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A SIMPLE PARAMETRIZATION OF ONE-PARTICLE TRANSFER FORM FACTORS FOR HEAVY-ION REACTIONS

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Abstract: A simple parametrization of the form factors needed to describe low-recoil one-particle transfer reactions in heavy-ion collisions is presented The parameters entering into the calculations can be directly read from figures

As an application we have used these form factors to calculate the imaginary part of the ion-ion potential due to transfer

1. Introduction

The form factors describing inelastic scattering and one-particle transfer processes are the building blocks of most descriptions of heavy-ion reactions. It is therefore convenient to have access to simple parametrizations of these quantities, at least in the surface region which is important for grazing reactions.

A compact expression, based on the collective model, already exists for the inelastic scattering form factors [cf. ref.[1]) and references therein]. In the present paper we work out a simple yet accurate parametrization of single-particle transfer form factors in the low-recoil limit based on the approximation introduced by Buttle and Goldfarb[2]). In sect. 2 the relevant expressions for the single-particle form factors are presented. It is also shown how these quantities are used in first-order perturbation theory. In sect. 3 the elements needed in the evaluation of the transfer form factors are displayed in graphical form and a few examples are worked out. The range of validity of the parametrization is also discussed. In sect. 4 the form factors are used in the calculation of the imaginary part of the optical potential due to transfer processes for a number of reactions The conclusions are presented in sect. 5.

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2. Single-particle form factors

2.1 FIRST-ORDER AMPLITUDES

The single-particle form factors enter into the calculation of both the quantal and the semiclassical transition amplitudes. For simplicity we show this explicitly for the case of first-order perturbation theory and for the case of the stripping reaction

$$a(b+1)+A \rightarrow b+B(A+1). \tag{1}$$

The formalism can also be used, with simple modifications, to describe pick-up processes.

The differential cross section for one-particle transfer reactions is

$$\left(\frac{d\sigma}{d\Omega}\right)_\beta = \frac{k_\alpha}{k_\beta}\frac{m_{aA}m_{bB}}{(2\pi\hbar^2)^2}\left[(2I_A+1)(2I_a+1)\right]^{-1}\sum_{\substack{M_A M_a\\ M_B M_b}}|T_{\beta\alpha}|^2, \tag{2}$$

where m_{aA} and m_{bB} are the reduced masses in entrance and exit channels, respectively, k_α and k_β being the corresponding wave numbers of relative motion. The T-matrix can be written with obvious notation [cf. e g ref [3])] as

$$T_{\beta\alpha} = \frac{4\pi}{k_\alpha k_\beta}\sum_{\substack{\lambda JJ'\\ \mu MM'}}\langle I_A M_A JM|I_B M_B\rangle \times\langle I_b M_b J'M'|I_a M_a\rangle\langle\lambda\mu JM|J'M'\rangle t_{\beta\alpha}(\lambda JJ'\mu), \tag{3}$$

in terms of the spherical tensors $t_{\beta\alpha}$ Performing a partial-wave expansion one finds in the distorted-wave Born approximation (DWBA)

$$t_{\beta\alpha}(\lambda JJ'\mu) = 4\pi\sum_{\substack{l_\alpha l_\beta\\ m_\alpha m_\beta}} i^{l_\alpha - l_\beta}\exp\left[i\left(\beta_{l_\alpha}+\beta_{l_\beta}\right)\right] \times Y_{l_\beta m_\beta}(\hat{\boldsymbol{k}}_\beta)Y^*_{l_\alpha m_\alpha}(\hat{\boldsymbol{k}}_\alpha)\langle l_\alpha m_\alpha\lambda\mu|l_\beta m_\beta\rangle I_{\beta\alpha}(\lambda JJ'). \tag{4}$$

The quantity $I_{\beta\alpha}$ is given by

$$I_{\beta\alpha}(\lambda JJ') = (2l_\beta+1)^{-1/2}\int dr\,\chi_{l_\beta}(r)\langle l_\beta\|f_\lambda^{JJ'}\|l_\alpha\rangle_r\chi_{l\alpha}(r), \tag{5}$$

where the functions χ_l are the radial wave functions.

The reduced matrix element appearing in eq. (5) has been worked out in ref. [3]). In the following we use for the variable r the average of the relative coordinates in entrance (α) and exit channel (β), i.e.

$$\boldsymbol{r} = \tfrac{1}{2}(\boldsymbol{r}_\alpha + \boldsymbol{r}_\beta). \tag{6}$$

For bombarding energies below about 10 MeV/nucleon one may use the approximate expression

$$\langle l_\beta\|f_\lambda^{JJ'}\|l_\alpha\rangle = \sqrt{2l_\alpha+1}\,\langle l_\alpha 0\lambda 0|l_\beta 0\rangle\tilde{f}_{\lambda 0}^{JJ'}(K_\parallel K_\perp r). \tag{7}$$

In the "prior" representation one finds

$$\tilde{f}^{JJ'}_{\lambda 0}(K_{\parallel}K_{\perp}r) = \mathrm{e}^{\mathrm{i}\bar{\sigma}(K_{\parallel})}\sqrt{\frac{2\lambda+1}{4\pi}}\,\delta(J,a_1)\delta(J,a_1') \times C(I_A a_1; I_B)C(I_b a_1'; I_a) f^{a_1 a_1'(\alpha)}_{\lambda}(r) \tag{8}$$

with[4])

$$f^{a_1 a_1'(\alpha)}_{\lambda}(r) = J4\pi^{3/2}\sqrt{2J_1+1}\,(-1)^{J_1'+\lambda+1/2} \times \begin{Bmatrix} l_1 & l_1' & \lambda \\ J_1' & J_1 & \frac{1}{2} \end{Bmatrix} \int_{y>0} y\,\mathrm{d}y\,\mathrm{d}z\,R^{(A)}_{a_1}(r_{1A})R^{(b)}_{a_1'}(r_{1b})\big(U_{1A}(r_{1A}) - \langle U_{1A}\rangle\big) \times \big[Y_{l_1}(\vartheta_A,0)Y_{l_2}(\vartheta_b,0)\big]_{\lambda 0}, \tag{9}$$

where

$$J = \big(4m_a m_b/(m_a+m_b)(m_A+m_B)\big)^3 = \big(m_a m_b/((m_a+m_A)m_0)\big)^3.$$

In (8) we have neglected the transverse recoil effect associated with $K_{\perp}$ and have extracted the main part of the longitudinal recoil effect associated with $K_{\parallel}$ through the phase[4])

$$\bar{\sigma}(K_{\parallel}) = K_{\parallel}\left(R_A - \frac{m_A+m_B}{2(m_a+m_A)}r\right). \tag{10}$$

The quantity $\hbar K_{\parallel} = \frac{1}{2}(m_d/m_0)(p_\alpha^{\parallel} + p_\beta^{\parallel})$ is the average value of the linear momentum of the transferred particle in entrance and exit channel (cf. eq. (16) below). The exponential factor in (8) may be incorporated[3]) in the argument of the distorted waves, by separating the operator $\exp(\mathrm{i}\bar{\sigma})$ into the product of two shift operators, leading to

$$I_{\beta\alpha} = (2l_\beta+1)^{-1/2} \times \int \chi_{l_\beta}\left(\frac{2m_a}{m_a+m_b}r - \frac{m_d}{2m_0}R_A\right)\langle l_\beta \| f^{JJ'} \| l_\alpha\rangle \times \chi_{l_\alpha}\left(\frac{2m_b}{m_a+m_b}r + \frac{m_d}{2m_0}R_A\right), \tag{11}$$

where the reduced matrix element is again given by eqs. (7) and (8) leaving out the phase factor $\exp(\mathrm{i}\bar{\sigma})$.

The indices $a_1 \equiv (n_1 l_1 J_1)$ and $a_1' \equiv (n_1' l_1' J_1')$ label the quantum numbers of the orbitals in which the transferred particle moves in target and projectile, respectively, U_{1A} being the corresponding shell-model potential associated with the target nucleus

A. The quantity $\langle U_{1A}\rangle$ is given by

$$\langle U_{1A}\rangle = U_{aA}^{N}(r) - U_{bA}^{N}(r_{bA}) + U_{aA}^{C}(r) - U_{bA}^{C}(r_{bA}), \tag{12}$$

where U^{N} and U^{C} are the nuclear and Coulomb part of the ion-ion potential.

The spectroscopic amplitudes appearing in (8) are the reduced matrix elements of the creation operator $a^{+}_{j_1 m_1}(a_1)$ of a nucleon in the orbital $a_1 m_1$, i.e.

$$C(I_A 0_1; I_B) = \frac{\langle I_B \| a^{+}_{j_1}(a_1) \| I_A\rangle}{\sqrt{2I_B+1}} \tag{13}$$

In the semiclassical description of heavy-ion reactions the first-order amplitude describing a transfer process of the type (1) is given by [cf. ref. [4])]

$$a^{NS}_{\alpha\to\beta}(t=+\infty) = -i \sum_{\substack{JJ'\lambda \\ MM'\mu}} \langle I_A M_A JM | I_B M_B\rangle \times \langle I_b M_b J'M' | I_a M_a\rangle \langle \lambda\mu JM | J'M'\rangle I_{\lambda\mu}, \tag{14}$$

where

$$I_{\lambda\mu} = D^{\lambda}_{\mu 0}(0, \tfrac{1}{2}\pi, \pi)\frac{1}{\hbar}\int_{-\infty}^{\infty} \mathrm{d}t \exp\{ i(\Delta E_{\beta\alpha} t + \gamma_{\beta\alpha}(t) + \mu\hbar\phi(t))/\hbar \} \times \tilde{f}^{JJ'}_{\lambda 0}(k_{\parallel}(t), k_{\perp}(t), r(t)) \tag{15}$$

The function $\tilde{f}^{JJ'}_{\lambda\mu'}$ in the low-recoil approximation is given by (8) The quantities $k_{\parallel}$ and $k_{\perp}$ are the components of the local momentum

$$\boldsymbol{k} = \frac{m_d}{\hbar}\dot{\boldsymbol{r}}(t), \tag{16}$$

where m_d is the mass of the transferred nucleon. The average position (cf eq. (6)) and velocity $\boldsymbol{r}$ of relative motion are both functions of time. The coordinate system used in working out (14) has the z-axis perpendicular to the plane of the trajectory, pointing in the direction of the angular momentum of relative motion The x-axis is directed towards the projectile at the point of closest approach and the D-function, including the factor $\exp(i\mu\phi(t))$ in (15) thus describes the rotation from this system to the intrinsic system with z-axis along $\boldsymbol{r}$

The formulae written above are also valid for pick-up reactions making the substitutions a ↔ A and b ↔ B and multiplying the resulting expressions by $(-1)^{\lambda+\mu}$.

One may prove [cf. ref [1])] that in the short-wavelength limit there exists a simple relation between the quantities (11) and (15), namely

$$I_{\beta\alpha}(\lambda JJ') \approx \tfrac{1}{4}\hbar v(-1)^{\mu} I_{\lambda\mu}, \tag{17}$$

with $\mu = l_\beta - l_\alpha$ and $v = \dot{r}(\infty)$.

The nuclear structure information appearing in (3) and (14) is contained in the spectroscopic amplitudes C and in the form factor (9). In what follows we present a

simple parametrization of this last quantity, which may, as we have seen, be used for either quantal or classical calculations

2 2 SCHEMATIC PARAMETRIZATION

In grazing collision the transfer process takes place at distances larger than the sum of the radii. For neutron transfer where the potential $\langle U_{1A} \rangle$ is small with respect to $U_{1A}(r_{1A})$, only the tail of the wave function $R_{a_1}(r_{1b})$ is then important. The radial wave function $R^{(b)}$ can thus be approximated by

$$R^{(b)}_{a_1'}(r_{1b}) = \bar{N}_{a_1'} \frac{k_{l_1'}(\kappa_{a_1'} r_{1b})}{k_{l_1'}(\bar{\kappa}_{a_1'} R_b)}, \tag{18}$$

where we use $R_b = 1.25A^{1/3}$ fm and

$$\kappa_{a_1'} = \sqrt{\frac{2m_{1b}}{\hbar^2} B_{a_1'}}, \tag{19}$$

and where k_l is a spherical Hankel function of order l.

We have also introduced the quantity

$$\bar{\kappa}_{a_1'} = \frac{2m_a}{m_a + m_b} \kappa_{a_1'} \approx \sqrt{\frac{2M}{\hbar^2} B_{a_1'}}, \tag{20}$$

where M is the nucleon mass.

Through the definition (18) it turns out that the quantity $\bar{N}_{a_1'}$ which is a measure of the value of the radial wave function at the radius R_b is essentially independent of the binding energy $B_{a_1'}$

Making use of the approximation (18) one obtains[2])

$$\begin{aligned} f_\lambda^{a_1 a_1'}(r) = & \left(\frac{2m_a}{m_a + m_b} \right)^3 \sqrt{4\pi(2J_1 + 1)}\, \delta(l_1 + l_1' + \lambda, \text{even}) \\ & \times (-1)^{J_1 - 1/2 + l_1'} \langle J \tfrac{1}{2} J_1' - \tfrac{1}{2} | \lambda 0 \rangle \bar{N}_{a_1} \frac{k_\lambda(\bar{\kappa}_{a_1'} r)}{k_{l_1'}(\bar{\kappa}_{a_1'} R_b)} \\ & \times G(\kappa_{a_1}, \kappa_{a_1'}) \end{aligned} \tag{21}$$

where

$$\begin{aligned} G(\kappa_{a_1}, \kappa_{a_1'}) = & \int r_{1A}^2 \, \mathrm{d}r_{1A} \, R^{(A)}_{a_1}(r_{1A}) U_{1A}(r_{1A}) \\ & \times \imath_{l_1}\left(\frac{m_A + m_B}{2m_B} \bar{\kappa}_{a_1'} r_{1A} \right), \end{aligned} \tag{22}$$

and where $\imath_l$ is the spherical Bessel function of imaginary argument. If the binding

energies of the neutron in the two nuclei fulfil the relation

$$\bar{\kappa}_{a_1} = \frac{2m_B}{m_A + m_B}\kappa_{a_1} = \sqrt{\frac{2M}{\hbar^2}B_{a_1}} = \bar{\kappa}_{a_1'}, \tag{23}$$

one may evaluate this integral explicitly and find [2])

$$f_\lambda^{a_1 a_1'}(r) = J\pi^{3/2}\sqrt{2j_1+1}\,\delta(l_1 + l_1' + \lambda, \text{even})$$
$$\times(-1)^{j_1+1/2+l_1'}\langle j_1 \tfrac{1}{2} j_1' - \tfrac{1}{2}|\lambda 0\rangle \frac{\hbar^2}{2M\bar{\kappa}}$$
$$\times \frac{\bar{N}_{a_1}\bar{N}_{a_1'}}{k_{l_1'}(\bar{\kappa}R_b)k_{l_1}(\bar{\kappa}R_A)} k_\lambda(\bar{\kappa}r), \tag{24}$$

with

$$J = \left(\frac{4m_a m_B}{(m_a + m_b)(m_A + m_B)}\right)^3.$$

In the calculation of $\bar{N}_{a_1}$ use was made of a similar asymptotic relation (18) for the wave function $R_{a_1}^{(A)}(r)$ describing the motion of the particle around the core A

Although the expression (24) was derived under the assumption (23) it turns out to be rather accurate also when the value of the wave numbers are different. In fact, numerical investigations (cf sect. 3) show that the integral (22) is rather stable with respect to changes in the binding energy of the single-particle states a_1 when this change is produced by changing the depth of the potential.

The expressions (21) and (24) can also be used for the transfer of a proton provided that the effective value of the wave number is used [2]), i.e.

$$\bar{\kappa}_{a_1'}^{\text{eff}} = \sqrt{\frac{2M}{\hbar^2}\left(B_{a_1'} + Z_b e^2/r_B(b)\right)} \tag{25}$$

The quantity $B_{a_1'}$ is the binding energy of the transferred proton in the projectile. For $r_B(i)$ we use

$$r_B(i) = 1.07\left(A_i^{1/3} + 1\right) + 2\ 72\ \text{fm} \tag{26}$$

It is noted that the form factors (21) and (24) have been calculated in the prior representation (cf. eq (9)) The post representation is obtained by making the replacements $\bar{\kappa}_{a_1'} \to \bar{\kappa}_{a_1}$ for neutrons and $\bar{\kappa}_{a_1'}^{\text{eff}} \to \bar{\kappa}_{a_1}^{\text{eff}}$ for protons, respectively.

To improve the radial dependence of the function (24) at distances close to the grazing distance one can make the substitution

$$k_\lambda(\bar{\kappa}r) \to \frac{k_\lambda(\bar{\kappa}(R_b + R_A))}{1 + k_\lambda(\bar{\kappa}(R_b + R_A))/k_\lambda(\bar{\kappa}r)}. \tag{27}$$

It is noted that for $\kappa R > \lambda(\lambda+1)$ so that k_λ can be approximated by $k_\lambda(x) = (\pi/2x)e^{-x}$, the form factor defined by eqs. (24) and (27) can be written as

$$f_\lambda^{a_1 a_1'}(r) \approx S\sqrt{2J_1+1}\,\delta(l_1+l_1'+\lambda, \text{even})J \times(-1)^{J_1+1/2+l_1'}\langle J_1\tfrac{1}{2}J_1'-\tfrac{1}{2}|\lambda 0\rangle \bar{N}_{a_1}\bar{N}_{a_1'} \times\left\{\frac{R_b R_A}{R_b+R_A}\frac{1}{1+(r/(R_b+R_A))\exp(\bar{\kappa}(r-(R_b+R_A)))}\right\} \tag{28}$$

where

$$S = 70\ \text{MeV}\cdot\text{fm}^2. \tag{28a}$$

The function f then has the same structure as the proximity form of the ion-ion potential[5]). As one may read off from figs. 3 and 4 one may, for orders of magnitude estimates, use

$$\bar{N}_a \approx \frac{1}{A}[(l+7)(n-1)+2(l+1)]\ \text{fm}^{-3/2} \tag{29}$$

both for neutrons and protons. This formula breaks down for loosely bound neutrons of high principal quantum number (n).

3. Numerical calculations and range of validity

The resulting expression for the one-nucleon stripping (NS) form factor is (in MeV)

$$\left(f_{\lambda 0}^{a_1 a_1'}(0,r)\right)^{(\text{NS})} = J\pi^{3/2}\sqrt{2J_1+1}\,\delta(l_1+l_1'+\lambda, \text{even}) \times(-1)^{J_1+1/2+l_1'}\langle J_1\tfrac{1}{2}J_1'-\tfrac{1}{2}|\lambda 0\rangle \bar{N}_{a_1}\bar{N}_{a_1'} \times\frac{k_\lambda(\bar{\kappa}(R_b+R_A))/\bar{\kappa}}{k_{l_1'}(\bar{\kappa}R_b)k_{l_1}(\bar{\kappa}R_A)}\frac{20}{1+k_\lambda(\bar{\kappa}(R_b+R_A))/k_\lambda(\bar{\kappa}r)}, \tag{30}$$

where

$$\bar{\kappa} = \begin{cases} \left.\begin{matrix}\bar{\kappa}_{a_1'} & \text{(neutron)}\\ \bar{\kappa}_{a_1'}^{\text{eff}} & \text{(proton)}\end{matrix}\right\} & \text{prior representation} \\ \left.\begin{matrix}\bar{\kappa}_{a_1} & \text{(neutron)}\\ \bar{\kappa}_{a_1}^{\text{eff}} & \text{(proton)}\end{matrix}\right\} & \text{post representation,}\end{cases} \tag{31}$$

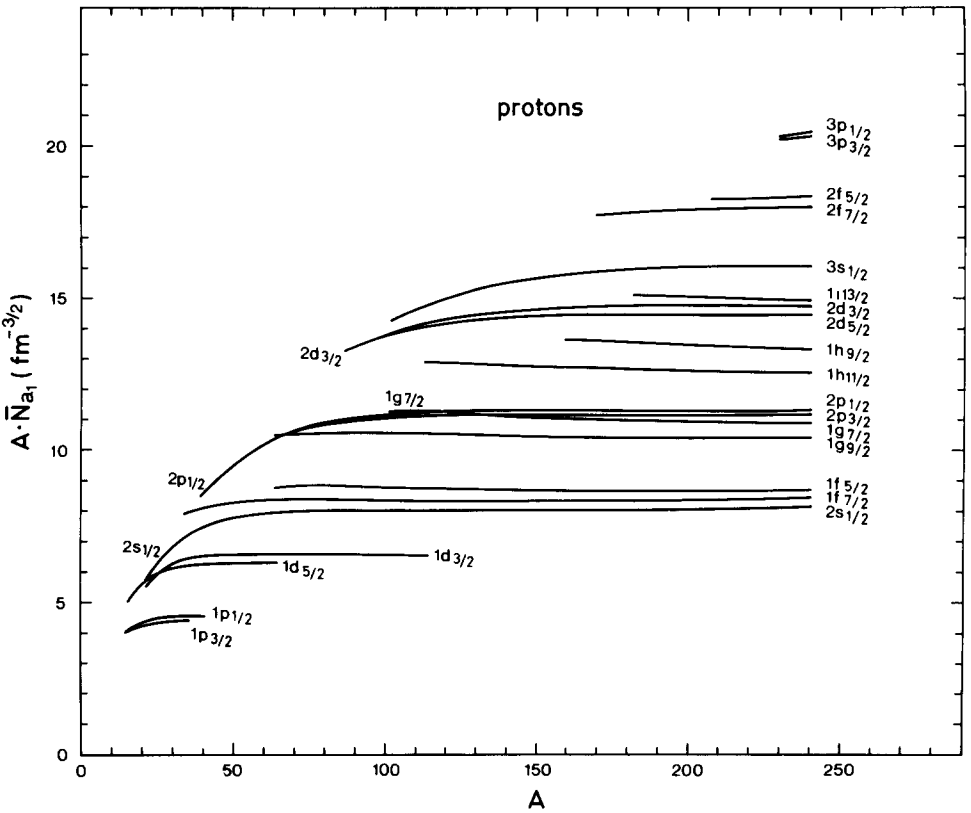

Fig 1 Normalization constants $\bar{N}_{a_1}$ for proton single-particle states as defined in eq (18) times the mass number A, as a function of A The quantum numbers $a_1 \equiv (n_1 l_1 j_1)$ associated with each bound orbital in each mass range label the different curves The calculation of the radial single-particle wave functions were carried out making use of a Saxon-Woods potential with parameters $r_0 = 1\ 25$ fm and $a = 0\ 65$ fm The depth was adjusted for each level individually to reproduce the binding energy obtained by a global fitting and reported in refs [8,9]) The calculation of $\bar{N}_{a_1}$ was done in two steps In the first the normalization N_{a_1} appearing in the relation $R^{(A)}_{a_1}(r_{1A}) = N_{a_1} k_{l_1}(\kappa^{\text{eff}}_{a_1} r_{1A})$ was calculated setting the variable r_{1A} which measures the distance of the particle 1 from the center of mass of nucleus A equal to the Coulomb radius of the A + 1 system (cf eq (26)) Making use of this normalization constant one calculates $\bar{N}_{a_1} = N_{a_1} k_l(\bar{\kappa}^{\text{eff}}_{a_1} R_A)$, where $R_A = 1\ 25 A^{1/3}$ fm The definition of $\bar{\kappa}^{\text{eff}}_{a_1}$ is given in eq (25)

The normalization constants are measured in fm$^{-3/2}$ while the wave number $\bar{\kappa}$ should be in fm^{-1}. The values of the radii used in (30) are to be calculated through the expression $R_i = 1\ 25 A_i^{1/3}$ fm (i = a, A)

The quantities $\bar{N}$ and κ^{eff} are displayed in figs 1–4 for both protons and neutrons as a function of the mass number A and for all the single-particle orbitals corresponding to bound states.

Making use of these functions the form factor (30) has been calculated for a variety of target–projectile combinations and compared with the results provided by

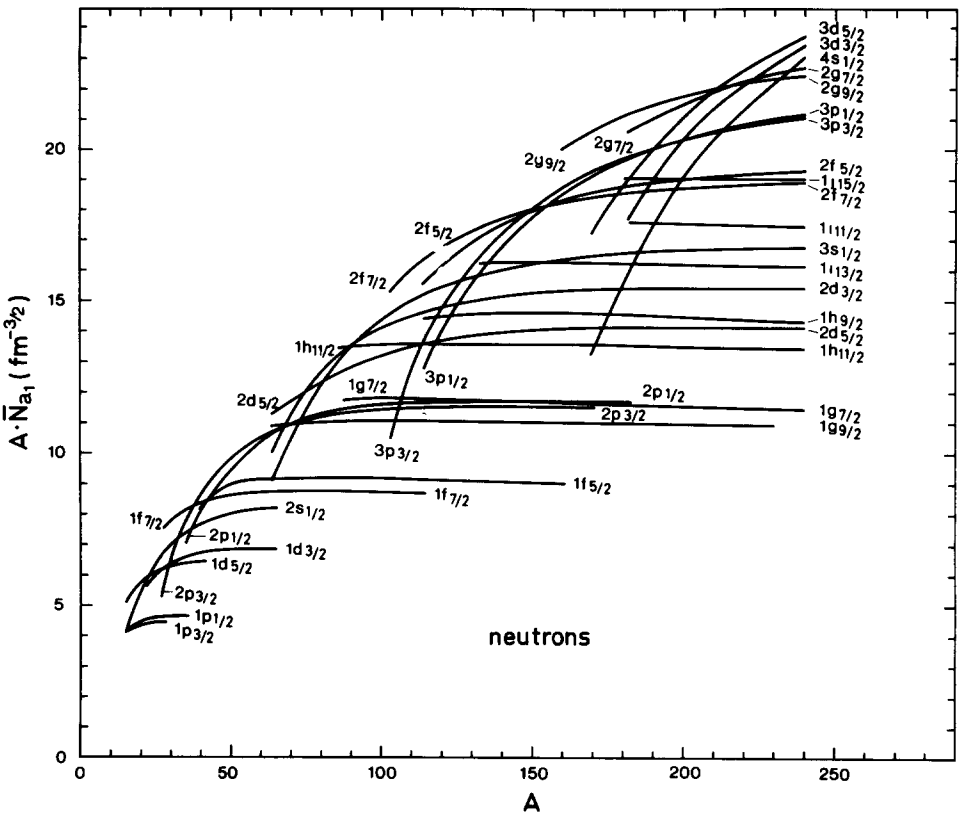

Fig 2 Normalization constant $\bar{N}_{a_1}$ for neutron single-particle states as defined in eq (18) times the mass number A, as a function of A The calculation of $\bar{N}_{a_1}$ follows the same steps as described in the caption to fig 1 but in this case r_{1A} was set equal to $1\,2A^{1/3}$ fm + 7 fm Once N_{a_1} is obtained the quantity $\bar{N}_{a_1}$ is calculated through the relation $\bar{N}_{a_1} = N_{a_1} k_l(\bar{\kappa}_{a_1} R_A)$ with $R_A = 1\,25A^{1/3}$

the "exact" expression (9). The results are displayed in fig. 5, and show that the approximate expression (30) is quite accurate for typical situations.

The observed discrepancies between the predictions of eqs. (9) and (30) are smaller than 20–30% at relative distances of the order of the grazing distance. This is true not only for the aligned transferred angular momentum leading to the largest values of $f_\lambda^{a_1 a_1'}(r)$ but also for smaller λ allowed values.

The largest discrepancies are connected with situations where the difference between κ_{a_1} and $\kappa_{a_1'}$ (or $\kappa_{a_1}^{\text{eff}}$ and $\kappa_{a_1'}^{\text{eff}}$) are largest and the final orbital in the transfer process has a small λ-value. Consistent with this result the approximation (30) is better for proton than for neutron transfer, as the Coulomb potential provides an effective barrier.

As mentioned in sect. 2 the validity of the expression (30) is based on the stability of the integral (22) to changes in the binding energy of the single-particle state a_1.

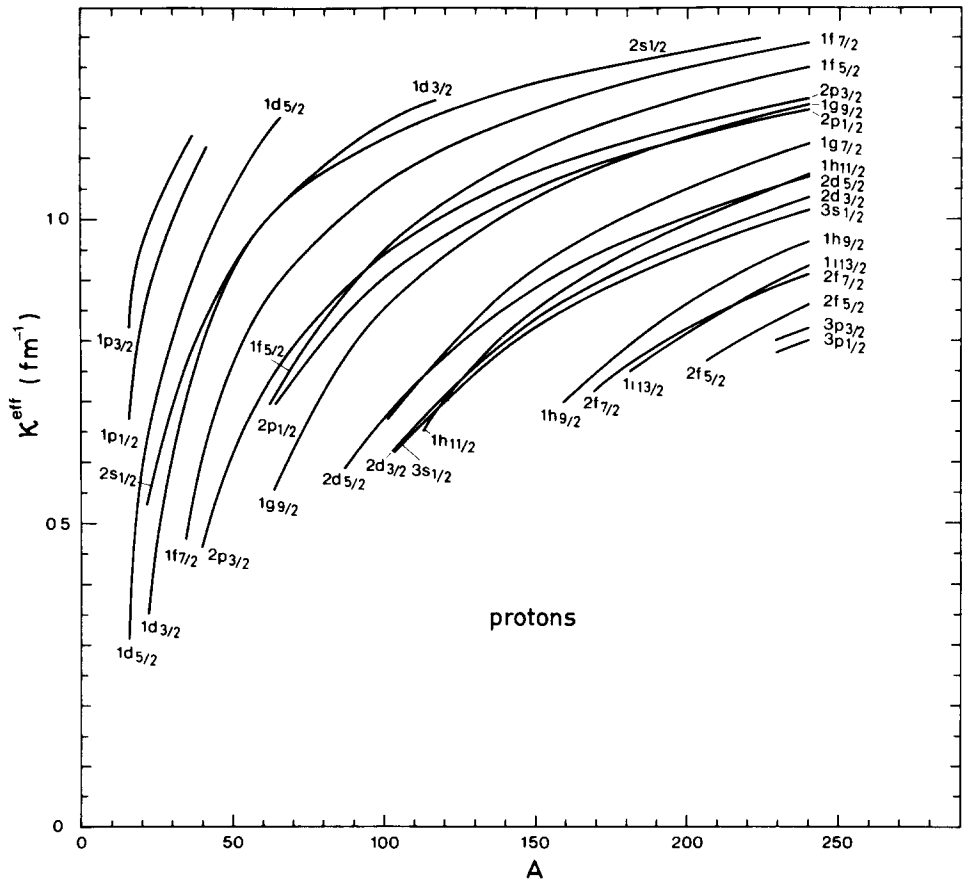

Fig 3 Effective wave number as a function of the mass number A associated with the bound single-particle proton states Eq (25) has been used with binding energies calculated from a global fit[8,9]) of single-particle levels The charge number Z_b appearing in this equation was calculated from the mass number making use of the relation $Z = 0\,487\,A/(1 + A^{2/3}/166)$

We have calculated the range of values $\kappa_{a_1}/\kappa_{a_1'}$ for which the ratio

$$R = \frac{G(\kappa_{a_1}, \kappa_{a_1'})}{G(\kappa_{a_1'}, \kappa_{a_1'})} \tag{32}$$

is unity within 30%. The calculations were done for the cases of $^{16}O + ^{208}Pb$ and $^{88}Sr + ^{208}Pb$, and for different values of $\kappa_{a_1'}$ (5 MeV $\lesssim B_{a_1'} \lesssim$ 16 MeV) For neutrons, the upper and lower limits of R are fulfilled for $\kappa_{a_1}/\kappa_{a_1'} = 0\,15l_1 + 1\,35$ and $\kappa_{a_1}/\kappa_{a_1'} = -0.15l_1 + 0.60$, respectively. The same calculations were done for protons interchanging κ_{a_1} and $\kappa_{a_1'}$ by $\kappa_{a_1}^{\text{eff}}$ and $\kappa_{a_1'}^{\text{eff}}$, respectively In this case the criterion of 30% stability for the ratio R implies value of B_{a_1} spanning the range 0–30 MeV

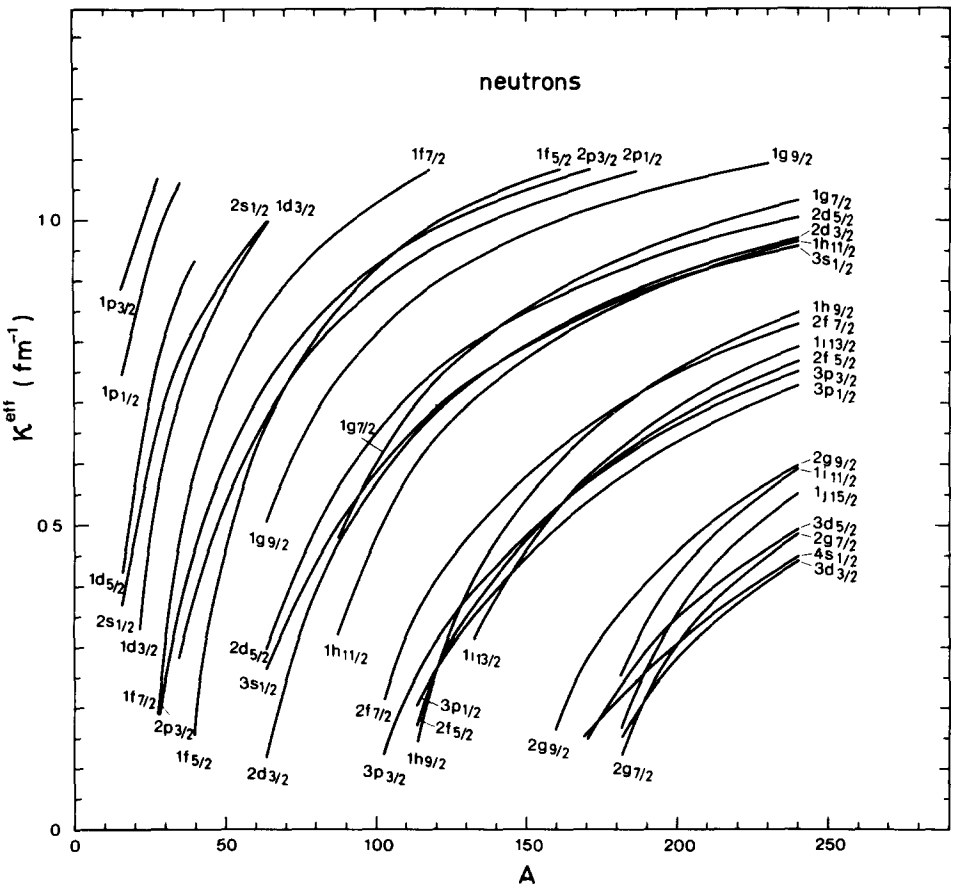

Fig 4 Wave number as a function of the mass number A associated with the bound single-particle neutron states Eq (19) has been used For more details cf caption to fig 3

4. Calculation of the absorptive potential due to particle transfer

As an application of the above parametrization of the form factors we give numerical results for the absorptive potential for a number of reactions and compare with the results of ref. [6]) As shown in ref [4]) the absorptive potentials due to transfer is proportional to the square of the corresponding form factors, i.e.

$$(W(r))_{\text{transf}} = \sum_{a_1 a_1' \lambda} \sqrt{\frac{a_{\text{tr}}(a_1, a_1')}{16\pi^2 |r_0| \hbar^2}}\, g_\lambda(Q) \times \Big\{ (2J_1' + 1) U^2(a_1 I_A) V^2(a_1' I_a) |f_\lambda^{a_1 a_1'(\text{NS})}(r)|^2 + (2J_1 + 1) V^2(a_1 I_A) U^2(a_1' I_a) |f_\lambda^{a_1 a_1'(\text{NP})}(r)|^2 \Big\}. \quad (33)$$

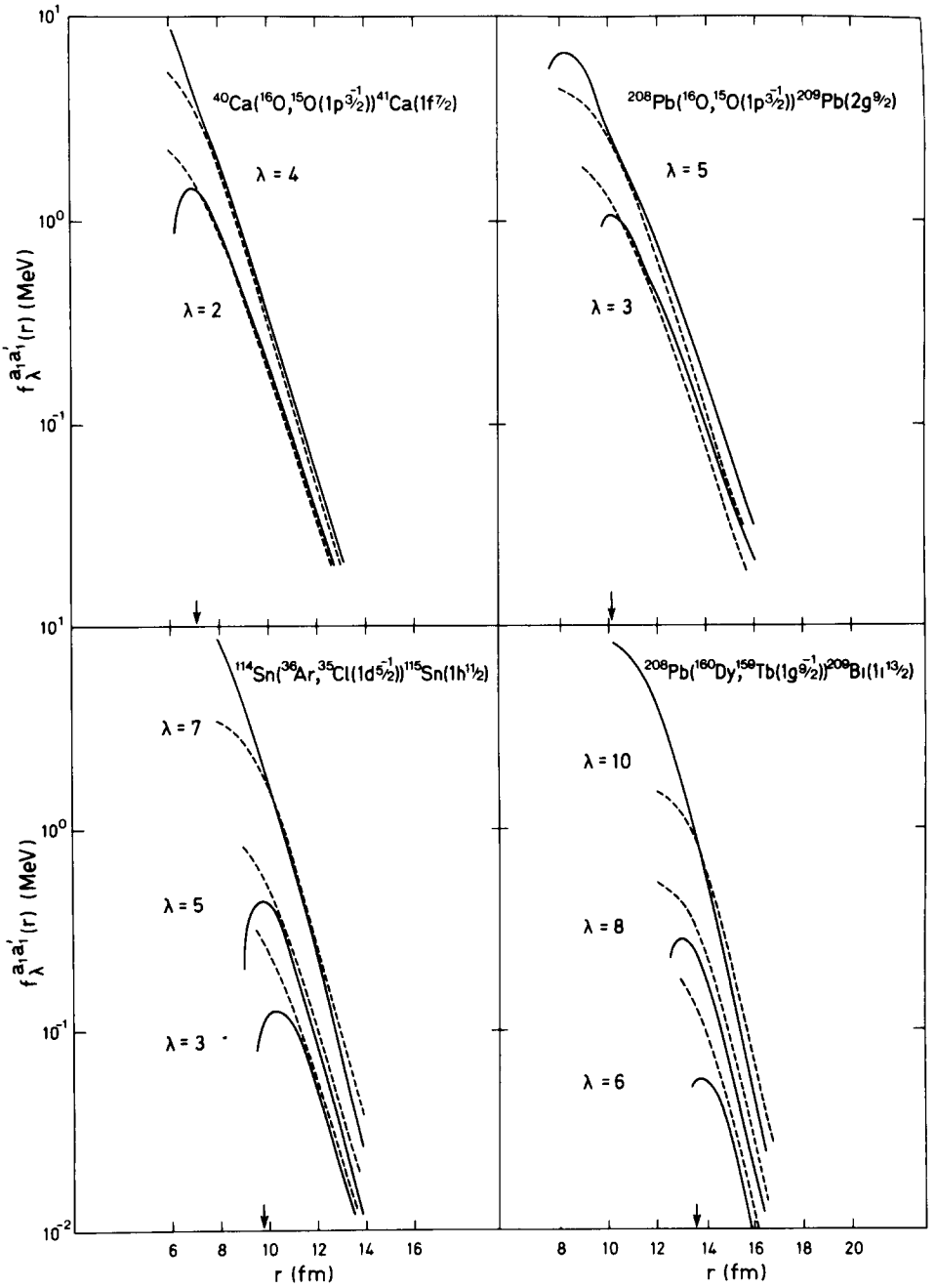

Fig 5 Single-particle form factors calculated making use of eq (30) and figs 1–4 (dashed curves), in comparison with the "exact" form factor given by eq (9) (continuous curves) The arrow on the ordinate indicates the value of the sum of the two radii $R_a + R_A$ where $R_i = 1\,2A^{1/3}$ fm ($i = a, A$) The transfer takes place as a rule at distances larger than $R_a + R_A$

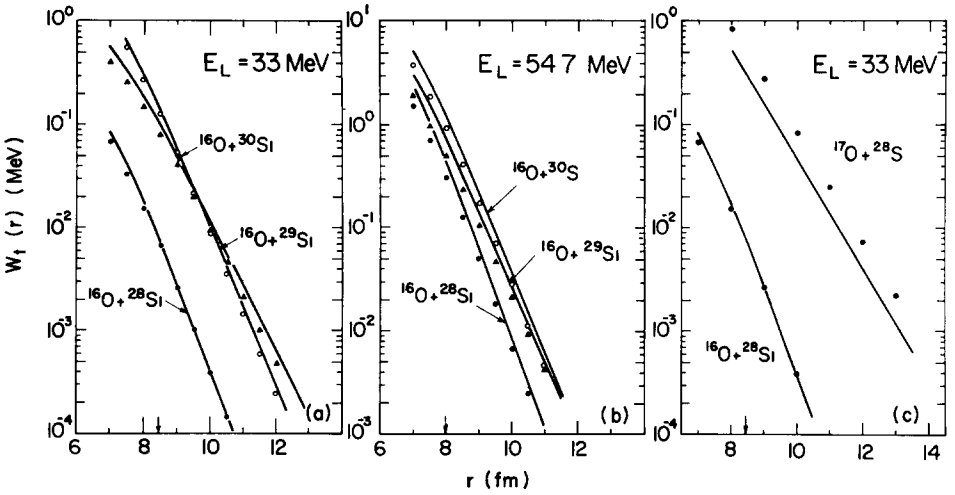

Fig 6 Absorptive potential (cf eq (33)) arising from particle transfer processes associated with the different reactions and bombarding conditions In (a) and (b) are shown the potentials associated with the scattering of ^{16}O on ^{28}Si, ^{29}Si and ^{30}Si at bombarding energies $E_{lab} = 33$ MeV and 54 7 MeV, respectively The results displayed with continuous curves were obtained making use of the approximated form factors given in eq (30) In both cases the results indicated by dots, open circles and crosses were obtained making use of the "exact" form factors given in eq (9) In (c) the absorptive potential associated with the reaction $^{17}O + ^{28}Si$ at $E_{lab} = 33$ MeV is shown in comparison with the one associated with the reaction $^{16}O + ^{28}Si$ [cf also ref [10])] It should be noted that for the latter reaction the assumptions underlying the evaluation of the absorptive potential are not fulfilled since only very few transitions contribute to the depopulation of the elastic channel

The form factor $f_\lambda^{a_1 a_1'(NS)}(r)$ describing stripping reactions coincides with (9), while that describing pick-up reactions is obtained from this making the substitutions a $\leftrightarrow$ A and b $\leftrightarrow$ B and changing $r \rightarrow -r$ (cf. subsect 2.1) The quantity $a_{tr}(a_1, a_1')$ is the diffuseness of the transfer form factors. It is equal to $1/\kappa_{a_1'}$ for $f^{(NS)}$ and to $1/\kappa_{a_1}$ for $f^{(NP)}$. These quantities are of the order of 1 2 fm. The acceleration r_0 at the distance of closest approach for a grazing trajectory can be estimated through the expression $r_0 = (2E - E_B)/(r_B m_{aA})$, where the quantity E_B is the Coulomb barrier. The function $g_\lambda(Q)$ which depends parametrically on the Q-value and on the angular momentum λ transferred in the reaction, determines the adiabatic cut-off [for more details cf. refs. [4,6])].

The quantities $V^2(a_1 I_A) = 1 - U^2(a_1, I_A)$ and $V^2(a_1', I_a) = 1 - U^2(a_1', I_a)$ are the occupation probabilities of the orbitals a_1 and a_1' in nuclei A and a, respectively. In all cases shown below they were given values equal to either 0 or 1.

In figs. 6 and 7 we compare the potential (33) for a variety of reactions making use of the "exact" form factors (9) as well as the approximate expression (30) The deviations around the grazing distance, indicated by an arrow in the figure, are typically of the order of 30%. For the $^{17}O + ^{28}Si$ system at $E_{lab} = 33$ MeV the difference is found to be considerably larger (50%) due to the small binding in ^{17}O.

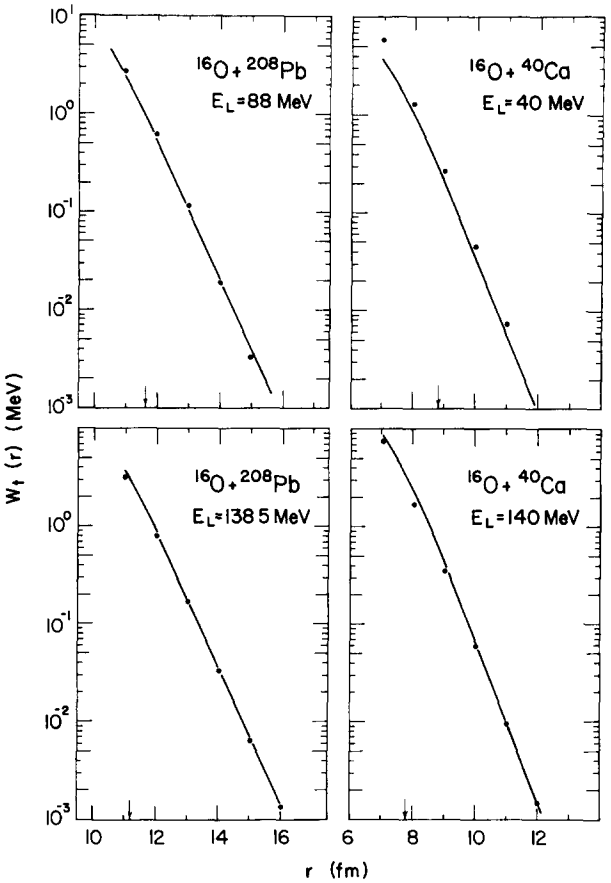

Fig 7 Absorptive potential associated with the reactions $^{16}O + ^{40}Ca$ and $^{16}O + ^{208}Pb$ for different bombarding energies As in the last figure, the dots correspond to the "exact" calculations, making use of eq (9), and the continuous curves are the results obtained using the parametrized form factors

In spite of these deviations, it is noted that the approximated expression reproduces quite accurately the changes of the absorptive potential with the number of nucleons and as a function of the bombarding energy. Such changes are expected to play an important role in the description of the different degrees of transparency shown by the system $^{16}O + ^{28,29,30}Si$ as evidenced by the backwards rise of the corresponding elastic cross sections[7]) Also shown in fig 7 are the absorptive potentials associated with the reactions $^{16}O + ^{40}Ca$ and $^{16}O + ^{208}Pb$ for different bombarding energies

Fig 8 shows the approximated and "exact" results for the $^{86}Kr + ^{208}Pb$ system at $E_{lab} = 695$ MeV. Because of the many orbitals implied in the transfer processes ($\gtrsim 500$ transitions), the calculation using the "exact" form factors is rather time

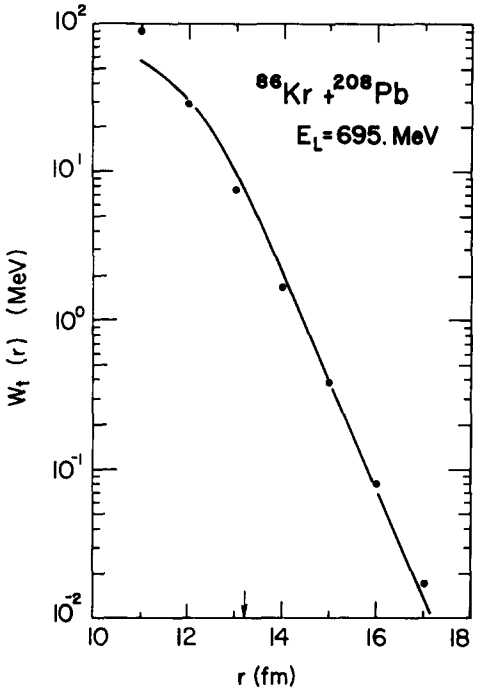

Fig 8 The absorptive potential associated with the reaction $^{86}Kr + ^{208}Pb$ at $E_{lab} = 695$ MeV For the significance of the signatures cf fig 7

consuming. The parametrized expression of the form factors is therefore in this case especially useful.

It is noted that the absorptive potential shown in fig. 8 is about a factor of 2 smaller than that quoted in ref. [11]).

5. Conclusions

A parametrization has been obtained for the low-recoil form factors describing one-particle transfer processes between heavy ions. The resulting functions are found to reproduce the "exact" form factor to better than 30% down to distances of the order of the sum of the two radii

One can envisage a variety of situations where it can be convenient to have access to a simple yet accurate parametrization of the form factors. For example, to describe transfer processes in deep-inelastic reactions and in the calculation of the absorptive potential associated with reactions between very heavy ions. In both cases a very large number of single-particle transfer channels participate in the process, and a detailed calculation of each form factor seems out of place.

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