# ON THE BACK-BENDING IN THE ROTATIONAL MOTION[‡]

R.A. BROGLIA[‡‡]
*Department of Physics, State University of New York at Stony Brook, Stony Brook, N.Y. 11790, USA*

A. MOLINARI, G. POLLAROLO
*Istituto di Fisica Teorica dell'Università di Torino, Torino, Italy*

and

T. REGGE
*The Institute for Advanced Study, Princeton, N.J. 08540, USA*

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The anomalous behavior of the moment of inertia, as a function of the angular momentum, for the ground and first excited rotational bands of $^{154}$Gd is accounted for by the mechanism of bands hybridization.

There exists now experimental evidence [1, 2] that the backbending phenomenon [e.g. 3] in the rotational motion of quadrupole deformed nuclei can be interpreted in terms of bands crossing [4, 5].

Utilizing this idea we have carried out an analysis of $^{64}_{90}\mathrm{Gd}^{154}$ which, to this purpose, is the best known case.

We use the Hamiltonian

$$H = H_0 + H_{\mathrm{coupl}} \tag{1}$$

where

$$H_0 = a(K)(I^2 - I_3^2) + b(K)I_3^2 + H_{\mathrm{intrinsic}}(K) \tag{2}$$

and

$$H_{\mathrm{coupl}} = a_{\mathrm{cor}}(K_i, K_f)(I^+ + I^-) + a_{\mathrm{cf}}(K_i, K_f)(I_1^2 + I_2^2). \tag{3}$$

The coefficients $a_{\mathrm{cor}}(K_i, K_f)$ and $a_{\mathrm{cf}}(K_i, K_f)$ are the strength of the Coriolis and centrifugal couplings among the bands. The labels $K_i$ and $K_f$ specify the $K$-quantum number of the pairs of bands interacting through (3); $I$ is the total angular momentum of the system.

We have found that $H_0$ has to be diagonalized in at least a three-dimensional vector space in order to have a satisfactory reproduction of the experimental data. The best choice of the basic eigen vectors of $H_0$ turned out to be

$$|\chi_g; I, K=0, M\rangle, \qquad |\chi_\beta; I, K=0, M\rangle, \qquad |\chi; I, K=1, M\rangle. \tag{4}$$

The corresponding eigen values are

$$P_{11} = Ax, \qquad P_{22} = Bx + C, \qquad P_{33} = Dx + E,$$

where $x = I(I+1)$ and the notation of ref. [4] has been used.

The first two rotational bands in (4a) are the experimentally observed ground state and $\beta$-vibrational (two-quasiparticle) bands. We will elaborate on the interpretation of the last one ($K=1$) later. Diagonalizing the Hamiltonian (1) in the basis (4) one gets the following cubic secular equation

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‡‡ On leave from Niels Bohr Institute, University of Copenhagen, Denmark.

Table 1

Experimental energies of the rotational levels in ${}^{90}_{64}Gd^{154}$ (columns 2 and 4). Columns 3, 5, 6 give our predicted lower (ground), intermediate ($\beta$) and upper trajectories.

| | Exp. L.T. | L.T. | Exp. I.T. | I.T. | U.T. |
|---|---|---|---|---|---|
| $I$ = 0 | 0.0 | 0.0 | 680.700 | – | 1658.357 |
| 2 | 123.000 | 110.228 | 815.400 | – | 1706.724 |
| 4 | 370.800 | 359.780 | 1047.400 | 1055.477 | 1822.380 |
| 6 | 717.400 | 724.945 | 1365.600 | 1344.981 | 2016.289 |
| 8 | 1144.100 | 1162.913 | 1756.200 | 1752.116 | 2328.403 |
| 10 | 1636.800 | 1639.865 | 2193.800 | 2198.503 | 2870.873 |
| 12 | 2184.500 | 2160.517 | 2621.600 | 2612.180 | 3710.951 |
| 14 | 2777.300 | 2741.776 | 3027.100 | 3035.551 | 4789.316 |
| 16 | 3404.500 | 3394.815 | 3490.600 | 3498.215 | 6065.199 |
| 18 | 4016.200 | 4013.278 | 4087.100 | 4121.828 | 7523.297 |

Table 2

Interband to intraband $B$(E2) ratio $R$(cf. eq. (12)). The approximation $\langle 0_g | M'(2,0) | 0_g\rangle = \langle 0_\beta | M'(2,0) | 0_\beta\rangle = \langle 0_\beta | M'(2,0) | 0_g\rangle = (5/16\pi)^{1/2} Q_0$ has been made in calculating the matrix element $\langle K_f M'(2, K_f - K_i) | K_i\rangle$ of the intrinsic quadrupole operator. The other off-diagonal matrix elements have been neglected.

| | Exp. | Th. |
|---|---|---|
| $I$ = 12 | 0.001 | 0.022 |
| 14 | <0.003 | 0.002 |
| 16 | 0.036 | 0.94 $10^{-4}$ |
| 18 | 1.4 | 2.46 |

$$\tilde{E}^3 + p\tilde{E} + q = 0 \tag{5}$$

where

$$\tilde{E} = E - \tfrac{1}{3}(P_{11} + P_{22} + P_{33}) \tag{6}$$

and

$$p = -\tfrac{1}{3}(P_{11}+P_{22}+P_{33})^2 + P_{11}P_{22} + P_{11}P_{33} + P_{22}P_{33} - a^2_{cf}(0_g, 0_\beta)\, x^2 - a^2_{cor}(0_g, 1)\, x - a^2_{cor}(0_\beta, 1)\, x. \tag{7}$$

$$q = \frac{P_{11}+P_{22}+P_{33}}{3}\left[p + \left(\frac{P_{11}+P_{22}+P_{33}}{3}\right)^2\right] +$$
$$+ P_{11} a^2_{cor}(0_\beta, 1)\, x + P_{22} a^2_{cor}(0_g, 1)\, x + P_{33} a^2_{cf}(0_g, 0_\beta)\, x^2 - 2a_{cf}(0_g, 0_\beta) a_{cor}(0_g, 1) a_{cor}(0_\beta, 1)\, x^2 - P_{11}P_{22}P_{33}. \tag{8}$$

The solutions for the energies are

$$E = \tfrac{1}{3}(P_{11}+P_{22}+P_{33}) + 2\sqrt{-\tfrac{1}{3}p}\, \cos[\tfrac{1}{3}(\theta + 2\pi l)] \tag{9}$$

where $l$ = 0, 1, 2 labels the three trajectories of the energy versus the angular momentum and

$$\cos\theta = \frac{3q}{2p}\sqrt{-\frac{3}{p}}. \tag{10}$$

Agreement with experimental data is obtained for the following values of the parameters

$A = 18.965$, $B = 12.413$, $C = 807.749$, $D = 7.196$, $E = 1658.358$,

$$a_{cf}(0_g, 0_\beta) = 4.338, \quad a_{cor}(0_g, 1) = 27.273, \quad a_{cor}(0_\beta, 1) = 17.675, \tag{11}$$

as it can be seen in table 1 and fig. 1. All magnitudes in (11) are expressed in keV. It is to be noted that the first two states of the $\beta$-band in ${}^{90}_{64}Gd^{154}$ (a nucleus on the border of the deformed rare earth region) have been left out from the present analysis being essentially vibrational in character.

For each value of the angular momentum $I$ the wave functions of the three trajectories are linear combinations of the states given in (4). With these wave functions we have calculated the $B$(E2) transition probabilities both for

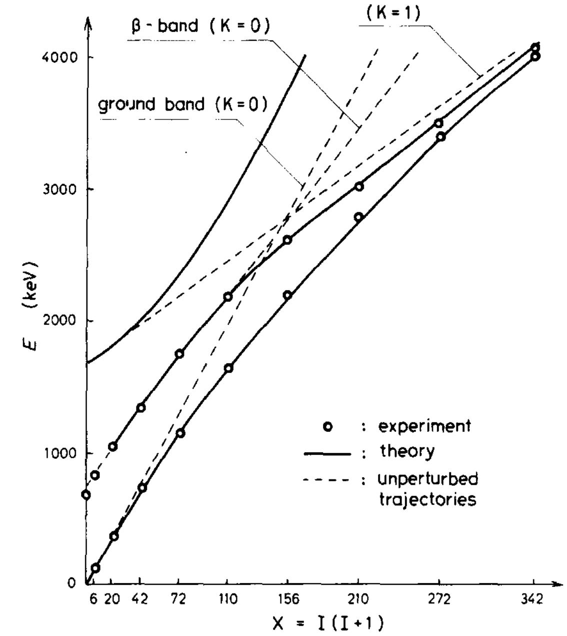


Fig. 1. Energy trajectories corresponding to the Hamiltonian (1) for $^{90}_{64}Gd^{154}$ in comparison with the experimental data. Also showed (dashed lines) the uncoupled bands.

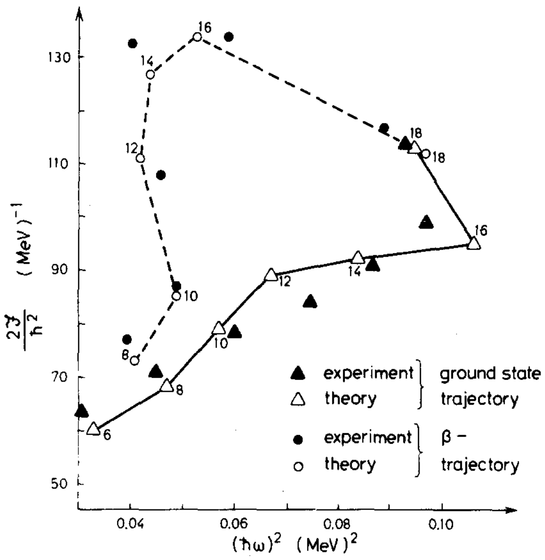


Fig. 2. The same as fig. 1 but in terms of different variables, i.e. $2\mathcal{J}/\hbar^2 = (4I-2)/[E(I)-E(I-2)]$ as a function of $(\hbar\omega)^2 = [(E(I)-E(I-2))/2]^2$. The continuous and dashed lines drawn through the predicted values are given as a help to follow the behaviour of the ground- and β-trajectories.

the interband as well as the intraband case. In table 2 we compare our results for the ratio

$$R = \frac{B(\mathrm{E2},(I)_\beta \to (I-2)_g)}{B(\mathrm{E2},(I)_\beta \to (I-2)_\beta)} \tag{12}$$

with the corresponding experimental values.

The analytic properties of our trajectories (9) in the complex plane of the square of the angular momentum are found by solving the algebraic equation

$$(\tfrac{1}{2}q)^2 + (\tfrac{1}{3}p)^3 = 0 \tag{13}$$

i.e. setting equal to zero the discriminant of (6). Since $p$ and $q$ are respectively of second and third order in $x$ we will have six roots two by two complex conjugate. The physically relevant ones are

$$x_1 = x_2^* = 84.98 + \mathrm{i}\, 49.44, \qquad x_3 = x_4^* = 306.1 + \mathrm{i}\, 5.17. \tag{14', 14''}$$

They correspond to the back-bending of the β- and ground-trajectory respectively. The real part of the branch cuts (14) gives information on the value of the angular momentum at which a substantial change of the moment of inertia occurs. The imaginary part on the other hand is related to the rate of change of the moment of inertia: to a sudden change corresponds a small imaginary part and conversely. This is best illustrated in $^{90}_{64}Gd^{154}$ (cf. fig. 2) where the change in the moment of inertia occurs gradually in the β-trajectory, and suddenly in the ground-trajectory‡.

‡ We point out that the existence of branch points singularities in the complex $x$-plane limits the validity of the usual expansion of $E$ as a power series in $x$.

Note that far from the branch points the ratio $R$ is exceedingly small while it becomes of order of unity near the branches (cf. fig. 1 and table 2).

The third band required by the experiments has $K = 1$, starts around 1.5 MeV of excitation energy and displays the moment of inertia of a rigid rotor, namely $J_{\mathrm{rig}} = \frac{2}{5}AMR^2(1+\delta/3)$ with the value $\delta = 0.3$ (cf. (11)).

It can thus be viewed as an effective band representing the average effect of all the $K^\pi = 1^+$ bands built on the two-quasiparticles states generated by the $j_+$ operator acting on the ground state (note that a particular combination of these states gives rise to the $K^\pi = 1^+$ spurious state). Among these excitations, the one based on the [642] 5/2 and [651] 3/2 Nilsson orbitals and with excitation energy $\sim 2$ MeV according to this model [6], is expected to play an important role [5] as it is strongly decoupled from the rotational motion. This intrinsic state will generate a band with a large moment of inertia. In fact, the ground state $K^\pi = 5/2^+$ band of $^{161}$Dy – which is based on the [642] 5/2 one-quasiparticle state – has a moment of inertia which is close to $J_{\mathrm{rig}}$.

The physical basis of the above interpretation of our third $K = 1$ band is quite simple. For high angular momenta the rotational energy becomes comparable to the energy of the intrinsic excitations and thus the adiabatic condition upon which the collective description is based is not valid any more. For these angular momenta a major change in the coupling scheme takes place, namely a phase transition.

The fact that no fitting was possible with three $K = 0$ bands seems to rule out the possibility of a superfluid to normal phase transition being the cause of the anomalous behaviour of the moment of inertia of the ground- and $\beta$-bands. Actually, even if the data could have been explained in terms of three $K = 0$ bands, the model based upon the analogy between the collective angular momentum acting on the nuclear condensate and the magnetic field acting on a superconductor [7], cannot account for the decrease of the moment of inertia that takes place in the $\beta$-trajectory after the backbending. On the other hand this, together with the occurrence of the backbending at relatively low values of the angular momentum ($\beta$-trajectory), is a natural result of the geometry of the many-band hybridization presented here and in ref. [4].

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## References


[1] D. Ward, R.L. Graham, F.S. Geiger and H.R. Andrews, Phys. Lett. 44B (1973) 39.
[2] T.L. Khoo, F.M. Bernthal, J.S. Boyno and R.A. Warner, Phys. Rev. Lett. 31 (1973) 1146.
[3] A. Johnson and Z. Szymanski, Phys. Rep. 7C (1973) 183;
R.A. Sorensen, Rev. Mod. Phys. 45 (1973) 353, and references therein.
[4] A. Molinari and T. Regge, Phys. Lett. 41B (1972) 257.
[5] F.S. Stephens and R.S. Simon, Nucl. Phys. A183 (1972) 257.
[6] S.G. Nilsson, Dan. Mat. Fys. Medd. 29 no. 16 (1955).
[7] B.R. Mo ttelson and J.G. Valatin, Phys. Rev. Lett. 5 (1960) 511.