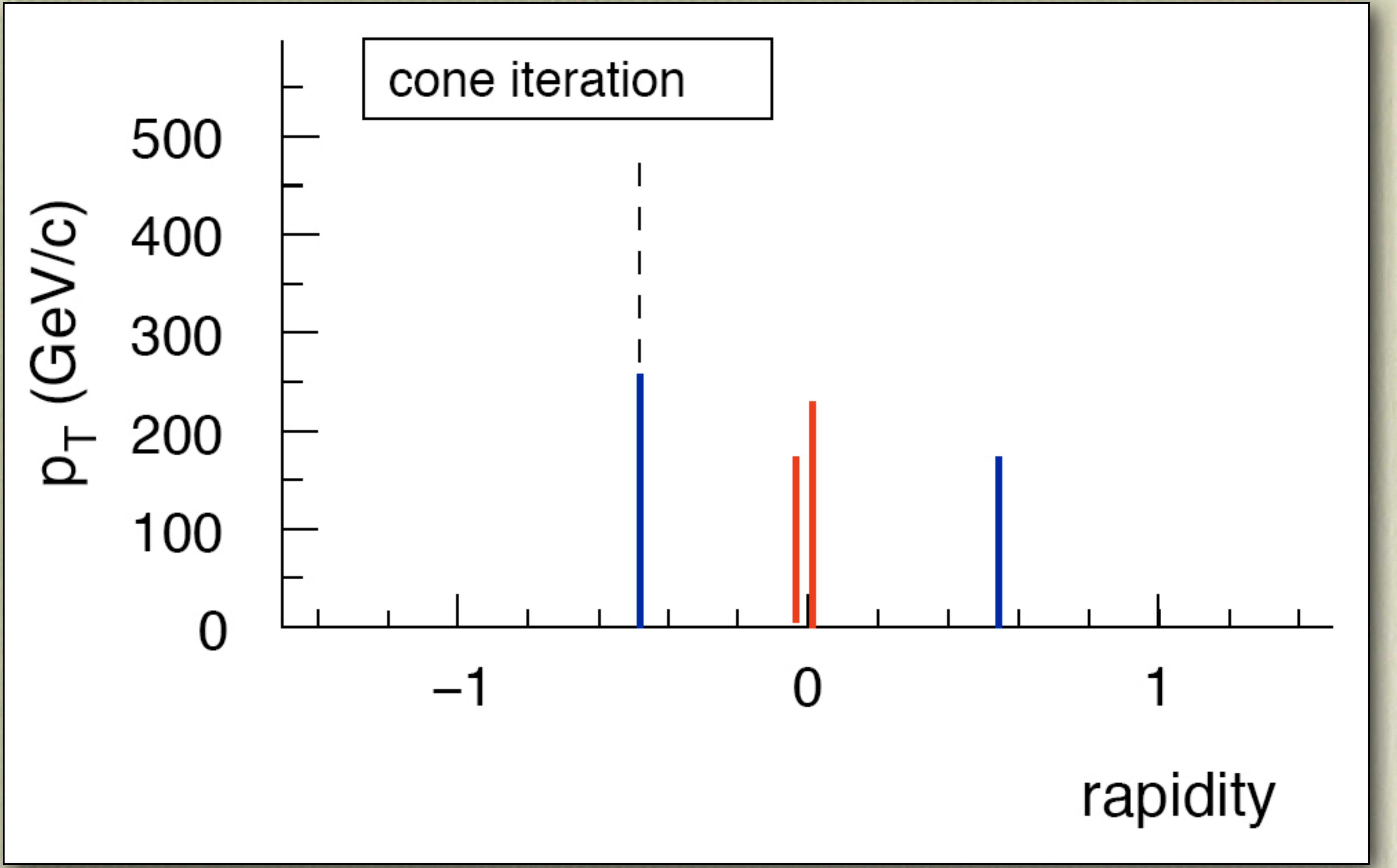
No more partons: algorithm ends

Safety of Jet Algorithms: a cartoon



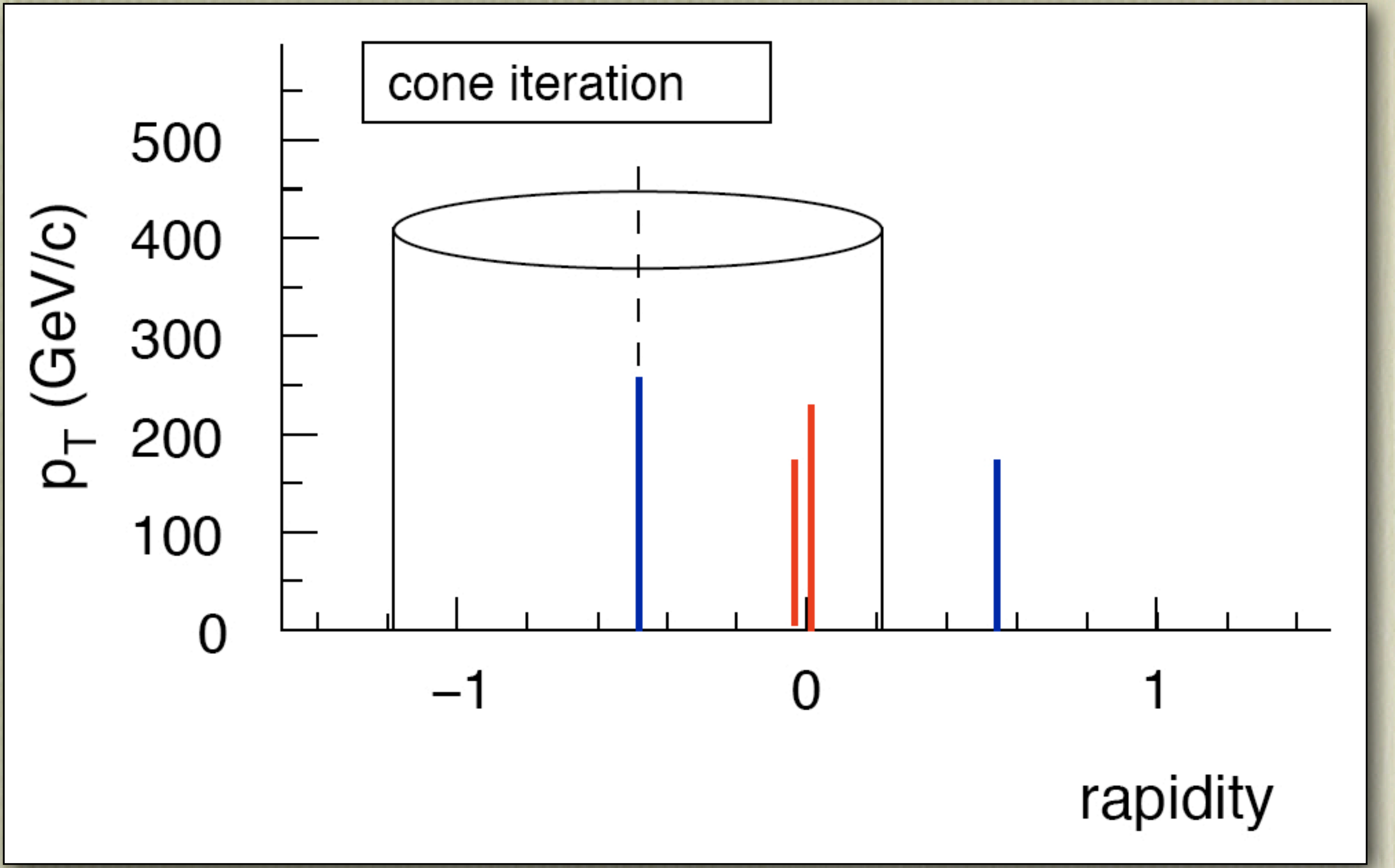
There was a collinear splitting!

Safety of Jet Algorithms: a cartoon



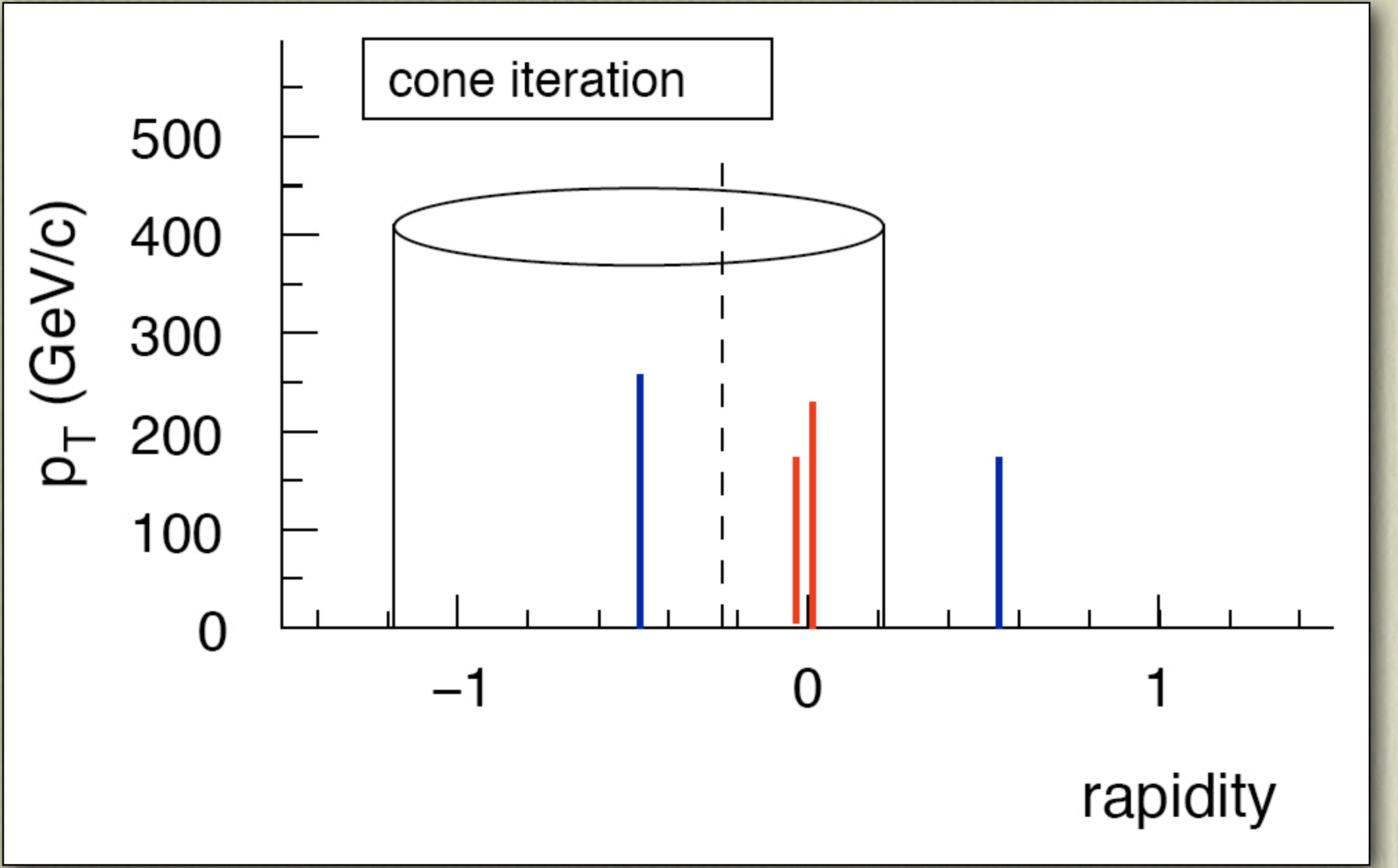
Pick the hardest parton as seed ...

Safety of Jet Algorithms: a cartoon



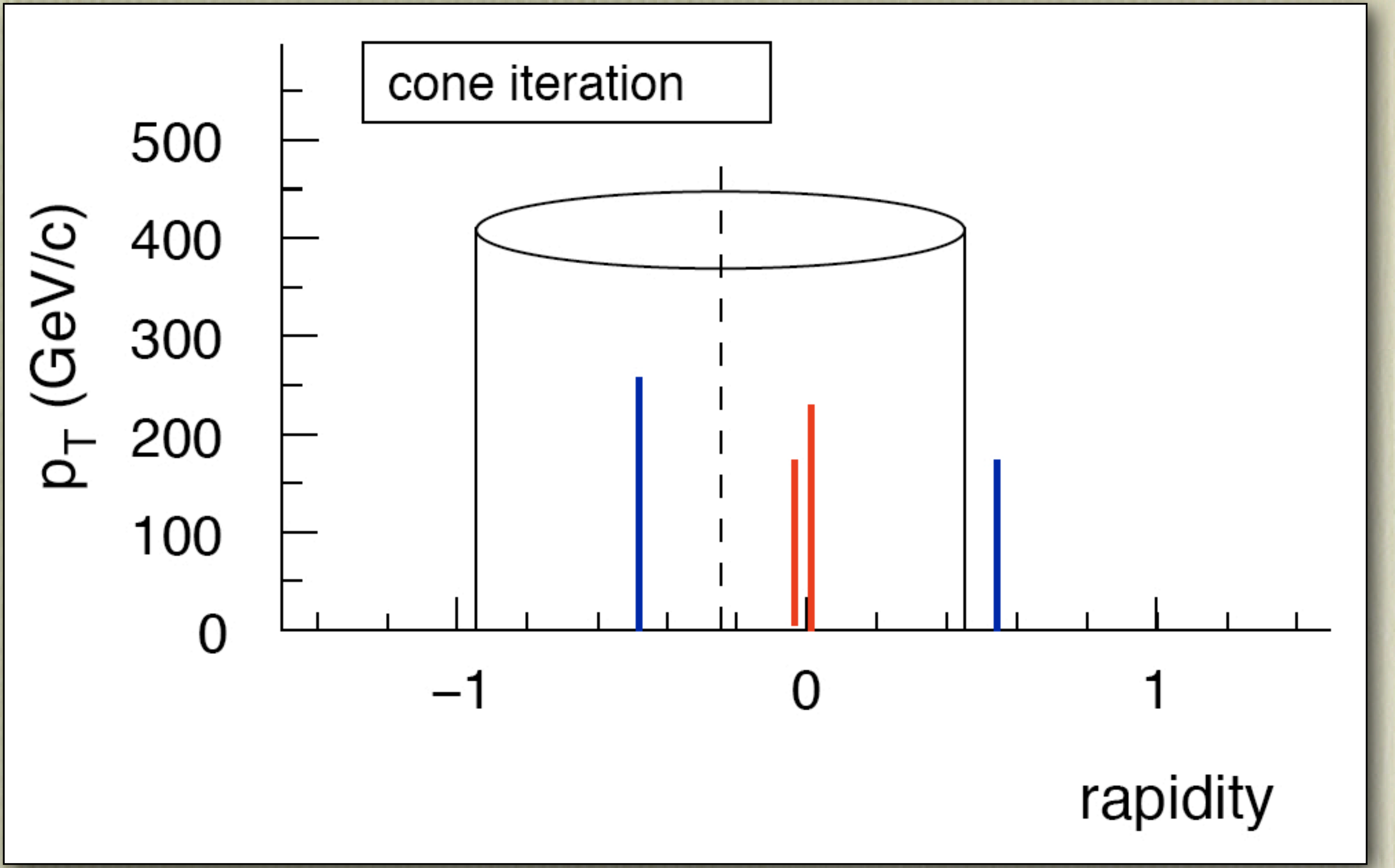
Draw a cone ...

Safety of Jet Algorithms: a cartoon



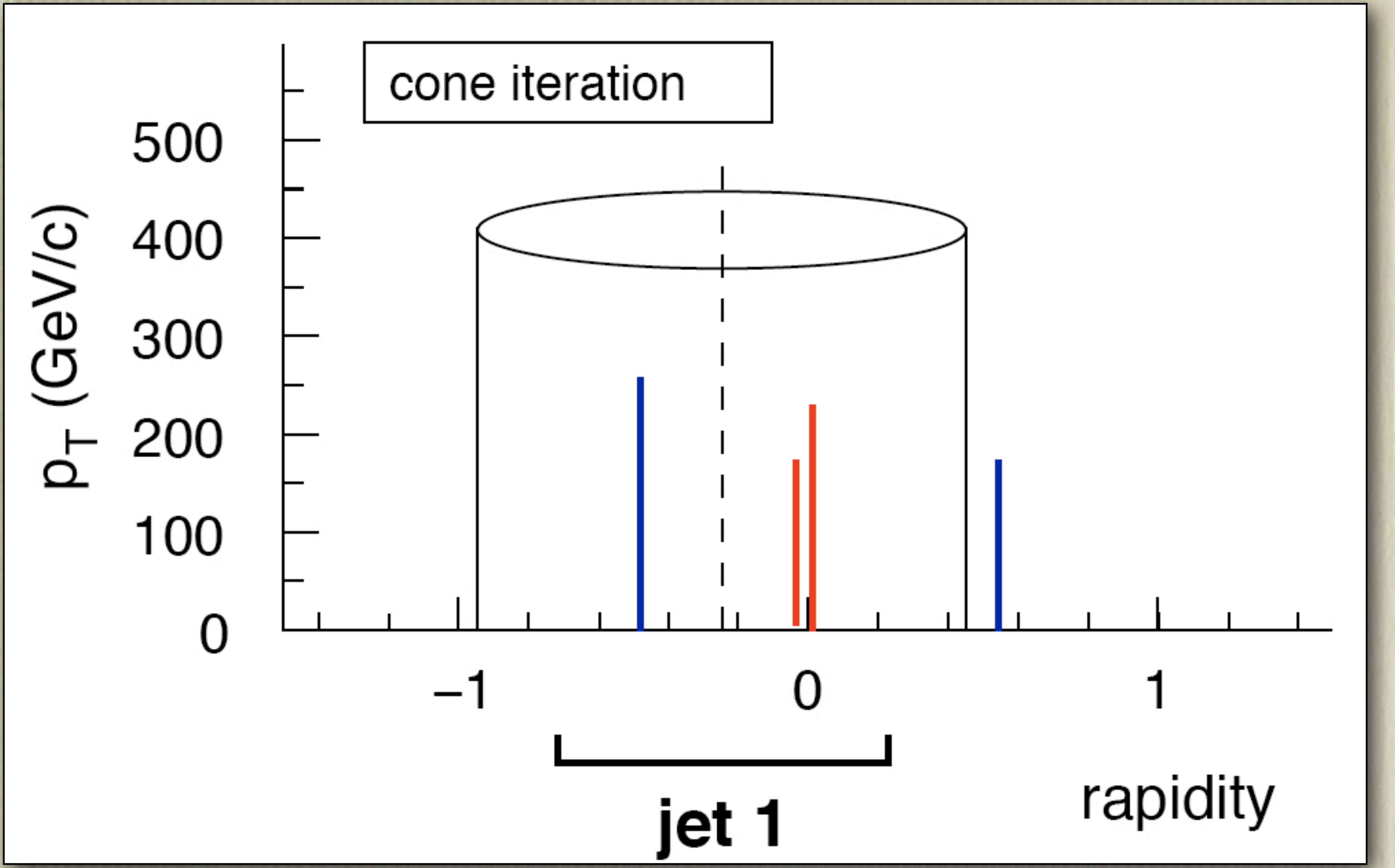
Sum the momenta to get a new seed ...

Safety of Jet Algorithms: a cartoon



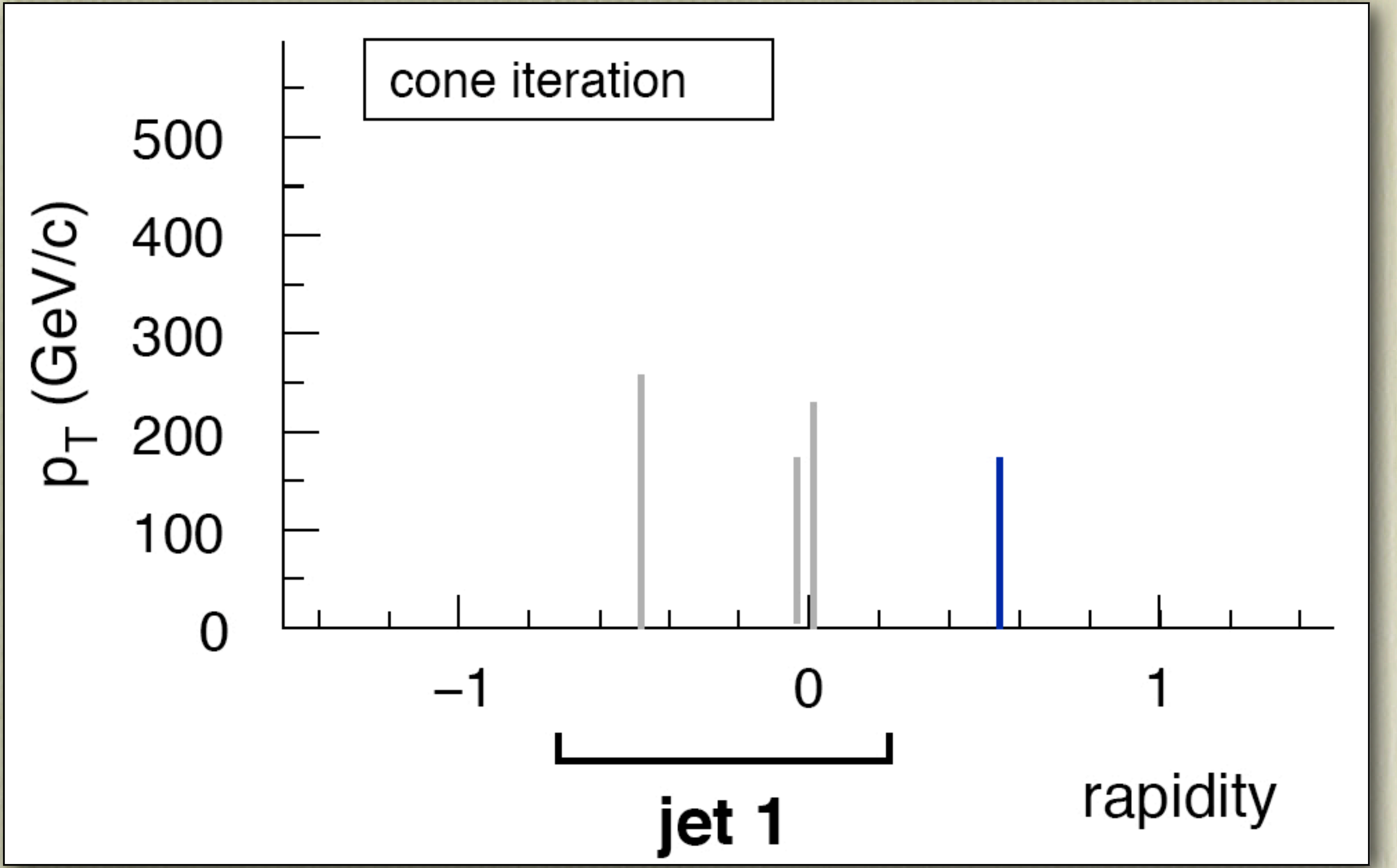
Draw a new cone ...

Safety of Jet Algorithms: a cartoon



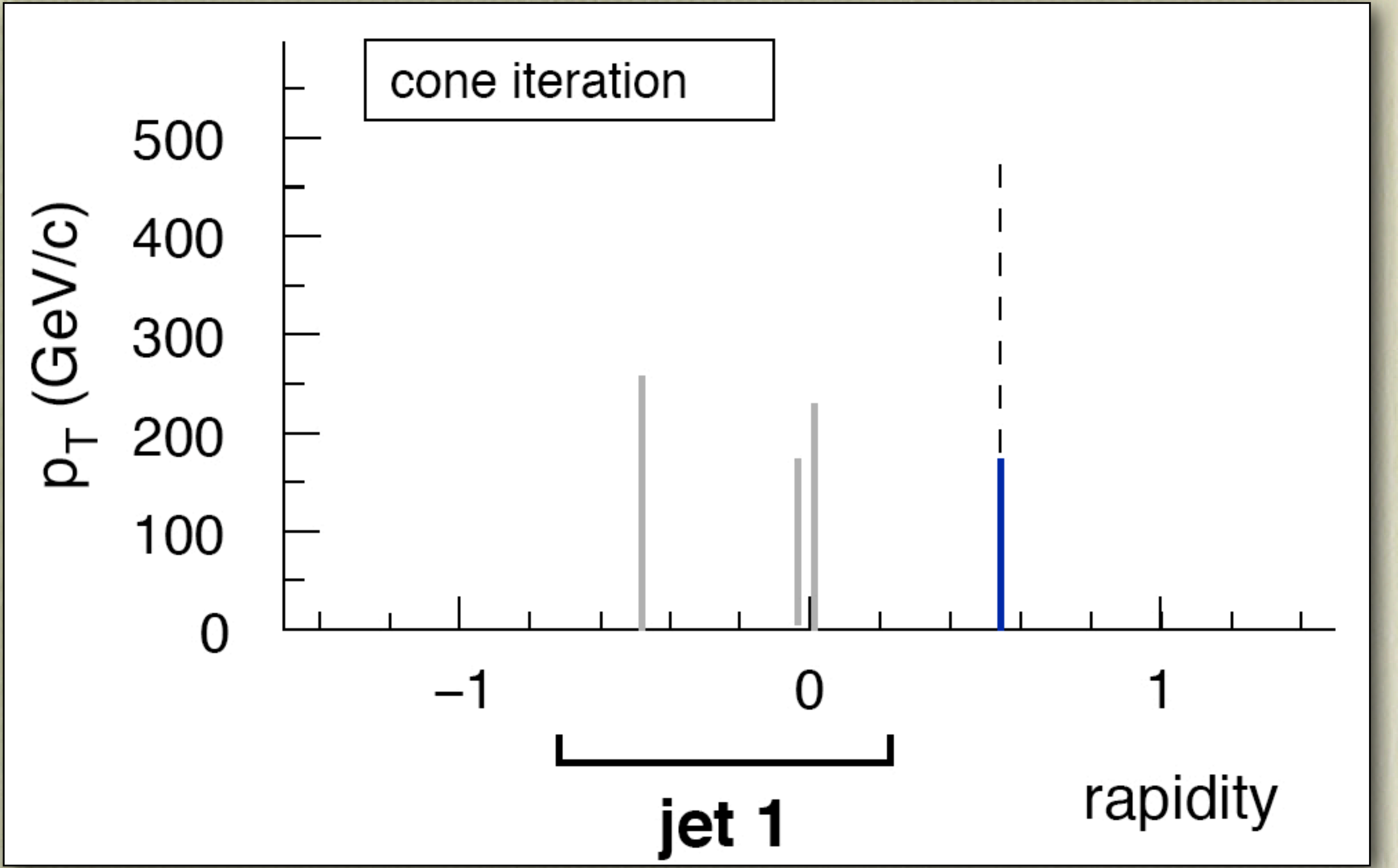
It is stable: call it a jet …

Safety of Jet Algorithms: a cartoon



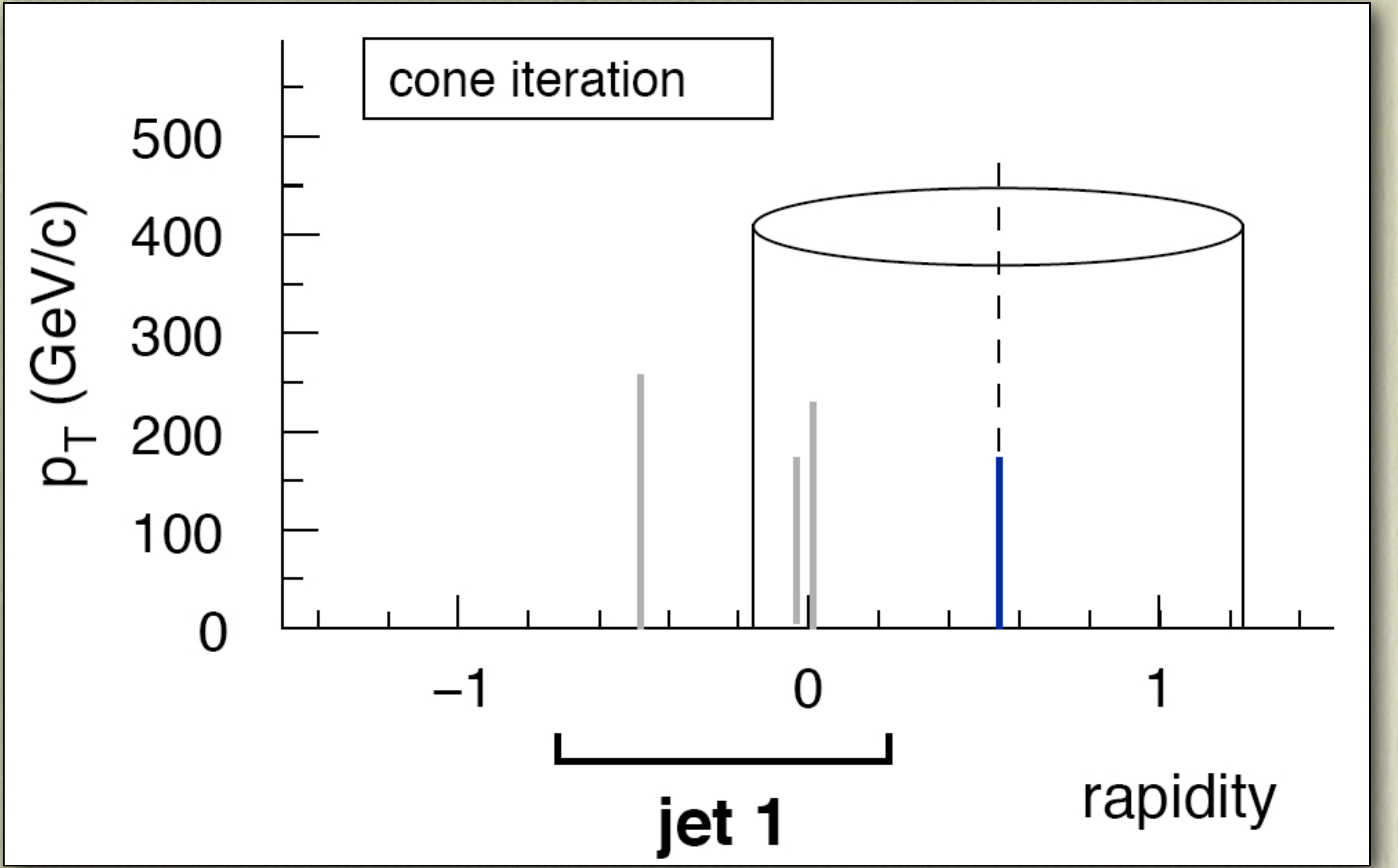
Erase the jet partons ...

Safety of Jet Algorithms: a cartoon



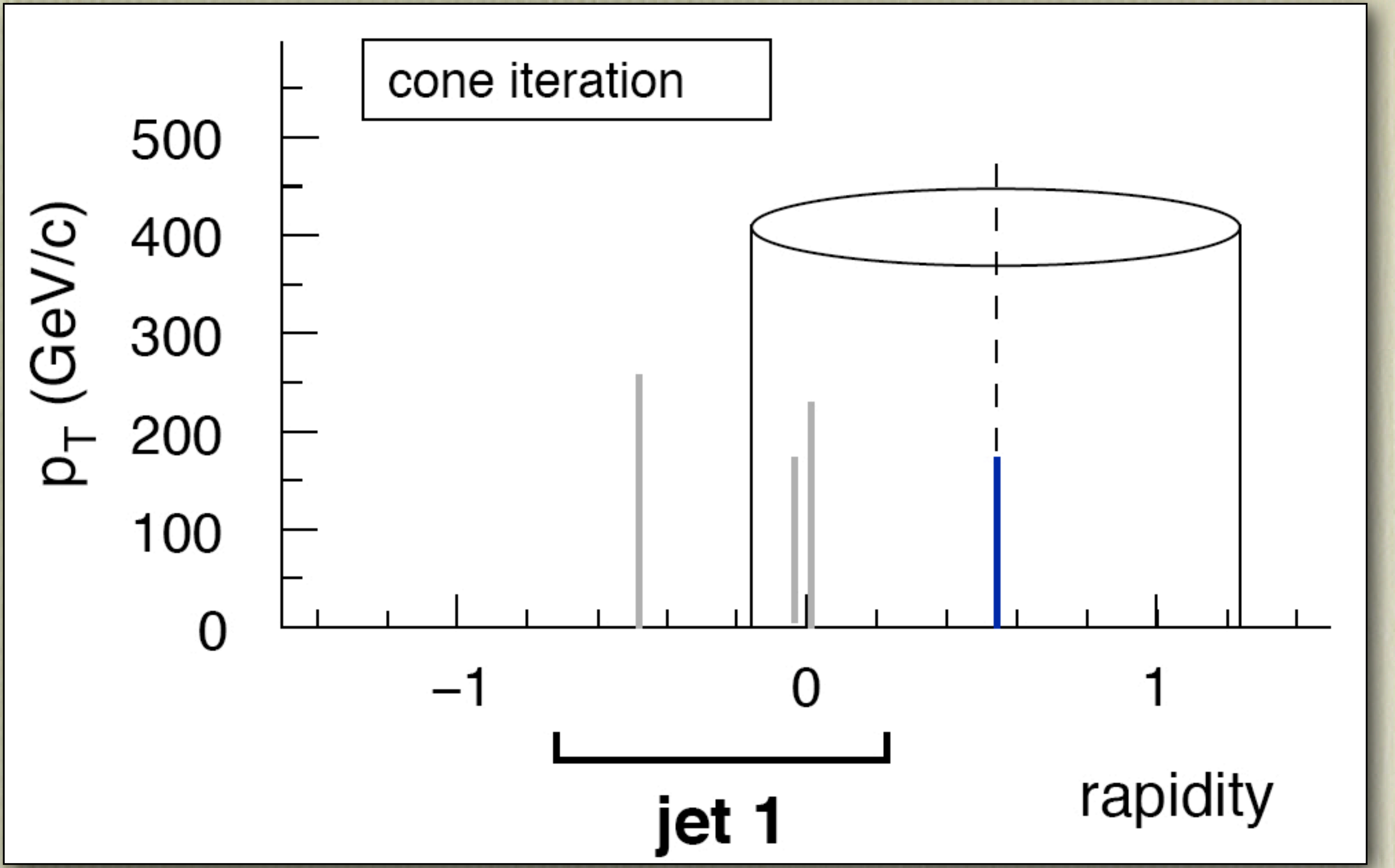
Pick the hardest remaining parton as seed ...

Safety of Jet Algorithms: a cartoon



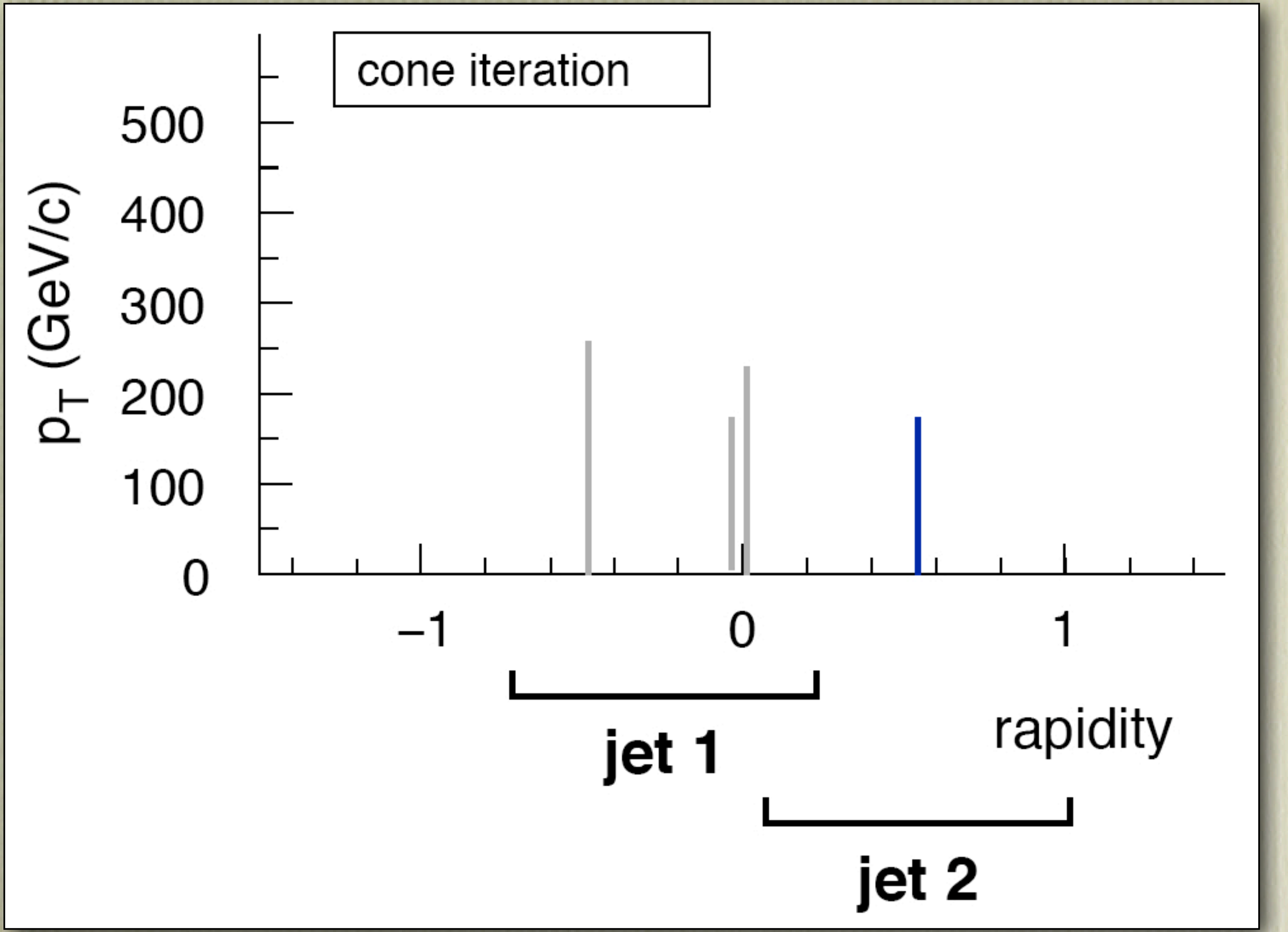
Draw a cone ...

Safety of Jet Algorithms: a cartoon



Sum the momenta to get a new seed ...

Safety of Jet Algorithms: a cartoon



It is stable: call it a jet

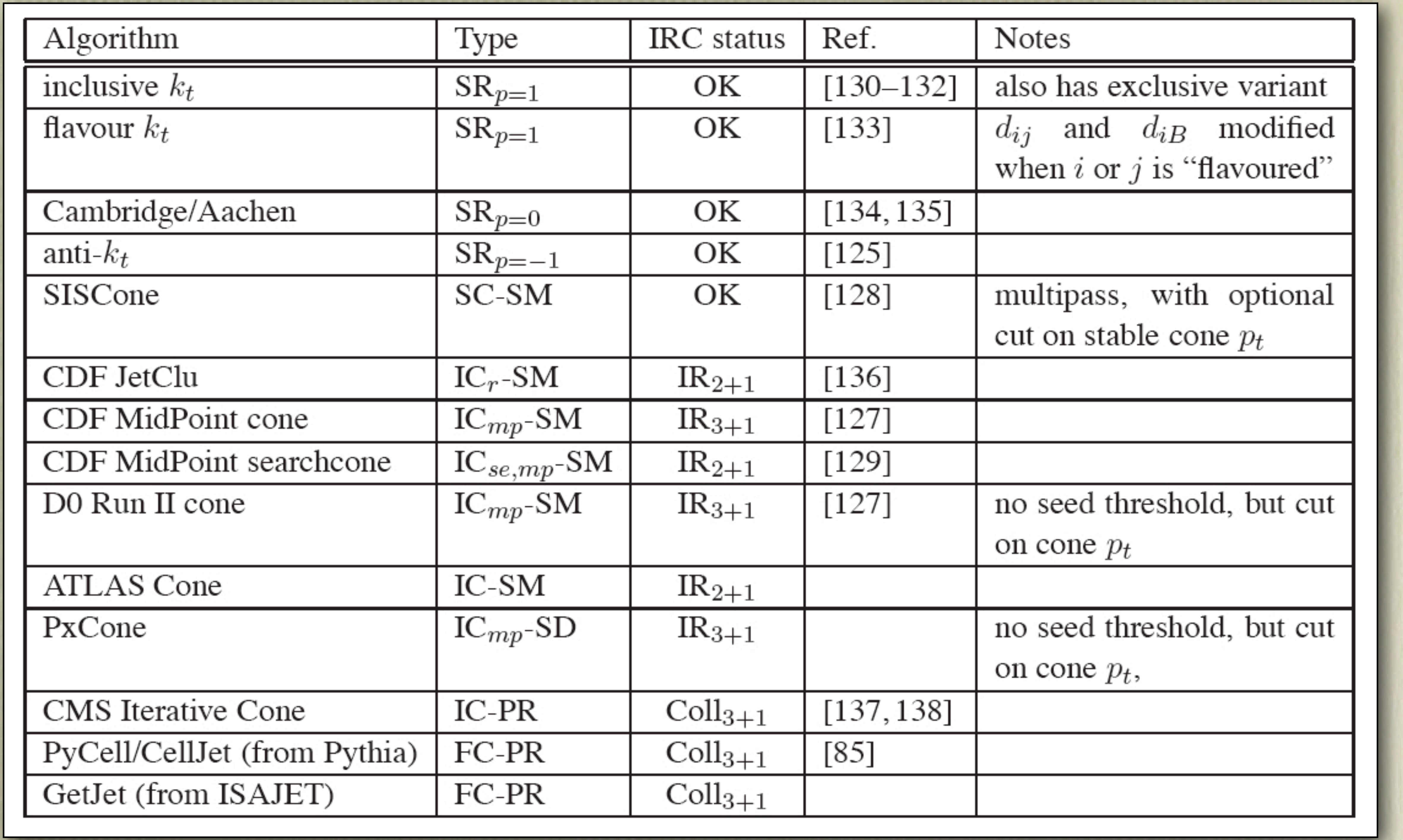
Unsafe Jet Algorithms

- For Theory: unsafe jet algorithms correspond to theoretical predictions that become meaningless beyond a given order.
- For Experiment: unsafe jet algorithms yield, event by event, a jet content that depends on emission of a soft pion or a highly collinear decay.

$$\begin{aligned} \sigma &= \sigma_0 \left(1 + c_1\alpha_s + c_2\alpha_s^2 + \ldots\right) \qquad \ldots \quad c_2 = \infty\,! \\ \sigma &= \sigma_0 \left(1 + c_1\alpha_s + K \log\left(\frac{\Lambda}{Q}\right)\alpha_s^2 + \ldots\right) = \sigma_0\ \left(1 + (c_1 + K)\alpha_s + \ldots\right)\,. \end{aligned}$$

- At a minimum, infrared/collinear sensitivity at N^pLO destroys the predictivity of a $N^{p-1}LO$ calculation.
- Impact depends on the specific algorithm and observable.
  - The single-inclusive jet cross section is least affected: $\delta\sigma/\sigma < 5\%$ comparing SISCone and MidPoint Cone algorithms.
  - Multi-Jet cross sections are severely affected.
    - ➡ W + n Jets existing NLO predictions (n = 2,3,4) are not applicable to MidPoint Cone algorithms.
    - ➡ For jet mass studies, the overall normalization is affected.

Comparing Jet Algorithms



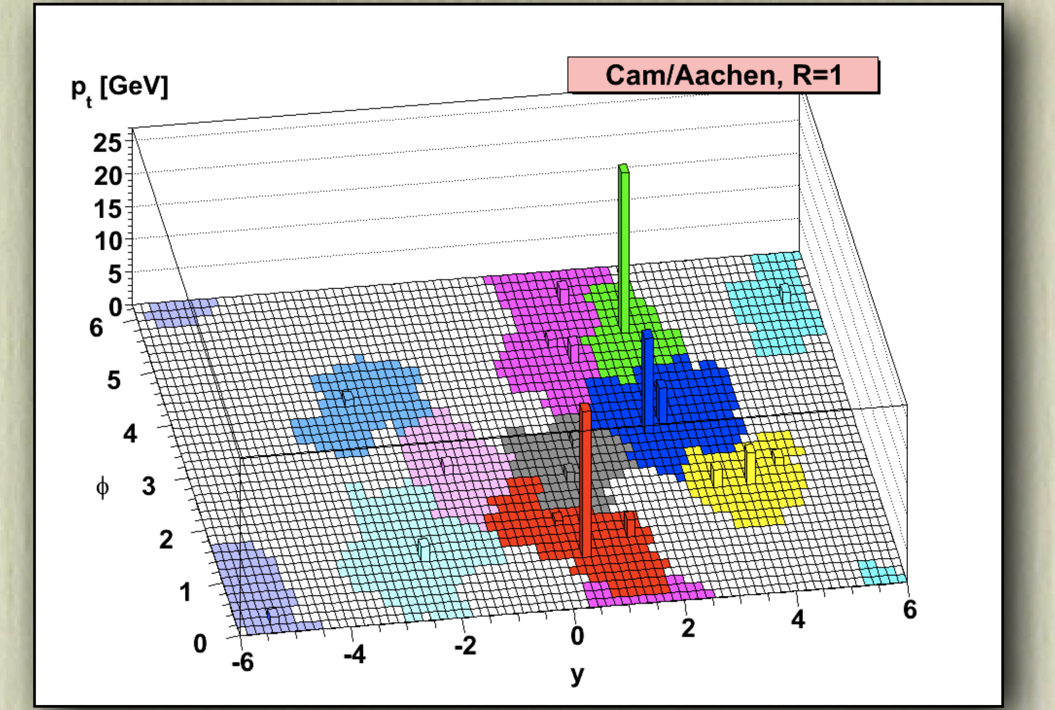
Algorithm	Type	IRC status	Ref.	Notes
inclusive k_t	SR$_{p=1}$	OK	[130–132]	also has exclusive variant
flavour k_t	SR$_{p=1}$	OK	[133]	d_{ij} and d_{iB} modified when i or j is "flavoured"
Cambridge/Aachen	SR$_{p=0}$	OK	[134, 135]	
anti-k_t	SR$_{p=-1}$	OK	[125]	
SISCone	SC-SM	OK	[128]	multipass, with optional cut on stable cone p_t
CDF JetClu	IC$_r$-SM	IR$_{2+1}$	[136]	
CDF MidPoint cone	IC$_{mp}$-SM	IR$_{3+1}$	[127]	
CDF MidPoint searchcone	IC$_{se,mp}$-SM	IR$_{2+1}$	[129]	
D0 Run II cone	IC$_{mp}$-SM	IR$_{3+1}$	[127]	no seed threshold, but cut on cone p_t
ATLAS Cone	IC-SM	IR$_{2+1}$		
PxCone	IC$_{mp}$-SD	IR$_{3+1}$		no seed threshold, but cut on cone p_t,
CMS Iterative Cone	IC-PR	Coll$_{3+1}$	[137, 138]	
PyCell/CellJet (from Pythia)	FC-PR	Coll$_{3+1}$	[85]	
GetJet (from ISAJET)	FC-PR	Coll$_{3+1}$		

A Les Houches compilation of jet algorithms, see arXiv:0803.0678

Soft gluon effects for jets

M. Dasgupta, LM, G. Salam

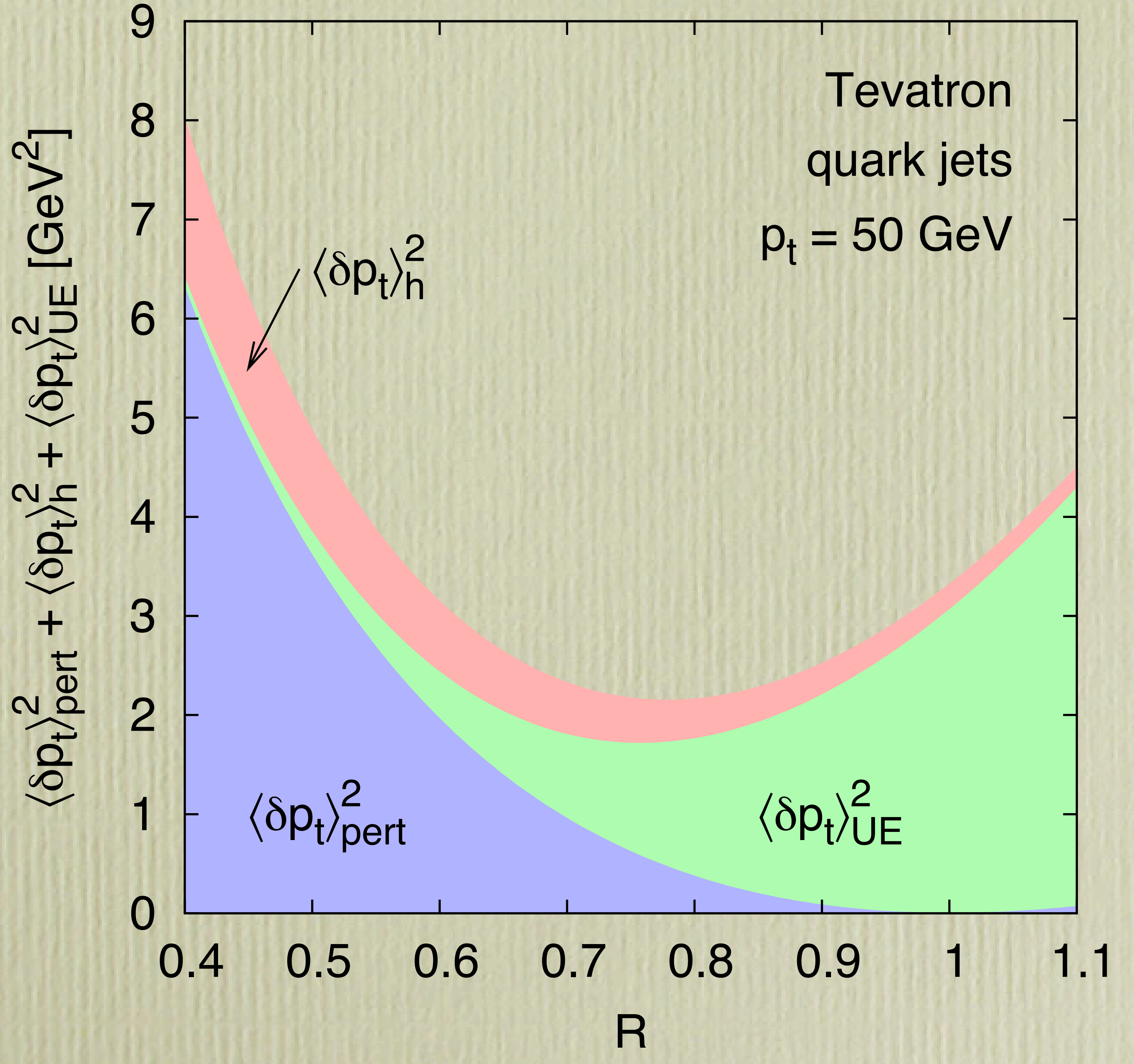
- Jet algorithms cluster particles into "cones" of radius R on the azimuth-rapidity cylinder.
- The jet energy is modified by "splash-in" effects due to the underlying event. They grow with R^2.
- The jet energy is modified by "splash-out" effects due to soft radiation. They can be estimated analytically using soft gluon resummation.
- Soft gluon effects grow at small radius, with a non-perturbative coefficient that can be measured in electron-positron collisions.
- The best R can be chosen to minimize unwanted effects, depending on the measurement.



Areas of jets as reconstructed with an infrared safe clustering algorithm (G. Salam et al.)

$$\boxed{\Delta p_t(R)|_{qq'\to qq'} = \mathcal{A}(\mu_f)\left[-\frac{2}{R}C_F + \frac{1}{8}R\left(5\,C_F - \frac{9}{N_c}\right) + \mathcal{O}\left(R^2\right)\right]}$$

Choosing the best jet radius



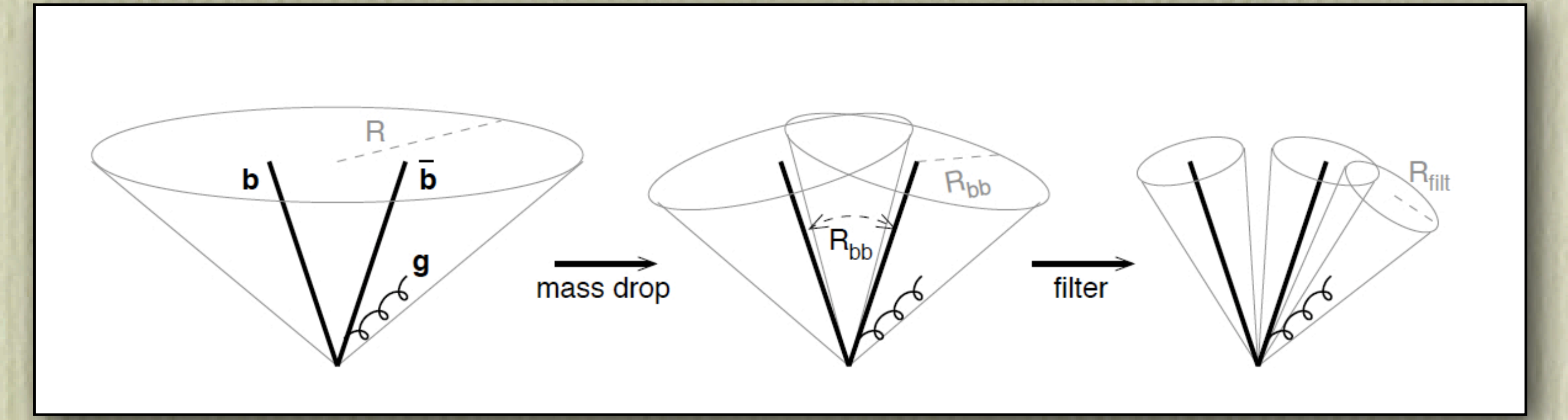
Shifts in transverse momentum for a 50 GeV quark jet, due to perturbative radiation, underlying event, and hadronization.

- Perturbative collinear radiation shifts the jet transverse momentum by computable terms of the form Log R.
- The underlying event contributes terms quadratic in R, estimated by Monte-Carlo simulations.
- Hadronization effects are proportional to 1/R, with a coefficient which can be experimentally determined.
- The best choice for R can be studied depending on machine energy, jet flavor, and on the goals of the measurement.
- Tuning the jet radius is important at LHC, where multijet events are commonplace, and are used for very different analyses.

Jet substructure

Detailed control of IR safe jet algorithms allows to study the way particles and energy are distributed inside a given jet: jet substructure.

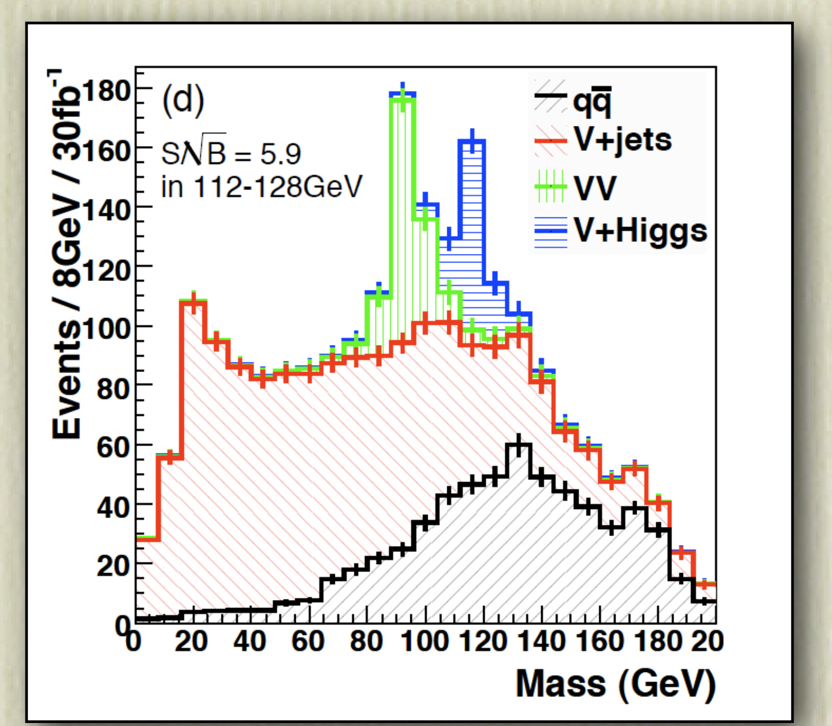
At LHC “heavy” particles such as Higgs bosons and top quarks can be produced in a highly boosted state, and decay products can form a single jet.



Jet substructure analysis with filtering (Butterworth et al. 2008)

- A light Higgs produced with a Z (W) and decaying to b quarks can be identified with 30 fb^{-1}.
- Boosted top tagging is also significantly improved

- Identify a candidate “fat” jet (R > 1).
- Undo the clustering until a steep mass drop is found: two subjets are identified.
- Recluster with smaller jet radius to drop extra radiation (“filtering”).
- Several similar techniques are used (“pruning”, “trimming”, “grooming”, ...).
- Event shapes controlling the energy flow inside jets (jet shapes, templates) are also used.



Boosted Higgs identification (Butterworth et al.)

Computing the hard scattering QCD cross section

Higher Orders

Order by order: LO

Or: when is a problem "solved"?

- Computing tree amplitudes in gauge theories is a nontrivial problem .

Njets	2	3	4	5	6	7	8
# diag's	4	25	220	2485	34300	$5\text{x}10^5$	10^7

- Quantum number management helps.

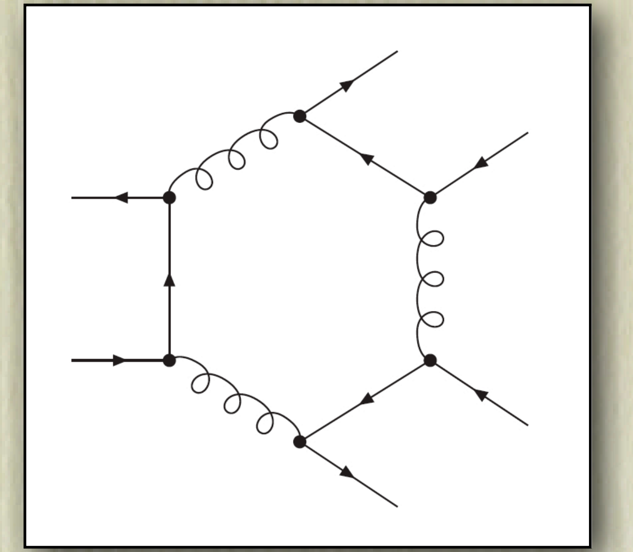
$$
\begin{aligned}
\mathcal{A}^{\mathrm{tree}}(1,2,\ldots,n) &= g^{n-2}\sum_{\mathrm{ncp}} \mathrm{Tr}\left(T_{a_1}T_{a_2}\ldots T_{a_n}\right) A^{\mathrm{tree}}(1,2,\ldots,n) \\
A^{\mathrm{tree}}(-,-,+,\ldots,+) &= \mathrm{i}\,\frac{\langle 12\rangle^4}{\langle 12\rangle\langle 23\rangle\ldots\langle n1\rangle}
\end{aligned}
$$

- The problem has a recursive solution.
  - Berends-Giele recursion relations 20 years old and still fastest.
  - Twistor-inspired methods lead to new insights, new recursion relations (BCFW).
  - Factorial complexity degraded to power law (t_n ~ n^4), except for color.

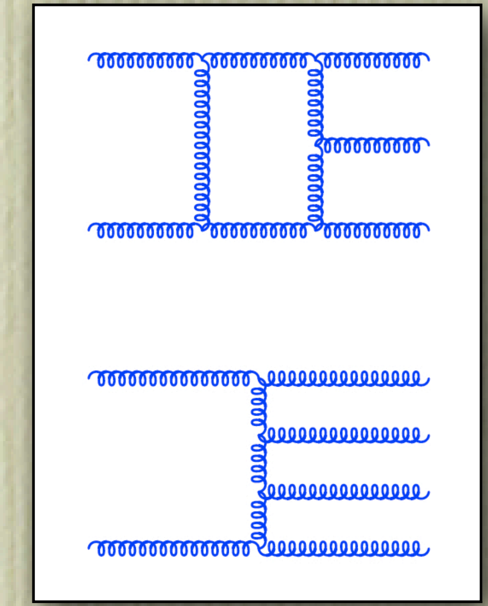
- LO calculations are clearly not enough for quantitative LHC phenomenology!

Order by order: NLO

Light after the bottlenecks



- Bottleneck #1: computing loop integrals.
  - Obstacles: analytic structure, tensor integral decomposition.
  - State of the art: 5 points "standard", 6 points "frontier".
  - Impressive progress with unitarity + "twistor" techniques.



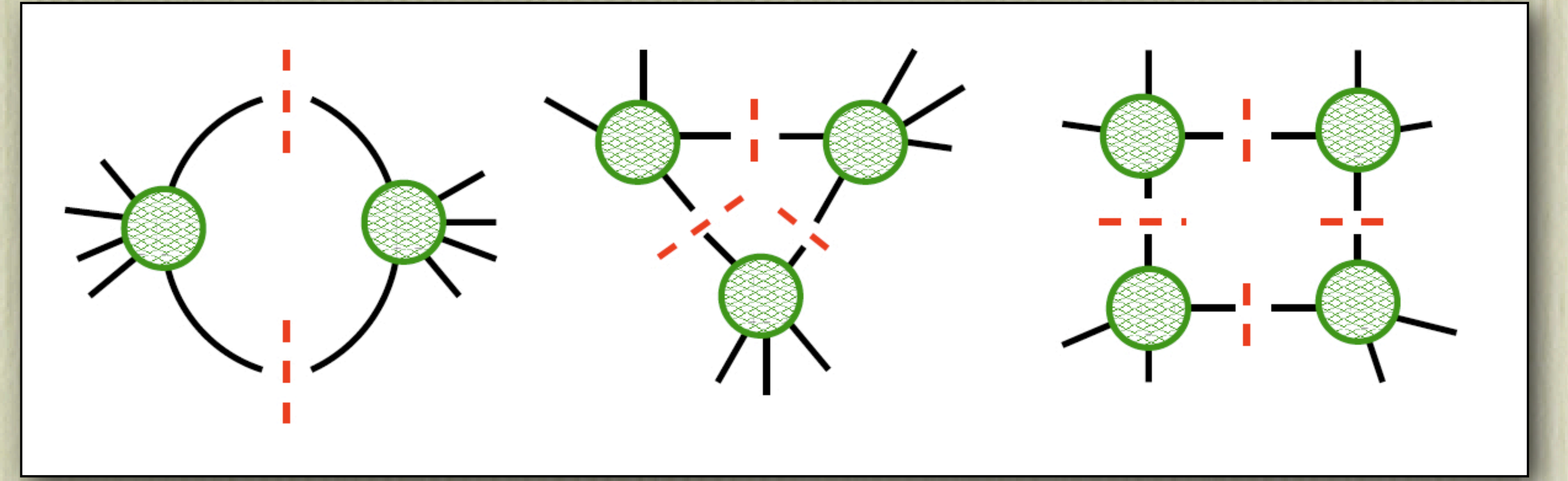
- Bottleneck #2: subtracting infrared and collinear poles.
  - Combine (n+1)-parton trees with n-parton one-loop amplitudes.
  - Compute singular phase space integrals for generic observables.
  - General methods exist: slicing, subtraction, dipole.

- Bottleneck #3: interfacing with parton shower MonteCarlo's.
  - Practical usage of a theory calculation requires four steps.

    ME → generator → shower → hadronization MC
  - New problem at NLO: double counting of first IR/C emission.
  - Methods are available (MC@NLO, POWHEG), implementations in progress.

Beyond Feynman diagrams: unitarity

- An old technique in QFT: reconstruct a g-loop amplitude from its imaginary part, which is given by (g-1)-loop amplitudes (Landau, Cutkosky, ...).
- New developments with modern techniques for massless gauge theories
  Bern Dixon, Dunbar, Kosower (94), Britto, Cachazo, Feng, Witten (05).

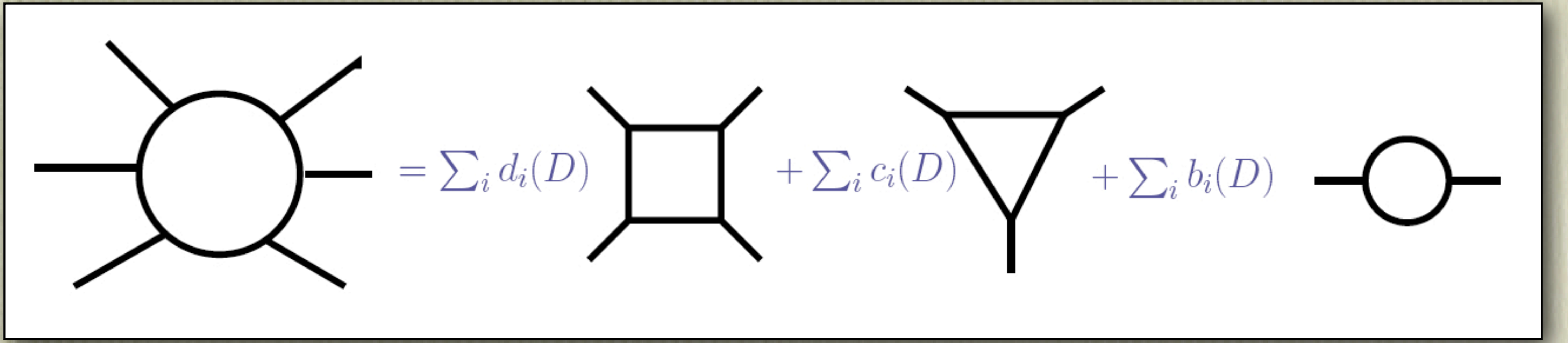


- A simple breakthrough: complex momenta allow for non-vanishing on-shell three-particle amplitude.
- Multiple unitarity cuts express discontinuities of the amplitude as products of lower-loop amplitudes.
- An iterative structure builds up linking loop amplitudes to Born amplitudes.

$$\begin{aligned} k_j^\mu &\to k_j^\mu(z) = k_j^\mu - \frac{z}{2}\langle j^-|\gamma^\mu|l^-\rangle, \\ k_l^\mu &\to k_l^\mu(z) = k_l^\mu + \frac{z}{2}\langle j^-|\gamma^\mu|l^-\rangle\,, \end{aligned}$$

Unitarity and automation at NLO

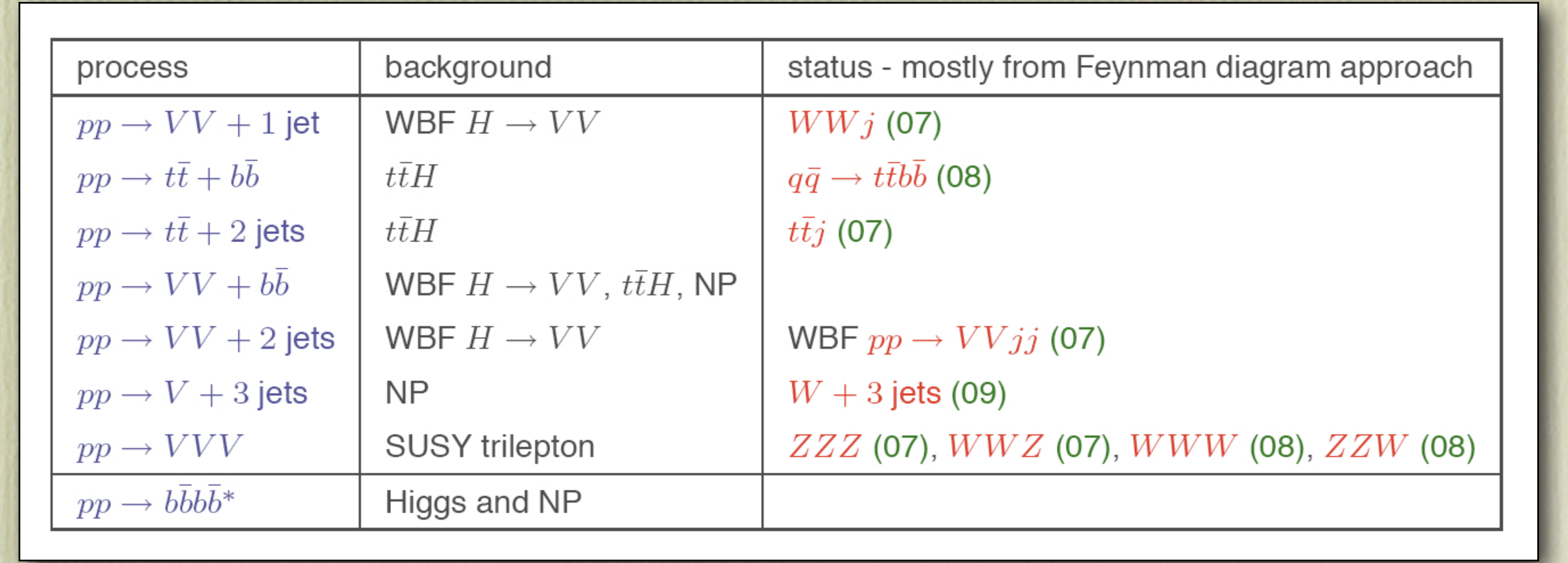
- Unitarity is not sufficient to solve the problem, even at NLO.
- A second cornerstone: a basis for scalar integrals is known.
  - For any number of particles, no polygons beyond boxes.
  - All relevant integrals are known analytically around d = 4.
  - The problem is now algebraic: compute the coefficients of the expansion.



- A third cornerstone: perform the reduction numerically at the integrand level Ossola, Papadopoulos, Pittau (06).
  - Decompose the integrand as in the basis by partial fractioning.
  - Compute coefficients numerically by selecting special values of momenta.
  - Completely algorithmic for general NLO calculations.

NLO factories

NLO calculations are know automated and being interfaced to event generators and parton showers: the effort has reached the industrial stage.



process	background	status - mostly from Feynman diagram approach
$pp \to VV + 1$ jet	WBF $H \to VV$	WWj (07)
$pp \to t\bar{t} + b\bar{b}$	$t\bar{t}H$	$q\bar{q} \to t\bar{t}b\bar{b}$ (08)
$pp \to t\bar{t} + 2$ jets	$t\bar{t}H$	$t\bar{t}j$ (07)
$pp \to VV + b\bar{b}$	WBF $H \to VV$, $t\bar{t}H$, NP	
$pp \to VV + 2$ jets	WBF $H \to VV$	WBF $pp \to VVjj$ (07)
$pp \to V + 3$ jets	NP	$W + 3$ jets (09)
$pp \to VVV$	SUSY trilepton	ZZZ (07), WWZ (07), WWW (08), ZZW (08)
$pp \to b\bar{b}b\bar{b}$*	Higgs and NP	

A partial Les Houches wishlist, on its way to being fulfilled

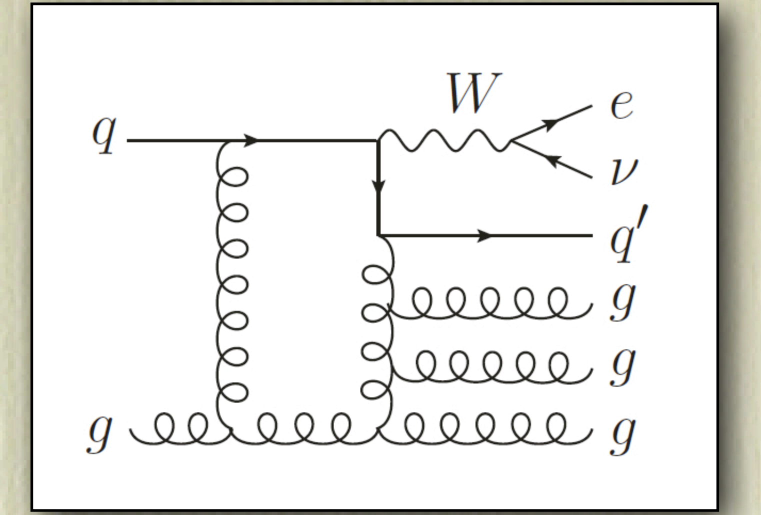
Brands competing on the market:

- Blackhat (d=4 unitarity + SHERPA)
- Rocket (d dimensional unitarity + OPP)
- Golem (Feynman diagrams)
- CutTools (OPP + HELAC)

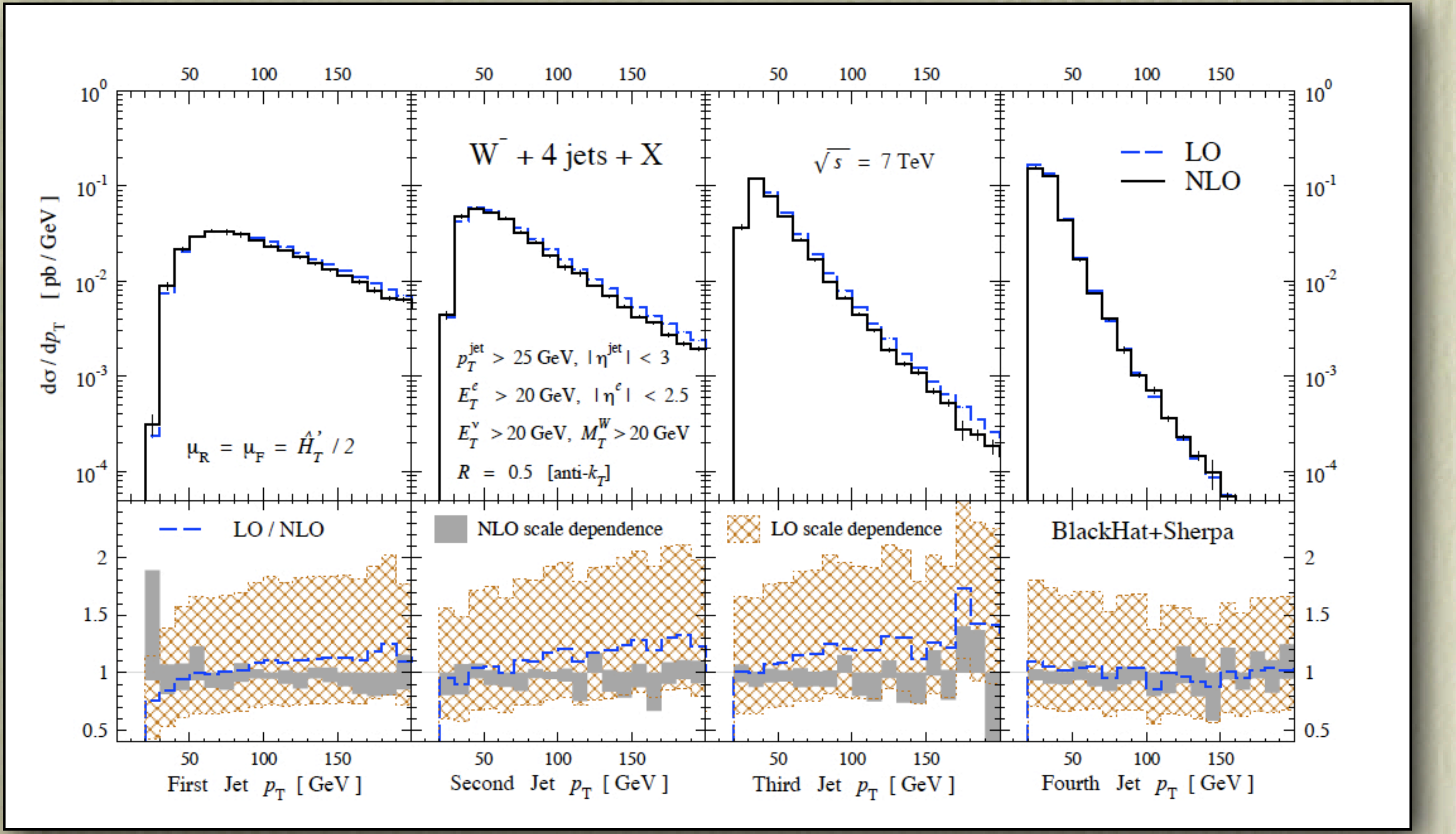
The NLO frontier: V + 4 jets

The first NLO QCD computation with a five-particle final state (leading color approximation) (Blackhat Collaboration 2010).

- Background to top production, SUSY searches ...
- Fully exclusive distributions available.
- Infrared subtractions performed with Sherpa.



A representative Feynman diagram



p_T distributions of the leading 4 jets at the LHC, with 7 TeV CM energy.

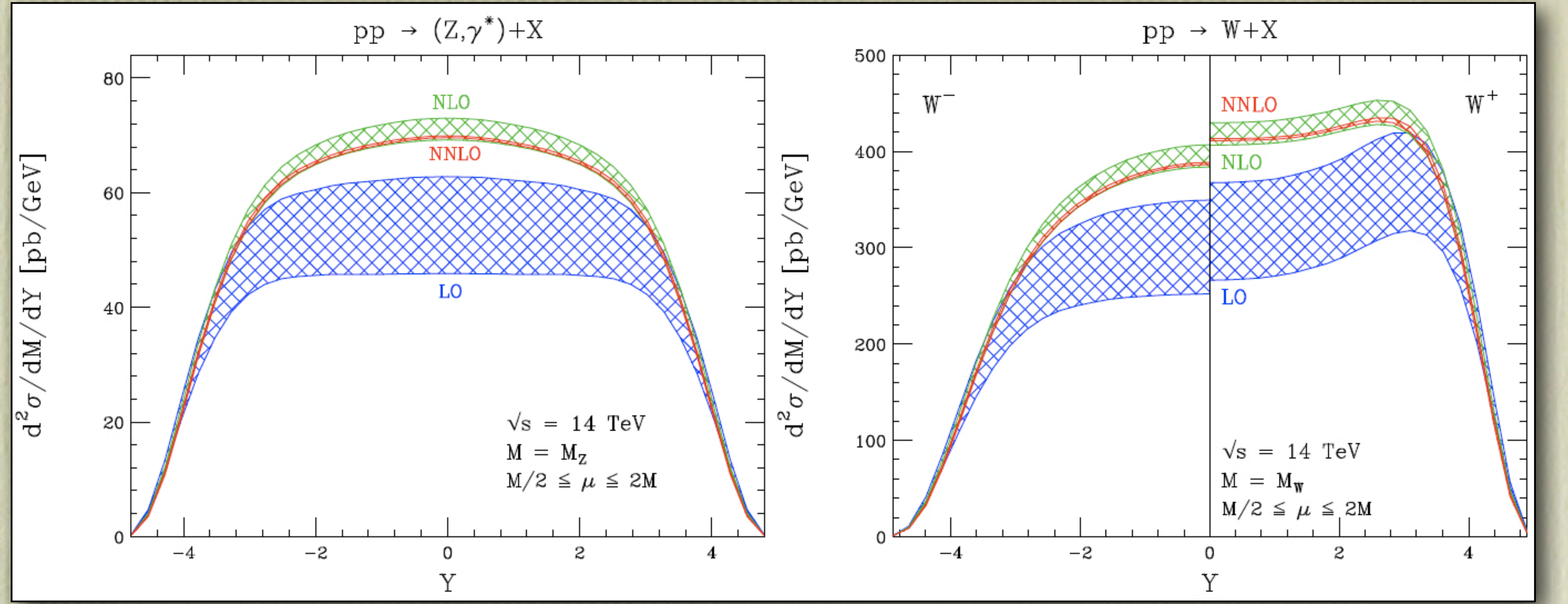
- A truly multi-scale QCD problem: needs a careful choice of renormalization and factorization scales.
- Scale dependence is significantly reduced
- NLO corrections at the 20-30% level for typical distributions

Order by order: NNLO

Deep in the dark bottlenecks

- Bottleneck #1: computing loop integrals.
  - Obstacles: analytic structure, tensor integral decomposition.
    However: a basis of scalar integrals is not known (Kosower et al., in progress!).
  - State of the art: “nearly massless” 4-point amplitudes.
    Ingredients for: jet production at NNLO.
  - Exclusive distributions at NNLO are known only for quantities with just one detected particle in the final state (DY, W-Z-H production).
- Bottleneck #2: subtracting infrared and collinear poles.
  - Combine (n+2)-parton trees with (n+1)-parton one-loop amplitudes and with n-parton two-loop amplitudes.
  - Several groups working on a general subtraction method.
  - Only one calculation completed to date: NNLO e+e- → 3 Jets.
    Gehrmann et al. (07), Weinzierl (08)
- Bottleneck #3: interfacing with parton shower MonteCarlo’s.
  - *Hic Sunt Leones . . .*

NNLO: a teaser



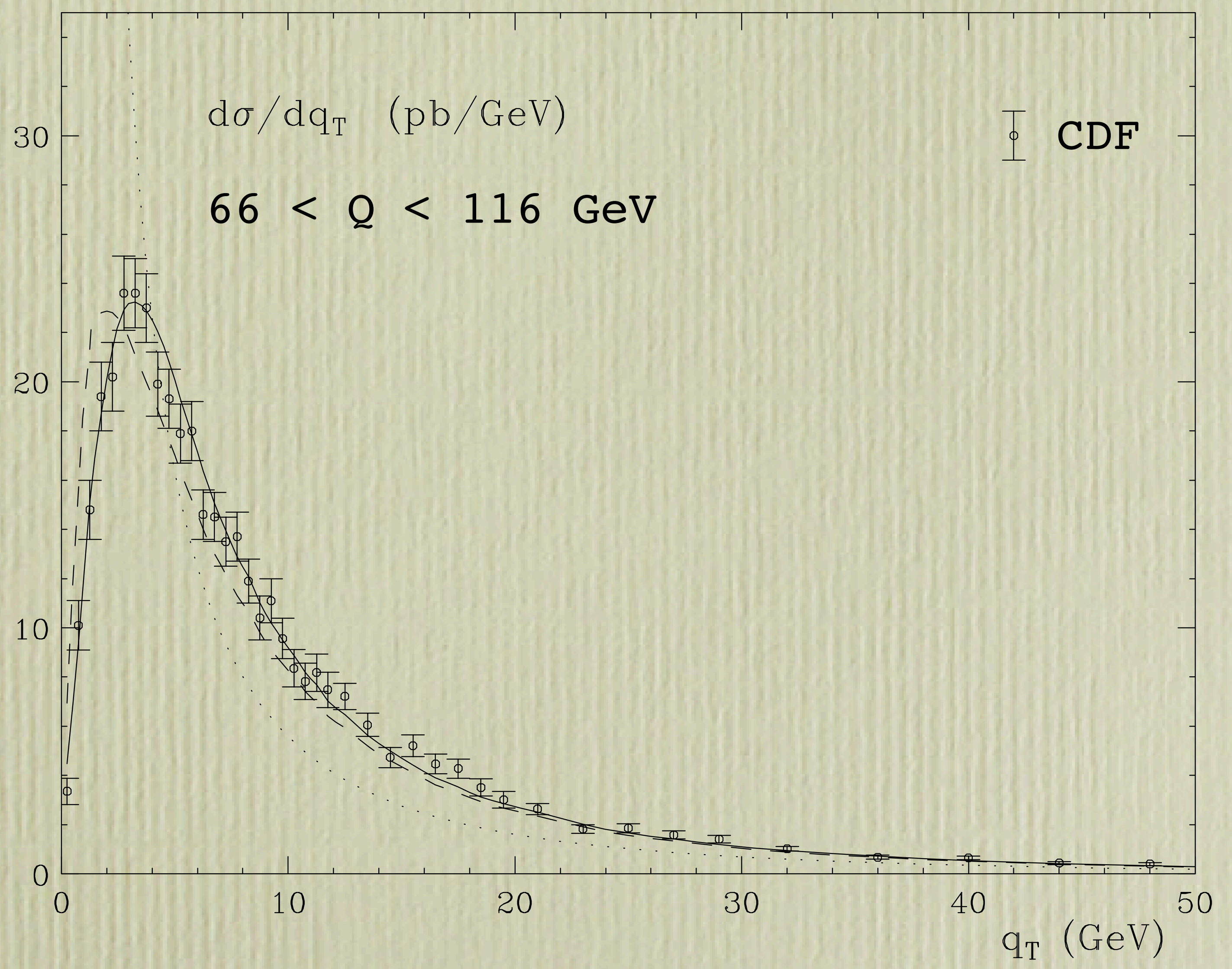
NNLO rapidity distributions for Z and W production at LHC (Anastasiou et al. (03))

- Even for **inclusive** quantities, 50-100% QCD corrections are **common**.
- "K factors" in general are not factors, distributions change shape.
- Theoretical uncertainties are greatly reduced.
- NNLO perturbative accuracy of order 1%: luminometry possible at LHC parton distributions dominate the uncertainty.

Computing the hard scattering QCD cross section

All Orders

Soft gluon phenomenology at colliders



Data for the transverse momentum distribution of Z bosons produced Tevatron, compared to QCD with soft gluon resummation and non-perturbative shift (A. Kulesza et al.).

- The cross section peaks in a region dominated by multiple soft gluon radiation.
- Soft gluon effects can be computed to all orders in perturbation theory.
- They are necessary to understand qualitatively and quantitatively many distributions near kinematic limits.
- Infrared and collinear singularities of amplitudes turn into logarithms of ratios of kinematic scales.
- Resummation of Sudakov (double) logarithms dominates for $q_T \ll Q$.
- Resummation points to power suppressed, non-perturbative corrections that shift the transverse momentum distribution.

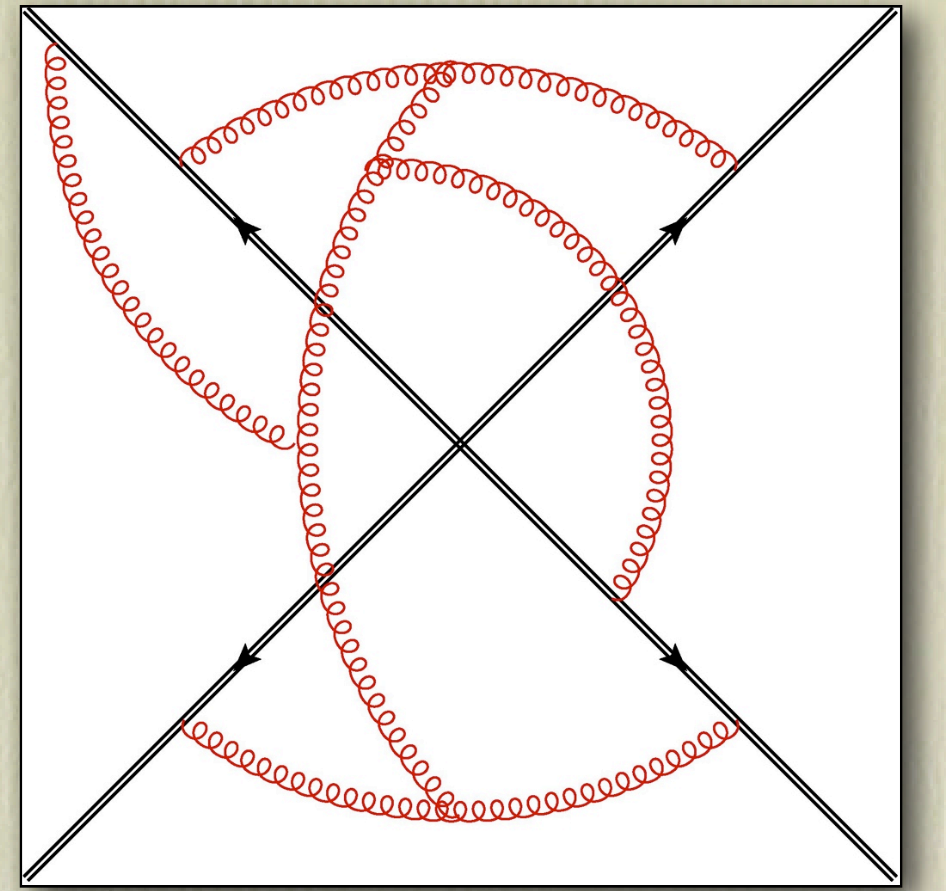
Infrared divergences to all orders

- Infrared singularities for any massless gauge theory amplitude are given by correlators of Wilson lines.
- Infrared singularities factor and exponentiate in terms of a matrix of anomalous dimensions.

$$\mathcal{M}\left(\frac{p_i}{\mu}, \alpha_s(\mu^2), \epsilon\right) = Z\left(\frac{p_i}{\mu_f}, \alpha_s(\mu_f^2), \epsilon\right)\ \mathcal{H}\left(\frac{p_i}{\mu}, \frac{\mu_f}{\mu}, \alpha_s(\mu^2), \epsilon\right),$$

$$Z\left(\frac{p_i}{\mu_f}, \alpha_s(\mu_f^2), \epsilon\right) = P \exp\left[-\frac{1}{2}\int_0^{\mu_f^2} \frac{d\lambda^2}{\lambda^2}\, \Gamma\left(\frac{p_i}{\lambda}, \alpha_s(\lambda^2, \epsilon)\right)\right],$$

- The soft anomalous dimension matrix can be computed directly in terms of special diagrams: “eikonal webs”.



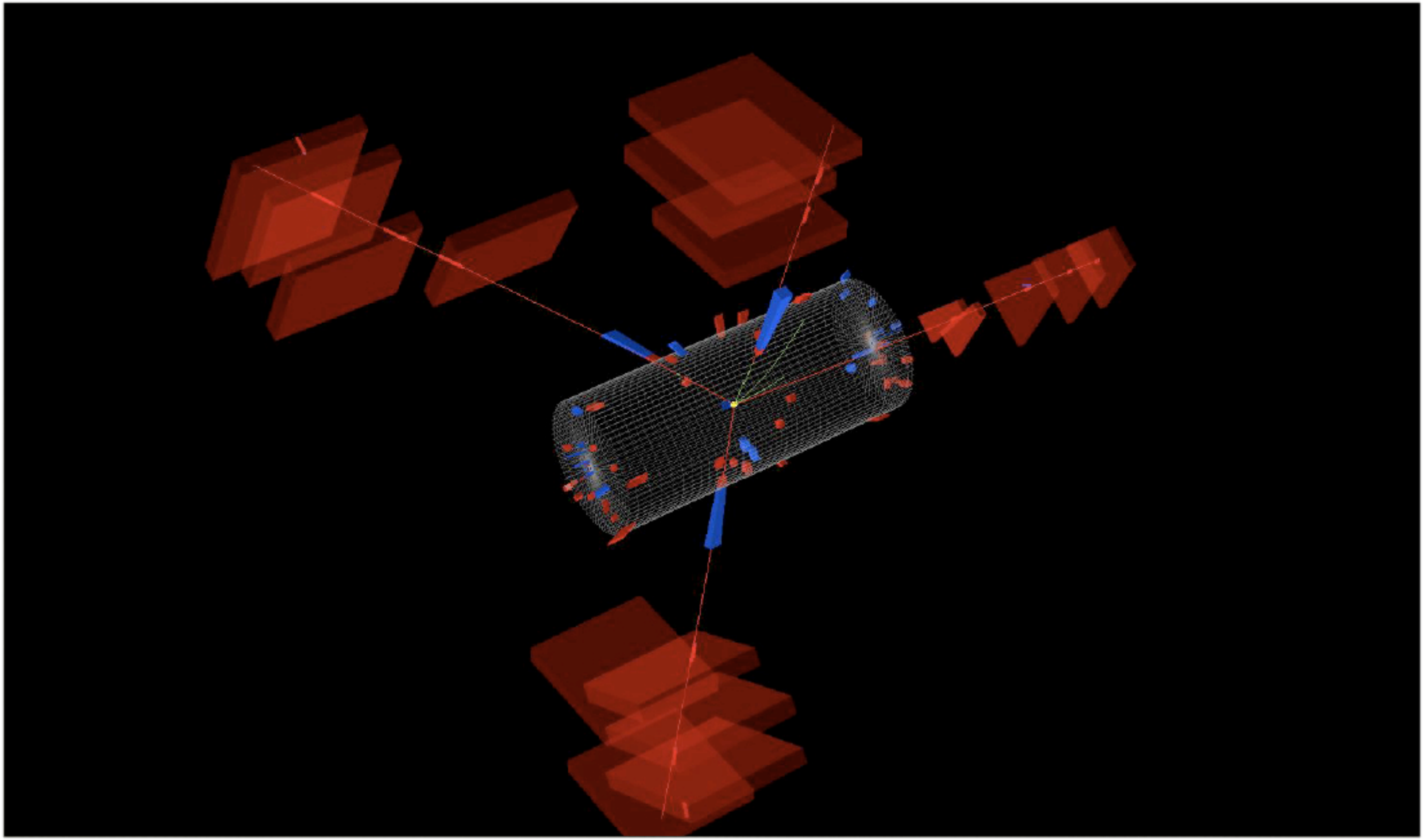
A “web” diagram contributing to the soft anomalous dimension matrix.

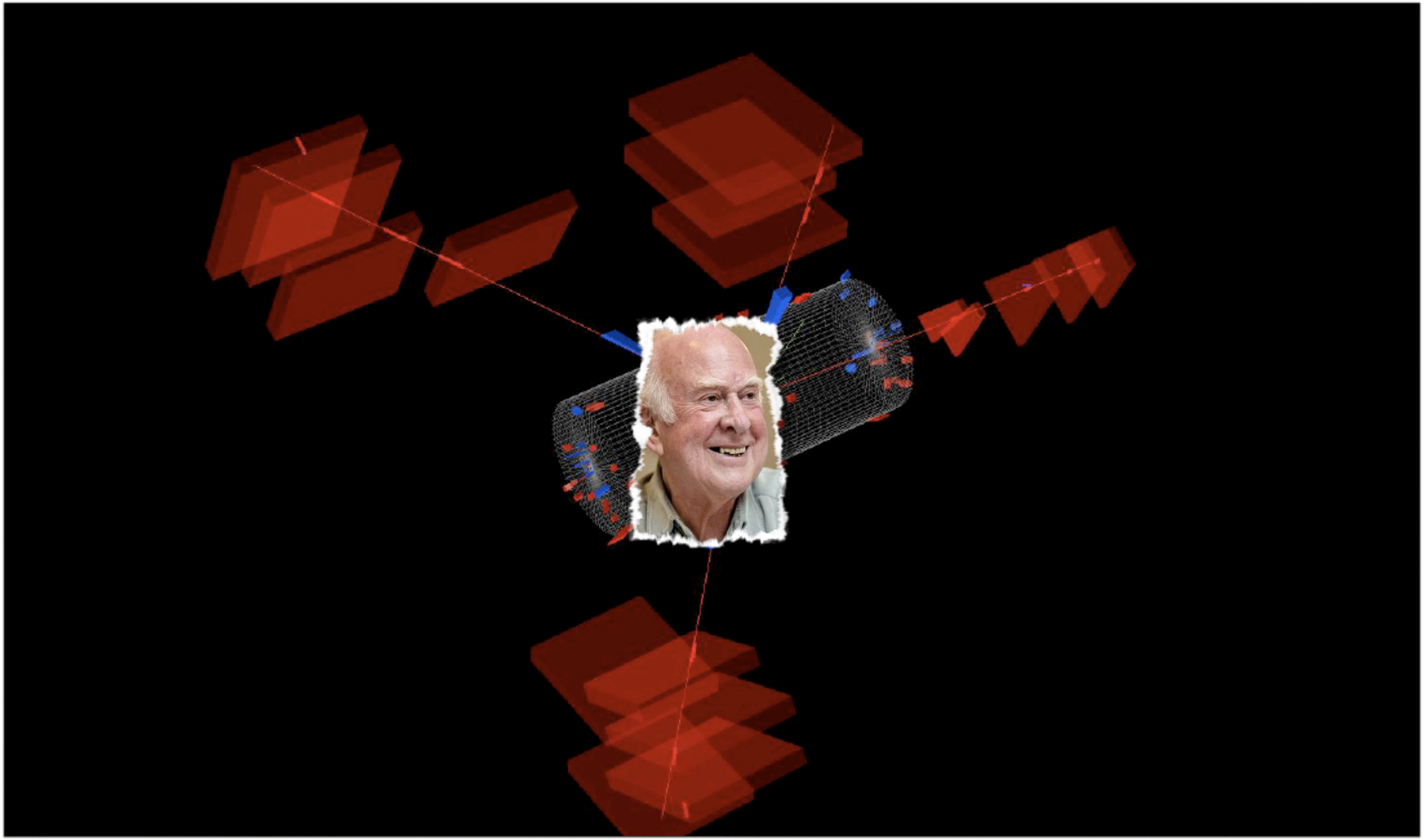
An all-order formula valid for any number of external legs and to all orders has been conjectured (Becher, Neubert 09; Gardi, LM 09).

$$\Gamma_{\rm dip}\left(\frac{p_i}{\lambda}, \alpha_s(\lambda^2)\right) = -\frac{1}{4}\,\widehat{\gamma}_K\left(\alpha_s(\lambda^2)\right) \sum_{j \neq i} \ln\left(\frac{-2\, p_i \cdot p_j}{\lambda^2}\right) \mathbf{T}_i \cdot \mathbf{T}_j + \sum_{i=1}^{n} \gamma_{J_i}\left(\alpha_s(\lambda^2)\right).$$

Perspective

- The motivation provided by LHC has lead to great progress
  - Expected Signal/Background ratios tell us we need total control of the SM
- Perturbative QCD is now predictive to a few % accuracy
  - A massive challenge for a confining non-abelian gauge theory has been met
- Many theorists have been converted to an industrial effort
  - Parton Distribution Factories, NLO Brands, Monte Carlo Marketing ...
- Surprising progress in QFT, not just phenomenology
  - Completely new techniques have emerged, and connections beyond QFT
- QCD is ready to meet the challenge of real data!





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