BAND HYBRIDIZATION IN ROTATIONAL MOTION[☆]

R.A. BROGLIA[*]
State University of New York, Stony Brook, New York 11790, USA
and *Brookhaven National Laboratory, Upton, New York 11973, USA*

A. MOLINARI and G. POLLAROLO
Istituto di Fisica Teorica dell'Universita di Torino, Torino, Italy
and *I.N.F.N. Sezione di Torino, Torino, Italy*

and

T. REGGE
The Institute for Advanced Study, Princeton, New Jersey 08540, USA

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The back-bending behaviour of the moment of inertia as a function of the square of the angular velocity, for the ground-, and β-rotational trajectories in ${}^{156}_{66}\mathrm{Dy}_{90}$, is described in terms of band hybridization.

The back-bending observed in the behaviour of the moment of inertia of quadrupole deformed nuclei as a function of the square of the angular velocity ω can be interpreted in terms of band hybridization [1, 2]. This requires the introduction of empirically determined [2] excited bands with large moment of inertia and $K \neq 0$. Strong support for the validity of the model would be provided by the experimental finding of at least some of these bands and by the compatibility between the empirically determined parameters characterizing the bands (and their couplings) with a proper microscopic description [3].

To further pursue this program we have analyzed ${}^{156}_{66}\mathrm{Dy}_{90}$. For this purpose we have diagonalized the Hamiltonian

$$H = H_0 + H_{\mathrm{coupl}} \tag{1}$$

with

$$H_0 = A(K)I^2 + H_{\mathrm{intrinsic}}(K) \tag{2}$$

and

$$H_{\mathrm{coupl}} = h_0(I_1^2 + I_2^2) + h_1 I_+ + \mathrm{h.c.} + h_2 I_+^2 + \mathrm{h.c.} \tag{3}$$

in different truncated vector spaces spanned by sets of eigenvectors of H_0 with energies $E = A(K)x + B(K)$ where $x = I(I+1)$ and $B(K)$ is the eigenvalue of the intrinsic Hamiltonian. The terms proportional to h_0, h_1 and h_2 represent the centrifugal, Coriolis and asymmetry interactions (cf. ref. [3]). The intrinsic part of the matrix elements of H_{coupl} as well as $A(K)$ and $B(K)$ are treated as free parameters to be determined by a least square search on the experimental data of ref. [4]. The results are reported in tables 1 and 2 and figs. 1 and 2. In the framework of our model the experimental levels as grouped in ref. [4], figs. 2 and 3a, cannot be reproduced. Our results refer instead to a grouping of the data such that the levels of the ground- and β-trajectories have been interchanged from $I = 16$ on (i.e., the choice (b) of fig. 3 in ref. [4] is the correct one for us). No acceptable fit is possible utilizing only $K = 0$ bands, thus making unlikely a pairing phase transition [5] (see also e.g., ref. [6]). The $K^\pi = 1^+$ band with a moment of inertia close to but smaller than the rigid value ($\hbar^2/2\mathcal{I}_{\mathrm{rig}} = 7.3$ keV for $\delta = 0.28$) and starting at about 1.6 MeV is required in all cases (cf. plots (a) and (c) of fig. 1).

In the low angular momentum region the data cannot be reproduced without the experimentally known

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* On leave from the Niels Bohr Institute, University of Copenhagen, Copenhagen, Denmark.

Table 1
Results of the best fit search for the experimental data of ref. [3]. Each band is characterized by the usual energy-angular momentum relationship $E(K) = A(K)x + B(K)$. In the first column we give the K quantum number, in the second and third columns the values of $A(K)$ and $B(K)$ in keV and in the last column the χ^2 values.

		K	$A(K)$	$B(K)$	χ^2
	a				
		0	23.2	0	
		0	11.5	967.1	18527
		0	30.7	4915.4	
I					
	a				
		0	20.5	0	
		0	11.7	909.0	7436
		1	7.3	1634.7	
	a				
		0	24.0	0	
		0	22.3	676.4	
		0	26.9	2971.8	5934
		0	80.5	12872.7	
II					
	a				
		0	21.8	0	
		0	14.7	795.0	
		1	7.9	1698.5	1016
		2	8.7	1611.5	
	b				
		0	28.7	0	
III		0	28.9	685.5	
		2	18.7	794.0	239
		1	8.8	1658.8	
	a				
		0	21.7	0	
		0	14.6	802.1	
IV		1	7.9	1712.8	1006
		2	8.7	1623.0	
		3	9.7	1780.0	
	c				
		0	21.4	0	
		0	14.1	842.7	
V		1	8.3	1699.4	116
		2	10.4	2219.9	
		3	9.6	2101.8	

a) Fit carried out in the range $0 \leq I \leq I_{max}$ ($I_{max}(gs) = 22$, $I_{max}(\beta) = 20$).
b) Fit carried out in the range $0 \leq I \leq 9$.
c) Fit carried out in the range $6 \leq I \leq I_{max}$.

γ-vibrational ($K^\pi = 2^+$) trajectory which however does not play an important role for higher angular momenta, close to the yrast trajectory. Because of computational limitations no fit with 6 bands was attempted, which would have allowed for a unified description of both regions of spin. However, the $K^\pi = 1^+$ band required by fits III and V (best fits for the low and high spin regions respectively) is essentially the same.

A sequence of rotational levels has recently been observed [7] whose energies agree with the corresponding ones of our $K^\pi = 1^+$ trajectory within 15% (cf. table 2). Utilizing the parameters of the best fit to the even angular momenta (i.e., fit V of table 1) we have calculated the three odd spin values trajectories of our model. They are reported in fig. 1c. For $I \geqslant 16$, i.e., around the backbending region, the lower even and odd trajectories are very close in energy and their spins alternate, implying a loss of axial symmetry of the nucleus.

Because of the failure of the fits with only $K = 0$ bands the large moment of inertia empirically found for our high lying bands is to be adscribed to the decoupling of pairs of particles from the rotating core. In fact, in this mass region, there are Nilsson orbitals with large j (= 13/2) approximately constant of motion, and Ω small. This implies particularly large Coriolis matrix elements. Assuming, according to ref. [1], that the configuration of the intrinsic states is $|gs\,(K^\pi = 0^+)\rangle = (\mathcal{A}/(2)^{1/2})\{[651]\,3/2^+, [651]\,3/2^+\} f(U, V)$, $|K^\pi = 1^+\rangle = (\mathcal{A}/(2)^{1/2})\{[651]\,3/2^+, [642]\,5/2^+\}$, $|K^\pi = 2^+\rangle = (\mathcal{A}/(2)^{1/2})\{[660]\,1/2^+, [642]\,5/2^+\}$ and $|K^\pi = 3^+\rangle = (\mathcal{A}/(2)^{1/2})\{[660]\,1/2^+, [642]\,5/2^+\}$ ($\mathcal{A}$ being the antisymmetrizing operator) one gets for the Coriolis matrix elements $\langle K_f | h_1 | K_i \rangle$ the values 61 keV ($K_i = 0, K_f = 1$), -34 keV ($K_i = 1, K_f = 2$) and 41 keV ($K_i = 2, K_f = 3$). The occupation probability $f(U, V)$ was chosen to be 0.5 (see ref. [1]).

The calculated matrix elements roughly agree with the empirical values displayed in the caption to table 2.

If the two quasiparticle interpretation of the $K^\pi = 1^+$ intrinsic state is valid then $E_{1^+} = ((\epsilon_{3/2} - \lambda)^2 + \Delta^2)^{1/2} + ((\epsilon_{5/2} - \lambda)^2 + \Delta^2)^{1/2} \approx 2((\epsilon_{3/2} - \lambda)^2 + \Delta^2)^{1/2}$ and, using the experimental single-particle levels one gets $\Delta = 730$ keV. The energy of the two quasiparticle states $K^\pi = 2^+$ and $K^\pi = 3^+$ is then $E = ((\epsilon_{1/2} - \lambda)^2 + \Delta^2)^{1/2} + ((\epsilon_{5/2} - \lambda)^2 + \Delta^2)^{1/2}$

Table 2

Experimental energies in keV of the rotational trajectories in ^{156}Dy. Also given are the predicted energies. The figures inside the frame correspond to fit III (cf. table 1). The rest to fit V. The intrinsic matrix elements corresponding to fit V are $\langle\beta|h_0|\mathrm{gs}\rangle = 4.9$, $\langle 1|h_1|\mathrm{gs}\rangle = 51.2$, $\langle 1|h_1|\beta\rangle = 17.4$, $\langle 2|h_1|1\rangle = -25.4$, $\langle 3|h_1|2\rangle = 73.8$, $\langle 2|h_2|\mathrm{gs}\rangle = -3.3$, $\langle 2|h_2|\beta\rangle = -1.7$ and $\langle 3|h_2|1\rangle = -0.3$. The intrinsic matrix elements corresponding to fit III are $\langle\beta|h_0|\mathrm{gs}\rangle = 13.5$, $\langle 1|h_1|\mathrm{gs}\rangle = 95.4$, $\langle 1|h_1|\beta\rangle = 74.8$, $\langle 2|h_1|1\rangle = -29.5$, $\langle\gamma|h_2|\mathrm{gs}\rangle = -6.3$, $\langle\gamma|h_2|\beta\rangle = -5.5$. All these matrix elements are expressed in keV.

	ground		beta		gamma		$K^\pi = 1^+$	
I	theory	exp.	theory	exp.	theory	exp.	theory	exp.
0	0.0	0.0	685.4	675.4				
2	132.9	137.8	819.7	828.7	904.5	890.8		
3					1007.4	1011.8		
4	403.8	404.1	1081.9	1088.6	1171.7	1168.8		
5					1315.9	1335.5		
6	762.8	770.3	1439.8	1437.4	1529.0	1525.8	2018.3	
7					1731.2	1728.9	2054.9	1810[a]
8	1219.5	1215.5	1357.5	1858.9	1975.7	1967.6	2180.1	1899
9					2187.4	2191.5	2258.6	2187
10	1726.3	1724.9	2317.2	2315.8			2401.9	2261
11							2529.6	2637
12	2282.2	2286.0	2695.8	2707.1			2798.9	2701
14	2885.3	2887.8	3066.6	3066.0				
16	3503.4	3498.8	3529.9	3523.5				
18	4034.4	4026.2	4176.1	4178.9				
20	4634.6	4636.1	4860.5	4859.8				
22	5311.3	5320.8						

[a] For the ambiguity concerning the quantum numbers of these levels see ref. [7]

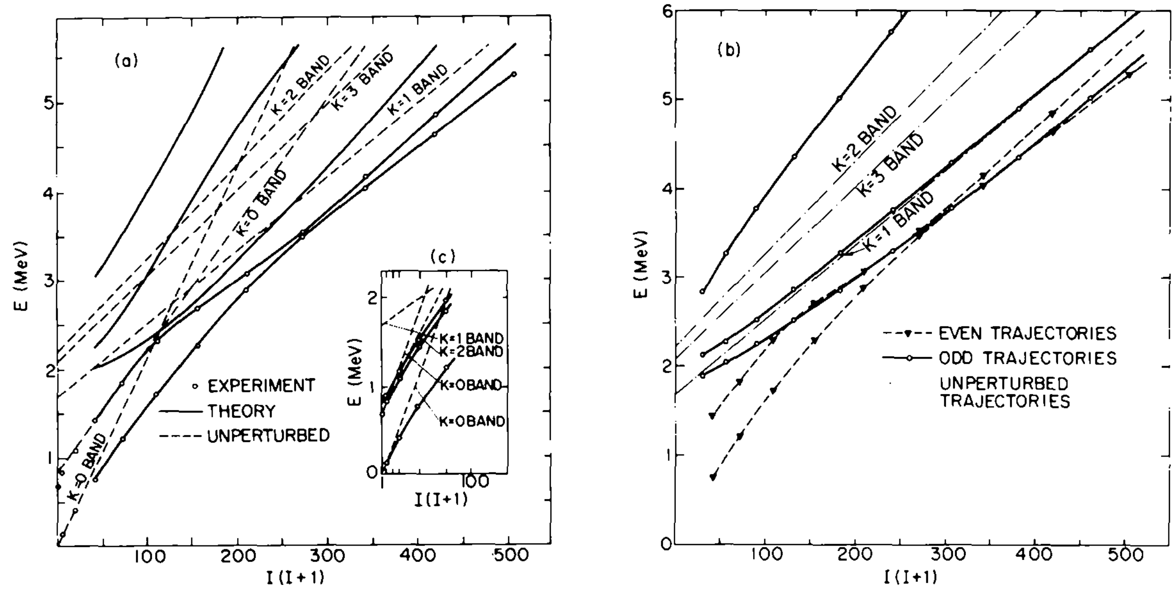

Fig. 1. (a) Energy trajectories resulting from the diagonalization of the Hamiltonian (1) (with the parameters V given in the caption to table 2) in the basis V of table 1. Only even values of spin were considered in the range $6 \leq I \leq I_{\max}$ ($I_{\max}(\mathrm{gs}) = 22$ and $I_{\max}(\beta) = 20$). (b) Same as (a) but for the odd values of spin. (c) Energy trajectories resulting from the diagonalization of the Hamiltonian (1) (with the parameters III given in the caption to table 2) in the basis III of table 1. Only even values of spin are displayed in the range $0 \leq I \leq 8$. In all cases the experimental [4] data utilized to determine the various parameters are plotted.

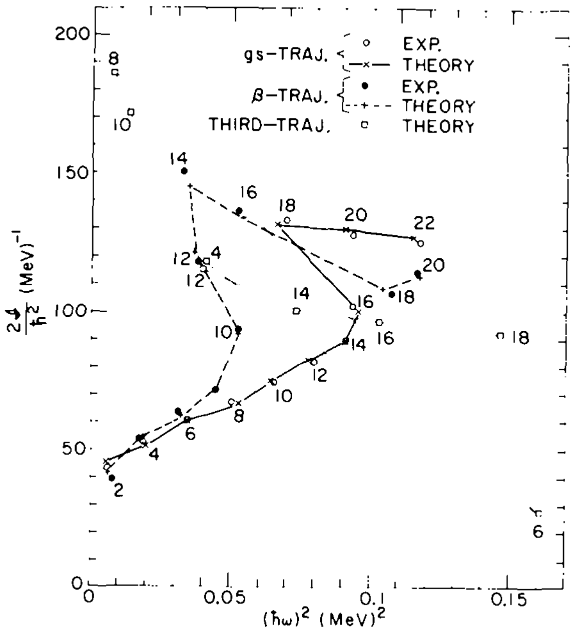

Fig. 2. Moment of inertia $2\mathcal{J}/\hbar^2 = (4I-2)/[E(I)-E(I-2)]$ as a function of $(\hbar\omega)^2 = [E(I)-E(I-2)]^2(I^2-I+1)/(2I-1)^2$ for the ground-, β-, and $K^\pi = 1^+$ trajectories.

= 2120 keV to be compared with the empirical value 2217 (or 2282) keV.

The accurate reproduction of the experimental data allowed by the band hybridization with intrinsic quantum numbers $K \neq 0$, strongly suggests that quadrupole deformed nuclei are superconductors of type II.

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