Angularities, flows, jets and other shapes

Lorenzo Magnea

Università di Torino – INFN, Sezione di Torino

LPTHE – 12/01/06

Outline

Event shapes

On event shape distributions

Resummation of Sudakov logarithms

Power corrections and shape functions

Dressed gluon exponentiation

Angularities

A family of event shapes

Resummation for angularities

Scaling of power corrections

Applications

Taming nonglobal logarithms

Hadron collisions

Perspective

1404 - 2004 Alma Universitas Taurinensis

On event shape distributions

Picturing the final state of high-energy collisions

- Event shape distributions probe QCD at *all scales* from the perturbative to the non-perturbative regime.

finite order $\longrightarrow$ resummation $\longrightarrow$ power corrections

- They provide a *global picture* of the final state of hard collisions.

energy flow $\longleftrightarrow$ hadronization $\longleftrightarrow$ mass effects

- A large amount of data is *available* (LEP, HERA ...)

better theory $\longleftrightarrow$ more analysis ?

- Studies are emerging for *hadron-hadron* collisions

impact at LHC ?

On event shape distributions

Examples

- Thrust: $T = \max_{\hat{n}} \frac{\sum_i |\vec{p}_i \cdot \hat{n}|}{Q}$; $\tau = 1 - T$.

  $\longrightarrow$ $\hat{n}$ is used to define several other shape variables.

- C-parameter: $C = 3 - \frac{3}{2} \sum_{i,j} \frac{(p_i \cdot p_j)^2}{(p_i \cdot q)\,(p_j \cdot q)}$.

  $\longrightarrow$ does not require maximization procedures.

- Angularity: $\tau_a = \frac{1}{Q} \sum_i (p_\perp)_i \mathrm{e}^{-|\eta_i|(1-a)}$.

  $\longrightarrow$ recently introduced, *one-parameter* family.

- Transverse Thrust: $T_\perp = \max_{\hat{n}_\perp} \frac{\sum_i |\vec{p}_{\perp i} \cdot \hat{n}_\perp|}{\sum_i \vec{p}_{\perp i}}$.

  $\longrightarrow$ defined for *hadron-hadron* collisions

1404-2004 Alma Universitas Taurinensis

Resumming Sudakov logarithms

Infrared and collinear emission dominates the two-jet limit

- Large *double* logarithms of the variable vanishing in the two-jet limit $(L = \log \tau \,; L = \log C \,; \ldots)$ enhance finite orders $\rightarrow$ *need to resum*.
- A pattern of *exponentiation* emerges

$$\sum_k \alpha_s^k \sum_p^{2k} c_{kp} L^p \rightarrow \exp\Big[L g_1(\alpha_s L) + g_2(\alpha_s L) + \alpha_s g_3(\alpha_s L) + \ldots \Big]$$

- In general the *Laplace transform* exponentiates. For thrust

$$\int_0^\infty d\tau \, e^{-\nu\tau} \frac{1}{\sigma} \frac{d\sigma}{d\tau} = \exp\left[\int_0^1 \frac{du}{u} \left(e^{-u\nu} - 1 \right) \left(B\left(\alpha_s\left(uQ^2\right)\right) + 2 \int_{u^2 Q^2}^{uQ^2} \frac{dq^2}{q^2} A\left(\alpha_s(q^2)\right) \right) \right] .$$

Reaching beyond perturbation theory

Exponentiating power corrections

- The exponent is *ill-defined* because of the *Landau pole*

regularization $\longrightarrow$ ambiguity $\longrightarrow$ power corrections

- Focus on *small* τ, *large* ν, set IR factorization scale μ, expand in powers of ν/Q (soft), *neglecting* ν/Q^2 (collinear).

$$\begin{aligned} S_{\rm NP}(\nu/Q,\mu) &= 2\int_0^{\mu^2}\frac{dq^2}{q^2}A\left(\alpha_s(q^2)\right)\int_{q^2/Q^2}^{q/Q}\frac{du}{u}\left({\rm e}^{-u\nu}-1\right) \\ &\simeq \sum_{n=1}^{\infty}\frac{1}{n!}\left(-\frac{\nu}{Q}\right)^n\lambda_n(\mu^2)\ , \end{aligned}$$

- *Non-perturbative* parameters

$$\lambda_n(\mu^2) = \frac{2}{n}\int_0^{\mu^2} dq^2\, q^{n-2} A\left(\alpha_s(q^2)\right)\ .$$

1404-2004 Alma Universitas Taurinensis

Parametrizing power corrections

Shape functions

- The parameters $\lambda_n(\mu^2)$ build up a *shape function*

$$\exp\left[S_{\rm NP}(\nu/Q,\mu)\right] \equiv \int_0^\infty d\epsilon\, {\rm e}^{-\nu\,\epsilon/Q}\, f_\tau(\epsilon,\mu)\ .$$

- The physical *distribution* is recovered via inverse transform

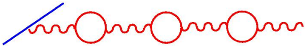
$$\sigma(\tau) \sim \int_0^{\tau Q} d\epsilon\, f_\tau(\epsilon,\mu)\, \sigma_{\rm PT}\,(\tau - \epsilon/Q)\ \ .$$

- One recovers the *perturbative* result *shifted* by the soft energy flow, and *smeared* by the shape function.
- *Universality* of power corrections is in general *lost*, however *specific* observables still *related* ($1 - T$, ρ_J, C, $\ldots$).
- *Assumption*: smooth transition to *nonperturbative* regime.

Dressed gluon exponentiation

It is possible to combine *renormalon* methods and *Sudakov resummation* to construct models of power corrections. One method is *dressed gluon exponentiation* (Gardi).

- *Step 1:* compute *characteristic function* $\mathcal{F}(k^2)$ of the dispersive method in the *Sudakov limit* (resum *"bubble graphs"*).



- *Step 2:* define the *Borel representation* of the *SDG* cross section.

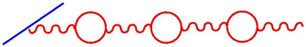
$$\frac{1}{\sigma}\frac{d\sigma}{d\tau}\Big|_{\mathrm{SDG}} = \frac{C_F}{2\beta_0}\int_0^\infty du\,\left(Q^2/\Lambda^2\right)^{-u} B(\tau,u)\,.$$

Note: the Borel integral is always left unperformed

Dressed gluon exponentiation

It is possible to combine *renormalon* methods and *Sudakov resummation* to construct models of power corrections. One method is *dressed gluon exponentiation* (Gardi).

- *Step 1:* compute *characteristic function* $\mathcal{F}(k^2)$ of the dispersive method in the *Sudakov limit* (resum *"bubble graphs"*).



- *Step 2:* define the *Borel representation* of the *SDG* cross section.

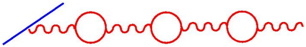
$$\frac{1}{\sigma}\frac{d\sigma}{d\tau}\Big|_{\rm SDG} = \frac{C_F}{2\beta_0}\int_0^\infty du\,\left(Q^2/\Lambda^2\right)^{-u} B(\tau,u)\,.$$

Note: the Borel integral is always left unperformed

Dressed gluon exponentiation

It is possible to combine *renormalon* methods and *Sudakov resummation* to construct models of power corrections. One method is *dressed gluon exponentiation* (Gardi).

- *Step 1:* compute *characteristic function* $\mathcal{F}(k^2)$ of the dispersive method in the *Sudakov limit* (resum *"bubble graphs"*).



- *Step 2:* define the *Borel representation* of the *SDG* cross section.

$$\frac{1}{\sigma}\,\frac{d\sigma}{d\tau}\Big|_{\rm SDG} = \frac{C_F}{2\beta_0}\,\int_0^\infty du\,\left(Q^2/\Lambda^2\right)^{-u}\,B(\tau,u)\,.$$

Note: the Borel integral is always left unperformed

1404-2004 Alma Universitas Taurinensis

- *Step 3:* exponentiate the *Laplace transform* of the distribution

$$\frac{1}{\sigma}\,\frac{d\sigma}{d\tau}\Big|_{\rm DGE} = \int_{k-{\rm i}\infty}^{k+{\rm i}\infty} \frac{d\nu}{2\pi{\rm i}}\, {\rm e}^{\nu\tau}\, \exp\left[S\left(\nu, Q^2\right)\right] \,,$$

using the *single gluon* result as kernel

$$S\left(\nu, Q^2\right) = \int_0^\infty d\tau\, \frac{1}{\sigma}\frac{d\sigma}{d\tau}\Big|_{\rm SDG}\left({\rm e}^{-\nu\tau} - 1\right) \,.$$

- *Step 4:* summarize results by *Borel exponent*

$$S\left(\nu, Q^2\right) = \frac{C_F}{2\beta_0}\int_0^\infty du\, \left(Q^2/\Lambda^2\right)^{-u} B_\tau(\nu, u)\,.$$

- *Example:* the Borel exponent for the *thrust*

$$B_\tau(\nu, u) = 2\,{\rm e}^{5u/3}\,\frac{\sin\pi u}{\pi u}\left[\Gamma(-2u)\left(\nu^{2u} - 1\right)\frac{2}{u} - \Gamma(-u)\left(\nu^u - 1\right)\left(\frac{2}{u} + \frac{1}{1-u} + \frac{1}{2-u}\right)\right].$$

- *Step 3:* exponentiate the *Laplace transform* of the distribution

$$\frac{1}{\sigma}\,\frac{d\sigma}{d\tau}\Big|_{\rm DGE} = \int_{k-{\rm i}\infty}^{k+{\rm i}\infty} \frac{d\nu}{2\pi {\rm i}}\, {\rm e}^{\nu\tau}\, \exp\left[S\left(\nu, Q^2\right)\right] \, ,$$

using the *single gluon* result as kernel

$$S\left(\nu, Q^2\right) = \int_0^\infty d\tau \;\, \frac{1}{\sigma}\frac{d\sigma}{d\tau}\Big|_{\rm SDG} \left({\rm e}^{-\nu\tau} - 1\right) \, .$$

- *Step 4:* summarize results by *Borel exponent*

$$S\left(\nu, Q^2\right) = \frac{C_F}{2\beta_0} \int_0^\infty du \, \left(Q^2/\Lambda^2\right)^{-u} B_\tau(\nu, u) \, .$$

- *Example:* the Borel exponent for the *thrust*

$$B_\tau(\nu, u) \;=\; 2\,{\rm e}^{5u/3}\,\frac{\sin \pi u}{\pi u}\left[\Gamma(-2u)\left(\nu^{2u} - 1\right)\frac{2}{u} \right.$$
$$\left. -\;\Gamma(-u)\left(\nu^{u} - 1\right)\left(\frac{2}{u} + \frac{1}{1-u} + \frac{1}{2-u}\right)\right] .$$

- *Step 3:* exponentiate the *Laplace transform* of the distribution

$$\frac{1}{\sigma}\,\frac{d\sigma}{d\tau}\Big|_{\rm DGE} = \int_{k-{\rm i}\infty}^{k+{\rm i}\infty} \frac{d\nu}{2\pi{\rm i}}\, {\rm e}^{\nu\tau}\, \exp\left[S\left(\nu, Q^2\right)\right]\,,$$

using the *single gluon* result as kernel

$$S\left(\nu, Q^2\right) = \int_0^\infty d\tau\ \frac{1}{\sigma}\frac{d\sigma}{d\tau}\Big|_{\rm SDG}\left({\rm e}^{-\nu\tau} - 1\right)\,.$$

- *Step 4:* summarize results by *Borel exponent*

$$S\left(\nu, Q^2\right) = \frac{C_F}{2\beta_0}\,\int_0^\infty du\,\left(Q^2/\Lambda^2\right)^{-u}\,B_\tau(\nu, u)\,.$$

- *Example:* the Borel exponent for the *thrust*

$$B_\tau(\nu, u) \ = \ 2\,{\rm e}^{5u/3}\,\frac{\sin\pi u}{\pi u}\left[\Gamma(-2u)\left(\nu^{2u} - 1\right)\frac{2}{u} - \ \Gamma(-u)\left(\nu^u - 1\right)\left(\frac{2}{u} + \frac{1}{1-u} + \frac{1}{2-u}\right)\right]\,.$$

1404-2004 Alma Universitas Taurinensis

Features of DGE

- *NLL* Sudakov resummation *reproduced* by using *"gluon bremsstrahlung"* definition of running coupling. *All* subleading logs computed in the "large n_f" limit.
- *Factorial growth* of subleading logs detected: a *handle* on the range of applicability of $N^p LL$ resummation.
- *Definite prescription* for merging resummed PT with power corrections.
- *Predictive phenomenology:* linked models of shape functions for thrust, jet masses, C-parameter, angularities. *Absence* of even power corrections.
- *Applications* to power corrections in the Sudakov region for DIS, Drell-Yan, fragmentation, B decays.

1404-2004 Alma Universitas Taurinensis

Angularities

- *Definition:* $\tau_a = \frac{1}{Q} \sum_i (p_\perp)_i \, e^{-|\eta_i|(1-a)}$.

  Also: $\tau_a = \frac{1}{Q} \sum_i \omega_i \, (\sin\theta_i)^a \, (1 - |\cos\theta_i|)^{1-a}$,

- *Some properties*
  - $\tau_0 = 1 - T \; ; \; \tau_1 = B$.
  - $a < 2$ for IR safety.
  - $a < 1$ for simplicity of resummation (*recoil* negligible).

- For *negative* a, high rapidity particles (*w.r.t.* the thrust axis) are weighted less: *better* collinear behavior.

- At *one loop*, with the thrust axis given by particle i,

$$\tau_a = \frac{(1-x_i)^{1-a/2}}{x_i} \left[(1 - x_j)^{1-a/2} (1 - x_k)^{a/2} + (j \leftrightarrow k) \right] .$$

1404-2004 ALMA UNIVERSITAS TAURINENSIS

Resummation for angularities

- Sudakov logs at one loop have *simple scaling* with a.

$$\left.\frac{d\sigma}{d\tau_a}\right|_{\log}^{(1)} = \frac{2}{2-a}\,\frac{2}{\tau_a}C_F\frac{\alpha_s}{\pi}\ln\left(\frac{1}{\tau_a}\right) = \frac{2}{2-a}\,\left.\frac{d\sigma}{d\tau}\right|_{\log}^{(1)} .$$

- Resummation is *intricate*. To *NLL* accuracy

$$\tilde{\sigma}_a(\nu) = \exp\left\{2\int_0^1 \frac{du}{u}\left[\int_{u^2Q^2}^{uQ^2}\frac{dq^2}{q^2}A\left(\alpha_s(q^2)\right)\left(\mathrm{e}^{-u^{1-a}\nu(q/Q)^a}-1\right)\right.\right.$$
$$\left.\left. + \frac{1}{2}B\left(\alpha_s(u\,Q^2)\right)\left(\mathrm{e}^{-u\,\nu^{2/(2-a)}}-1\right)\right]\right\}.$$

- General a-dependence of Sudakov logs is *nontrivial*.

$$g_1(x,a) = -\frac{4}{\beta_0}\,\frac{2-a}{1-a}\,\frac{A^{(1)}}{x}\left[\frac{1-x}{2-a}\ln(1-x)\right.$$
$$\left. - \left(1-\frac{x}{2-a}\right)\ln\left(1-\frac{x}{2-a}\right)\right].$$

Scaling for the shape function

An analysis of power corrections for angularities using the *shape function* approach (Berger, Sterman) shows a remarkable *scaling*.

- As done for *thrust*, focus on *small* τ_a, *large* ν, set IR factorization scale μ, expand in powers of ν/Q (soft), *neglecting* ν/Q^2 (collinear). In this case

$$S_{\rm NP}^{(a)}(\nu/Q,\mu) \;=\; 2\int_0^{\mu^2}\frac{dq^2}{q^2}A\left(\alpha_s(q^2)\right)\int_{q^2/Q^2}^{q/Q}\frac{du}{u}\left({\rm e}^{-u^{1-a}\nu(q/Q)^a}-1\right)$$

$$\simeq \;\frac{1}{1-a}\sum_{n=1}^{\infty}\frac{1}{n!}\left(-\frac{\nu}{Q}\right)^n\lambda_n(\mu^2)\;,$$

- The *full result* suggested by the resummation can be expressed in terms of *two* shape functions

$$\tilde\sigma_a\left(\nu\right)=\tilde\sigma_{a,{\rm PT}}\left(\nu,\mu\right)\,\tilde f_{a,{\rm NP}}\left(\frac{\nu}{Q},\mu\right)\,\tilde g_{a,{\rm NP}}\left(\frac{\nu}{Q^{2-a}},\mu\right)\;,$$

Alma Universitas Taurinensis 1404-2004

- *Leading* power corrections are described by $\tilde{f}_{a,\mathrm{NP}}$ and obey

$$\tilde{f}_{a,\mathrm{NP}}\left(\frac{\nu}{Q},\mu\right)=\left[\tilde{f}_{0,\mathrm{NP}}\left(\frac{\nu}{Q},\mu\right)\right]^{1/(1-a)} \quad .$$

- *Scaling* can be traced to *boost invariance* in the eikonal limit. A *renormalon* calculation breaks boost invariance but *scaling survives* in the Sudakov limit. *DGE* (Berger, LM) yields

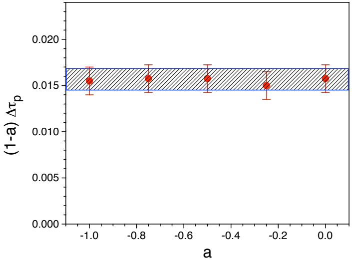
$$B_a^{\mathrm{soft}}(\nu,u)=\frac{1}{1-a}\left[2\,\mathrm{e}^{5u/3}\,\frac{\sin\pi u}{\pi u}\,\Gamma(-2u)\left(\nu^{2u}-1\right)\frac{2}{u}\right]$$

- *Collinear* contribution shows an *intricate* structure of *fractional* power corrections in DGE, but they are suppressed by ν/Q^{2-a}, consistent with resummation.
- *Scaling* is a *testable prediction* with *existing* LEP data!

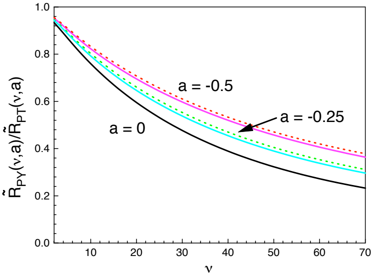
1404-2004 Alma Universitas Taurinensis

Testing the scaling rule

The scaling rule is a *prediction* waiting for data *analysis* ... in the meantime, it can be compared with PYTHIA output (Berger).

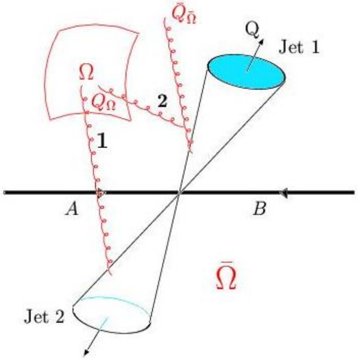


Shift in the position of the peak of τ_a distribution, between NLL result and PYTHIA, after rescaling by $1 - a$, vs. shift for $a = 0$ computed from data.



The leading shape function for different a, PYTHIA output (solid) vs. scaled result (dashed).

On the dangers of slicing phase space



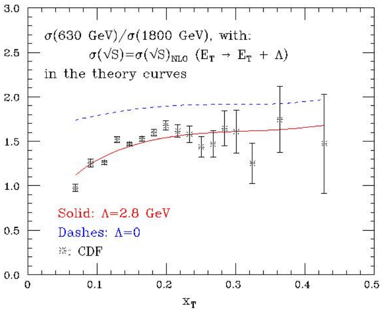
Energy flow into Ω and origin of nonglobal logs

- Gluon 1: $\log(Q_\Omega/Q)$.
- Gluon 2: $\log(Q_\Omega/Q_{\bar{\Omega}})$.
- *Resummation* of nonglobal logs possible in the *large* N_c limit.
- Non-global $\longrightarrow$ Non-Sudakov $\longrightarrow$ Non-linear.
- Can one *suppress* them?
- Study soft radiation *without* hard antenna?

Event shape/energy flow correlations

- In e^+e^- annihilation *suppress* nonglobal logs via BKS joint distribution
$$\sigma\left(\epsilon,\tau_a\right) = \frac{1}{2s}\sum_N \overline{\left|M(N)\right|^2}\,\delta(\epsilon - f_\Omega(N))\,\delta\left(\tau_a - \tau_a(N)\right)$$
with $f_\Omega(N) = \left(\sum_{i\in\Omega}\omega_i\right)/s$.
- At small ϵ, τ_a, with $\epsilon \sim \tau_a$ radiation into $\bar{\Omega}$ is forced to the *two-jet limit*. Logarithms of ϵ and τ_a *factor* and can be separately *resummed*.
- When $\epsilon \ll \tau_a$ nonglobal logs *reappear* in the eikonal function describing wide-ange soft gluons. They can be *resummed* in the N_c limit as before. The nonglobal radiator is *evaluated* at the *reduced scale* $\tau_a Q$ (DM).

What impact at Tevatron/LHC?



Fit of CDF data with NLO QCD assuming E_T-independent shift Λ in jet energy (Mangano, hep-ph/9911256).

- Cross section ratio should *scale* up to *PDF* ad α_s effects.
- Data can be fitted with *shift* in distribution.
- *Small* Λ has impact at *high* E_T.
- $\sigma(E_T) \sim E_T^{-n} \to \frac{\delta\sigma}{\sigma} \sim -n\delta E_T$
- *Several* sources of energy flow *in* and *out* of jets.

Power corrections and other problems ...

- *Sources* of power corrections
  - Soft radiation from *hard antenna* ⇒ *resummation*.
    - ∗ *Calculable* in perturbative QCD.
    - ∗ Partly *localized* in phase space.
  - Soft radiation from *underlying event* ⇒ *models*.
    - ∗ *Not calculable* in perturbative QCD.
    - ∗ *Fills* phase space (*minijets?*)
- Experimental *issues*.
  - Detector *coverage* and event *cuts* ⇒ *constraints* on global event shapes.
  - *Observable-specific* problems ⇒ *jet* algorithms, *non-global* logarithms.
- Need *discriminating* observables ...

1404–2004 Alma Universitas Taurinensis

Hadronic angularities and more

- In hard *hadron collisions* there are *at least four* jets, and measurements *cannot* be *fully inclusive* in the beam region.
- *Angularities* can be defined *w.r.t. the beam* direction and measured *jointly* with a hard distribution to *suppress beam remnants* (Berger).
  $\sigma_{AB}(\tau_a, p_\perp) = \sum_{a,b} \int dx_A dx_B \; f_{a/A}(x_A) \, f_{b/B}(x_B) \; \hat{\sigma}_{ab}(\tau_a, p_\perp) \, .$
  NOTE: Vanishing variable is $\tau_a - \tau_a^{(J)}$, depends on jet algorithm.
- Further *generalization*: introduce *auxiliary shape variable* v_j or parameter $\bar{a}$ to constrain 'current' jets. Combinations of $\{\epsilon, \tau_a, v_j(\bar{a})\}$ serve as *handles* to *tune* soft radiation.
- The *hunt* for *perfect hadronic event shapes* is on ...

Perspective

- *Event shape distributions* map the transition between perturbative and non-perturbative QCD.
- *Theoretical advances* lead to testable QCD-motivated models of power corrections (*shape functions*).
- *Angularities* can tune jet sizes using the parameter a. They obey a *simple scaling rule* testable on existing data.
- *Joint distributions* for angularities and *energy flows* outside jets enhance control on nonglobal logs.
- Extensions to *hadron collisions* are desirable, flexible and targeted observables required.

1404-2004 ALMA UNIVERSITAS TAURINENSIS