All-order results in QCD (and some of their applications)

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Outline

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- All-order QCD versus data
- The strategy of perturbative QCD
- The borders of perturbative QCD

Resummations
- Factorization leads to Resummation
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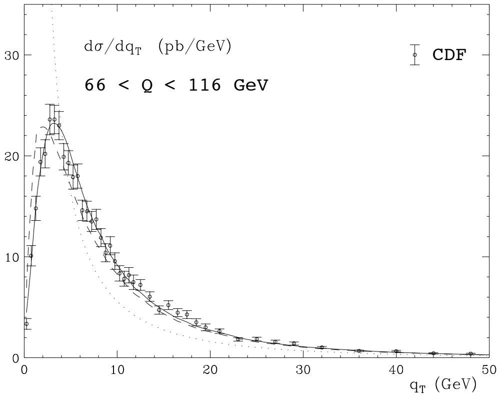
- Angularities

Developments

Soft gluons versus data

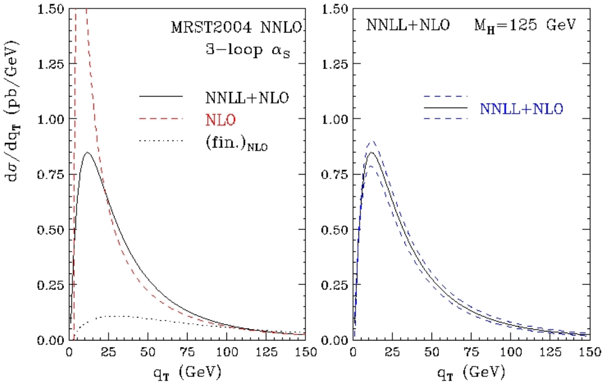
Z boson spectrum at Tevatron (A. Kulesza et al., hep-ph/0207148)



CDF data on Z production compared with QCD predictions at fixed order (dotted), with resummation (dashed), and with the inclusion of power corrections (solid).

Soft gluons versus data

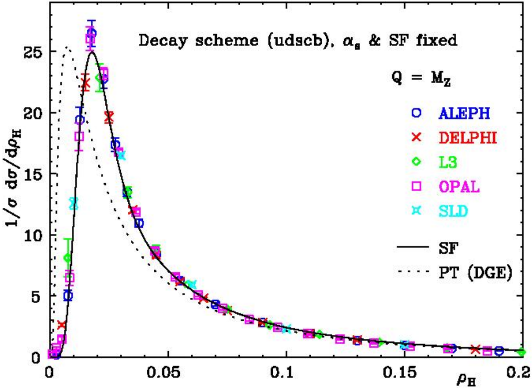
Higgs boson spectrum at LHC *(M. Grazzini,* hep-ph/0512025*)*



Predictions for the q_T spectrum of Higgs bosons produced via gluon fusion at the LHC, with and without resummation.

Soft gluons versus data

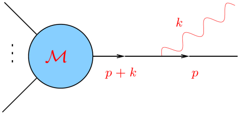
Jet shape distributions *(E. Gardi and J. Rathsmann,* `hep-ph/0201019`*)*



LEP data on the Heavy Jet Mass distribution, compared with resummed QCD prediction, and with power corrections treated by Dressed Gluon Exponentiation.

How can PQCD work at all?

- In a world of *hadrons*, we compute with *quarks* and *gluons*, which do not exist in the true asymptotic states of QCD.
- Perturbatively: the QCD *S-matrix* does not exist in the Fock space of quarks and gluons, due to *mass singularities*.
- *Example*: a massless fermion emits a massless gauge boson



$$\rightarrow \quad - \mathrm{i} g \overline{u}(p) \not{\epsilon}(k) t_a \frac{\mathrm{i}(\not{p} + \not{k})}{(p+k)^2 + \mathrm{i}\varepsilon} \mathcal{M} \, ,$$

$$2p \cdot k = 2 \, p_0 \, k_0 \, (1 - \cos\theta_{pk}) = 0 \, ,$$

$$k_0 = 0 \quad (IR); \quad \cos\theta_{pk} = 0 \quad (C).$$

- QCD is *worse than QED*: the *KLN theorem* cannot be applied, the true asymptotic states are not close enough to the Fock states.

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The Strategy of Perturbative QCD

Infrared Safety: *cancelling mass divergences.*

- *Compute* partonic cross sections with IR regulator

$$\sigma_{\rm part} = \sigma_{\rm part}\left(\frac{Q^2}{\mu^2}, \alpha_s(\mu^2), \left\{\frac{m^2(\mu^2)}{\mu^2}, \epsilon\right\}\right) .$$

- *Identify IR–safe* cross sections, having a finite limit as regulators are removed ($\epsilon \to 0$, $m^2(\mu^2) \to 0$).

$$\sigma_{\rm part} = \sigma_{\rm part}\left(\frac{Q^2}{\mu^2}, \alpha_s(\mu^2), \{0, 0\}\right) + \mathcal{O}\left(\left\{\left(\frac{m^2}{\mu^2}\right)^p, \epsilon\right\}\right) .$$

- *Interpret* $\sigma_{\rm part}$ as perturbative estimate of hadronic cross section valid up to corrections $\mathcal{O}\left((\Lambda_{QCD}/Q)^p\right)$

The Strategy of Perturbative QCD

Factorization: *neutralizing mass divergences.*

- *Quantum incoherence* in the presence of different scales coupled with *gauge invariance* implies, to all orders in PT for inclusive cross sections,

$$\sigma_{\rm part} = f \left(\frac{m^2}{\mu_F^2} \right) * \widehat{\sigma}_{\rm part} \left(\frac{Q^2}{\mu^2}, \frac{\mu_F^2}{\mu^2} \right) + \mathcal{O} \left(\left(\frac{m^2}{\mu_F^2} \right)^p \right) .$$

- *Combine* $\widehat{\sigma}_{\rm part}$ (perturbatively *finite* but *process–dependent*) with universal f (*non-perturbative*, measured, *universal*) to derive hadronic cross section.
- Use the *arbitrariness* of μ_F to derive *evolution equations* for the μ_F dependence of $f(m^2/\mu_F^2)$, *computable* in perturbation theory.

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The borders of perturbative QCD

Power Corrections

- Factorization theorems apply up to *nonperturbative corrections* suppressed by $\mathcal{O}\left(\left(\Lambda^2/Q^2\right)^p\right)$.
- In the presence of *several hard scales*, power corrections can be *enhanced*.

  Example: DIS as $x \sim 1 \quad \Rightarrow \quad \mathcal{O}\left(\Lambda^2/\left(Q^2(1-x)\right)\right)$.
- *Power corrections* can be *phenomenologically significant* even at LHC. They *compete* with NLO (at LEP) or NNLO (at LHC) *perturbative* corrections.
- *All-order results* in perturbation theory *encode* information on the *parametric size* of power corrections.

  Techniques: *OPE*, *Renormalons*,
  Sudakov resummations.

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The borders of perturbative QCD

Large Logarithms

Multi–scale problems can have *large perturbative corrections* of the general form $\alpha_s^n \log^k \left(Q_i^2/Q_j^2\right)$, with $k \leq n$ *(single logs)* or $k < 2n$ *(double logs)*. Examples include

- *Renormalization* logs: $\alpha_s^n \log^n \left(Q^2/\mu_R^2\right)$.
- *Collinear factorization* logs: $\alpha_s^n \log^n \left(Q^2/\mu_F^2\right)$.
- *High–energy* logs: $\alpha_s^n \log^{n-2} (s/t)$.
- *Sudakov* logs in *DIS*: $\alpha_s^n \log^{2n-1} \left(Q^2/W^2\right)$.
  in *Higgs* production: $\alpha_s^n \log^{2n-1} \left(1 - M_H^2/\hat{s}\right)$.
- *Transverse momentum* logs: $\alpha_s^n \log^{2n-1} \left(Q_\perp^2/Q^2\right)$.

Note: Sudakov logs originate from *mass singularities*: they are *universal* and can/*must* be resummed.

Factorization leads to Resummation

All *factorizations* separating dynamics at different energy scales lead to *resummation* of logarithms of the ratio of scales.

- *Renormalization group* logarithms.
  Renormalization *factorizes* cutoff dependence

$$G_0^{(n)}\left(p_i, \Lambda, g_0\right) = \prod_{i=1}^{n} Z_i^{1/2}\left(\Lambda/\mu, g(\mu)\right)\ G_R^{(n)}\left(p_i, \mu, g(\mu)\right) \ ,$$

$$\frac{dG_0^{(n)}}{d\mu} = 0 \quad \rightarrow \quad \frac{d\log G_R^{(n)}}{d\log\mu} = -\sum_{i=1}^{n}\gamma_i\left(g(\mu)\right) \ .$$

- *RG* evolution *resums* $\alpha_s^n(\mu^2)\log^n\left(Q^2/\mu^2\right)$ into $\alpha_s(Q^2)$.

Note: *Factorization* is the *difficult* step!

Factorization leads to Resummation

- *Collinear factorization* logarithms.
  Mellin moments of partonic *DIS structure functions* factorize

$$\widetilde{F}_2\left(N, \frac{Q^2}{m^2}, \alpha_s\right) = \widetilde{C}\left(N, \frac{Q^2}{\mu_F^2}, \alpha_s\right) \widetilde{f}\left(N, \frac{\mu_F^2}{m^2}, \alpha_s\right)$$

$$\frac{d\widetilde{F}_2}{d\mu_F} = 0 \quad \to \quad \frac{d\log\widetilde{f}}{d\log\mu_F} = \gamma_N\left(\alpha_s\right) \; .$$

- *Altarelli-Parisi* evolution *resums* collinear logarithms into *evolved PDF's*.

Note: *Double* logarithms are more difficult. Ordinary renormalization group is not sufficient. Gauge invariance plays a key role. *Or:* use effective filed theory (SCET).

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Tools: dimensional regularization

Nonabelian *exponentiation* of IR poles requires d-*dimensional* evolution equations. The *running coupling* in $d = 4 - 2\epsilon$ obeys

$$\mu \frac{\partial \overline{\alpha}}{\partial \mu} \equiv \beta(\epsilon, \overline{\alpha}) = - 2 \, \epsilon \, \overline{\alpha} + \hat{\beta}(\overline{\alpha}) \, , \quad \hat{\beta}(\overline{\alpha}) = - \frac{\overline{\alpha}^2}{2\pi} \sum_{n=0}^{\infty} b_n \left(\frac{\overline{\alpha}}{\pi} \right)^n \, .$$

The *one-loop* solution is

$$\overline{\alpha} \left(\mu^2, \epsilon \right) = \alpha_s(\mu_0^2) \left[\left(\frac{\mu^2}{\mu_0^2} \right)^\epsilon - \frac{1}{\epsilon} \left(1 - \left(\frac{\mu^2}{\mu_0^2} \right)^\epsilon \right) \frac{b_0}{4\pi} \, \alpha_s(\mu_0^2) \right]^{-1} \, .$$

Note: $d \, \overline{\alpha}(\mu^2, \epsilon)/d\mu_0^2 = 0$; $\overline{\alpha}(\mu^2, 0)$ is the usual *finite* $\overline{\alpha}(\mu^2)$.

At *two loops* one can expand

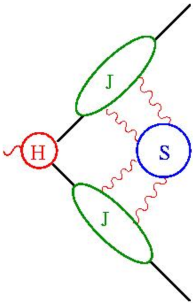
$$\begin{aligned} \overline{\alpha} \left(\xi^2, \epsilon \right) \quad &= \quad \alpha_s \, \xi^{-2\epsilon} + \alpha_s^2 \, \xi^{-4\epsilon} \, \frac{b_0}{4\pi\epsilon} \left(1 - \xi^{2\epsilon} \right) \\ &+ \alpha_s^3 \, \xi^{-6\epsilon} \, \frac{1}{8\pi^2\epsilon} \left[\frac{b_0^2}{2\epsilon} \left(1 - \xi^{2\epsilon} \right)^2 + b_1 \left(1 - \xi^{4\epsilon} \right) \right] \, . \end{aligned}$$

The quark form factor

Consider as an example the *quark form factor*

$$\Gamma_\mu(p_1,p_2;\mu^2,\epsilon) = \langle p_1,p_2|J_\mu(0)|0\rangle = -\mathrm{i}ee_q\,\overline{u}(p_1)\gamma_\mu v(p_2)\,\Gamma\left(Q^2,\epsilon\right)\ .$$

In dimensional regularization *IR* and *collinear* singularities in the form factor *factorize* as



- $\Gamma\left(Q^2,\epsilon\right) = J\left(\frac{(p_i\cdot n)^2}{\mu^2 n^2}\right)\mathcal{S}\left(\beta_i\cdot n\right)H\left(\frac{(p_i\cdot n)^2}{\mu^2 n^2}\right)\ .$
- *Gauge invariance* implies $\frac{\partial\log\Gamma}{\partial\log(p_i\cdot n)} = 0$.
- $\frac{\partial\log J_i}{\partial\log(p_i\cdot n)} = -\frac{\partial\log H}{\partial\log(p_i\cdot n)} - \frac{\partial\log\mathcal{S}}{\partial\log(\beta_i\cdot n)}\ .$

Note: the *r.h.s* is *sum* of a *finite* function $G_J(Q^2,\epsilon)$ and a *pure counterterm* $K_J(\epsilon)$.

The quark form factor

- A similar equation holds for the *full form factor*

$$Q^2 \frac{\partial}{\partial Q^2} \log \left[\Gamma \left(\frac{Q^2}{\mu^2}, \alpha_s(\mu^2), \epsilon \right) \right] = \frac{1}{2} \left[K \left(\epsilon, \alpha_s(\mu^2) \right) + G \left(\frac{Q^2}{\mu^2}, \alpha_s(\mu^2), \epsilon \right) \right] ,$$

- *Renormalization group invariance* of the form factor requires

$$\mu \frac{dG}{d\mu} = -\mu \frac{dK}{d\mu} = \gamma_K \left(\alpha_s(\mu^2) \right) ,$$

Note: $\gamma_K(\alpha_s)$ is the *cusp anomalous dimension* of the Wilson line representing the quark pair trajectory in the *eikonal approximation*.

- *Dimensional regularization* provides a *trivial initial condition* for evolution if $\epsilon < 0$ (for *IR* regularization).

$$\overline{\alpha}(\mu^2 = 0, \epsilon < 0) = 0 \quad \rightarrow \Gamma \left(0, \alpha_s(\mu^2), \epsilon \right) = \Gamma \left(1, \overline{\alpha}\left(0, \epsilon\right), \epsilon \right) = 1 \ .$$

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Results for the Sudakov form factor

- In dimensional regularization $(\epsilon < 0 \to d > 4)$ one can solve the evolution equation in *pure exponential* form

$$\log\left[\Gamma\left(Q^2,\epsilon\right)\right] = \frac{1}{2}\int_0^{-Q^2}\frac{d\xi^2}{\xi^2}\left[K\left(\epsilon\right) + G\left(\overline{\alpha}\left(\xi^2,\epsilon\right),\epsilon\right) + \frac{1}{2}\int_{\xi^2}^{\mu^2}\frac{d\lambda^2}{\lambda^2}\gamma_K\left(\overline{\alpha}\left(\lambda^2,\epsilon\right)\right)\right]$$

- *All mass singularities* are generated through *integration* over the scale of the d-dimensional *running coupling*
- For $\epsilon < -b_0\alpha_s(Q^2)/(4\pi)$, the *Landau pole* moves *away from the real axis*. $\Gamma(Q^2,\epsilon)$ can be *analytically evaluated* to the desired order in *resummed* perturbation theory.
- The *ratio* of the *timelike* to the *spacelike* form factor admits a simple representation

$$\log\left[\frac{\Gamma(Q^2,\epsilon)}{\Gamma(-Q^2,\epsilon)}\right] = \mathrm{i}\frac{\pi}{2}K(\epsilon) + \frac{\mathrm{i}}{2}\int_0^{\pi}\left[G\left(\overline{\alpha}\left(\mathrm{e}^{\mathrm{i}\theta}Q^2\right),\epsilon\right) - \frac{\mathrm{i}}{2}\int_0^{\theta}d\phi\,\gamma_K\left(\overline{\alpha}\left(\mathrm{e}^{\mathrm{i}\phi}Q^2\right)\right)\right]$$

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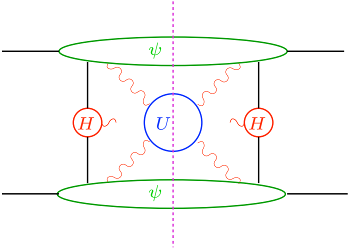
Parton level Drell-Yan factorization

Cross sections for *electroweak annihilation* can be similarly *factorized* near threshold



Up to $1/N$ corrections

- $\omega(N,\epsilon) = |H_{\rm DY}|^2 \, \psi(N,\epsilon)^2 \, U(N)$.
- $\psi(N,\epsilon) = \mathcal{R}(\epsilon) \, \psi_R(N,\epsilon)$,
- $U(N) = U_V(\epsilon) \, U_R(N,\epsilon)$,

Virtual contributions reconstruct the *form factor*

$$\omega(N,\epsilon) = \left| H_{\rm DY} \, \mathcal{R}(\epsilon) \sqrt{U_V(\epsilon)} \right|^2 \psi_R(N,\epsilon)^2 \, U_R(N,\epsilon)$$
$$= \left| \Gamma(Q^2,\epsilon) \right|^2 \psi_R(N,\epsilon)^2 \, U_R(N,\epsilon) \; .$$

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Collinear factorization in the $\overline{\mathrm{MS}}$ scheme

For *collinear factorization* one needs the $\overline{\mathrm{MS}}$ quark distribution.

- Up to $1/N$ corrections, it *exponentiates*

$$\phi_{\overline{\mathrm{MS}}}(N,\epsilon) = \exp\left[\int_0^{\mu_F^2}\frac{d\xi^2}{\xi^2}\left(B_\delta\left(\overline{\alpha}(\xi^2,\epsilon)\right) + \int_0^1 dz\,\frac{z^{N-1}-1}{1-z}A\left(\overline{\alpha}(\xi^2,\epsilon)\right)\right)\right].$$

Note: $A(\alpha_s)$ and $B_\delta(\alpha_s)$ are the singular parts of the AP kernels.

- A *virtual contribution* can be defined to cancel *virtual poles*.

$$\phi_V(\epsilon) = \exp\left\{\frac{1}{2}\int_0^{\mu_F^2}\frac{d\xi^2}{\xi^2}\left[K(\epsilon) + \widetilde{G}\left(\overline{\alpha}(\xi^2,\epsilon)\right) + \frac{1}{2}\int_{\xi^2}^{\mu^2}\frac{d\lambda^2}{\lambda^2}\gamma_K\left(\overline{\alpha}(\lambda^2,\epsilon)\right)\right]\right\},$$

- The function $\widetilde{G}(\alpha_s)$ can be defined *recursively*.

$$G(\alpha_s,\epsilon) = \sum_{n=1}^{\infty}\sum_{m=0}^{\infty} G_n^{(m)}\epsilon^m\left(\frac{\alpha_s}{\pi}\right)^n,$$

$$\widetilde{G}_{M+1} = G_{M+1}^{(0)} - \frac{b_0}{4}G_M^{(1)} - \frac{b_1}{4}G_{M-1}^{(1)} + \frac{b_0^2}{16}G_{M-1}^{(2)} + \ldots$$

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The $\overline{\rm MS}$ scheme Drell-Yan cross section

The *factorized* Drell–Yan cross section in the $\overline{\rm MS}$ scheme is the product of *separately finite* real and virtual terms.

$$\widehat{\omega}_{\overline{\rm MS}}(N) = \left|\frac{\Gamma(Q^2,\epsilon)}{\phi_V(Q^2,\epsilon)}\right|^2 \left[U_R(N,\epsilon)\left(\frac{\psi_R(N,\epsilon)}{\phi_R(N,\epsilon)}\right)^2\right] ,$$

One recovers a *generalization* of the usual Drell-Yan resummation, *including* N-independent terms (E. Laenen, LM)

$$\widehat{\omega}_{\overline{\rm MS}}(N) = \left|\frac{\Gamma(Q^2,\epsilon)}{\phi_V(Q^2,\epsilon)}\right|^2 \exp\left[F_{\overline{\rm MS}}(\alpha_s) + \int_0^1 dz\, \frac{z^{N-1}-1}{1-z} \left\{2\int_{Q^2}^{(1-z)^2Q^2} \frac{d\mu^2}{\mu^2} A\left(\alpha_s(\mu^2)\right) + D\left(\alpha_s\left((1-z)^2Q^2\right)\right)\right\}\right] .$$

Similar results hold for *all EW annihilation processes*.

Relating EW annihilation and DIS

- *All* threshold logarithms for *Drell-Yan* are determined by
  - The quark *form factor*.
  - Virtual contributions to the *quark splitting function*.
  - *Lower order* singular terms in the Drell-Yan cross section.

$$
\begin{aligned}
A(\alpha_s) &= \gamma_K(\alpha_s)/2 \, , \\
D(\alpha_s) &= 4\, B_\delta(\alpha_s) - 2\, \widetilde{G}(\alpha_s) + \hat{\beta}(\alpha_s)\, \frac{d}{d\alpha_s} F_{\overline{\rm MS}}\,(\alpha_s) \, .
\end{aligned}
$$

- An *identical* formula applies to *Higgs production* via *gluon fusion*, after *integration* of the *top loop*.
  - The *gluon* form factor and splitting kernel *replace* the quark.
  - Up to *three loops* the result is obtained simply by replacing $C_F \leftrightarrow C_A$.

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Finite order results

- At *one loop:* $A^{(1)} = C_F \,, \quad D^{(1)} = 0 \,.$
- At *two loops:* (E. Laenen *et al.*, A. Vogt)

$$A^{(2)} = \frac{1}{2} C_A C_F \left(\frac{67}{18} - \zeta(2) \right) - \frac{5}{18} n_f C_F$$

$$D^{(2)} = \left(-\frac{101}{27} + \frac{11}{3}\zeta(2) + \frac{7}{2}\zeta(3) \right) C_A C_F + \left(\frac{14}{27} - \frac{2}{3}\zeta(2) \right) n_f C_F \,,$$

- At *three loops:* (S. Moch and A. Vogt, E. Laenen and LM)

$$A^{(3)} = C_F C_A^2 \left(\frac{245}{96} - \frac{67}{36}\,\zeta_2 + \frac{11}{24}\,\zeta_3 + \frac{11}{20}\,\zeta_2^2 \right) + C_F^2 n_f \left(-\frac{55}{96} + \frac{1}{2}\zeta_3 \right) + C_F C_A n_f \left(-\frac{209}{436} + \frac{5}{18}\,\zeta_2 - \frac{7}{12}\,\zeta_3 \right) - \frac{1}{108}\,C_F n_f^2 \,.$$

$$D^{(3)} = \left(-\frac{297029}{23328} + \frac{6139}{324}\zeta(2) - \frac{187}{60}\zeta^2(2) + \frac{2509}{108}\zeta(3) - \frac{11}{6}\zeta(2)\zeta(3) - 6\zeta(5) \right) C_A^2 C_F + \left(\frac{31313}{11664} - \frac{1837}{324}\zeta(2) + \frac{23}{30}\zeta^2(2) - \frac{155}{36}\zeta(3) \right) n_f C_A C_F + \left(\frac{1711}{864} - \frac{1}{2}\zeta(2) - \frac{1}{5}\zeta^2(2) - \frac{19}{18}\zeta(3) \right) n_f C_F^2 + \left(-\frac{58}{729} + \frac{10}{27}\zeta(2) + \frac{5}{27}\zeta(3) \right) n_f^2 C_F \,.$$

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Features of Sudakov resummation

- Non–trivial. *Reorganizes perturbation theory* in a predictive way. For *threshold* resummation, let $L = \log N$. Then

$$\sum_k \alpha_s^k \sum_p^{2k} c_{kp} L^p \to \exp\left[\sum_k \alpha_s^k \sum_p^{k+1} d_{kp} L^p\right] \quad .$$

- Predictive. Resummation *extends the range* of perturbative methods. Fixed order: $\alpha_s L^2 \ll 1$. NLL resummation: $\alpha_s \ll 1$ suffices. *Scale dependence* is *reduced*.
- Widespread. NLL soft gluon resummations *exist* for *most inclusive cross sections* of interest at colliders (NNLL now available for processes which are *EW* at tree level).
- Non–perturbative aspects of QCD become *accessible*. Integrals in the exponent run into the *Landau pole*.

On event shape distributions

Picturing the final state of high-energy collisions

- *Thrust:* $T = \max_{\hat{n}} \frac{\sum_i |\vec{p}_i \cdot \hat{n}|}{Q}$; $\tau = 1 - T$.

  $\longrightarrow$ $\hat{n}$ is used to define several other shape variables.

- *C-parameter:* $C = 3 - \frac{3}{2} \sum_{i,j} \frac{(p_i \cdot p_j)^2}{(p_i \cdot q)\,(p_j \cdot q)}$.

  $\longrightarrow$ does not require maximization procedures.

- *Angularity:* $\tau_a = \frac{1}{Q} \sum_i (p_\perp)_i \mathrm{e}^{-|\eta_i|(1-a)}$.

  $\longrightarrow$ recently introduced, *one-parameter* family.

- *Transverse Thrust:* $T_\perp = \max_{\hat{n}_\perp} \frac{\sum_i |\vec{p}_{\perp i} \cdot \hat{n}_\perp|}{\sum_i \vec{p}_{\perp i}}$.

  $\longrightarrow$ defined for *hadron-hadron* collisions

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Resumming Sudakov logarithms

Infrared and collinear emission dominates the two-jet limit

- Large *double* logarithms of the variable vanishing in the two-jet limit $(L = \log\tau \;; L = \log C \;; \ldots)$ *enhance* finite orders → *need to resum*.
- As before, a pattern of *exponentiation* emerges

$$\sum_k \alpha_s^k \sum_p^{2k} c_{kp} L^p \to \exp\left[L\, g_1(\alpha_s L) + g_2(\alpha_s L) + \alpha_s\, g_3(\alpha_s L) + \ldots\right]$$

- In general the *Laplace transform* exponentiates. For thrust

$$\int_0^\infty d\tau\, \mathrm{e}^{-\nu\tau} \frac{1}{\sigma}\frac{d\sigma}{d\tau} = \exp\left[\int_0^1 \frac{du}{u}\left(\mathrm{e}^{-u\nu} - 1\right)\left(B\left(\alpha_s\left(uQ^2\right)\right) + 2\int_{u^2Q^2}^{uQ^2} \frac{dq^2}{q^2} A\left(\alpha_s(q^2)\right)\right)\right].$$

Reaching beyond perturbation theory

Exponentiating power corrections

- The exponent is *ill-defined* because of the *Landau pole*

  regularization $\longrightarrow$ ambiguity $\longrightarrow$ power corrections

- Focus on *small* τ, *large* ν, set IR factorization scale μ, expand in powers of ν/Q (soft), *neglecting* ν/Q^2 (collinear).

$$S_{\rm NP}(\nu/Q,\mu) = 2\int_0^{\mu^2}\frac{dq^2}{q^2}A\left(\alpha_s(q^2)\right)\int_{q^2/Q^2}^{q/Q}\frac{du}{u}\left({\rm e}^{-u\nu}-1\right)$$
$$\simeq \sum_{n=1}^{\infty}\frac{1}{n!}\left(-\frac{\nu}{Q}\right)^n\lambda_n(\mu^2)\ ,$$

- *Non-perturbative* parameters

$$\lambda_n(\mu^2) = \tfrac{2}{n}\int_0^{\mu^2} dq^2\, q^{n-2} A\left(\alpha_s(q^2)\right)\ .$$

Parametrizing power corrections

Shape functions

- The parameters $\lambda_n(\mu^2)$ build up a *shape function*

$$\exp\left[S_{\rm NP}(\nu/Q,\mu)\right] \equiv \int_0^\infty d\epsilon\, {\rm e}^{-\nu\,\epsilon/Q}\, f_\tau(\epsilon,\mu)\ .$$

- The physical *distribution* is recovered via inverse transform

$$\sigma(\tau) \sim \int_0^{\tau Q} d\epsilon\, f_\tau(\epsilon,\mu)\, \sigma_{\rm PT}\left(\tau - \epsilon/Q\right)\ .$$

- One recovers the *perturbative* result *shifted* by the soft energy flow, and *smeared* by the shape function.
- *Universality* of power corrections is in general *lost*, however *specific* observables still *related* ($1 - T$, ρ_J, C, $\ldots$).
- *Assumption*: smooth transition to *nonperturbative* regime.

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Angularities

(C.F. Berger, G. Sterman)

- *Definition:* $\tau_a = \frac{1}{Q} \sum_i (p_\perp)_i \, \mathrm{e}^{-|\eta_i|(1-a)}$.

  Also: $\tau_a = \frac{1}{Q} \sum_i \omega_i \, (\sin\theta_i)^a \, (1 - |\cos\theta_i|)^{1-a}$,
- *Some properties*
  - $\tau_0 = 1 - T \; ; \; \tau_1 = B$.
  - $a < 2$ for IR safety.
  - $a < 1$ for simplicity of resummation (*recoil* negligible).
- For *negative* a, high rapidity particles (*w.r.t.* the thrust axis) are weighted less: *better* collinear behavior.
- At *one loop*, with the thrust axis given by particle i,

$$\tau_a = \frac{(1-x_i)^{1-a/2}}{x_i} \left[(1 - x_j)^{1-a/2} (1 - x_k)^{a/2} + (j \leftrightarrow k) \right] .$$

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Resummation for angularities

- Sudakov logs at one loop have *simple scaling* with a.

$$\left.\frac{d\sigma}{d\tau_a}\right|_{\log}^{(1)} = \frac{2}{2-a}\,\frac{2}{\tau_a} C_F \frac{\alpha_s}{\pi} \ln\left(\frac{1}{\tau_a}\right) = \frac{2}{2-a}\,\left.\frac{d\sigma}{d\tau}\right|_{\log}^{(1)} .$$

- Resummation is *intricate*. To *NLL* accuracy

$$\tilde{\sigma}_a(\nu) = \exp\left\{ 2\int_0^1 \frac{du}{u}\left[\int_{u^2Q^2}^{uQ^2} \frac{dq^2}{q^2} A\left(\alpha_s(q^2)\right)\left(\mathrm{e}^{-u^{1-a}\nu(q/Q)^a} - 1\right) + \frac{1}{2} B\left(\alpha_s(u\,Q^2)\right)\left(\mathrm{e}^{-u\,\nu^{2/(2-a)}} - 1\right)\right]\right\}.$$

- General a-dependence of Sudakov logs is *nontrivial*.

$$g_1(x,a) = -\frac{4}{\beta_0}\,\frac{2-a}{1-a}\,\frac{A^{(1)}}{x}\left[\frac{1-x}{2-a}\ln(1-x) - \left(1-\frac{x}{2-a}\right)\ln\left(1-\frac{x}{2-a}\right)\right].$$

Scaling for the shape function

- As done for *thrust*, focus on *small* τ_a, *large* ν, set IR factorization scale μ, expand in powers of ν/Q (soft), *neglecting* ν/Q^2 (collinear). In this case

$$
\begin{aligned}
S_{\rm NP}^{(a)}(\nu/Q,\mu) &= 2\int_0^{\mu^2}\frac{dq^2}{q^2}A\left(\alpha_s(q^2)\right)\int_{q^2/Q^2}^{q/Q}\frac{du}{u}\left(e^{-u^{1-a}\nu(q/Q)^a}-1\right) \\
&\simeq \frac{1}{1-a}\sum_{n=1}^{\infty}\frac{1}{n!}\left(-\frac{\nu}{Q}\right)^n\lambda_n(\mu^2)\,,
\end{aligned}
$$

- The *full result* suggested by the resummation can be expressed in terms of *the shape function for thrust*

$$
\tilde{\sigma}_a(\nu) = \tilde{\sigma}_a^{\rm PT}(\nu,\mu)\ \tilde{f}_a^{\rm NP}\left(\frac{\nu}{Q},\mu\right) = \left[\tilde{f}_0^{\rm NP}\left(\frac{\nu}{Q},\mu\right)\right]^{1/(1-a)}\ .
$$

- The *scaling rule* can already be tested with LEP data.

Recent developments

- *Joint* resummation (p_T and threshold).
- *Automatic* resummation (Caesar).
- *Non-global* logarithms.
- *Subleading* logarithms ($(\log^p N)/N$).
- Resummed *parton distributions*.
- *Dressed gluon* exponentiation (DGE).
- Resummation with *effective field theories* (SCET).
- *Power corrections* in *hadron collisions*.
- Resummation in *Susy theories*, towards *AdS/CFT*.
- .....

Resummations, supersymmetry and strings

- *Supersymmetric versions* of *Yang-Mills theory* and *QCD* have remarkable properties.

  Example: $\mathcal{N} = 4$ SYM is *conformal invariant*: $\beta_{\mathcal{N}=4}(\alpha_s) = 0$.

- *Exponentiation* of IR/C poles in QCD amplitudes *simplifies*

$$\log\left[\Gamma\left(Q^2,\epsilon\right)\right] = -\frac{1}{2}\sum_{n=1}^{\infty}\left(\frac{\alpha_s(Q^2)}{\pi}\right)^n e^{-i\pi n\epsilon}\left[\frac{\gamma_K^{(n)}}{2n^2\epsilon^2}+\frac{G^{(n)}(\epsilon)}{n\epsilon}\right],$$

  Note: at most *double* poles in the exponent.

- Thanks to *AdS/CFT*, $\mathcal{N} = 4$ SYM must 'be simple' *at strong coupling*. Can this be seen in *perturbation theory*?

- *Exponentiation* has been observed for amplitudes with up to *three loops* or *five legs* (Z. Bern *et al.*).

- A *stringy* calculation at *strong coupling* is consistent with the *perturbative result* (L. Alday and J. Maldacena).

Perspective

- *Sudakov resummations* are a very active and *rapidly progressing* field of study in QCD.
- They are *necessary* for phenomenological analysis of *data* in a variety of processes.
- They provide a *window* into *nonperturbative* contributions to many high energy cross-sections.
- The *boundaries* of IR/C *nonabelian exponentiation* are still being *probed*.
- *Dimensional regularization* is a powerful tool. Bypassing the *Landau pole* it links to *non-perturbative* effects.
- *Remarkable results* in quantum field theory *link* resummations in *super Yang-Mills* with *string theory*.

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