Nuclear Physics **A361** (1981) 307–325; © *North-Holland Publishing Co., Amsterdam*
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ON THE ABSORPTIVE POTENTIAL IN HEAVY ION SCATTERING

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Received 21 August 1980

Abstract: A preliminary investigation of the nuclear imaginary potential to be used for the analysis of elastic scattering data of heavy ions is presented. The derivation is carried out in the framework of the semiclassical description. The resulting potential is angular momentum independent and shows two components. A long range part due to transfer reactions and a short range part due to nuclear inelastic scattering. Coulomb excitation has not been taken into account. Simple closed expressions are derived for the transition amplitudes associated with the transfer and inelastic processes, including the Q-value dependence which can be used for the analysis of reaction data.

1. Introduction

In the analysis of elastic scattering of nucleons on nuclei one has introduced, besides the average (real) potential, U, an imaginary potential, W. It accounts for the depopulation of the elastic channel due to residual interactions. A part of this absorption is associated with the mean free path of nucleons in nuclear matter. This part of the absorption, W_v, is constant over the nuclear volume and vanishes in the surface region more or less proportional to the density of the nucleus. The volume part, W_v, has been determined experimentally to have a smooth dependence on the bombarding energy and is rather well understood [1]).

Besides the interaction of the nucleon with the bulk of the nucleus, depopulation of the entrance channel will take place through the coupling to specific channels like the excitation of surface modes and the pick-up of nucleons. Attempts to calculate a surface absorptive potential, W_s, which describes these effects have not been successful. In fact the potentials calculated from the interpretation of W_s in terms of a position dependent mean free path are strongly non-local functions [2]).

Nevertheless it has been possible to describe elastic scattering by empirically adjusting a suitably parametrized function $W = W_v + W_s$, which is independent of angular momentum. The surface contribution to the absorption is usually larger than the volume contribution for bombarding energies below 40–50 MeV [ref. [3])].

The concept of an imaginary potential has been used also in the analysis of elastic scattering of heavy ions with similar success [4]). It is expected that the surface

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absorption is in this case completely dominant in that all exit channels including fusion can hardly be reached except via channels excited during the approach of the nuclear surfaces.

In this note we show how one may understand the empirically found absorptive potential, when the strict interpretation of $W_s(r)$ in terms of a mean free path is discarded. Explicit expressions for W_s are given.

2. Semiclassical treatment

The semiclassical equations of motion for the amplitudes associated with the different reaction channels β are [5])

$$i\hbar\dot{c}_\beta(t) = \sum_\gamma (\omega_\beta, (V_\gamma - U_\gamma)\psi_\gamma) e^{i(E_\beta - E_\gamma)t/\hbar} c_\gamma(t), \tag{1}$$

where $E_\beta = E_b + E_B$ is the sum of the energies of the two nuclei in channel β, while E_γ is similarly defined in channel γ. The quantities ψ_γ and ω_β are related to the product wave functions associated with the channels γ and β respectively. The ion-ion potential U_γ is the expectation value of the interaction V_γ in channel γ. The matrix elements are functions of the relative position of the ions and are functions of time as the ions move on the classical trajectory of relative motion. For each term the relative motion is chosen as the average between the two channels connected by the matrix element. Eqs. (1) are to be solved for a given impact parameter with the initial condition that $c_\beta(-\infty) = \delta(\beta, \alpha)$, where α is the entrance channel (cf. appendix A).

The question we want to address is, whether one can solve the set of equations (1) in terms of a small number, S, of active channels, including the entrance channel, introducing at the same time an imaginary potential, i.e.

$$i\hbar\dot{c}_\beta(t) = \sum_{\gamma \in S} (\omega_\beta, (V_\gamma + iW_\gamma^{(S)} - U_\gamma)\psi_\gamma) e^{i(E_\beta - E_\gamma)t/\hbar} c_\gamma(t). \tag{2}$$

The index S indicates that W would depend on the size of the subspace of states which are explicitly treated by eqs. (2). In fact W would vanish if S included all channels. In so far as the imaginary potential $W_\gamma^{(S)}$ has only diagonal matrix elements independent of the channel label one may solve (2) in terms of the solutions c' of the correspondingly truncated set of equations (1), i.e.

$$c_\beta(t) = c'_\beta(t) \exp\left\{\frac{1}{\hbar}\int_{-\infty}^{t} W(\boldsymbol{r}(t))\mathrm{d}t\right\}, \tag{3}$$

where $\boldsymbol{r}(t)$ indicates the classical trajectory of relative motion which we assumed to be similar within the group of channels S. The total probability of finding the

system in one of the channels S after the collision is

$$P_S = \sum_{\beta \in S} |c'_\beta(t)|^2 \exp\left\{\frac{2}{\hbar}\int_{-\infty}^{\infty} W^{(S)}(r(t))\mathrm{d}t\right\} = \exp\left\{-\frac{2}{\hbar}\int_{-\infty}^{\infty} W^{(S)}(r(t))\mathrm{d}t\right\}, \quad (4)$$

where we used the unitarity of any truncated set of equations (1).

As the subset S we first consider only the entrance channel α. In order to evaluate W we view the total depopulation of the entrance channel as due to elementary transitions like excitation of collective modes and transfers of single nucleons. In this picture multinucleon transfer is the result of the successive transfer of nucleons. We shall assume (i) that these elementary transitions are independent of each other, and (ii) that the probability, P_n, that a specific transition n occurs during the collision is a small number. In order to evaluate in this model the probability, P_0, of remaining in the entrance channel after the collision, we must envisage that there are so many possible transitions n that $p = \sum_n p_n > 1$ although all p_n are small. The probability P_0 can thus not be estimated by $1-p$, but one should use the expression

$$P_0 = \prod_n (1-p_n) \approx \exp\{-\sum_n p_n\}. \quad (5)$$

In the first expression we used assumption (i), while to obtain the final form we also used assumption (ii).

Although these two assumptions may seem crude, they may be rather well fulfilled in actual cases. The first assumption of independence, essentially amounts to describe the nuclear states as product states of fermion excitations, particle-hole phonons and pairing phonons. The limitation of this description is associated with the overcounting of the degrees of freedom as well as with the effects of the residual interactions, which couple e.g. the transferred nucleons into pairing and particle-hole phonons. The second assumption is rather well fulfilled except for the collective surface vibrational or rotational states. To the extent that one can describe the vibrations as purely harmonic, one can substitute a collective vibrator by a large number of independent vibrators each of them being excited only weakly. Formula (5) is actually an exact expression for the probability of staying in the ground state of a harmonic oscillator. For the excitation of rotational states this argument does not apply and the expression (5) will lead to a poor approximation for the imaginary potential associated with the elastic scattering on deformed nuclei.

In order to evaluate the exponent in (5) we should calculate the transition probabilities p_n for stripping and pick-up of single nucleons as well as for the excitation of surface and pairing vibrations in target and projectile. We use the expression

$$p_n = |c_{\alpha\to\beta}|^2, \quad (6)$$

where $c_{\alpha\to\beta}$ is the transition amplitude

$$c_{\alpha\to\beta} = \frac{1}{i\hbar}\int_{-\infty}^{\infty} (\psi_\beta, (V_\alpha - U_\alpha)\psi_\alpha) \exp i(E_\beta - E_\alpha)t/\hbar \mathrm{d}t, \tag{7}$$

in perturbation theory (cf. appendix B).

Inserting (6) into (5) and comparing to (4) we obtain the following equation which is to be satisfied by the imaginary potential

$$\int_{-\infty}^{\infty} W^{(0)}(r(t))\mathrm{d}t = \tfrac{1}{2}\hbar \sum_\beta |c_{\alpha\to\beta}|^2. \tag{8}$$

Since this equation does not provide a prescription to determine the function $W(r)$, additional constraints have to be imposed.

If one adhers to the concept of a depopulation local in time one should enforce (8) for an arbitrary upper limit t. One then finds

$$(W^{(0)}(r(t)))_{\mathrm{I}} = \hbar \,\mathrm{Re} \sum_\beta \dot{c}^*_{\alpha\to\beta}(t) c_{\alpha\to\beta}(t), \tag{9}$$

where $c_{\alpha\to\beta}(t)$ is given by (7) with t as an upper limit instead of ∞. This prescription leads to a $W(r(t))$ which is different for $t = \pm|t|$, although $r(t) = r(-t)$. Compromising on the locality in time one may enforce the locality in r by the prescription

$$(W^{(0)}(r))_{\mathrm{II}} = \tfrac{1}{2}\hbar \,\mathrm{Re} \sum_n (\dot{c}^*_{\alpha\to\beta}(t) c_{\alpha\to\beta}(t) + \dot{c}^*_{\alpha\to\beta}(-t) c_{\alpha\to\beta}(-t)), \tag{10}$$

which also satisfies (8). This prescription leads to a function which depends strongly on the energy and the angular momentum, and which coincides, for the case of Coulomb excitation, with the expressions given in ref. [12]).

Since the imaginary potential used in the analysis of experimental data is angular momentum independent, it seems more natural to use this criterion as the subsidiary condition to determine $W(r)$ from (8). This can be done by interpreting (8) as an identity in the angular momentum. For nuclear interactions, where the $c_{\alpha\to\beta}$'s in (8) are mainly determined by the distance of closest approach r_0, the condition in l is largely equivalent to an interpretation of (8) as an identity in r_0.

Using a parabolic expansion of the trajectory around the turning point (cf. appendix B, eq. (10)), one finds

$$\sqrt{\frac{2r_0}{\ddot{r}_0}} \int_1^\infty \frac{W(r_0 x)}{\sqrt{x-1}} \mathrm{d}x = \tfrac{1}{2}\hbar \sum_\beta |c_{\alpha\to\beta}|^2, \tag{11}$$

$\ddot{r}_0$ being the acceleration at the turning point. As shown in appendix B, the amplitudes c can be parametrized as

$$c_{\alpha\to\beta} \approx K \mathrm{e}^{-r_0/a_\beta}. \tag{12}$$

In this case, the integral equation (11) for W can be solved leading to the result

$$W(r) = \sum_{\beta} \sqrt{\frac{\ddot{r}_0 \hbar^2}{4\pi a_\beta}} (|c_{\alpha \to \beta}|^2)_{r_0 = r}. \tag{13}$$

For the transfer channels, the quantity α_β is of the order of 1.2 fm, while for nuclear inelastic scattering it is of the order of 0.6 fm. This implies that the total absorptive potential has two components; a long range part with the difuseness of $\sim$ 0.6 fm due to transfer, and a short range with a difuseness of $\sim$ 0.3 fm due inelastic processes. Some experimental evidence for such two-component potential have been found [13]).

When Coulomb excitation is included, the transition amplitudes for inelastic scattering cannot be parametrized as in (12). In fact the amplitudes would consist of two terms with opposite sign, leading to a short range potential due to inelastic scattering, a long range potential due to Coulomb excitation, and an interference contribution which is in fact a source term.

In the present paper we shall neglect Coulomb excitation and the results are therefore valid for relatively light ion scattering.

Since Coulomb excitation, at bombarding energies of few MeV per nucleon, leads only to states of low excitation energy, such reactions are often counted experimentally as elastic events. Because Coulomb excitation does not change the trajectory of relative motion to any appreciable extent, one might, for the analysis of such "elastic" experiments, use an absorptive potential which consists merely of the W due to transfer plus a W due to inelastic scattering to high-lying states which can only be excited by nuclear interaction.

3. Calculation of the absorptive potential

In this section we evaluate the absorptive potential $W(r)$ defined in eq. (13), utilizing the approximate expressions for the transition amplitudes given in appendix B. The contributions to W from nucleon stripping and pick-up are given by

$$(W(r))_{\text{transf}} = \frac{1}{(2I_a+1)(2I_A+1)} \sum \sqrt{\frac{\ddot{r}_0 \hbar^2}{4\pi a_\beta}} \, (|(c^{\text{NS}}_{\alpha \to \beta})_{r_0 = r}|^2 + |(c^{\text{NP}}_{\alpha \to \beta})|^2_{r_0 = r}), \tag{14}$$

where the summation is to be carried over the quantum numbers associated with the index β, that is, $I_b M_b$, I_B and M_B. Furthermore, the sum also implies an averaging over the initial orientation, that is, a sum over M_a and M_A.

Utilizing the expressions (B.19), (B.29), (B.44) and (B.47) we can write (14) in

the form

$$(W(r))_{\text{transf}} = \sum_{a_1 a_1' \lambda} \sqrt{\frac{\ddot{r}_0 \hbar^2}{4\pi a_{\text{tr}}(a_1, a_1')}} \times \left\{ \frac{2j_1'+1}{2\lambda+1} U^2(a_1, I_A) V^2(a_1', I_a) \sum_{\mu} |(I^{\text{NS}}_{\lambda\mu}(a_1 a_1'))|^2_{r_0=r} + \frac{2j_1+1}{2\lambda+1} \times U^2(a_1', I_a) V^2(a_1, I_A) \sum_{\mu} |(I^{\text{NP}}_{\lambda\mu}(a_1' a_1))_{r_0=r}|^2. \quad (15)$$

The orbital integrals over the single-particle form-factor are given by (B.28) and (B.45) for nucleon stripping and nucleon pick-up respectively except that the $f^{JJ'}_{\lambda\mu}$ form factors are substituted by (B.30) and (B.48) respectively. In deriving (15) we have neglected the dependence of ΔE on I_B and I_b for a given fixed single-particle orbital, that is, the variation of the Q-value within the energy interval over which the single-particle strength is spread. The quantities V^2 are the occupation probabilities

$$\begin{aligned} V^2(a_1', I_a) &= \frac{1}{2I_a+1} \sum_{M_a} \langle I_a M_a | a^+_{j_i m_i}(a_1') a_{j_i m_i}(a_1') | I_a M_a \rangle \\ &= \frac{1}{2j_1'+1} \sum_{m_i} \langle I_a M_a | a^+_{j_i m_i}(a_1') a_{j_i m_i}(a_1') | I_a M_a \rangle, \end{aligned} \quad (16)$$

that the single-particle orbitals a_1' are occupied in the nucleus a. The quantity $U^2 = 1 - V^2$ is the corresponding probability that the state is empty.

Using the approximate expression (B.41) for the orbital integral we find

$$(W(r))_{\text{transf}} = \sum_{a_1 a_1' \lambda} \sqrt{\frac{a_{\text{tr}}(a_1 a_1')}{16\pi |\ddot{r}_0| \hbar^2}} \{(2j_1'+1) U^2(a_1 I_A) V^2(a_1' I_a) | f^{a_1 a_1'(\text{NS})}_{\lambda 0}(a, r)|^2 + (2j_1+1) U^2(a_1' I_a) V^2(a_1 I_A) | f^{a_1' a_1(\text{NP})}_{\lambda 0}(0, r)|^2 \} g_\lambda(Q), \quad (17)$$

where we assumed $\mu' = 0$ and define

$$g_\lambda(Q) = \sum_{\mu=-\lambda}^{\lambda} |D^\lambda_{\mu 0}(0, \tfrac{1}{2}\pi, 0)|^2 \exp\{-\text{Re}\,(a - \mu b/\lambda)^2\}. \quad (18)$$

The function g determines the adiabatic cut-off. The quantities a and b are related to the quantity q in eq. (B.42). They are given by

$$a = \sqrt{\frac{a_{\text{tr}}(a_1 a_1')}{\ddot{r}_0 \hbar^2}} (Q - Q_{\text{opt}}), \quad (19)$$

where the optimum Q-value is

$$Q_{\text{opt}} = \left(\frac{Z_d}{Z_A} - \frac{Z_d}{Z_b} \right) E_B + \left(\frac{m_d}{m_b} - \frac{m_d}{m_A} \right) (E - E_B) + \frac{m_d \ddot{r}_0}{m_a + m_A} (R_A m_b - R_a m_B), \quad (20)$$

and

$$b = \sqrt{\frac{a_{tr}(a_1 a_1')}{\ddot{r}_0 \hbar^2}} \frac{\hbar^2(l_g + \frac{1}{2})}{m_{aA} r_0^2} \lambda. \tag{21}$$

For small collision times the quantity (18) is unity. One can estimate the function g by utilizing the approximation

$$|D^{\lambda}_{\mu 0}(0, \tfrac{1}{2}\pi, 0)|^2 = \frac{2}{\pi} \frac{1}{\sqrt{\lambda^2 - \mu^2}} \tag{22}$$

and substituting the sum over μ by an integral. One thus obtains

$$g_{\lambda}(a, b) = \frac{1}{\pi} \int_0^{\pi} \exp\left[-\mathrm{Re}\,(a - b\cos\theta)^2\right] \mathrm{d}\theta. \tag{23}$$

This function has been evaluated numerically for real values of a and b and the result is given in fig. 1.

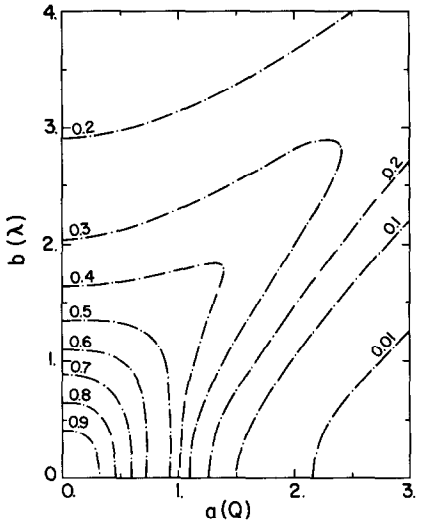

Fig. 1. The adiabatic cut-off function $g_{\lambda}(Q)$ which describes the ratio of the actual transition probability to the same quantity in the sudden approximation. It is defined in (18) and is given as a function of the dimensionless parameters a and b. The parameter a which depends on the Q-value is defined in (19), while b which depends on the angular momentum transfer λ, is defined in (21).

The contributions to W from nuclear inelastic scattering is similarly given by

$$W_{\mathrm{inel}}(r) = \sum \sqrt{\frac{\ddot{r}_0 \hbar^2}{4\pi a}} \left(\frac{1}{2\lambda + 1} \sum_{\mu} |I^{\mathrm{TE}}_{\lambda\mu}|^2 + \frac{1}{2\lambda + 1} \sum_{\mu} |I^{\mathrm{PE}}_{\lambda\mu}|^2 \right), \tag{24}$$

where the orbital integrals for target and projectile excitation are given by (B.3) and

(B.7) with the form factors (B.4a) and the nuclear part of (B.8) respectively. Inserting the result (B.12) we find

$$W_{\text{inel}}(r) = \sum_{\lambda} \sqrt{\frac{a}{16\pi|\ddot{r}_0|\hbar^2}} (|f_\lambda^{N(TE)}(r)|^2 + |f_\lambda^{N(PE)}(r)|^2) g_\lambda(Q). \tag{25}$$

The adiabatic cut-off function g is given by the same expression as (18) with $Q_{\text{opt}} = 0$ and with a diffuseness parameter a instead of a_{tr}. The sum of the form factors appearing in (25) can be written in terms of the total zero-point fluctuation σ of the two nuclear surfaces defined by

$$\sigma^2 = \sum_{\lambda} (2\lambda+1) \left(\frac{\hbar\omega_\lambda^{\text{a}}}{2C_\lambda^{\text{a}}} (R_{\text{a}}^{(0)})^2 + \frac{\hbar\omega_\lambda^{\text{A}}}{2C_\lambda^{\text{A}}} (R_{\text{A}}^{(0)})^2 \right) g_\lambda(\hbar\omega_\lambda), \tag{26}$$

of all states below the adiabatic cut-off, i.e.

$$W_{\text{inel}}(r) = \sqrt{\frac{a}{16\pi|\ddot{r}_0|\hbar^2}} \sigma^2 \left(\frac{\partial \tilde{U}_{\text{aA}}}{\partial r} \right)^2. \tag{27}$$

The total absorption is thus equal to

$$W(r) = W_{\text{transf}}(r) + W_{\text{inel}}(r). \tag{28}$$

In the derivation of this potential we assumed (cf. eq. (12)) that the transition amplitudes were exponential functions of the distance of closest approach. This ansatz seems reasonable, because although the transition amplitudes depends also on $\ddot{r}_0$, this dependence appears as a multiplicative factor under the square root. However the expressions (17) and (27) are singular for $\ddot{r}_0 = 0$ which in the classical description happens when the angular momentum of relative motion is equal to the grazing value l_{g}. This is however a fictitious problem since the existence of an imaginary potential means that the classical trajectories should be complex. In terms of the effective potential for the radial motion U_{eff}, the turning point determined from

$$E = U_{\text{eff}}(r_0) + iW(r_0), \tag{29}$$

is complex and the acceleration

$$\ddot{r}_0 = -\frac{1}{m_{\text{aA}}} \frac{\partial}{\partial r} (U_{\text{eff}} + iW)_{r=r_0} \tag{30}$$

is in general non-vanishing. In fact the use of the expression (28) for $W(r)$ requires that $\ddot{r}_0$ and W should be determined self-consistently from eqs. (28)–(30).

For simple estimates one may use for the modulus of the acceleration an estimate based on a Coulomb trajectory which is

$$m_{\text{aA}}\ddot{r}_0 = \frac{2E}{r_0} - \frac{Z_{\text{a}}Z_{\text{A}}e^2}{r_0^2} \approx \frac{2E - E_{\text{B}}}{r_{\text{B}}}, \tag{31}$$

where E_B is the height of the Coulomb barrier and r_B the radius of the barrier

$$(r_B)_{fm} = 1.07(A_a^{\frac{1}{3}} + A_A^{\frac{1}{3}}) + 2.72. \tag{32}$$

The absorptive potential determined self-consistently from eqs. (28)–(30) is local as far as is independent of l. It does show however an energy dependence through the parameter $\bar{r}_0$. Besides entering as a multiplicative factor, $\bar{r}_0$ enters in the adiabatic cut-off parameters (19) and (21). For increasing bombarding energies it is expected that $\bar{r}_0$ increases. The corresponding decrease in W, because of the square root factor, is more than compensated by the change in the adiabatic cut-off which implies that more states contribute to the depopulation of the entrance channel.

Preliminary numerical calculations of the imaginary potential on the basis of the expressions given above are in good agreement with the experimental data. Systematic calculations are in progress and will be presented elsewhere.

4. Conclusions

In this paper we have derived explicit expressions for the imaginary potential to be used in the analysis of elastic scattering data. This was done by imposing the condition that W should not depend on the angular momentum, and making use of the assumption that the different channels contributing to W are independent. Only the effects due to nuclear interaction were included. In the derivation we used the semiclassical description and the corresponding expressions for the transition probabilities. The explicit expressions for these quantities are usefull for the discussion of the Q-value dependence of transfer reactions and inelastic scattering of heavy ions. The resulting W has a long range component due to single-particle transfer, and a short range part due to inelastic scattering.

Appendix A

SEMICLASSICAL EQUATIONS OF MOTION

The coupled equations (1) are complicated by the fact that the channel wave functions ψ_γ describing various exit channels in transfer reactions are non-orthogonal, i.e. the overlap matrix

$$\alpha_{\beta\gamma} = (\psi_\beta, \psi_\gamma) \tag{A.1}$$

is only diagonal at $t = \pm\infty$.

From the time-dependent Schrödinger equation the coupled equations are directly obtained in the form

$$i\hbar \sum_\gamma \alpha_{\beta\gamma} \dot{c}_\gamma e^{i(E_\beta - E_\gamma)t/\hbar} = \sum_\gamma (\psi_\beta, (V_\gamma - U_\gamma)\psi_\gamma) e^{i(E_\beta - E_\gamma)t/\hbar} c_\gamma. \tag{A.2}$$

In order to write them in the simple form (1) we introduce the adjoint channel wave functions ω_γ as

$$\omega_\gamma = \sum_\beta (\alpha^{-1})_{\beta\gamma}\psi_\beta, \tag{A.3}$$

where α^{-1} is the reciprocal matrix to (A.1). For small overlap (grazing collisions) where

$$\alpha = 1+\varepsilon, \tag{A.4}$$

with $\varepsilon \ll 1$ we may use

$$\alpha^{-1} = 1-\varepsilon+\varepsilon^2+\ldots. \tag{A.5}$$

Defining adjoint amplitudes

$$\bar{c}_\beta = \sum_\gamma \alpha_{\beta\alpha} c_\gamma e^{i(E_\beta - E_\gamma)t/\hbar} \tag{A.6}$$

as the expansion of the state vector on the adjoint channel wave functions (A.3) we find that they satisfy the equations

$$\begin{aligned} i\hbar\dot{\bar{c}}_\beta &= \sum_\gamma (\psi_\beta, (V_\beta - U_\beta)\psi_\gamma) e^{i(E_\beta - E_\gamma)t/\hbar} c_\gamma \\ &= \sum_\gamma (\psi_\beta, (V_\beta - U_\beta)\omega_\gamma) e^{i(E_\beta - E_\gamma)t/\hbar} \bar{c}_\gamma. \end{aligned} \tag{A.7}$$

We used here the post-prior symmetry relation

$$\begin{aligned} &[(\psi_\beta, (V_\beta - U_\beta)\psi_\gamma) - (\psi_\beta, (V_\gamma - U_\gamma)\psi_\gamma)] e^{i(E_\beta - E_\gamma)t/\hbar} \\ &\qquad = i\hbar \frac{d}{dt}(\alpha_{\beta\gamma} e^{i(E_\beta - E_\gamma)t/\hbar}), \end{aligned} \tag{A.8}$$

derived in ref. [5]) (cf. ibid, eq. (4.2)).

While the quantities $|c_\beta|^2$ or $|\bar{c}_\beta|^2$ at time $\pm\infty$ signify the probability of being in channel β they do not satisfy unitarity at $t \approx 0$. We may at intermediate times rather use the quantity

$$P_\beta(t) = \mathrm{Re}\{\bar{c}^*_\beta(t) c_\beta(t)\}, \tag{A.9}$$

for which it is easy to prove unitarity, i.e.

$$\frac{d}{dt}\sum_\beta P_\beta(t) = 0. \tag{A.10}$$

Appendix B

SEMICLASSICAL FIRST ORDER PERTURBATION THEORY

In this appendix we quote the results of the semiclassical perturbation theory of transfer and inelastic reactions between heavy ions [cf. refs. [5-7])].

We consider a collision between projectile a and target A. We shall consider spherical nuclei and assume that excitations only take place in either target or projectile. For target excitation the first order amplitude is

$$c^{\mathrm{TE}} = \frac{1}{i\hbar}\int_{-\infty}^{\infty} \mathrm{d}t \exp\left[i(E'_{\mathrm{A}} - E_{\mathrm{A}})t/\hbar\right] \langle I'_{\mathrm{A}} M'_{\mathrm{A}} | V_{\mathrm{aA}} - U_{\mathrm{aA}} | I_{\mathrm{A}} M_{\mathrm{A}} \rangle, \tag{B.1}$$

where $I_{\mathrm{A}} M_{\mathrm{A}}$ and $I'_{\mathrm{A}} M'_{\mathrm{A}}$ are the spin quantum numbers of the target states before and after the excitation, while E_{A} and E'_{A} are the corresponding energies. We may write (B.1) in the form

$$c^{\mathrm{TE}} = -i\sum_{\lambda\mu} \langle I_{\mathrm{A}} M_{\mathrm{A}} \lambda - \mu | I'_{\mathrm{A}} M'_{\mathrm{A}} \rangle \frac{(-1)^{\lambda-\mu}}{\sqrt{2\lambda+1}} I_{\lambda\mu}, \tag{B.2}$$

with

$$I_{\lambda\mu} = \frac{1}{\hbar}\int_{-\infty}^{\infty} \mathrm{d}t \exp\left[i\omega_{\lambda}^{\mathrm{A}} t\right] f_{\lambda}^{(\mathrm{A})}(r(t)) Y_{\lambda\mu}(\hat{r}(t)), \tag{B.3}$$

where $r(t)$ indicates the time-dependent position vector of the projectile with respect to the target and $\hbar\omega_{\lambda}^{\mathrm{A}}$ is the excitation energy. The form factor $f_{\lambda}^{(\mathrm{A})}(r)$ is to a good approximation [7]) given by

$$f_{\lambda}^{(\mathrm{A})}(r) = f_{\lambda}^{\mathrm{N(A)}}(r) + f_{\lambda}^{\mathrm{C(A)}}(r),$$

with

$$f_{\lambda}^{\mathrm{N(A)}}(r) = -(-1)^{\lambda} \sqrt{\frac{\hbar\omega_{\lambda}^{\mathrm{A}}}{2C_{\lambda}^{\mathrm{A}}}} R_{\mathrm{A}}^{(0)} \frac{\partial U_{\mathrm{aA}}}{\partial r} \sqrt{2\lambda+1}, \tag{B.4a}$$

$$f_{\lambda}^{\mathrm{C(A)}}(r) = (-1)^{\lambda} \frac{4\pi Z_{\mathrm{a}} e}{(2\lambda+1)} \langle \lambda || \mathcal{M}_{\mathrm{A}}(\mathrm{E}\lambda) || 0 \rangle r^{-\lambda-1}, \tag{B.4b}$$

where $\sqrt{\hbar\omega_{\lambda}^{\mathrm{A}}/2C_{\lambda}^{\mathrm{A}}}$ is the zero point amplitude, $\langle 1_{\lambda\mu} | \alpha_{\lambda\mu} | 0 \rangle$, of the vibration, while $U_{\mathrm{aA}}(r)$ is the ion-ion potential and $R_{\mathrm{A}}^{(0)}$ the radius of nucleus A. The second term is the Coulomb excitation part of the interaction, Z_{a} indicating the charge number of the projectile. The reduced matrix element of the electric multipole moment may be written

$$\langle \lambda || \mathcal{M}(\mathrm{E}\lambda) || 0 \rangle = \frac{3Z_{\mathrm{A}} e (R_{\mathrm{A}}^{\mathrm{c}})^{\lambda}}{4\pi} \sqrt{\frac{\hbar\omega_{\lambda}^{\mathrm{A}}}{2C_{\lambda}^{\mathrm{A}}}} \sqrt{2\lambda+1}, \tag{B.5}$$

where $R_{\mathrm{A}}^{\mathrm{c}}$ is the charge radius of nucleus A, while $Z_{\mathrm{A}} e$ is its charge.

For projectile excitation one finds a similar expression

$$c^{\mathrm{PE}} = -i\langle I_{\mathrm{a}} M_{\mathrm{a}} \lambda - \mu | I'_{\mathrm{a}} M'_{\mathrm{a}} \rangle \frac{(-1)^{\mu}}{\sqrt{2\lambda+1}} I_{\lambda\mu}, \tag{B.6}$$

with

$$I_{\lambda\mu} = \frac{1}{\hbar}\int_{-\infty}^{\infty} \mathrm{d}t \exp\left[i\omega_\lambda^{\mathrm{a}} t\right] f_\lambda^{(\mathrm{a})}(r(t))\, Y_{\lambda\mu}(\hat{r}(t)), \tag{B.7}$$

and

$$f_\lambda^{(\mathrm{a})}(r) = (-1)^\lambda \left[-\sqrt{\frac{\hbar\omega_\lambda^{\mathrm{a}}}{2C_\lambda^{\mathrm{a}}}}\sqrt{2\lambda+1}\, R_{\mathrm{a}}^{(0)} \frac{\partial U_{\mathrm{aA}}}{\partial r} + \frac{4\pi Z_{\mathrm{A}} e}{2\lambda+1} \langle \lambda \| \mathscr{M}_{\mathrm{a}}(\mathrm{E}\lambda) \| 0 \rangle r^{-\lambda-1} \right]. \tag{B.8}$$

The difference in sign between (B.6) and (B.2) is due to the fact that $\hat{r}$ is oriented from A to a.

The orbital integrals (B.3) or (B.7) are most easily evaluated in a coordinate system where the z-axis is perpendicular to the plane of the trajectory along the angular momentum of relative motion, and the x-axis is pointing towards the projectile at the point of closest approach. In this case

$$Y_{\lambda\mu}(\hat{r}(t)) = Y_{\lambda\mu}(\tfrac{1}{2}\pi, 0)\, \mathrm{e}^{i\mu\phi(t)}, \tag{B.9}$$

where ϕ is the azimuthal angle.

Since the nuclear part of the form factor has a short range we may evaluate the orbital integrals by a series expansion around the time ($t = 0$) of closest approach, i.e.

$$\begin{aligned} r(t) &= r_0 + \tfrac{1}{2}\ddot{r}_0 t^2, \\ \phi(t) &= \dot{\phi}_0 t, \end{aligned} \tag{B.10}$$

where the acceleration $\ddot{r}_0$ and the angular velocity $\dot{\phi}_0$ at the closest distance r_0 are positive. Utilizing an exponental form for U_{aA}, i.e.

$$\frac{\partial U_{\mathrm{aA}}}{\partial r} \approx U'(r_0)\, \mathrm{e}^{-(r-r_0)/a}, \tag{B.11}$$

with $a \approx 0.6$ fm, we find

$$\begin{aligned} I_{\lambda\mu}^N &= Y_{\lambda\mu}(\tfrac{1}{2}\pi, 0) f_\lambda^{\mathrm{N}}(r_0) \frac{1}{\hbar}\int_{-\infty}^{\infty} \exp\left[-\tfrac{1}{2}\ddot{r}_0 t^2 + i\omega_\lambda t^2/a + i\omega_\lambda t + i\mu\dot{\phi}_0 t\right] \mathrm{d}t \\ &= Y_{\lambda\mu}(\tfrac{1}{2}\pi, 0) \sqrt{\frac{2\pi a}{\ddot{r}_0 \hbar^2}} f_\lambda^N(r_0) \exp\left[-(\mu\dot{\phi}_0 + \omega_\lambda)^2 a/2\ddot{r}_0\right]. \end{aligned} \tag{B.12}$$

The quantity $\sqrt{a/\ddot{r}_0}$ indicates the collision time, τ_{inel}, and in the absence of the exponential factor the orbital integral is the product of this time and the form factor at the distance of closest approach in units of $\hbar$. The exponential function gives rise to the adiabatic cut-off at nuclear frequencies ω_λ which exceed the inverse collision time. We may estimate the quantity $m_{\mathrm{aA}}\ddot{r}_0$ (m_{aA} being the reduced mass) by the bombarding energy in the c.m. system divided by r_0 [cf. (31)]. The adiabatic cut-off

energy is

$$\frac{\hbar}{\tau_{\text{inel}}} = \sqrt{\frac{\hbar^2 \ddot{r}_0}{a}} \approx \sqrt{\frac{70\varepsilon_{\text{MeV}}}{(r_0)_{\text{fm}}}}\,\text{MeV}, \tag{B.13}$$

where ε_{MeV} is the laboratory bombarding energy per nucleon measured in MeV and $(r_0)_{\text{fm}} \approx (R_a + R_A)_{\text{fm}}$ is the distance of closest approach in fm. The quantity $\hbar\dot{\phi}_0$ is similarly given by

$$\hbar\dot{\phi}_0 = \frac{\hbar L}{m_{aA} r_0^2} = \frac{40(A_a + A_A)}{A_a A_A} \frac{l_g}{(r_{0\text{fm}})^2}\,\text{MeV}, \tag{B.14}$$

where l_g is the grazing angular momentum L, in units of $\hbar$.

Nuclear states can thus only be excited if the excitation energy $\Delta E = \hbar\omega$ satisfies the inequality

$$\Delta E + (\Delta L)_z \dot{\phi}_0 \lesssim \frac{\hbar}{\tau_{\text{inel}}}, \tag{B.15}$$

where $(\Delta L)_z = +\mu\hbar$ indicates the angular momentum, which is gained by the relative motion in the direction of the relative angular momentum $\boldsymbol{L}$. In a purely classical description with a continuous energy and angular momentum loss, the two quantities on the left-hand side of (B.15) would be equal since

$$\frac{\boldsymbol{L}}{L} \cdot \Delta\boldsymbol{L} \approx \int \mathrm{d}t\, (\boldsymbol{r} \times \boldsymbol{F}(t)) \frac{\boldsymbol{L}}{L} \approx \frac{m_{aA} r_0^2}{L} \int \boldsymbol{v} \cdot \boldsymbol{F}(t) \mathrm{d}t = -\Delta E/\dot{\phi}_0,$$

where $\boldsymbol{F}$ is the effective dissipative force acting on the projectile.

The expression (B.12) has been compared [8]) to the results of the WKB approximation for the radial matrix elements in a DWBA treatment of inelastic scattering. If one uses for r_0, $\ddot{r}_0$ and $\dot{\phi}_0$ the average values of the corresponding quantities in entrance and exit channel, which are in general complex numbers, one finds quite accurate agreement even for large Q-values.

The orbital integrals (B.3) for the Coulomb excitation part have been evaluated numerically and are given e.g. in ref. [9]).

Next we consider a single particle stripping reaction, where the residual nucleus B contains one nucleon more than the target A while the scattered nucleus b has lost a nucleon.

The first order amplitude for this reaction is

$$c^{\text{NS}} = \frac{1}{i\hbar} \int_{-\infty}^{\infty} \mathrm{d}t \langle I_b M_b I_B M_B | V_{aA} - U_{aA} | I_a M_a I_A M_A \rangle \times \exp i[(E_b + E_B - E_a - E_A)t + \gamma_{\beta\alpha}(t)]/\hbar, \tag{B.16}$$

where we have indicated the nuclear states by their spin quantum numbers. The phase $\gamma_{\beta\alpha}(t)$ is given by [cf. ref. [5]) eq. (2.52)]

$$\gamma_{\beta\alpha}(t) = \int_0^t (\mathscr{L}_\alpha(t) - \mathscr{L}_\beta(t))\mathrm{d}t, \tag{B.17}$$

where $\mathscr{L}(t)$ is the lagrangian of relative motion, i.e.

$$\mathscr{L}_\alpha(t) = \tfrac{1}{2}m_{\mathrm{aA}}v^2 - U_{\mathrm{aA}}(r), \tag{B.18}$$

the (average) position and velocity of relative motion being $\boldsymbol{r}(t) = \frac{1}{2}(\boldsymbol{r}_{\mathrm{aA}} + \boldsymbol{r}_{\mathrm{bB}})$ and $v(t) = \dot{r}(t)$, respectively.

We shall write the amplitude (B.16) in the form [cf. ref. [6]) sects. 2 and 3]

$$c^{\mathrm{NS}}_{\mathrm{aA}\to\mathrm{bB}} = -i \sum_{\substack{JJ'\lambda \\ MM'\mu}} \langle I_{\mathrm{A}} M_{\mathrm{A}} JM | I_{\mathrm{B}} M_{\mathrm{B}} \rangle \langle I_{\mathrm{b}} M_{\mathrm{b}} J'M' | I_{\mathrm{a}} M_{\mathrm{a}} \rangle \langle \lambda\mu JM | J'M' \rangle I_{\lambda\mu}, \tag{B.19}$$

with

$$I_{\lambda\mu} = \frac{1}{\hbar}\int_{-\infty}^{\infty} \mathrm{d}t \exp\left[i(\Delta E t + \gamma_{\beta\alpha}(t))/\hbar\right] f^{JJ'}_{\lambda\mu}(\boldsymbol{k}(t), \boldsymbol{r}(t)). \tag{B.20}$$

The dependence of the form factor on the velocity of relative motion through

$$\boldsymbol{k}(t) = \frac{m_{\mathrm{d}}}{\hbar}\dot{\boldsymbol{r}}(t), \tag{B.21}$$

is associated with the recoil effect, $m_{\mathrm{d}} = m_{\mathrm{a}} - m_{\mathrm{b}}$ being the mass of the transferred nucleon.

The form factors can be expressed in terms of the intrinsic form factors through the relation

$$f^{JJ'}_{\lambda\mu}(\boldsymbol{k}, \boldsymbol{r}) = \sum_{\mu'} D^{\lambda}_{\mu\mu'}(\vartheta_i) f^{JJ'}_{\lambda\mu'}(k_3, k_2, r). \tag{B.22}$$

The eulerian angles ϑ_i are defined by the rotation from the laboratory system to the intrinsic (1, 2, 3) system where the 3-axis is along $\boldsymbol{r}$ and the 1-axis is in the direction of the orbital angular momentum. If we chose to evaluate the amplitude (B.19) in the coordinate system (A) used above where

$$(\vartheta_1, \vartheta_2, \vartheta_3) = (\phi(t), \tfrac{1}{2}\pi, \pi), \tag{B.23}$$

we may write

$$I_{\lambda\mu} = \sum_{\mu'} D^{\lambda}_{\mu\mu'}(0, \tfrac{1}{2}\pi, \pi) \times \frac{1}{\hbar}\int_{-\infty}^{\infty} \mathrm{d}t \exp\left[i(\Delta E t + \gamma_{\beta\alpha}(t) + \hbar\mu\phi(t))/\hbar\right] f^{JJ'}_{\lambda\mu'}(k_3(t), k_2(t), r(t)). \tag{B.24}$$

Examples of the intrinsic form factors are shown in ref. [6]), figs. 6–8. The main part of the longitudinal recoil effect associated with k_3 may be extracted as an average phase factor

$$f^{JJ'}_{\lambda\mu'}(k_3, k_2, r) \approx e^{i\bar{\sigma}} f^{JJ'}_{\lambda\mu'}(0, k_2, r) \approx e^{i\bar{\sigma}} \sqrt{\frac{2\lambda+1}{4\pi}} (rk_2)^{\mu'} f^{JJ'}_{\lambda\mu'}(0, r). \tag{B.25}$$

In the latter equation we used an approximation for the transverse recoil effect which is valid for not too high bombarding energies [cf. ref. [6]) eq. (4.36)]. The phase $\bar{\sigma}$ is given by (cf. ref. [5]), eq. (3.16))

$$\bar{\sigma} = k_3(t)\left(R_A - \frac{m_B}{m_b+m_B} r\right) \approx \frac{1}{\hbar} \frac{m_d}{m_a+m_A} \dot{r}(t)(R_A m_b - R_a m_B), \tag{B.26}$$

where we used that $r \approx R_a + R_A$. The transverse momentum k_2 can be expressed in terms of the angular momentum L. The product

$$k_2(t)r(t) = -\frac{m_d}{m_{aA}} \frac{L}{\hbar} \tag{B.27}$$

is thus a constant. Inserting these results in (B.24) we find

$$I_{\lambda\mu} = \sum_{\mu'} D^{\lambda}_{\mu\mu'}(0, \tfrac{1}{2}\pi, 0) \sqrt{\frac{2\lambda+1}{4\pi}} \left(\frac{m_d}{m_{aA}} \frac{L}{\hbar}\right)^{\mu'} \times \frac{1}{\hbar} \int_{-\infty}^{\infty} dt \exp\left[i(\Delta E t + \gamma_{\beta\alpha}(t) + \hbar\bar{\sigma}(t) + \hbar\mu\phi(t))/\hbar\right] f^{JJ'}_{\lambda\mu'}(0, r(t)). \tag{B.28}$$

The radial form factors can be expressed in terms of form factors describing the transition between the single-particle configuration $a'_1 \equiv (n'_1 l'_1 j'_1)$ in a and $a_1 \equiv (n_1 l_1 j_1)$ in A, i.e.

$$f^{JJ'}_{\lambda\mu'}(0, r) = \sum_{a_1 a'_1} C^*(I_A a_1; I_B) C(I_b a'_1; I_a) \times f^{a_1 a'_1}_{\lambda\mu'}(0, r) \delta(j_1, J) \delta(j'_1, J'), \tag{B.29}$$

where the C's are spectroscopic amplitudes and where the single particle form factors for $\mu' \geqq 0$ are given by [cf. ref. [6]), eqs. (6.18) and (6.9)]

$$f^{a_1 a'_1}_{\lambda\mu'}(0, r) = \frac{4\pi^{\frac{3}{2}}}{2^{\mu'}\mu'!} \sqrt{2j_1+1}(-1)^{j'_1+\lambda+\frac{1}{2}} \begin{Bmatrix} l_1 & l'_1 & \lambda \\ j'_1 & j_1 & \frac{1}{2} \end{Bmatrix} \left(\frac{1}{2}\left(\frac{m_B}{m_A} + \frac{m^a}{m_b}\right)\right)^3 \times \int_{y>0} y dy dz \left(\frac{y}{r}\right)^{\mu'} R^{(A)}_{a_1}(r_{1A})(U_{1A}(r_{1A}) - \langle U_{1A}\rangle) R^{(b)}_{a'_1}(r_{1b}) \times \sum_{m_1 m'_1} \langle l_1 m_1 l'_1 m'_1 | \lambda\mu' \rangle Y_{l_1 m_1}(\vartheta_A, 0) Y_{l'_1 m'_1}(\vartheta_b, 0). \tag{B.30}$$

The angles ϑ_A and ϑ_b are defined by

$$\cos\vartheta_A = \frac{z}{\sqrt{y^2+z^2}}, \qquad \sin\vartheta_A = \frac{y}{\sqrt{y^2+z^2}},$$
$$\cos\vartheta_b = \frac{z-r}{\sqrt{y^2+(z-r)^2}}, \qquad \sin\vartheta_b = \frac{y}{\sqrt{y^2+(z-r)^2}}, \tag{B.31}$$

while

$$r_{1A} = \frac{2m_B}{m_A+m_B}\sqrt{y^2+z^2},$$
$$r_{1b} = \frac{2m_a}{m_a+m_b}\sqrt{y^2+(z-r)^2}. \tag{B.32}$$

In (B.30) we included the Jacobian J_ρ in the definition of the form factor. The potential $U_{1A}(r_{1A})$ is the shell model potential in A while $\langle U_{1A}\rangle$ is defined by [compare ref. [6]), eq. (5.17)]

$$\langle U_{1A}\rangle = U^N_{aA}(r_{aA}) - U^N_{bA}(r_{bA}) + U^C_{aA}(r_{aA}) - U^C_{bA}(r_{bA}), \tag{B.33}$$

where U^N and U^C are the nuclear and Coulomb part of the ion-ion interaction. The quantity (B.33) is a function of $r_1 = (y, z)$ since [cf. ref. [5]), eq. (2.35)]

$$r_{aA} = r + \frac{m_d}{2m_{aB}} r_{1\alpha}, \tag{B.34}$$

or

$$r_{aA} \approx \left(1 - \frac{m_d}{2m_a}\right) r + \tfrac{1}{2} m_d \frac{m_B+m_a}{m_B m_a} z$$

and

$$r_{bA} = \frac{m_a+m_B}{m_a+m_A}\left(r + \tfrac{1}{2} m_d \left(\frac{1}{m_B} - \frac{1}{m_a}\right) r_{1\alpha}\right) \tag{B.35}$$

or

$$r_{bA} \approx \left(1 + \frac{m_d}{2m_a}\right) r - \tfrac{1}{2} m_d \frac{m_B-m_a}{m_B m_a} z.$$

The form factors for $\mu' < 0$ are found from the relation

$$f^{a_1 a i}_{\lambda -\mu'}(0, r) = (-1)^{\lambda+\pi} f^{a_1 a' i}_{\lambda\mu'}(0, r), \tag{B.36}$$

which shows that $f^{a_1 a i}_{\lambda\mu'}$ vanishes for $\mu' = 0$ if $\lambda+\pi$ is odd.

Several examples of radial form factors have been evaluated in ref. [6]) (cf. ibid.

fig. 11 and 12) in the post representation where $V_{aA}-U_{aA}$ in (B.16) is substituted by $V_{bB}-U_{bB}$. The form factors for $\mu'=1$ are usually a factor of ten smaller than the ones corresponding to $\mu'=0$. For not too high bombarding energies where the factor

$$\frac{m_d L}{m_{aA}\hbar} \approx \frac{l_g(A_A+A_a)}{A_a A_A} \lesssim \sqrt{\frac{\varepsilon_{MeV}}{20}}(r_0)_{fm} \tag{B.37}$$

is much smaller than 10, we may neglect the terms with $\mu' \neq 0$.

Outside the sum of the nuclear radii ($r > R_a+R_A$) the form factors have an exponential dependence with a range a_{tr} related to the separation energy of a nucleon in a. We may therefore use

$$f^{JJ'}_{\lambda\mu'}(0, r) \approx f^{JJ'}_{\lambda\mu'}(0, r_0)\, e^{-(r-r_0)/a_{tr}}, \tag{B.38}$$

where $a_{tr} \approx 1.2$ fm. With this approximation and the expansion (B.10) of the trajectory we find

$$k_3(t) \approx \ddot{r}_0 \frac{m_d}{\hbar} t, \tag{B.39}$$

$$\gamma_{\beta\alpha}(t) = (\mathscr{L}_\alpha(0)-\mathscr{L}_\beta(0))t$$

$$= \left[\left(\frac{m_d}{m_b}-\frac{m_d}{m_A}\right)\frac{1}{2}\frac{m_b m_A}{(m_a+m_A)}v^2(0)+U_{bB}(r_0)-U_{aA}(r_0)\right]t. \tag{B.40}$$

We may thus evaluate the orbital integral with the result

$$I_{\lambda\mu} = \sum_{\mu'} D^{\lambda}_{\mu\mu'}(0, \tfrac{1}{2}\pi, 0)\sqrt{\frac{2\lambda+1}{4\pi}}\left(\frac{m_d}{m_{aA}}\frac{L}{\hbar}\right)^{\mu'}$$

$$\times \sqrt{\frac{2\pi a_{tr}}{\ddot{r}_0\hbar^2}}\, f^{JJ'}_{\lambda\mu'}(0, r_0)\exp\left[-q^2 a_{tr}/2\ddot{r}_0\hbar^2\right], \tag{B.41}$$

where

$$q = -Q+\mu\hbar\dot{\phi}_0+\left(\frac{Z_d}{Z_A}-\frac{Z_d}{Z_b}\right)E_B$$

$$+\left(\frac{m_d}{m_b}-\frac{m_d}{m_A}\right)(E-E_B)+\frac{m_d\ddot{r}_0}{m_a+m_A}(R_A m_b-R_a m_B) \tag{B.42}$$

with $Q = E_a+E_A-E_b-E_B$, being the Q-value for the reaction, and $Z_d = Z_a-Z_b$ the charge number of the transferred nucleon. In the last expression we have estimated the difference in potential energy by the Coulomb energy only, E_B being the average height of the Coulomb barrier. The quantity E is similarly the average total energy of relative motion in the c.m. system between initial and final state.

The structure of the expression (B.41) is similar to the corresponding expression (B.12) for inelastic scattering and the accuracy has similarly been checked [10]) against the corresponding WKB matrix elements. The characteristic collision time $(a_{tr}/\ddot{r}_0)^{\frac{1}{2}} = \tau_{trans}$ is slightly longer than τ_{inel} i.e.

$$\frac{\hbar}{\tau_{trans}} \approx \sqrt{\frac{40\,\varepsilon_{MeV}}{(r_0)_{fm}}}\,\text{MeV}. \tag{B.43}$$

The effective Q-value, q, contains besides the term $\mu\hbar\dot{\phi}_0$ which also appears in (B.12), terms related to the transfer of mass, m_d, and charge, Z_d. The charge dependence is associated with the fact that the effective Q-value is to be measured at the time of contact where the energies of nuclear states are changed by the magnitude of the Coulomb field at this distance (cf. e.g. ref. [11]) fig. 2). Additional energy is provided for by the kinetic energy of relative motion. The magnitude of the last term in (B.42) depends on the use of the average trajectory $r(t) = \frac{1}{2}(r_\alpha(t)+r_\beta(t))$.

For the pick-up reaction, where a nucleon d is removed from the target A to form B we find

$$c^{NP}_{aA\to bB} = -i\sum_{JJ'\lambda}\langle I_B M_B JM|I_A M_A\rangle\langle I_a M_a J'M'|I_b M_b\rangle\langle\lambda\mu J'M'|JM\rangle I_{\lambda\mu}, \tag{B.44}$$

with

$$I_{\lambda\mu} = (-1)^\pi \sum_{\mu'} D^\lambda_{\mu\mu'}(0, \tfrac{1}{2}\pi, \pi) \times \frac{1}{\hbar}\int_{-\infty}^{\infty} dt \exp\left[i(\Delta E t+\gamma_{\beta\alpha}(t)+\hbar\mu\phi(t))/\hbar\right] f^{J'J}_{\lambda\mu}(k_3(t), k_2(t), r(t)). \tag{B.45}$$

With this definition the form factor is the same as it would be for the stripping reaction with A as projectile and a as target. The phase factor depending on the (even or odd) parity change $\Delta\pi$ arises from the fact that the intrinsic 3-axis is oriented towards a.

Using the approximation (B.26) we find for pick-up

$$(\bar{\sigma})_{NP} = -\bar{\sigma}. \tag{B.46}$$

The expansion (B.29) takes the form

$$f^{J'J}_{\lambda\mu'}(0, r) = \sum_{a_1 a_1'} C^*(I_a a_1'; I_b) C(I_B a_1; I_A) f^{a_1' a_1}_{\lambda\mu'}(0, r)\delta(j_1, J)\delta(j_1', J'), \tag{B.47}$$

while the single-particle form factor is given by

$$f^{a_1' a_1}_{\lambda\mu'}(0, r) = \frac{4\pi^{\frac{3}{2}}}{2^{\mu'}\mu'!}\sqrt{2j_1'+1}(-1)^{j_1+\pi+\frac{1}{2}}\begin{Bmatrix} l_1 & l_1' & \lambda \\ j_1' & j_1 & \frac{1}{2}\end{Bmatrix}\left(\frac{1}{2}\left(\frac{m_b}{m_a}+\frac{m_A}{m_B}\right)\right)^3 \times \int_{y>0} y\,dy\,dz\left(\frac{y}{r}\right)^{\mu'} R^{(a)}_{a_1'}(r_{1a})(U_{1a}(r_{1a})-\langle U_{1a}\rangle)R^{(B)}_{a_1}(r_{1B}) \times \sum_{m_1 m_1'}\langle l_1 m_1 l_1' m_1'|\lambda\mu'\rangle Y_{l_1 m_1}(\vartheta_B, 0) Y_{l_1' m_1'}(\vartheta_a, 0), \tag{B.48}$$

where we used $\pi = l_1 + l'_1$, and where

$$\langle U_{1_a} \rangle = U^{N}_{aA}(r_{aA}) - U^{N}_{aB}(r_{aB}) + U^{c}_{aA}(r_{aA}) - U^{c}_{aB}(r_{aB}). \tag{B.49}$$

It is interesting to compare the amplitude $c^{NP}_{bB\to aA}$ for the pick-up reaction $b+B \to a+A$ with the amplitude $c^{NS}_{aA\to bB}$ for the stripping reaction $a+A \to b+B$. By the substitution of $a \leftrightarrow b$ and $A \leftrightarrow B$ in (B.44)–(B.49) one finds that $\bar{\sigma}$ is unchanged while ΔE, γ and μ will change sign. The single-particle form factors are related by

$$(f^{a'_1 a_1(\alpha)}_{\lambda\mu'}(0,r))^{NP}_{bB\to aA} = (-1)^{\mu'+j_1-j'_1+\lambda}\sqrt{\frac{2j'_1+1}{2j_1+1}}\,(f^{a_1 a'_1(\beta)}_{\lambda\mu'}(0,r))^{NS}_{aA\to bB}. \tag{B.50}$$

By the indices (α) and (β) we specify whether the form factor is given in prior or post representation respectively. Comparing the expressions for the two amplitudes (B.19) and (B.44) we see that

$$\begin{aligned} c^{NS(\alpha)}_{aA\to bB} &= -(c^{NP\ (\beta)}_{bB\to aA})^* \\ &= -(\bar{c}^{NP(\alpha)}_{bB\to aA})^*, \end{aligned} \tag{B.51}$$

where we introduced the adjoint amplitude (cf. eq. (A.7)). At time $t = \infty$ the amplitudes and the adjoint amplitudes are identical and the probabilities for the two transfers are therefore the same

$$P^{NS}_{aA\to bB} = P^{NP}_{bB\to aA}. \tag{B.52}$$

At intermediate times where, the two amplitudes are different, the symmetry is reconstituted by the use of the definition (A.9) of the probability.

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