Exploring the borders of perturbative QCD

Lorenzo Magnea
Università di Torino
I.N.F.N. Torino

magnea@to.infn.it

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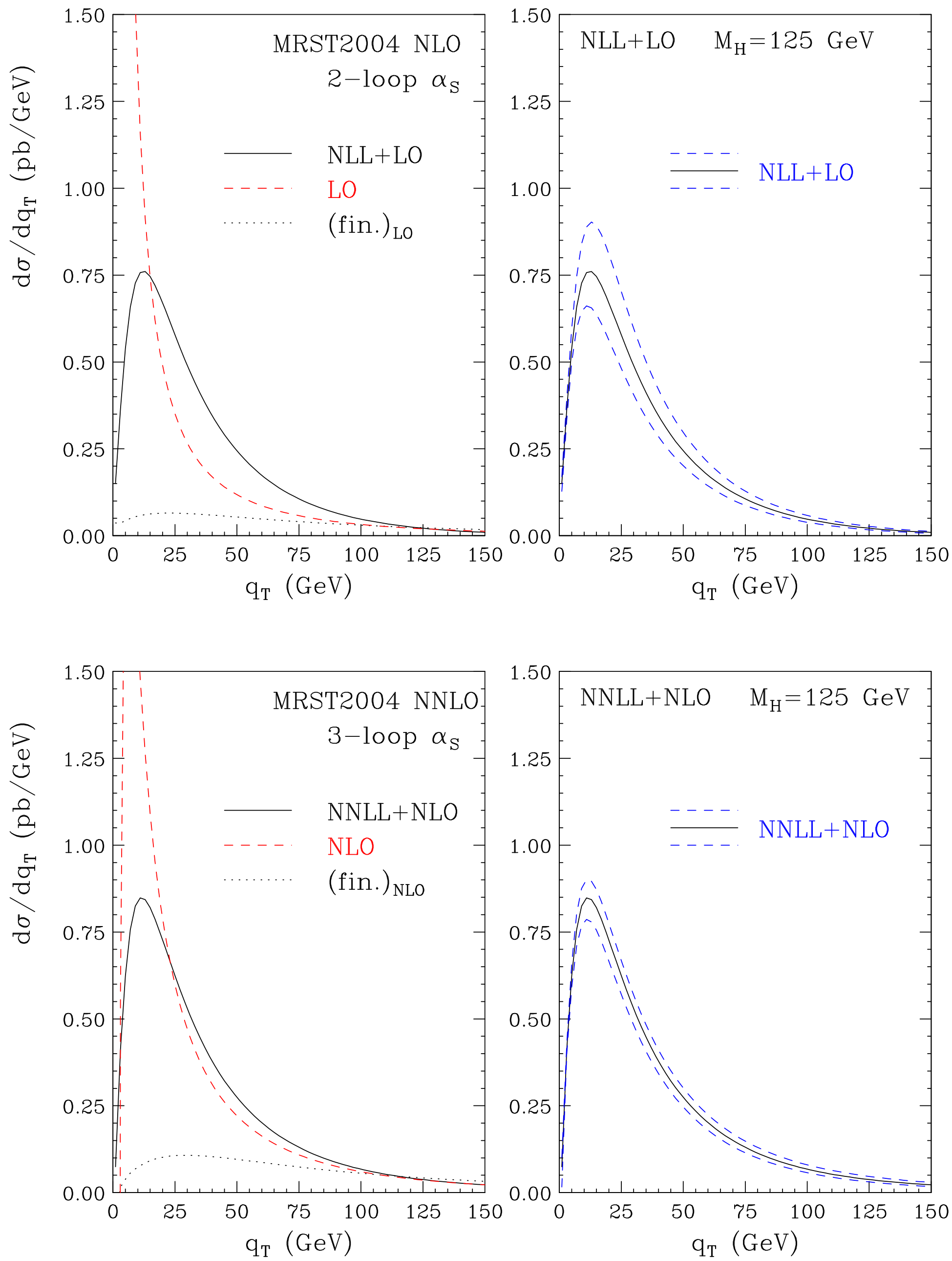
Abstract

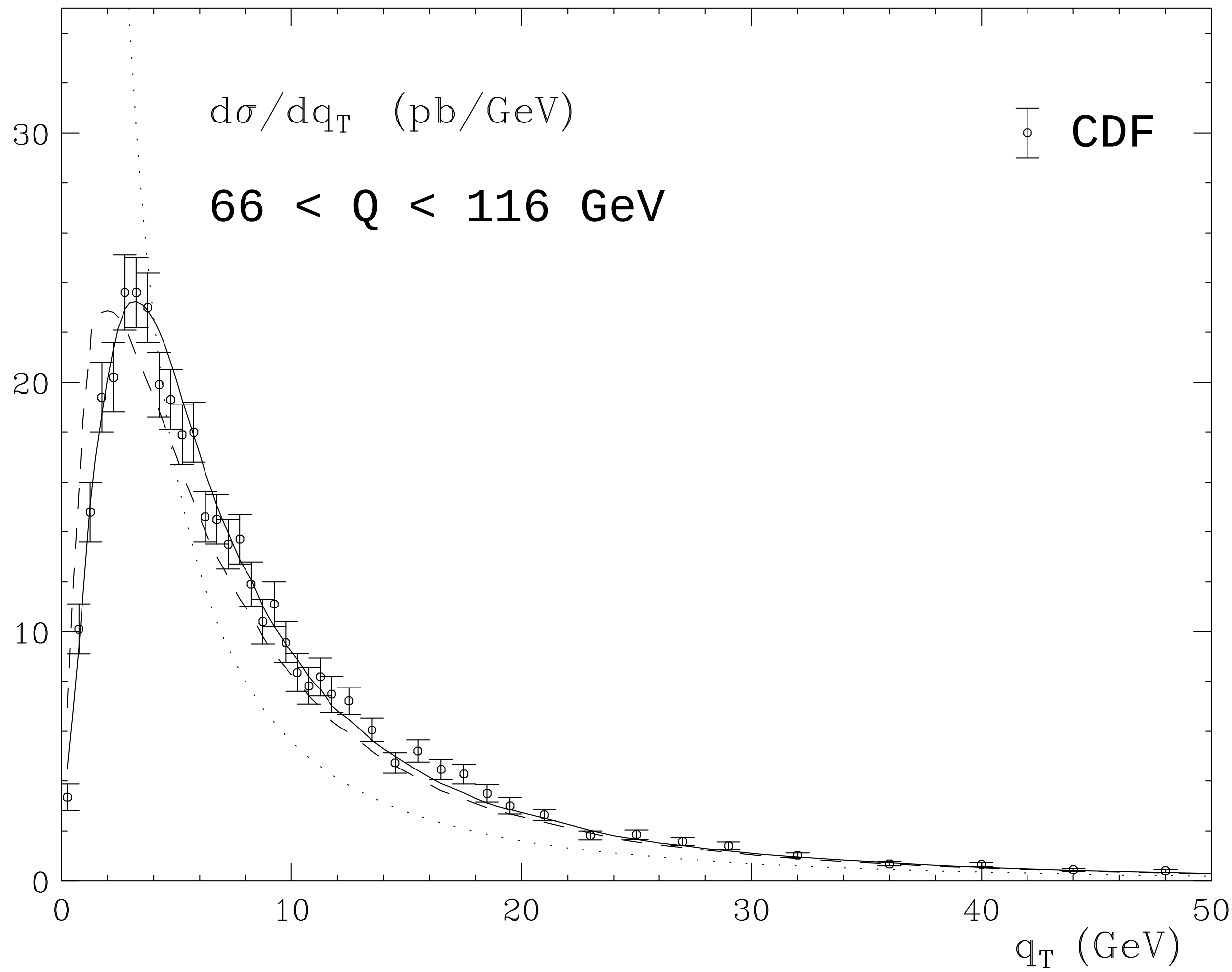
I will review recent progress in the study of soft gluon effects by means of perturbative QCD. This includes improvements and extensions of the techniques of soft gluon resummation, applications to modeling of nonperturbative corrections, and a study of resummation effects on parton distribution fits.

Outline

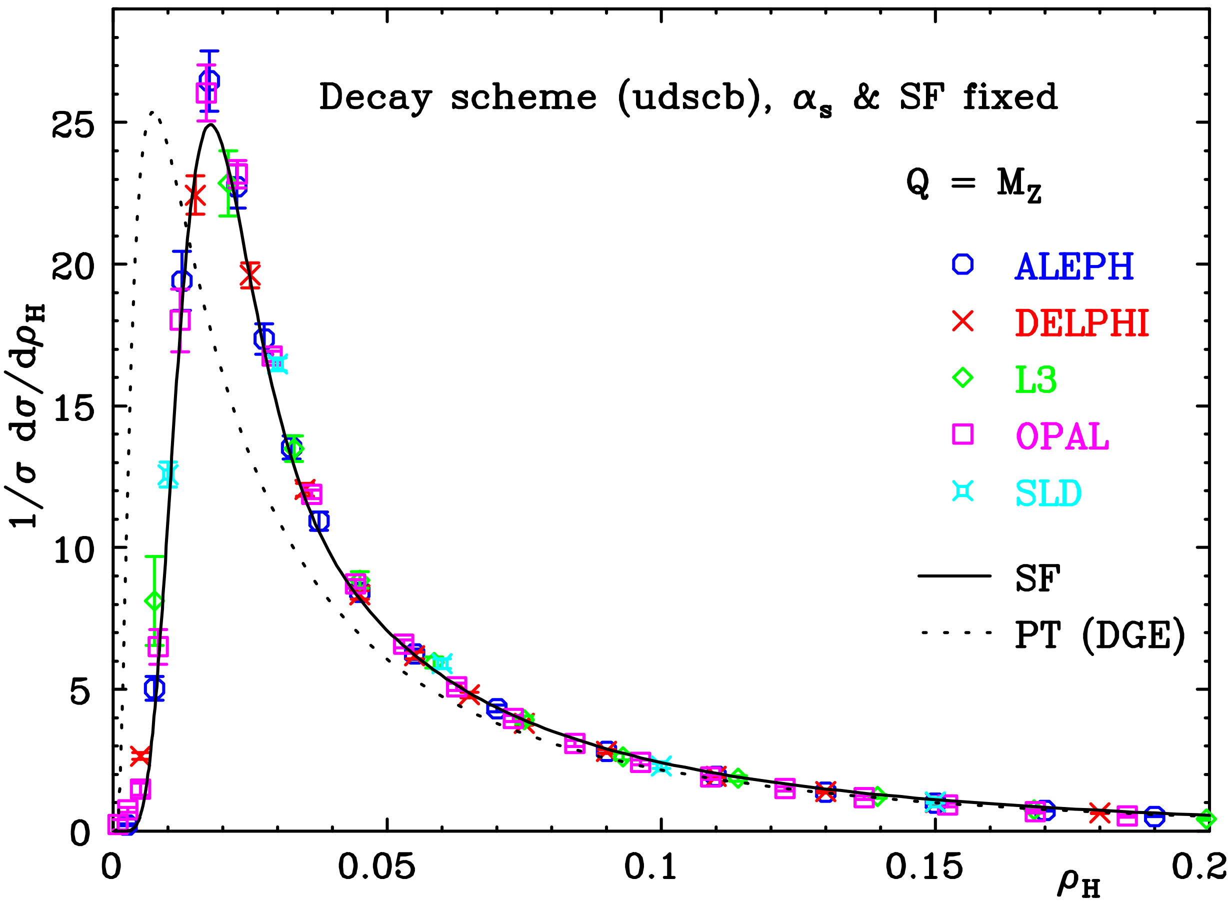
- Strength and Weakness of PQCD
  - Why does PQCD work at all?
  - Limits to the applicability of PQCD.
- From Factorization to Resummation
  - One-scale problems: RG, AP.
  - Multi-scale problems: Sudakov logarithms.
- On Sudakov resummation
  - Examples: EW annihilation, event shapes.
  - From resummation to power corrections.
- Recent Developments
  - More logs for old observables.
  - More observables for old logs.
  - New logs: the non-global movement.
  - Joint logs: joint resummation.
- Resummation effects on parton distributions
  - Motivations and feasibility.
  - A simplified fit: results.
- Perspective



Predictions for the q_T spectrum of Higgs bosons produced via gluon fusion at the LHC, with and without resummation. From M. Grazzini, hep-ph/0512025.



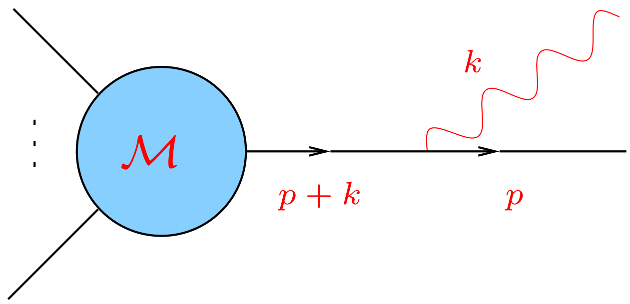
CDF data on Z production compared with QCD predictions at fixed order (dotted), with resummation (dashed), and with the inclusion of power corrections (solid). From A. Kulesza, G. Sterman and W. Vogelsang, hep-ph/0207148.



LEP data on the Heavy Jet Mass distribution, compared with resummed QCD prediction (dotted), and with power corrections treated by Dressed Gluon Exponentiation (solid), with parameters fixed by fitting the thrust distribution. From E. Gardi and J. Rathsmann, hep-ph/0201019.

Why does PQCD work at all?

- In a world of hadrons, we compute cross sections involving quarks and gluons, which do not exist in the true asymptotic states of the theory.
- This inconsistency is visible in perturbation theory: the QCD S-matrix does not exist in the Fock space of quarks and gluons, due to mass singularities.
- Example: a massless fermion emits a gauge boson in the final state



$$\rightarrow \quad - \mathrm{i} g \overline{u}(p) \not{\epsilon}(k) t_a \frac{\mathrm{i}(\not{p} + \not{k})}{(p+k)^2 + \mathrm{i}\varepsilon} \mathcal{M} \, ,$$

Mass singularities: $2p \cdot k = 2\, p_0\, k_0\, (1 - \cos\theta_{pk}) = 0 \, ,$

$\rightarrow k_0 = 0$ (IR); $\cos\theta_{pk} = 0$ (C).

- The situation is worse than QED: the KLN theorem cannot be directly applied, the true asymptotic states are not close enough to the Fock states.

Strategy of Perturbative QCD

- **Infrared Safety**: cancelling mass divergences.
  - Compute partonic cross sections with IR regulator

$$\sigma_{\rm part} = \sigma_{\rm part}\left(\frac{Q^2}{\mu^2}, \alpha_s(\mu^2), \left\{\frac{m^2(\mu^2)}{\mu^2}, \epsilon\right\}\right) \; .$$

  - Identify IR–safe cross sections, having a finite limit as regulators are removed ($\epsilon \to 0$, $m^2(\mu^2) \to 0$).

$$\sigma_{\rm part} = \sigma_{\rm part}\left(\frac{Q^2}{\mu^2}, \alpha_s(\mu^2), \{0,0\}\right) + \mathcal{O}\left(\left\{\left(\frac{m^2}{\mu^2}\right)^p, \epsilon\right\}\right) \; .$$

  - Interpret $\sigma_{\rm part}$ as perturbative estimate of hadronic cross section valid up to corrections $\mathcal{O}\left((\Lambda_{QCD}/Q)^p\right)$

- **Factorization**: neutralizing mass divergences.
  - Quantum incoherence in the presence of different scales implies, to all orders in PT for inclusive cross sections,

$$\sigma_{\rm part} = f\left(\frac{m^2}{\mu_F^2}\right) * \widehat{\sigma}_{\rm part}\left(\frac{Q^2}{\mu^2}, \frac{\mu_F^2}{\mu^2}\right) + \mathcal{O}\left(\left(\frac{m^2}{\mu_F^2}\right)^p\right) \; .$$

  - Combine the partonic, perturbative, process–dependent $\widehat{\sigma}_{\rm part}$ with measured, nonperturbative, universal f to derive hadronic cross section.

The Borders of Perturbative QCD

- Power corrections
  - All factorization theorems are valid up to nonperturbative corrections suppressed by powers of the hard scale, $\mathcal{O}\left(\left(\Lambda^2/Q^2\right)^p\right)$.
  - In the presence of several hard scales, power corrections can be enhanced. In DIS as $x \to 1$, for example, as $\mathcal{O}\left(\left(\Lambda^2/\left(Q^2(1-x)\right)\right)^p\right)$.

- Large logarithms

  Multi–scale problems can have large perturbative corrections of the general form $\alpha_s^n \log^k\left(Q_i^2/Q_j^2\right)$, with $k \leq n$ (single logs) or $k < 2n$ (double logs). Examples include
  - Renormalization logs, $\alpha_s^n \log^n\left(Q^2/\mu_R^2\right)$.
  - Factorization logs, $\alpha_s^n \log^n\left(Q^2/\mu_F^2\right)$.
  - High–energy logs, $\alpha_s^n \log^{n-2}(s/t)$.
  - Sudakov logs in DIS, $\alpha_s^n \log^{2n-1}\left(Q^2/W^2\right)$.
  - Sudakov logs in Higgs production, $\alpha_s^n \log^{2n-1}\left(1 - M_H^2/\hat{s}\right)$.
  - Transverse momentum logs, $\alpha_s^n \log^{2n-1}\left(Q_\perp^2/Q^2\right)$.

- Sudakov logs originate from mass singularities, thus they are universal and can be resummed. All-order expressions contain nonperturbative information.

Factorization leads to Resummation

All factorizations separating dynamics at different energy scales lead to resummation of logarithmic dependence on the ratio of scales.

- Renormalization group logarithms.

  Renormalization factorizes cutoff dependence

$$G_0^{(n)}\left(p_i, \Lambda, g_0\right) = \prod_{i=1}^{n} Z_i^{1/2}\left(\Lambda/\mu, g(\mu)\right)\; G_R^{(n)}\left(p_i, \mu, g(\mu)\right) \;,$$

$$\frac{dG_0^{(n)}}{d\mu} = 0 \quad \to \quad \frac{d\log G_R^{(n)}}{d\log\mu} = -\sum_{i=1}^{n} \gamma_i\left(g(\mu)\right) \;.$$

  RG evolution resums $\alpha_s^n(\mu^2)\log^n\left(Q^2/\mu^2\right)$ into $\alpha_s(Q^2)$.

- Altarelli–Parisi logarithms.

  Partonic DIS structure functions factorize as

$$\widetilde{F}_2\left(N, \frac{Q^2}{m^2}, \alpha_s\right) = \widetilde{C}\left(N, \frac{Q^2}{\mu_F^2}, \alpha_s\right) \widetilde{f}\left(N, \frac{\mu_F^2}{m^2}, \alpha_s\right)$$

$$\frac{d\widetilde{F}_2}{d\mu_F} = 0 \quad \to \quad \frac{d\log\widetilde{f}}{d\log\mu_F} = \gamma_N\left(\alpha_s\right) \;.$$

  AP evolution resums collinear logarithms into evolved PDF's.

- Double logarithms are more difficult. Renormalization group is not sufficient, gauge invariance plays a key role.

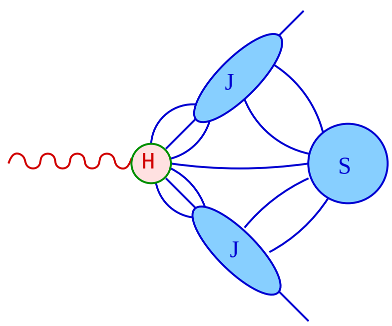
Simplest Sudakov: the quark form factor

At the amplitude level, resummation leads to exponentiation of IR and collinear poles. Consider the EM quark form factor

$$\Gamma_\mu(p_1, p_2; \mu^2, \epsilon) \equiv \langle p_1, p_2 | J_\mu(0) | 0 \rangle = -ieq_f \, \overline{u}(p_1) \gamma_\mu v(p_2) \, \Gamma\left(\frac{Q^2}{\mu^2}, \alpha_s(\mu^2), \epsilon\right)$$

Power counting of singular regions in Feynman diagrams and application of Ward identities lead to a diagrammatic factorization of collinear and soft effects

$$\Gamma_\nu\left(p_1, p_2; \mu^2, \epsilon\right) =$$



Schematically, in an axial gauge, this leads to

$$\Gamma\left(\frac{Q^2}{\mu^2}, \alpha_s(\mu^2), \epsilon\right) = J\left(\frac{(p \cdot n)^2}{\mu^2 n^2}\right) \mathcal{S}\left(u_i \cdot n\right) H\left(\frac{(p_i \cdot n)^2}{\mu^2 n^2}\right).$$

with H finite. Gauge invariance then implies

$$\frac{\partial \log \Gamma}{\partial p_1 \cdot n} = 0 \quad \rightarrow \quad \frac{\partial \log J_1}{\partial \log(p_1 \cdot n)} = -\frac{\partial \log H}{\partial \log(p_1 \cdot n)} - \frac{\partial \log \mathcal{S}}{\partial \log(u_1 \cdot n)}.$$

Again, the two functions on the right hand side depend on different arguments. The equation is of the form

$$\frac{\partial \log J}{\partial \log Q^2} = K_J \left(\alpha_s(\mu^2), \epsilon \right) + G_J \left(\frac{Q^2}{\mu^2}, \alpha_s(\mu^2), \epsilon \right) \ .$$

with all Q^2 dependence in the finite function G_J, and all divergences in the Q^2-independent function K_J. It is easy to show that an identical equation is obeyed by the full form factor. Then one can exploit RG invariance of the form factor to write

$$\mu \frac{dG}{d\mu} = - \mu \frac{dK}{d\mu} = \gamma_K(\alpha_s(\mu)) \ ,$$

Solving the equations leads to the exponentiation (LM, G. Sterman)

$$\Gamma \left(\frac{Q^2}{\mu^2}, \alpha_s(\mu^2), \epsilon \right) \ = \ \exp \left\{ \frac{1}{2} \int_0^{-Q^2} \frac{d\xi^2}{\xi^2} \left[K \left(\epsilon, \alpha_s(\mu^2) \right) \ + \right. \right.$$

$$\left. \left. G \left(-1, \overline{\alpha} \left(\frac{\xi^2}{\mu^2}, \alpha_s(\mu^2), \epsilon \right), \epsilon \right) + \frac{1}{2} \int_{\xi^2}^{\mu^2} \frac{d\lambda^2}{\lambda^2} \gamma_K \left(\overline{\alpha} \left(\frac{\lambda^2}{\mu^2}, \alpha_s(\mu^2), \epsilon \right) \right) \right] \right\} .$$

where singularities at $\xi = 0$ are regulated by the d-dimensional running coupling

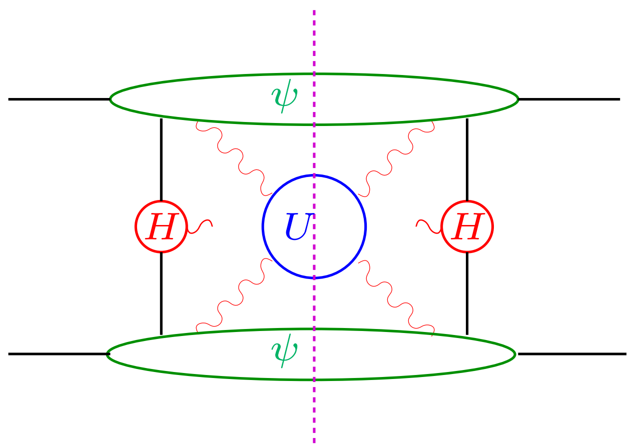
$$\overline{\alpha} \left(\frac{\mu^2}{\mu_0^2}, \alpha_s(\mu_0^2), \epsilon \right) = \frac{\alpha_s(\mu_0^2)}{\left(\frac{\mu^2}{\mu_0^2} \right)^{\epsilon} - \frac{1}{\epsilon} \left(1 - \left(\frac{\mu^2}{\mu_0^2} \right)^{\epsilon} \right) \frac{b_0}{4\pi} \alpha_s(\mu_0^2)} \ ,$$

Double poles in Γ are replaced by single poles in the exponent.

Example: electroweak annihilation

The cross sections for Drell-Yan, W, Z and Higgs production receive large QCD corrections.

- Threshold logarithms, $\log(1 - Q^2/\hat{s})$
- Transverse momentum logarithms $\log(p_t^2/Q^2)$.



To resum threshold logarithms one works with the Mellin transform of the partonic cross section. It factorizes as

$$\omega(N, \epsilon) = |H_{\rm DY}|^2 \, \psi(N, \epsilon)^2 \, U(N) + \mathcal{O}(1/N) \, .$$

After subtracting collinear poles, in the $\overline{\rm MS}$ scheme

$$\widehat{\omega}_{\overline{\rm MS}}(N) = \exp \left[\int_0^1 dz \, \frac{z^{N-1} - 1}{1 - z} \left\{ 2 \int_{Q^2}^{(1-z)^2 Q^2} \frac{d\mu^2}{\mu^2} \, A\left(\alpha_s(\mu^2)\right) \right. \right.$$

$$\left. \left. + \; D\left(\alpha_s\left((1-z)^2 Q^2\right)\right) \right\} + \mathcal{F}_{\overline{\rm MS}}(\alpha_s) \right] + \mathcal{O}\left(\frac{1}{N}\right) .$$

Example: event shape distributions

- Examples
  - Thrust: $T = \max_{\vec{n}} \frac{\sum_i |\vec{p}_i \cdot \vec{n}|}{Q}$; $\quad t = 1 - T$.

    $\rightarrow$ $\vec{n}$ is used to define several other shape variables.
  - C-parameter: $C = 3 - \frac{3}{2} \sum_{i,j} \frac{(p_i \cdot p_j)^2}{(p_i \cdot q)\,(p_j \cdot q)}$.

    $\rightarrow$ does not require maximization procedures.
  - Angularity: $\tau_a = \frac{1}{Q} \sum_i (p_\perp)_i \mathrm{e}^{-|\eta_i|(1-a)}$.

    $\rightarrow$ recently introduced (C. Berger, G. Sterman)

- Two-jet limit: infrared and collinear emission dominates
  - Double logarithms here involve the variable vanishing in the two-jet limit: $\log(1-T)$, $\log C$.
  - The Laplace transform exponentiates. For thrust

$$\int_0^\infty d\,\tau \mathrm{e}^{-\nu\tau} \frac{1}{\sigma} \frac{d\sigma}{d\tau} \;=\; \exp \left[\int_0^1 \frac{du}{u} \left(\mathrm{e}^{-u\nu} - 1 \right) \right.$$

$$\left. \times \left(B\left(\alpha_s\left(uQ^2\right)\right) + \int_{u^2Q^2}^{uQ^2} \frac{dq^2}{q^2} A\left(\alpha_s(q^2)\right) \right) \right] .$$

  - $A(\alpha_s) = \sum A_n (\alpha_s/\pi)^n$, determining leading logs, is a universal anomalous dimension, with $A_1 = C_F$, $A_2 = C_A C_F (67/18 - \zeta(2))/2 - 5 n_f C_F/18$.

Features of Sudakov resummation

- Non–trivial. Reorganizes perturbation theory in a predictive way. For threshold resummation, let $L = \log N$. Then

$$\sum_k \alpha_s^k \sum_p^{2k} c_{kp} L^p \to \exp\left[\sum_k \alpha_s^k \sum_p^{k+1} d_{kp} L^p\right] .$$

- Predictive. Resummation extends the range of perturbative methods. Fixed order: $\alpha_s L^2 \ll 1$. NLL resummation: $\alpha_s \ll 1$ suffices. Scale dependence is reduced.
- Widespread. NLL soft gluon resummations exist for most inclusive cross sections of interest at colliders (NNLL now available for processes which are electroweak at tree level).
- Non–perturbative aspects of QCD become accessible. Integrals in the exponent run into the Landau pole.

  A variety of regularizations have been proposed
  - Principal value/cutoff (G. Korchemsky and G. Sterman, E. Gardi)
  - Regular IR coupling (Y. Dokshitzer, G. Marchesini and B. Webber)
  - Dimensional regularization (LM)
  - Minimal prescription, (S. Catani *et al.*)

  * Result: answer ambiguous by a power–suppressed amount.
  * Conclusion: a matching ambiguity must be provided by power–suppressed, nonperturbative contributions.
  * Phenomenology: models of power corrections built using information from resummations.

More logarithms ....

- $(\ln N)^0$ terms exponentiate in all processes which are electroweak at tree level (TEW) (T. Eynck, E. Laenen and LM).

  For Drell-Yan in the $\overline{\mathrm{MS}}$ scheme

$$\widehat{\omega}_{\overline{\mathrm{MS}}}(N) = \left(\frac{|\Gamma(Q^2,\epsilon)|^2}{\phi_V(\epsilon)^2}\right) \left[\frac{(\psi_R(N,\epsilon))^2\, U_R(N,\epsilon)}{(\phi_R(N,\epsilon))^2}\right] + \mathcal{O}\left(\frac{1}{N}\right) .$$

  - Real and virtual contributions can be made separately finite.
  - Virtual contributions are given exactly by finite terms in the Sudakov form factor
  - Not as predictive as resummation of logarithms, but ...

- Three loops are now available for the nonsinglet splitting function (S. Moch, J. Vermaseren and A. Vogt).

$$A_3 = 16\, C_F C_A^2 \left(\frac{245}{24} - \frac{67}{9}\zeta_2 + \frac{11}{6}\zeta_3 + \frac{11}{5}\zeta_2^2\right) + 16\, C_F^2 n_f \left(-\frac{55}{24} + 2\zeta_3\right)$$
$$+\ 16\, C_F C_A n_f \left(-\frac{209}{108} + \frac{10}{9}\zeta_2 - \frac{7}{3}\zeta_3\right) + 16\, C_F n_f^2 \left(-\frac{1}{27}\right) .$$

  - A_3 was accurately estimated numerically from approximate calculations (A. Vogt).
  - n_f-dependence was independently computed with different methods (J. Gracey, C. Berger).
  - It has a small numerical effect on tested cross sections.
  - Complete NNLL threshold resummation now available for all inclusive TEW processes.

... and more ....

- For N^kLL resummation, $A^{(k+1)}$ and $D^{(k)}$ (or $B^{(k)}$) are needed.
- With the three-loop calculation of DIS coefficient functions by MVV, only $A^{(4)}$ is missing to perform N^3LL resummation for DIS. The effect of $A^{(4)}$ is tiny and can be estimated.
- Approximate N^3LL resummation tests convergence of logarithmic as well as fixed order expansions.
- The same degree of accuracy can be reached for all TEW processes. (MV, E. Laenen & LM, A. Idilbi et al.).
- The function $D(\alpha_s)$ for Drell-Yan and Higgs production via gluon fusion can be computed to k loops using
  - Sudakov and splitting function data at k loops.
  - Constant terms for Drell-Yan (Higgs) at $(k-1)$ loops.

$$D(\alpha_s) = 4\, B_\delta(\alpha_s) - 2\, \widetilde{G}(\alpha_s) + \beta(\alpha_s)\, \frac{d}{d\alpha_s} F_{\overline{\mathrm{MS}}}\,(\alpha_s)\ .$$

- At three loops

$$\begin{aligned} D_R^{(3)} = \ & \left(-\frac{297029}{23328} + \frac{6139}{324}\zeta_2 - \frac{187}{60}\zeta_2^2 + \frac{2509}{108}\zeta_3 - \frac{11}{6}\zeta_2\zeta_3 - 6\zeta_5\right) C_A^2 C_R \\ + \ & \left(\frac{31313}{11664} - \frac{1837}{324}\zeta_2 + \frac{23}{30}\zeta_2^2 - \frac{155}{36}\zeta_3\right) n_f C_A C_R \\ + \ & \left(\frac{1711}{864} - \frac{1}{2}\zeta_2 - \frac{1}{5}\zeta_2^2 - \frac{19}{18}\zeta_3\right) n_f C_F C_R \\ + \ & \left(-\frac{58}{729} + \frac{10}{27}\zeta_2 + \frac{5}{27}\zeta_3\right) n_f^2 C_R\ . \end{aligned}$$

… and even more?

- Do suppressed logs exponentiate? In the $\overline{\rm MS}$ scheme

$$\gamma_{\rm ns}^{(n)}(N) \;=\; A_n(\ln N + \gamma_e) - B_n - C_n \, \frac{\ln N}{N} + \mathcal{O}\left(\frac{1}{N}\right) \; .$$

$$C_1 = 0 \; , \qquad C_2 = 4 C_F A_1 \; , \qquad C_3 = 8 C_F A_2 \; .$$

  … a "suggestive relation" (MVV).

- Mixed evidence for exponentiation at $(\ln N)/N$ level.
  - Leading $\alpha_s^k (\ln N)^{(2k-1)}/N$ terms appear to exponentiate.
  - They have collinear origin (nonsingular terms in the splitting function).
  - Subleading $\alpha_s^k (\ln N)^{(2k-2)}/N$ terms do not exponentiate according to the conventional pattern.

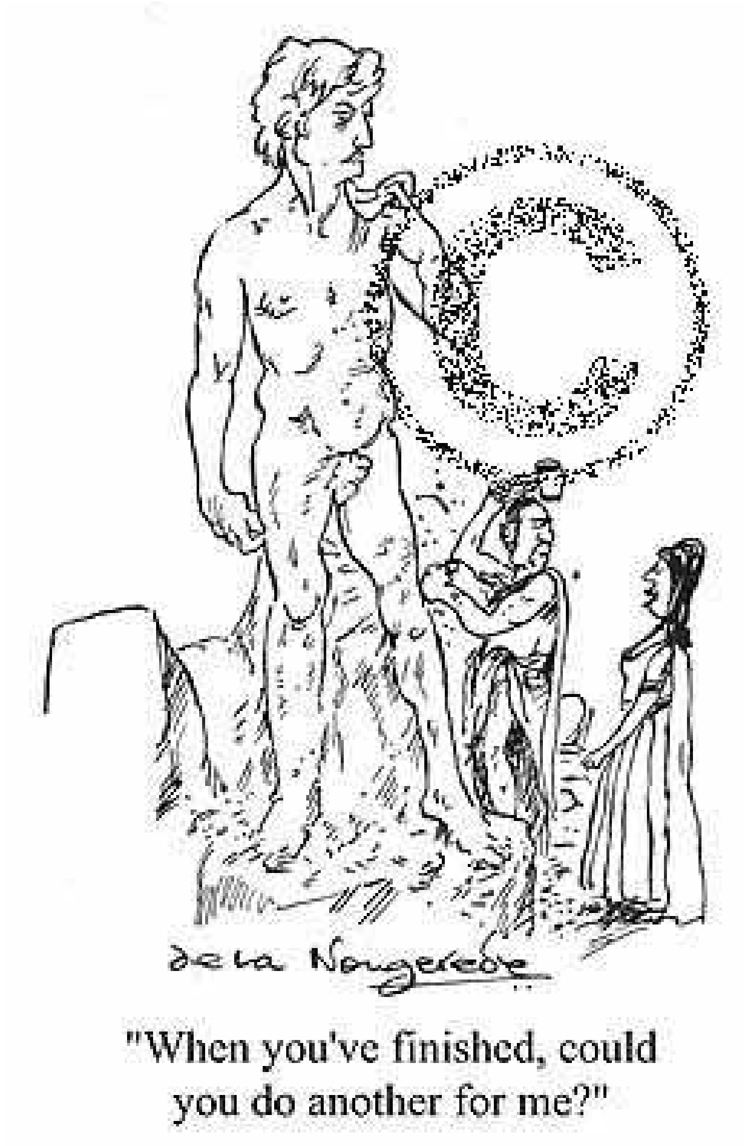
- An unconventional pattern?
  Parton evolution can be modified at large x (Y. Dokshitzer, G. Marchesini, G. Salam)

$$\partial_t D(x,Q^2) = \int_0^1 \frac{dz}{z} P\left(z, \alpha_s(z^{-1} Q^2)\right) \, D\left(\frac{x}{z}, z^\sigma Q^2\right)$$

  - Restores symmetry between PDF and fragmentation function evolution $(\sigma = \pm 1)$.
  - Provides an explanation for the "suggestive relation" of (MVV).

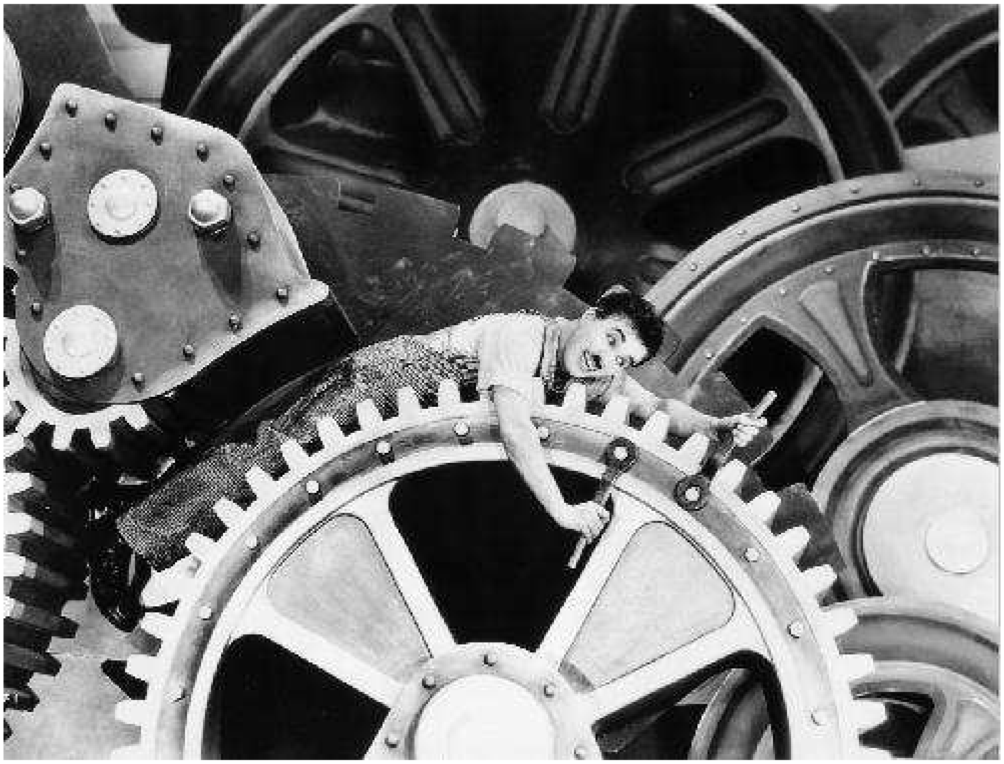
  Note: impact of $(\ln N)/N$ exponentiation can be sizeable.

More observables?



Resummation in Classical Times

Sure ...



Resummation in Modern Times

... here they are!

Automated resummation procedures are being developed for a vast class of observables and processes, including hadronic collisions (A. Banfi, G. Salam and G. Zanderighi).

- Observables: with up to 4 hard partons, must vanish when a softer parton becomes collinear to a hard one.

$$V(\{p_i\}, k) = d_i \left(\frac{k_t}{Q}\right)^a \mathrm{e}^{-b_i \eta} \, g_i(\phi) \ .$$

  Example: $a = d_i = g_i = 1$, $b_i = 0 \longrightarrow$ thrust.

- Requirements
  - Recursive IRC safety: slightly stronger than conventional IRC safety, it requires that the observable behave uniformly under the addition of a hierarchy of soft/collinear partons.
  - Continuous globality: the observable must be sensitive to emissions in the whole phase space without discontinuities, to avoid non-global logs.

- NLL Master Equation

$$\ln \Sigma(v) = -\sum_{i=1}^{n} C_i \left[R_i(a, b_i) + v \frac{\partial R_i}{\partial v} f\left(d_i, g_i\right) + B_i \, T\left(\frac{\log v}{a + b_i}\right)\right]$$

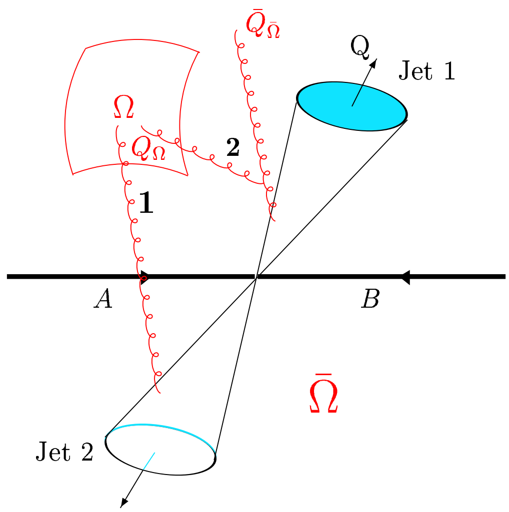
$$+ \sum_{i=1}^{n_i} \ln \frac{f_i(x_i, v^{\frac{2}{a+b_i}} \mu_f^2)}{f_i(x_i, \mu_f^2)} + \ln\left(S\left[T\left(\frac{\log v}{a}\right)\right]\right) + \ln\left[\mathcal{F}_{\mathrm{num}}\left(R_i\right)\right] .$$

- Phenomenology: in progress.

New logs

``‘‘Thou shall not cut up your phase space!’’``

- Consider radiation into a fixed angular region Ω, in the presence of a hard event at scale Q.
- Measure cross section for radiation into Ω to carry energy $E < Q_\Omega << Q \longrightarrow$ get $\alpha_s \log(Q_\Omega/Q)$.



- Primary radiation: hard partons emit gluons into Ω. Standard soft gluon techniques apply.
- Secondary radiation: a primary "semihard" gluon carrying energy $\bar{Q}_{\bar{\Omega}}$ into $\bar{\Omega}$ emits softer gluons into Ω.
- With no restriction on radiation into $\bar{\Omega}$, get $\log(Q_\Omega/\bar{Q}_{\bar{\Omega}}) \sim \log(Q_\Omega/Q)$ (M. Dasgupta and G. Salam).

The Non-Global movement

The rise of non-global logarithms has triggered considerable theoretical activity. Two approaches have been considered.

- Define observables that minimize the impact of non-global logs and apply standard tecniques.
  - R. Appleby and M. Seymour, hep-ph/0211426: rapidity gap events at HERA. Constrain final state by clustering algorithm.
  - C. Berger, T. Kucs, G. Sterman, hep-ph/0303051: event-shape energy-flow correlations. Constrain final state by focusing on two-jet limit.
- Resum non-global logarithms.
  - A. Banfi, G. Marchesini and G. Smye, 0206076: leading non-global logs obey an evolution equation, valid at large N_c, and can be resummed.
  - Y. Dokshitzer and G. Marchesini, hep-ph/0303101: in event-shape energy-flow correlations leading non-global logs factorize and exponentiate.
  - G. Marchesini and A. Mueller hep-ph/0308284: intriguing connection with BFKL dynamics, for a somewhat exotic observable.
  - H. Weigert, hep-ph/0312050: analogy with small-x dynamics pursued beyond the large N_c limit.

Joint resummation

- Phenomenology requires applying resummation techniques to more differential distributions. More soft logarithms appear.
- Resummed logs in differential distributions may leave non-logarithmic but large remainders in integrated distributions.

Sudakov resummation techniques can treat simultaneously p_t and threshold logarithms (E. Laenen, G. Sterman and W. Vogelsang). For weak boson production

$$\frac{d\sigma_{AB}^{\rm res}}{dQ^2\,dQ_T^2} = \sum_a \sigma_a^{(0)} \int_{C_N} \frac{dN}{2\pi i}\,\tau^{-N} \int \frac{d^2b}{(2\pi)^2}\, e^{i\vec{Q}_T\cdot\vec{b}}$$

$$\times\, \mathcal{C}_{a/A}(Q,b,N,\mu,\mu_F)\ \exp\left[\, E_{a\bar{a}}\,(N,b,Q,\mu)\,\right]\ \mathcal{C}_{\bar{a}/B}(Q,b,N,\mu,\mu_F)\ .$$

$E_{a\bar{a}}$ is similar to the Sudakov exponent for p_t resummation

$$E_{a\bar{a}}\,(N,b,Q,\mu) = -\int_{Q^2/\chi^2}^{Q^2} \frac{dk_t^2}{k_t^2}\left[A_a\,(\alpha_s\,(k_t))\,\ln\left(\frac{Q^2}{k_t^2}\right) + B_a\,(\alpha_s\,(k_t))\right].$$

$\mathcal{C}_{a/A}$ act as generalized parton distributions.

- Landau pole is handled with minimal prescription.
- Phenomenology is under way: electroweak annihilation, prompt photon, heavy quark production, (A. Banfi, A. Kulesza, E. Laenen, G. Sterman, W. Vogelsang).
- Higgs: $p_\perp$ distribution very important at LHC. Competing approach: (S. Catani, M. Grazzini), NNLL $p_\perp$ resummation.

A case for resummed PDF's

- Phenomenology
  - Resummation justifies including more data in PDF fits.
    $W^2 \sim Q^2(1-x) \longrightarrow$ close to resonance region
  - Large-x quarks influence large-x gluons and smaller-x partons via sum rules and evolution.
    Q^2 evolution of partons at x_0 determined by partons at $x > x_0$.
  - Light Higgs@LHC (made at small x) should not be unique focus: large-x is new physics region.

    t-channel exchange of heavy particles? High-E_T jets?

- Theory
  - Consistency requires matching accuracy for parton distributions and hard cross sections.
  - The boundary between perturbative and nonperturbative must be defined.
    Leading Twist $\leftrightarrow$ NLO $\leftrightarrow$ $\overline{\mathrm{MS}}$ do not mix well!
  - Lattice determinations of PDF's use different, precise definition of leading twist ... comparison?
  - Resummation provides a gateway to nonperturbative corrections.
    * Define resummed exponent $\leftrightarrow$ define power correction.
    * QCD models for power corrections to structure functions can be tested.

- Resummed global PDF fits?

  Soft gluon resummation to NLL is now standard in all simple QCD cross sections.

  – DIS. The best understood cross section in QCD.
    NNNLO, (N)NNLL cross section, OPE, proposed non perturbative factorization (E. Gardi *et al.*).
  – Drell-Yan. Next best. NNLO, (N)NNLL cross section, NNLO rapidity distribution.
  – Prompt photon. Problematic phenomenology.
    NLO, NLL, joint resummation, fragmentation component? Power corrections? Data?
  – Jet production. Incomplete.
    NLO, formal NLL, non-global logs! `Caesar?`

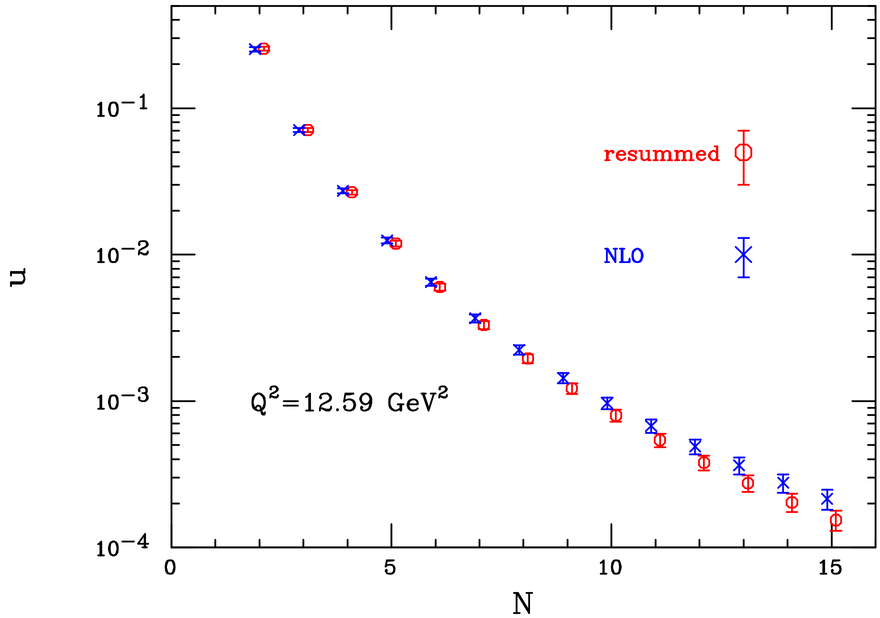
  A global resummed fit is theoretically achievable.

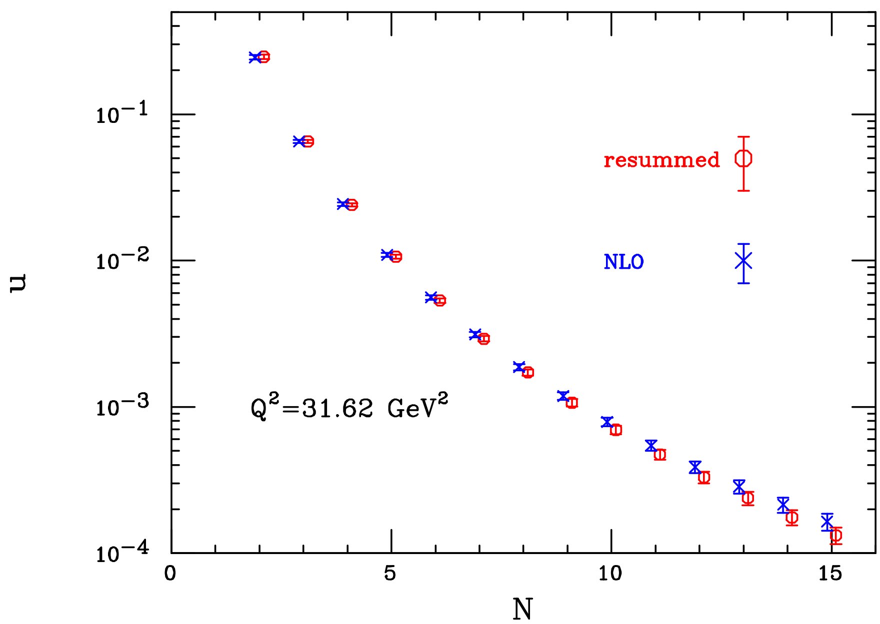
- A toy large-x parton fit (G. Corcella, LM)

  We consider NuTeV and NMC/BCDMS data.

  – Data are parametrized at different fixed values of Q^2
  – Moments of data can be computed with uncertainties.
    NOTE: resummation takes place in moment space
  – Extract moments of linear combinations of PDF's, solve for valence quarks with assumptions on gluon and sea.
  – Fit x-space functional forms to moments.

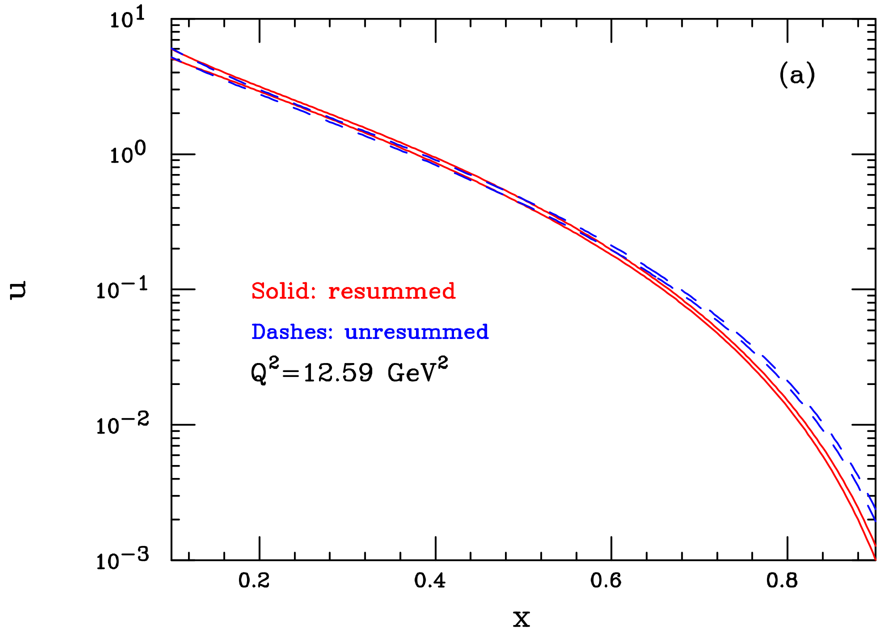
Results for moments

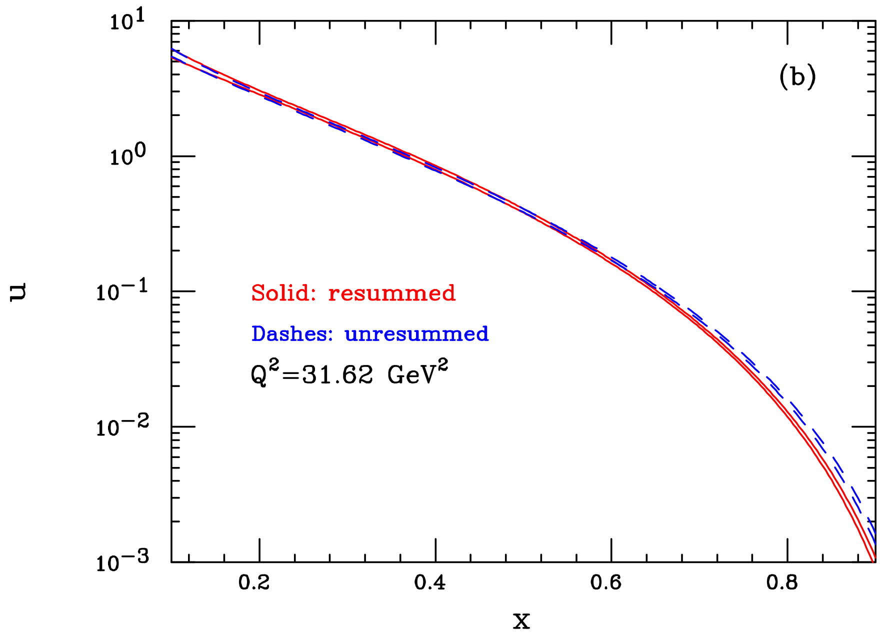




NLO and resummed moments of the up quark distribution at $Q^2 = 12.59$ and 31.62 GeV^2.

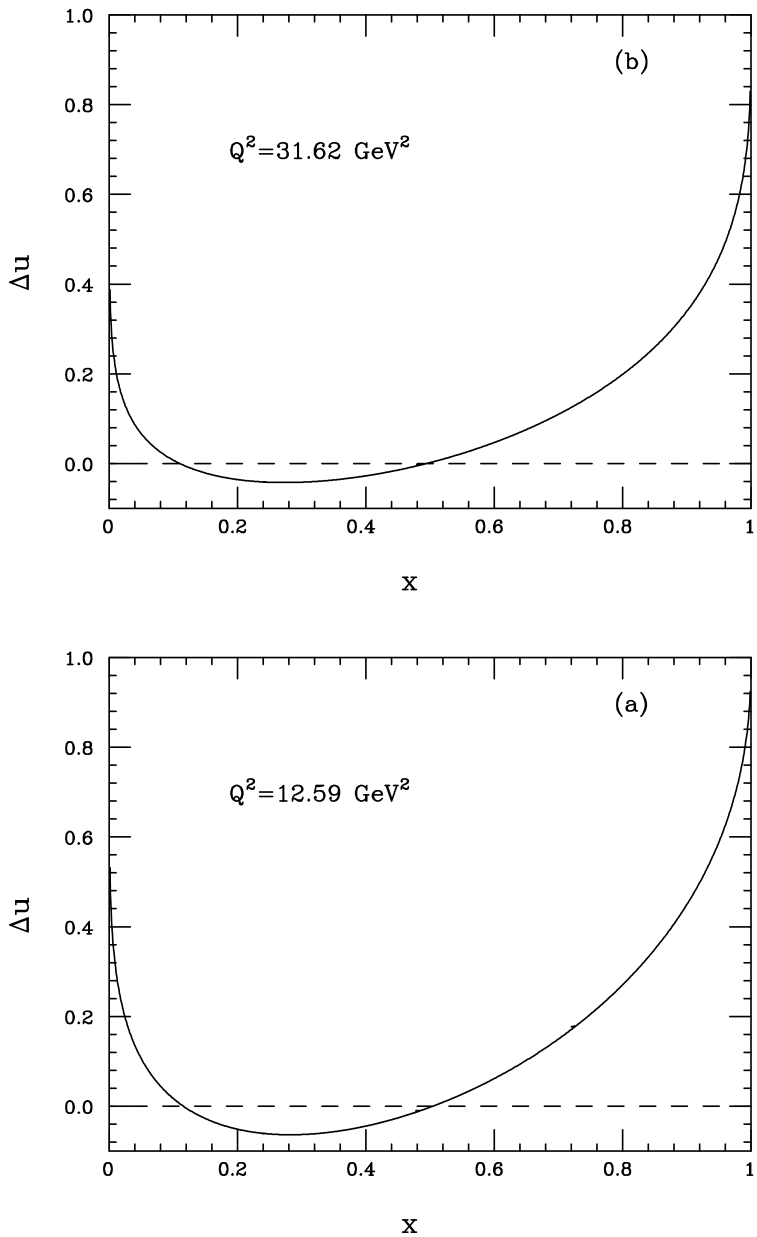
Results in x-space





NLO and resummed up quark distribution at $Q^2 = 12.59$ and 31.62 GeV^2.

Variation



Normalized deviation between NLO and resummed up quark distribution at $Q^2 = 12.59$ and 31.62 GeV^2.

Perspective

- Sudakov resummations are a very active and rapidly progressing field of study in QCD.
- They are necessary for phenomenological analysis of data in a variety of processes.
- They provide a window into nonperturbative contributions to high energy cross-sections.
- They are available for inclusive processes with high logarithmic accuracy.
- They are becoming a practical tool, directly applicable to many measurable cross sections, not only fully inclusive ones.
  - more differential cross sections can be resummed, for example via joint resummation.
  - realistic cuts begin to be implemented, and the associated non-global logs can also be resummed.
- Impact on PDF's
  - Most cross sections used in the extraction of PDF's are known in resummed form, to high accuracy (NLL, NNLL), often with a QCD-motivated parametrization of power corrections.
  - Fully resummed observables require resummed parton distributions.
  - A toy fit shows that impact may be sizeable at large x, with possible cancellations of hard effects.

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