Refining Threshold Resummations

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Outline

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- Resumming long-distance singularities
- The Sudakov form factor

N-independent terms

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- The $\overline{\mathrm{MS}}$ scheme Drell-Yan cross section

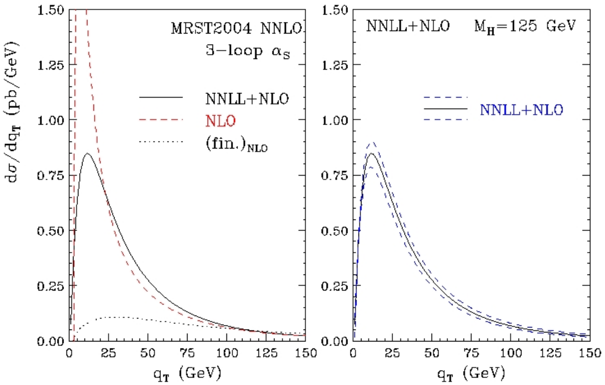
EW Annihilation from DIS data

- Constraints of finiteness
- Three-loop and all-order results

Perspective

Soft gluons versus data

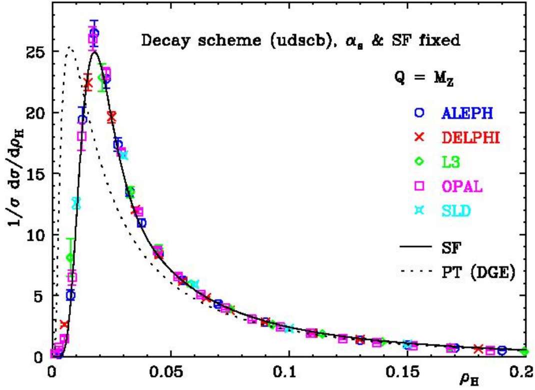
Higgs boson spectrum *(M. Grazzini)*



Predictions for the q_T spectrum of Higgs bosons produced via gluon fusion at the LHC, with and without resummation.

Soft gluons versus data

Jet shape distributions *(E. Gardi and J. Rathsmann)*



LEP data on the Heavy Jet Mass distribution, compared with resummed QCD prediction, and with power corrections treated by Dressed Gluon Exponentiation.

Resummation of Sudakov logarithms

Poles lead to logarithms in finite observables

- *Double poles* of infrared-collinear origin $(\alpha_s^k/\epsilon^{2k})$ cancel in IRC-safe observables but leave behind *double logarithms* of ratios of kinematic scales *e.g.* $\left(\log^{2k-1}(1-x)/(1-x)\right)_+$.
- Nonabelian *exponentiation* of long-distance singularities

$$\sum_k \alpha_s^k \sum_p^{2k} c_{kp}\epsilon^{-p} \to \exp\left[\sum_k \alpha_s^k \sum_p^{k+1} d_{kp}\epsilon^{-p}\right]$$

leads to a nontrivial *resummation* of logarithms in the Mellin (Laplace) transform of the cross section

$$\sum_k \alpha_s^k \sum_p^{2k} \hat{c}_{kp} L^p \quad \to \quad \exp\Big[L\, g_1(\alpha_s L) + g_2(\alpha_s L) + \alpha_s\, g_3(\alpha_s L) + \ldots\Big]$$

Tools: dimensional regularization

Nonabelian exponentiation of IR poles requires *d-dimensional* evolution equations. The *running coupling* in $d = 4 - 2\epsilon$ obeys

$$\mu \frac{\partial \overline{\alpha}}{\partial \mu} \equiv \beta(\epsilon, \overline{\alpha}) = -2\,\epsilon\,\overline{\alpha} + \hat{\beta}(\overline{\alpha}) \quad , \quad \hat{\beta}(\overline{\alpha}) = -\frac{\overline{\alpha}^2}{2\pi} \sum_{n=0}^{\infty} b_n \left(\frac{\overline{\alpha}}{\pi} \right)^n \quad .$$

The *one-loop* solution is

$$\overline{\alpha}\left(\mu^2\right) = \alpha_s(\mu_0^2) \left[\left(\frac{\mu^2}{\mu_0^2} \right)^{\epsilon} - \frac{1}{\epsilon} \left(1 - \left(\frac{\mu^2}{\mu_0^2} \right)^{\epsilon} \right) \frac{b_0}{4\pi} \alpha_s(\mu_0^2) \right]^{-1} \quad .$$

At *two loops* one can expand

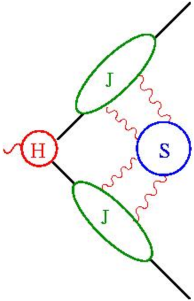
$$\begin{aligned} \overline{\alpha}\left(\xi^2, \alpha_s, \epsilon\right) \quad &= \quad \alpha_s\, \xi^{-2\epsilon} + \alpha_s^2\, \xi^{-4\epsilon} \frac{b_0}{4\pi\epsilon} \left(1 - \xi^{2\epsilon}\right) \\ &+ \alpha_s^3\, \xi^{-6\epsilon} \frac{1}{8\pi^2 \epsilon} \left[\frac{b_0^2}{2\epsilon} \left(1 - \xi^{2\epsilon}\right)^2 + b_1 \left(1 - \xi^{4\epsilon}\right) \right] \quad . \end{aligned}$$

Tools: factorization and evolution

Consider as an example the *quark form factor*

$$\Gamma_\mu(q,\epsilon) = \langle p_1, p_2 | J_\mu(0) | 0 \rangle = -\mathrm{i} e e_q \, \overline{u}(p_1)\gamma_\mu v(p_2) \, \Gamma\left(Q^2,\epsilon\right) \; .$$

In dimensional regularization *IR* and *collinear* singularities in the form factor *factorize* as



- $\Gamma\left(Q^2,\epsilon\right) = J\left(p\cdot n,\epsilon\right) S\left(u_i \cdot n,\epsilon\right) H\left(Q^2\right) \; .$
- $Q^2 \frac{\partial}{\partial Q^2} \log\left[\Gamma\left(Q^2,\epsilon\right)\right] = \frac{1}{2}\left[K\left(\epsilon\right) + G\left(Q^2,\epsilon\right)\right]$
- with $K(\epsilon)$ a *pure counterterm* and $G(Q^2,\epsilon)$ *finite*.

Results for the Sudakov form factor

- In dimensional regularization $(\epsilon < 0)$ one has the *boundary value* $\Gamma(0,\epsilon) = 1$. Using *RG invariance*, one then gets

$$\log\left[\Gamma\left(Q^2,\epsilon\right)\right] = \frac{1}{2}\int_0^{-Q^2}\frac{d\xi^2}{\xi^2}\left[K\left(\epsilon\right) + G\left(\overline{\alpha}\left(\xi^2\right),\epsilon\right) + \frac{1}{2}\int_{\xi^2}^{\mu^2}\frac{d\lambda^2}{\lambda^2}\gamma_K\left(\overline{\alpha}\left(\lambda^2\right)\right)\right]$$

  with γ_K the *cusp anomalous dimension* of the Wilson line representing the quarks.

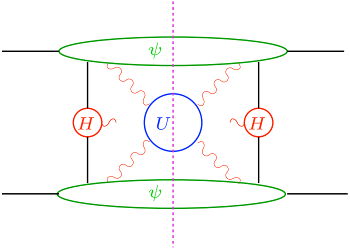
- The *ratio* of the *timelike* to the *spacelike* form factor admits a simple representation

$$\log\left[\frac{\Gamma(Q^2,\epsilon)}{\Gamma(-Q^2,\epsilon)}\right] = \mathrm{i}\frac{\pi}{2}K(\epsilon) + \frac{\mathrm{i}}{2}\int_0^{\pi}\left[G\left(\overline{\alpha}\left(\mathrm{e}^{\mathrm{i}\theta}Q^2\right),\epsilon\right) - \frac{\mathrm{i}}{2}\int_0^{\theta}d\phi\,\gamma_K\left(\overline{\alpha}\left(\mathrm{e}^{\mathrm{i}\phi}Q^2\right)\right)\right]$$

  which is *physically relevant* for resummed EW annihilation processes.

Parton level Drell-Yan factorization

Cross sections for *electroweak annihilation* can be similarly *factorized* near threshold



Up to $1/N$ corrections

- $\omega(N,\epsilon) = |H_{\rm DY}|^2 \, \psi(N,\epsilon)^2 \, U(N)$.
- $\psi(N,\epsilon) = \mathcal{R}(\epsilon) \, \psi_R(N,\epsilon)$,
- $U(N) = U_V(\epsilon) \, U_R(N,\epsilon)$,

Virtual contributions reconstruct the *form factor*

$$\begin{aligned} \omega(N,\epsilon) &= \left| H_{\rm DY} \, \mathcal{R}(\epsilon) \sqrt{U_V(\epsilon)} \right|^2 \psi_R(N,\epsilon)^2 \, U_R(N,\epsilon) \\ &= \left| \Gamma(Q^2,\epsilon) \right|^2 \psi_R(N,\epsilon)^2 \, U_R(N,\epsilon) \, . \end{aligned}$$

Parton level Drell-Yan exponentiation

Evolution equations lead to *exponentiation* also for *real emission*

- $\psi_R(N,\epsilon) = \exp\left[\int_0^1 dz \, \frac{z^{N-1}}{1-z} \int_z^1 \frac{dy}{1-y} \, \kappa_\psi \left(\overline{\alpha}\left((1-y)^2 Q^2\right), \epsilon\right)\right]$.
- $U_R(N,\epsilon) = \exp\left[-\int_0^1 dz \, \frac{z^{N-1}}{1-z} \, g_U \left(\overline{\alpha}\left((1-z)^2 Q^2\right), \epsilon\right)\right]$.

The functions κ_ψ and g_U are *finite* and *RG invariant*.
All poles arise integrating the running coupling in $d = 4 - 2\epsilon$.

- $\left(\mu \frac{\partial}{\partial \mu} + \beta(\epsilon, \alpha_s) \frac{\partial}{\partial \alpha_s}\right) \kappa_\psi \left(\frac{(1-y)Q}{\mu}, \alpha_s(\mu^2), \epsilon\right) = 0$,
- $\kappa_\psi^{(1)}(\epsilon) = 2 C_F \frac{\Gamma(2-\epsilon)}{\Gamma(2-2\epsilon)} \quad ; \quad g_U^{(1)}(\epsilon) = -2 C_F \frac{\Gamma(1-\epsilon)}{\Gamma(2-2\epsilon)}$.

The *parton level* Drell-Yan cross section *exponentiates* up to $1/N$ corrections.

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$\overline{\mathrm{MS}}$ *quark distribution*

For *collinear factorization* one needs the $\overline{\mathrm{MS}}$ quark distribution.

- Up to $1/N$ corrections, it *exponentiates*

$$\phi_{\overline{\mathrm{MS}}}(N,\epsilon) = \exp\left[\int_0^{Q^2} \frac{d\xi^2}{\xi^2}\left(B_\delta\left(\overline{\alpha}(\xi^2)\right) + \int_0^1 dz\, \frac{z^{N-1}-1}{1-z} A\left(\overline{\alpha}(\xi^2)\right)\right)\right].$$

- A *virtual contribution* can be defined to cancel *virtual poles*.

$$\phi_V(\epsilon) = \exp\left\{\frac{1}{2}\int_0^{Q^2}\frac{d\xi^2}{\xi^2}\left[K(\epsilon) + \widetilde{G}\left(\overline{\alpha}(\xi^2)\right) + \frac{1}{2}\int_{\xi^2}^{\mu^2}\frac{d\lambda^2}{\lambda^2}\gamma_K\left(\overline{\alpha}(\lambda^2)\right)\right]\right\},$$

- The function $\widetilde{G}(\alpha_s)$ can be defined *recursively*.

$$G(\alpha_s,\epsilon) = \sum_{n=1}^{\infty}\sum_{m=0}^{\infty} G_n^{(m)}\epsilon^m\left(\frac{\alpha_s}{\pi}\right)^n,$$

$$\widetilde{G}_{M+1} = G_{M+1}^{(0)} - \frac{b_0}{4}G_M^{(1)} - \frac{b_1}{4}G_{M-1}^{(1)} + \frac{b_0^2}{16}G_{M-1}^{(2)} + \ldots$$

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The $\overline{\mathrm{MS}}$ scheme Drell-Yan cross section

The *factorized* Drell–Yan cross section in the $\overline{\mathrm{MS}}$ scheme is the product of *separately finite* real and virtual terms.

$$\widehat{\omega}_{\overline{\mathrm{MS}}}(N) = \left|\frac{\Gamma(Q^2,\epsilon)}{\phi_V(Q^2,\epsilon)}\right|^2 \left[U_R(N,\epsilon)\left(\frac{\psi_R(N,\epsilon)}{\phi_R(N,\epsilon)}\right)^2\right] ,$$

One recovers a *generalization* of the usual Drell-Yan resummation, *including* N-independent terms (E. Laenen, LM)

$$\widehat{\omega}_{\overline{\mathrm{MS}}}(N) = \left|\frac{\Gamma(Q^2,\epsilon)}{\phi_V(Q^2,\epsilon)}\right|^2 \exp\left[F_{\overline{\mathrm{MS}}}(\alpha_s) + \int_0^1 dz\, \frac{z^{N-1}-1}{1-z} \left\{2\int_{Q^2}^{(1-z)^2Q^2} \frac{d\mu^2}{\mu^2} A\left(\alpha_s(\mu^2)\right) + D\left(\alpha_s\left((1-z)^2Q^2\right)\right)\right\}\right] .$$

Similar results hold for *all EW annihilation processes*.

Outlook on N-independent terms

- *Threshold resummation* implies *complete exponentiation* of N–independent terms for *all* EW annihilation processes.
- Exponentiation for N–independent terms does *not* have the same predictive power as for *threshold logarithms*, however
  - detailed factorization *links* different processes;
  - reexpanding *gauges impact* of higher order corrections
  - All-order structure *simplifies computations* of specific terms.
- *What else* exponentiates? Empirical evidence suggests some $\log(N)/N$ terms do in the Drell–Yan cross section.
  - *Note*: numerical impact of $\log(N)/N$ terms can be sizeable.
- *Other processes?* Sudakov factorization applies to cross sections with *multiple colored particles*.
  How far does exponentiation reach?

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Constraints of finiteness

- Collinear *factorization* gives a *master formula* to compute resummation coefficients

$$\lim_{\epsilon \to 0} \left[\frac{\left(\psi_R(N,\epsilon)\right)^2 \, U_R(N,\epsilon)}{\left(\phi_R(N,\epsilon)\right)^2} \right] = \exp \left[F_{\overline{\rm MS}}(\alpha_s) + \int_0^1 dz \, \frac{z^{N-1} - 1}{1 - z} \right.$$
$$\left. \left\{ 2 \int_{Q^2}^{(1-z)^2 Q^2} \frac{d\mu^2}{\mu^2} A\left(\alpha_s(\mu^2)\right) + D\left(\alpha_s\left((1-z)^2 Q^2\right)\right) \right\} \right] + \mathcal{O}\left(\frac{1}{N}\right) .$$

- All *scale dependence* is determined by *RG arguments*.

$$\begin{aligned} \kappa_\psi^{(1)}(\xi,\epsilon) &= \kappa_\psi^{(1)}(1,\epsilon)\,\xi^{-2\epsilon} , \\ \kappa_\psi^{(2)}(\xi,\epsilon) &= \kappa_\psi^{(2)}(1,\epsilon)\,\xi^{-4\epsilon} + \frac{b_0}{4\epsilon}\kappa_\psi^{(1)}(1,\epsilon)\,\xi^{-2\epsilon}\left(\xi^{-2\epsilon} - 1\right) , \quad \ldots \end{aligned}$$

- All *scale integrals* can be performed yielding *moments of distributions*.

$$\mathcal{D}_k(N) \equiv \int_0^1 x^{N-1} \left(\frac{\log^k(1-x)}{1-x}\right)_+ \sim \frac{(-1)^{k+1}}{k+1} \log^{k+1} N ,$$

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Constraints of finiteness

Finiteness, dictated by the factorization theorem, links the coefficients of DIS and DY functions. At *one loop*

$$\lim_{\epsilon\to 0}\left\{\frac{1}{2\epsilon^2}\left(\kappa^{(1)}_{\psi,0}-\gamma^{(1)}_K\right)+\frac{1}{\epsilon}\left[\frac{g^{(1)}_{U,0}+\kappa^{(1)}_{\psi,1}}{2}+2B^{(1)}_\delta-\widetilde{G}^{(1)}+\left(2A^{(1)}-\kappa^{(1)}_{\psi,0}\right)\mathcal{D}_0(N)\right]\right.$$
$$\left.+\,2\,\kappa^{(1)}_{\psi,0}\,\mathcal{D}_1(N)-\left(g^{(1)}_{U,0}+\kappa^{(1)}_{\psi,1}\right)\mathcal{D}_0(N)+\frac{g^{(1)}_{U,1}+\kappa^{(1)}_{\psi,2}}{2}\right\}=F^{(1)}_{\overline{\mathrm{MS}}}+D^{(1)}\mathcal{D}_0(N)+4A^{(1)}\mathcal{D}_1(N)\,.$$

Considering $\left\{\kappa^{(j)}_{\psi,i},g^{(j)}_{U,i},D^{(j)},A^{(j)}\right\}$ as *unknowns*, one finds

$$\begin{aligned}
A^{(1)} &= \frac{\kappa^{(1)}_{\psi,0}}{2}=\frac{\gamma^{(1)}_K}{2}=C_F\,,\\
D^{(1)} &= -\left(g^{(1)}_{U,0}+\kappa^{(1)}_{\psi,1}\right)=4B^{(1)}_\delta-2\widetilde{G}^{(1)}=0\,,\\
F^{(1)}_{\overline{\mathrm{MS}}} &= \frac{g^{(1)}_{U,1}+\kappa^{(1)}_{\psi,2}}{2}=-\frac{3}{2}\zeta(2)C_F\,.
\end{aligned}$$

All DY logarithms are determined by *form factor* and *splitting function* data.

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Finite order results

The *pattern* of cancellation is repeated at higher orders. At *two loops* one recovers (E. Laenen *et al.*, A. Vogt)

$$\begin{aligned} D^{(2)} &= 4B_\delta^{(2)} - 2\widetilde{G}^{(2)} - \frac{b_0}{2} F_{\overline{\rm MS}}^{(1)} \\ &= \left(-\frac{101}{27} + \frac{11}{3}\zeta(2) + \frac{7}{2}\zeta(3)\right) C_A C_F + \left(\frac{14}{27} - \frac{2}{3}\zeta(2)\right) n_f C_F \,, \end{aligned}$$

At *three loops* one finds (S. Moch and A. Vogt, E. Laenen and LM, A. Idilbi *et al.*, V. Ravindran)

$$\begin{aligned} D^{(3)} &= 4B_\delta^{(3)} - 2\widetilde{G}^{(3)} - b_0 F_{\overline{\rm MS}}^{(2)} - \frac{b_1}{2} F_{\overline{\rm MS}}^{(1)} \\ &= \left(-\frac{297029}{23328} + \frac{6139}{324}\zeta(2) - \frac{187}{60}\zeta^2(2) + \frac{2509}{108}\zeta(3) - \frac{11}{6}\zeta(2)\zeta(3) - 6\zeta(5)\right) C_A^2 C_F \\ &+ \left(\frac{31313}{11664} - \frac{1837}{324}\zeta(2) + \frac{23}{30}\zeta^2(2) - \frac{155}{36}\zeta(3)\right) n_f C_A C_F \\ &+ \left(\frac{1711}{864} - \frac{1}{2}\zeta(2) - \frac{1}{5}\zeta^2(2) - \frac{19}{18}\zeta(3)\right) n_f C_F^2 \\ &+ \left(-\frac{58}{729} + \frac{10}{27}\zeta(2) + \frac{5}{27}\zeta(3)\right) n_f^2 C_F \,. \end{aligned}$$

$D^{(3)}$ (together with $A^{(4)}$) determines *single* logarithms at *three loops* and N^3L logarithms to *all orders*.

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An all-order Ansatz

- *All* threshold logarithms for *Drell-Yan* are determined by
  - The quark *form factor*.
  - Virtual contributions to the *quark splitting function*.
  - *Lower order* singular terms in the Drell-Yan cross section.

$$A(\alpha_s) = \gamma_K(\alpha_s)/2 \, ,$$

$$D(\alpha_s) = 4\, B_\delta(\alpha_s) - 2\, \widetilde{G}(\alpha_s) + \hat{\beta}(\alpha_s) \frac{d}{d\alpha_s} F_{\overline{\rm MS}}(\alpha_s) \, .$$

- An *identical* formula applies to *Higgs production* via *gluon fusion*
  - The *gluon* form factor and splitting kernel *replace* the quark.
  - Up to *three loops* the result is obtained simply by replacing $C_F \leftrightarrow C_A$.

Perspective

- The *boundaries* of IRC *nonabelian exponentiation* are still being *probed*. Dimensional regularization is a powerful tool.
- *N-independent* terms exponentiate in color singlet processes.
- *Resummation* for *EW annihilation* is lead by *DIS data*.
    - *All-order ansatz* needs splitting functions and form factor.
    - N^3LL accuracy *achieved* up to (small, computable) $A^{(4)}$.
    - Resummed PT *converges well*: reduced scale dependence for Higgs production (S. Moch, A. Vogt)
    - Applications are *possible* also to p_t resummation.
- For the *future*.
    - *Mixed evidence* for $\log(N)/N$ terms, *modifed evolution* needed? (Y.L. Dokshitzer, G. Marchesini, G, Salam)
    - *Truly remarkable results* in $\mathcal{N} = 4$ *SYM* motivate further study for *multicolored amplitudes*.

      (Z. Bern *et al.*, M. Staudacher *et al.*).