Heavy ion fusion reactions and the nucleus-nucleus potential

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Abstract. Several open questions on heavy-ion fusion reactions will be discussed by using a semi-classical model (GRAZING) that incorporate well known properties of the interacting nuclei like single particle levels and surface vibrations. It will be shown that the behavior of the fusion excitation function at very low energies is sensitive to the actual shape of the ion-ion potential at distances shorter then the position of the Coulomb barrier and that the high energy hindrance to fusion may be reconciled by taking into account the flux that will remain in all binary reactions.

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INTRODUCTION

The collision between two heavy ions is described, in its simplest model, as the collision of two hard spheres moving in a central potential consisting of a short range nuclear attraction and a long range Coulomb repulsion. In the partial-waves expansion formalism the fusion cross section is:

$$\sigma(E) = \frac{\pi\hbar^2}{2mE}\sum_{\ell}(2\ell+1)T_{\ell}(E,B) \tag{1}$$

where m is the reduced mass of the system and E the center of mass energy. The transmission coefficient through the potential barrier $T_{\ell}(E,B)$ is usually calculated in the inverse parabolic approximation (B is the potential barrier). Since this model, and all the ones mentioned below, is not able to follow the evolution of the system toward the formation of the compound nucleus the above formula gives, more appropriately, the capture cross section. For many projectile and target combinations the above description reproduce quite accurately the fusion process for energies above the Coulomb barrier but underestimate the cross section by orders of magnitude [1] at the lower energies.

H. Esbensen[2] was the first to recognized that the actual position of the barrier is influenced by the coupling of the relative motion with the intrinsic states of the two nuclei. He introduced the concept of barrier distribution $D(B)$

$$T_{\ell}(E) = \int_0^{\infty} T_{\ell}(E,B)D(B)dB \tag{2}$$

and in the Zero Point Motion (ZPM) approximation he was able to calculate the effect of the excitation of the surface modes on the fusion process.

Later it has been shown [3] that the barrier distribution can be extracted directly from precise measurments of the fusion excitation function as:

$$D(E) = \frac{1}{\pi R^2}\frac{d^2 E\sigma(E)}{dE^2} \tag{3}$$

being R the interaction radius. The importance of the barrier distribution $D(E)$ lies in the fact that it retains structures that may be related to the eigenvalues of the coupling Hamiltonian while the excitation function is usually structurless. However this relation is true only if the coupling Hamiltonian has eigenvalues that are energy independent.

From a theoretical point of view the study of fusion reactions led to the development of very sophisticated coupled-channels programs that incorporate the coupling to surface vibrations, static deformations (rotations) and particle transfer [5, 6, 7, 8]. Despite these models have to be build considering approximations, all done to reduce the number of effective channels, they have been quite successful in clarifying the mechanism of fusion process but many questions remain un-answered. For instance the role of the transfer channels is still not clearly understood, the behavior of the fusion cross section at high energies is not well reproduced; these calculations, in fact, lead always to over-predictions of the fusion cross section. This hindrance to fusion questioned the use of a single potential for the description of elastic scattering and fusion or even questioned the couple-channel models in that they may be missing important interactions [9].

More recently the measurements of the fusion excitation function have been extended at very low energies [10, 11]. Here it has been found that the fusion excitation function drop more rapidly then the expected exponential behavior. All coupled channels calculations fail to reproduce this energy dependence indicating that also in this energy region the theoretical models are inadequate or that the shape of the nuclear potential has to be somewhat modified [12] from the normally adopted Wood-Saxon shape.

In this contribution some of the above difficulties will be addressed and, hopefully clarified, by showing several calculations of heavy ion reactions done with a semiclassical model [13, 14, 15] that includes, on the same footing, surface and particle transfer degrees of freedom. What it is essential is that this model is able calculate how the total reaction cross section is divided among the different final states.

THE THEORY

The semi-classical theory that is used to analyze the data will not be summarized here, for details refer to the works in ref. [13, 14, 17], here suffice to remember that the cross section are calculated by solving in an approximate way the well known system of coupled equations

$$i\,\hbar\dot{c}_\beta(t) = \sum_\alpha c_\alpha(t) < \beta|H_{int}|\alpha > e^{\frac{i}{\hbar}(E_\beta - E_\alpha)t + i(\delta_\beta - \delta_\alpha)} \tag{4}$$

obtained from the time dependent Schrödinger equation

$$i\,\hbar\dot{\Psi}(t) = (H_0 + H_{int})\Psi(t) \tag{5}$$

by expanding the total wave function of the system in term of channel wave functions $\psi_\alpha = \psi^a(t)\psi^A(t)e^{i\delta(\vec{R})}$ corresponding to the asymptotic mass partitions. The coefficient $c_\beta(t)$ gives the amplitude for the system to be in channel β.

The residual interaction H_{in} is constructed by using the well known form-factors for the inelastic excitation and for the trasfer of one-nucleon (neutron and proton, stripping and pick-up). The time dependence of the interaction is obtained, by solving the classical equations of motion in a nuclear plus Coulomb field. For the nuclear potential U_{aA} it is used the simple Wood Saxon parameterization [16] whose parameters come from the knowledge of the nuclear densities and have been slightly adjusted through a systematic comparison of elastic scattering data. To obtain the right position of the barrier one allows for a small modification ($\delta R \sim 0.1$ fm) of the nuclear radius (cfr. ref. [17]).

For illustration it is convenient to discuss a toy model, a forced linear harmonic oscillator, that can be solved explicitly and that is ideal to put forward the main physics of the processes we are dealing with. The Hamiltonian is here very simple:

$$\hat{H} = \hbar\omega\, a^\dagger a + f(t)(a^\dagger + a) \tag{6}$$

where $a^\dagger(a)$ are operators for the creation(annihilation) of a phonon of energy $\hbar\omega$ and $f(t)$ is the time dependent external force. The distribution of internal energy $\mathscr{E}(t)$ can be calculated exactly, its average value and standard deviation being:

$$\langle \mathscr{E}(t) \rangle = \frac{1}{i}\frac{d}{d\beta}\ln Z(\beta)\bigg|_{\beta=0} = \hbar\omega|\eta(t)|^2 - \frac{f(t)^2}{\hbar\omega} \tag{7}$$

$$\sigma^2_{\mathscr{E}} = \frac{1}{i^2}\frac{d^2}{d\beta^2}\ln Z(\beta)\bigg|_{\beta=0} = (\hbar\omega)^2|\eta(t)|^2 \tag{8}$$

where:

$$\eta(t) = \frac{1}{i\,\hbar}\int_{-\infty}^{t} f(t')e^{i\omega t'}dt' + \frac{f(t)}{\hbar\omega}e^{i\omega t} \quad . \tag{9}$$

The solution of the forced linear harmonic oscillator have been written in term of the characteristic function $Z(\beta)$ to illustrate the procedure used in the true model for the calculation of the probabilities to reach the different final states.

The distribution in internal energy may, of course, be translated in a distribution of relative motion energy and this, at the turning point were the tunneling probability has to be calculated, is more conveniently seen as a distribution of barriers. If more than one mode in present the barrier distribution is obtained by the convolution of the different distributions. Depending on the properties of the modes this may acquire a structure. Notice that the average value and standard deviation are function of the bombarding energy E through the time integral along the classical trajectory. The barrier distributions are thus energy dependent. It is also important to point out the contribution of the force in the expression of the average energy (cfr. Eq. 7), this illustrate quite well how the residual interaction modify directly the barrier.

To illustrate the energy dependence of the barrier distribution in Fig. 1 are shown, for several bombarding energies, the barrier distributions for the ^{26}S + ^{90}Zr system. At energy below the Coulomb barrier the barrier distributions are almost identical while

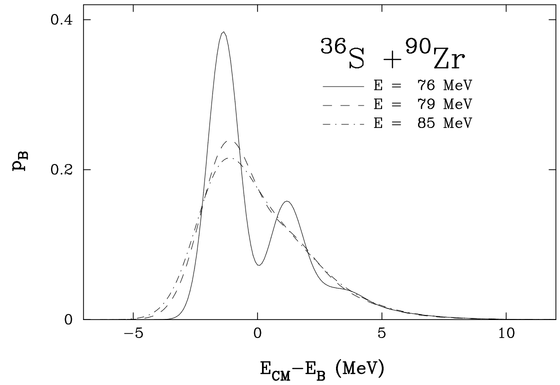

FIGURE 1. Barrier distributions for the ^{26}S + ^{90}Zr system calculated at the indicated center of mass energies.

they become smoother and wider at energy above. This behaviour come from the transfer of augular momentum and from the opening of the particle transfer channels. At energy above the barrier the transfer of angular momentum is important and this may be seen as leading to a shift of the energy of the mode ($\hbar\omega \rightarrow \hbar\omega + \mu\dot{\phi}_o$) proportional to the transferred angular momentum μ and to the angular acceleration at the turning point $\dot{\phi}_o$. The exchange of mass and charge will also contribute to the smearing of the barrier distribution due to the large number of contributing channels. For more details and examples on the application of the semiclassical model to fusion reactions refer to [17].

FUSION HINDRANCE

The analysis of the fusion reactions for the ^{16}O + ^{208}Pb system reviled [18] that it was impossible to fit at the same time the excitation function and the barrier distribution by using a standard Wood-Saxon nuclear potential with a diffuseness derived from elastic scattering data. The cross sections at the higher energies were alway over-estimated. By concentrating on the high energy part of the fusion excitation functions Newton et al. quantized [9] the over-estimation of the fusion cross section by introducing an hindrance factor as the ratio between the predicted fusion cross section with a standard potential and the measured one. They suggested that possible work-around may be found or by introducing a ”fusion potential“ with a diffuseness much larger then the one derived from from elastic scattering data or by realizing that the coupled-channels models are inadequate for fusion in that they are missing important couplings.

As mentioned in the introduction the semi-classical code GRAZING estimates, together with the fusion cross section, the cross sections for all binary events, quasi-elastic and deep-inelastic. In Fig. 2, for some systems, are shown, together with the excitation functions for fusion the energy dependence of the total reaction and binary (quasi-elastic plus deep-inelastic) cross sections. While at the lowest energies the total cross section is dominated, for all systems, by the quasi-elastic processes at the higher energies the fusion is the dominant process but only for system with $A_aA_A < 5000$ being. This is clearly seen in in Fig. 3 where is shown the ratio between the fusion cross section and the cor-

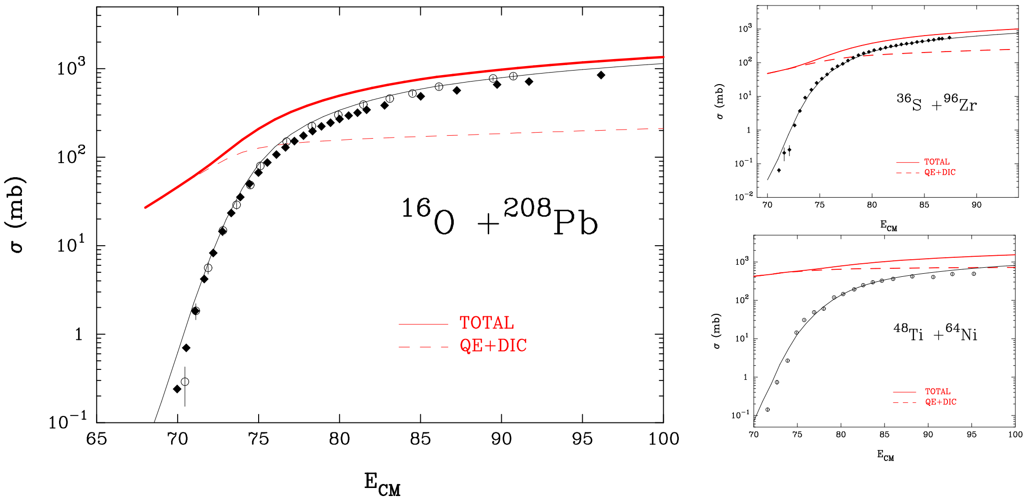

FIGURE 2. Fusion excitation function for the indicated systems in comparison with the total reaction cross section (thick line) and the total binary (QE+DIC) cross section (dash-line).

responding total reaction cross section (notice that this ratio is not the hindrance factor as defined in ref. [9]) calculated at the higher energy. These calculations clearly indicate that the high-energy hindrance may be explained by taking into account the contribution of the deep-inelastic events. Unfortunately these channels cannot be explicitly included in quantal coupled channel calculations but they should be taken into account through an imaginary potential. For the real part of the potential one can use the standard potentials derived from the analysis of elastic scattering data.

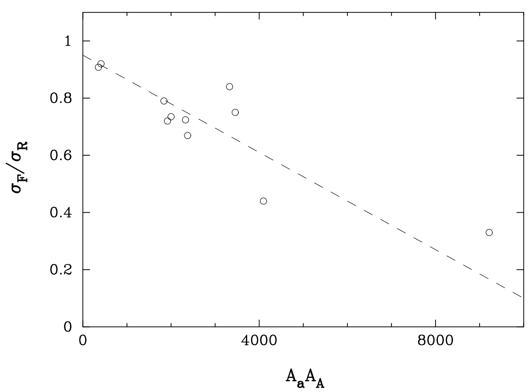

FIGURE 3. Ratio of the total cross section to the total binary cross section (Quasi-Elastic and deep inelastic) for several system as a function of the product of the charge of the two reactant.

The inverse parabolic approximation of the transmission coefficient suggest that at very low energies the fusion cross section decays like an exponential with the slope determined by the frequency characterizing the parabola fitting the effective potential at the barrier. As the experiments of ref. [10, 11] indicate this is not the case, the data show a much faster drop of the fusion cross sections. A simple inspection to the effective potential show that this, beside the well known barrier, has also a pocket located at

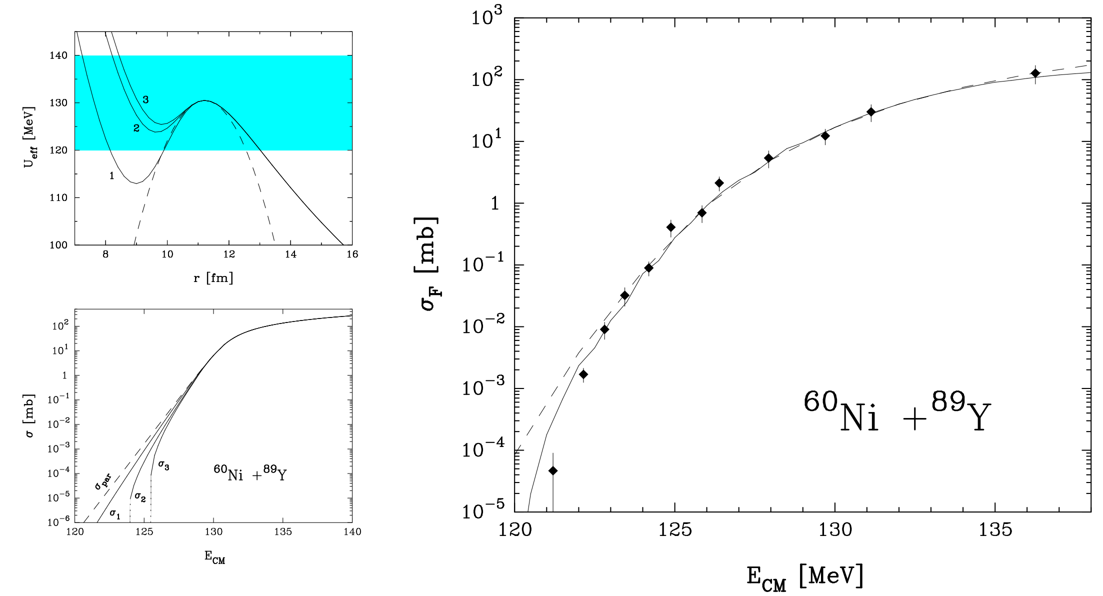

FIGURE 4. In the lower part of the left hand-side of the figure are shown the fusion cross section in the no-coupling limit for the three potentials shown on the top. The calculation refer to the ^{60}Ni + ^{89}Y system. In the right hand side, in comparison with the experimental data, are shown the calculations of the fusion cross sections for the ”original” nuclear potential (dotted line) e for the modified potential (full line). Notice that all the potential share the same behavior at large distances.

somewhat smaller distances. If this pocket exist (it is indeed a common feature of all the potentials) it defines naturally the lower limit of the energy range for the fusion to occur. In ref. [12] it is suggested that the above experiments determine the position of this pocket.

To demonstrate that fusion is sensitive to the potential at so short distances one has to use models that do not rely on the parabolic approximation for the tunneling and/or do not impose incoming wave boundary conditions at the location of the pocket (this is done in most quantal calculations for numerical stability).

This sensitivity may be shown in absence of couplings with the internal degrees of freedom since, it is well known, that the final results always inherit whatever characteristics are already present in the simple barrier-penetration formulation of the problem.

By taking a family of hypothetical potentials (cfr, top of the left panel of Fig. 4) that are identical for large values of r, share the same barrier (position r_B and value V_B) and are approximated by the same parabola but differ somewhat inside of the barrier one obtains un-coupled fusion excitation functions. for the potentials 2 and 3, that display clearly the the quenching of the fusion cross sections (see bottom part of the same panel). The drastic cut-off occurs, in these examples, for cross sections in the range of microbarns to nanobarns in reasonable agreement with the values reported in the Argonne experiment. For the potential labeled 3 a full calculation has been done and the results are shown in the right-hand side of Fig. 4 in comparison with the experimental data if ref. [10]. The calculations include the couplings to the low lying 2^+ and 3^- states of projectile and target and the exchange of particles.

A similar calculation has been performed for the $^{64}Ni+^{64}Ni$ system recently [11]

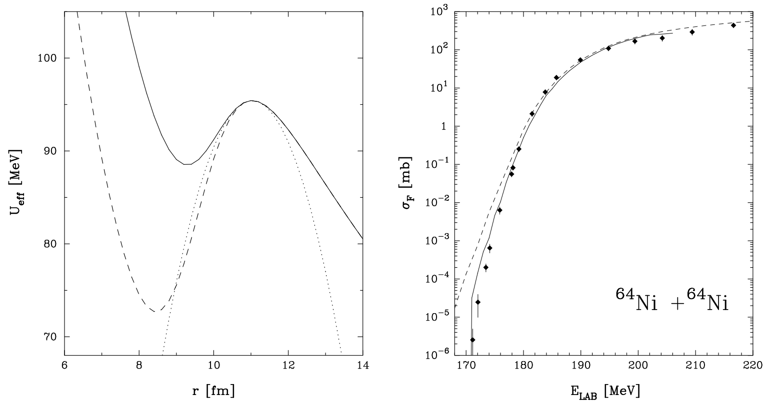

FIGURE 5. For ^{64}Ni + ^{64}Ni system is shown the potential used to calculated the fusion excitation function (right) is shown on the left panel in comparison with the empirical potential of ref. [16] and the corresponding parabolic approximation. The data are from ref. [11].

extended to very low energies. The result is shown in Fig. 5, in this case beside modifying the inner part of the nuclear potential the radius of the empirical potential has been increased by 0.1 fm as in ref [11]. The figure show also the excitation function obtained with the default potential, it agrees quite well with the quantum mechanical calculations of ref. [11].

The modification of the inner pocket of the potential allows also the understanding of the fusion excitation function for the Zirconium isotopes [19]. Here the measured fusion excitation function could not be fitted with calculations using standard potentials and couplings. The fusion cross section was strongly overestimated at the higher energies. In Fig. 6 the preliminary calculations for the ^{92}Zr + ^{90}Zr system, done by modifying the inner pocket of the potential, indicate that by making shallower the nuclear potential one non only reduces the capture at the lowest energies by increase the cross section of binary processes, like deep-inelastic collisions, reduces also the fusion cross section at the highest energies.

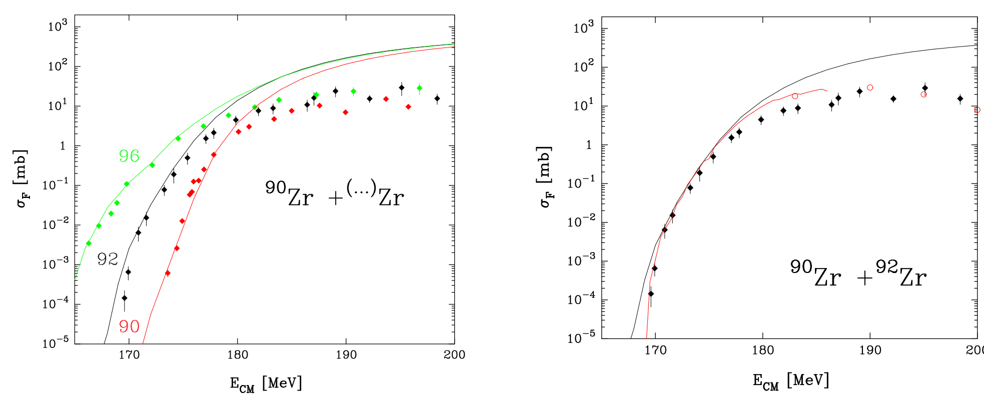

FIGURE 6. Fusion excitation function for the indicated system. In the left panel the data are compared with semiclassical calculation that use the standard empirical potential. In the right panel for the ^{92}Zr + ^{90}Zr the comparison is done modifying the potential in the inner pocket region.

CONCLUSION

The semi-classical theory offer a very powerful tool for the analysis of heavy ion reactions since it allows a clear separation between the relative motion degrees of freedom and the one for the intrinsic states. With this theory it has been possible to show that the fusion reactions are dominated by the dynamic of the nuclear surfaces. The transfer channels acting mostly has absorber of the entrance flux are very important for the explanation of the high-energy fusion hindrance since they constitute the building blocks for the deep-inelastic reactions. It has also been shown that the very fast decay of the fusion cross section at the low energies may be used to learn about the shape of the ion-ion potential at distances shorted then the Coulomb barrier.

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