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ON THE NUCLEAR POLARIZATION POTENTIAL FOR HEAVY-ION SCATTERING

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Abstract. The real polarization potential ΔV due to transfer reactions is studied using a microscopic semiclassical formalism. It is found that ΔV is typically of the same order of magnitude as the corresponding absorptive potential W. Different types of possible energy dependences for ΔV and W are explored qualitatively. Specific calculations for $^{16}O+{}^{208}Pb$ and $^{16}O+{}^{60}Ni$ scattering show features similar to those which have been deduced empirically.

1. Introduction

The optical model is widely used for describing the elastic scattering of nuclei. The imaginary part, W, of this potential takes into account the presence of other states that couple to the elastic one and leads to a flux loss into the open reaction channels. The couplings also give rise to a real polarization potential ΔV which renormalizes the "bare" real part V of the potential. Thus the optical potential has the general form

$$U = V + \Delta U,$$

and

$$\Delta U = \Delta V + iW.$$

Experimental estimates of ΔV can be masked by the uncertainties in V and W which are inherent in optical model calculations. However some recent analyses of low-energy heavy-ion collisions indicate that sizeable polarization effects are occurring [1-5]. It is the aim of the present paper to calculate ΔU within a semiclassical description of the collision process, focusing on the role played by intermediate transfer reaction channels.

There has been recent progress towards a microscopic understanding of the imaginary part of the heavy-ion optical potential [see ref. [6]) and references therein]. In particular, it has been shown that transfer reaction channels determine the long-range part of W [refs. [6,7])]. It follows that they should also be the processes that determine ΔV at large distances. Previous work on the real nuclear polarization potential for heavy ions has been mainly concerned with effects of couplings to inelastic excitation channels, which become important at relatively smaller distances (long-range polarization and absorption effects due to Coulomb excitation have also been studied extensively). A review is given in ref. [8]).

In the next section we derive a microscopic expression for ΔV using semiclassical perturbation theory. After examining the structure of this result we present in sect. 3 some detailed numerical calculations of ΔV for specific cases. A summary of the main results of this work is given in sect. 4.

2. A semiclassical expression for the polarization potential

Our approach is based on using semiclassical perturbation theory to obtain the amplitudes for second-order transitions. The correction to the elastic potential can be obtained as a special case of the formalism developed in ref. [9]) for inelastic couplings by setting the final channel equal to the initial one. This can be shown in the most convenient way by considering a system of coupled inelastic channels which carry no spins.

The elastic scattering amplitude for an even system can be written in general as

$$f_{\alpha\alpha}(\theta) = (i/2k)\sum_{l}(2l+1)P_l(\cos\theta)(1-e^{i2\beta_l}\,a^l_{\alpha\alpha})\,,$$

where β_l is the real partial wave phase shift due to the bare elastic potential and $a^l_{\alpha\alpha}$ is the reaction amplitude for the elastic channel. Up to second-order semiclassical perturbation theory one obtains for $a^l_{\alpha\alpha}$ the expression [10])

$$a^l_{\alpha\alpha} = 1-(1/\hbar^2)\sum_{\gamma}\int_{-\infty}^{+\infty} dt\, V_{\alpha\gamma}(r(t))\, e^{-i\omega_\gamma t}\int_{-\infty}^{t} dt'\, V_{\gamma\alpha}(r(t'))\, e^{i\omega_\gamma t'}\,.$$

Here $V_{\gamma\alpha}(r)$ specifies the coupling, i.e. the form factor, to an intermediate state with excitation energy $E_\gamma = \hbar\omega_\gamma$ ($E_\alpha \equiv 0$) and the integration is carried out along the trajectory for the impact parameter $\rho = l/k$. The effects of the coupling can be taken into account implicitly by a change $\Delta U = \Delta V + iW$ in the elastic potential. This amounts to incorporating the reaction amplitude into a new phase shift $\tilde{\beta}_l$ in the partial wave summation. Treating ΔU to leading order and using the semiclassical approximation for its contribution to the phase shift [10]), one obtains

$$e^{i2\tilde{\beta}_l} \cong e^{i2\beta_l}\left\{1-(i/\hbar)\int_{-\infty}^{+\infty} dt\, \Delta U\right\}.$$

Thus we can identify the integral of ΔU with the second-order contribution to the reaction amplitude.

At this point we can directly take over the procedure from ref. [9]) for localizing the right hand side of the preceding equation to obtain ΔU. Thus by assuming an exponential radial dependence for the coupling, $V_{\gamma\alpha} \sim \exp(-r/a_\gamma)$, and expanding about the turning point, $r = r_0 + \frac{1}{2}\ddot{r}_0 t^2$, we are led to the result

$$\Delta U(r) = (-i/\hbar)\Sigma_\gamma |V_{\alpha\gamma}(r)|^2 (1/\sqrt{\pi\sigma_\gamma^2}) I(\omega_\gamma)\,,$$

$$I(\omega_\gamma) = \int_{-\infty}^{+\infty} dt\, e^{-(t^2/2\sigma_\gamma^2) - i\omega_\gamma t} \int_{-\infty}^{t} dt'\, e^{-(t'^2/2\sigma_\gamma^2) + i\omega_\gamma t'}\,.$$

Here $\sigma_\gamma^2 = a_\gamma/\ddot{r}_0$. We will be considering one class of intermediate couplings characterized by a common diffuseness parameter a_γ. Notice that $\tau = 2.3\sigma_\gamma$ gives a measure of the collision time; i.e., the duration of the coupling along the trajectory. In this work we concentrate on the effects of one-nucleon transfer reactions. They usually carry the largest cross sections. Their form factors $V_{\alpha\gamma}(r)$ are relatively long-ranged with diffuseness parameters of about $a = 1.3$ fm [refs. [6,9])]. The corresponding induced elastic potential therefore has $a = 0.65$ fm, which is typical for empirical optical potentials. For transfer reactions the quantity $\hbar\omega$ is the difference, $Q - Q_{\mathrm{opt}}$, between the Q-value and the optimum Q-value. In general, it also depends on the transfer of angular momentum, as indicated in sect. 3.

The real and imaginary parts of the double integral $I(\omega)$ are given by

$$\mathrm{Re}\, I(\omega) = \int_{-\infty}^{+\infty} dt \int_{-\infty}^{t} dt'\, e^{-(t^2+t'^2)/2\sigma^2} \cos\omega(t'-t) = \pi\sigma^2 e^{-\omega^2\sigma^2}\,,$$

$$\mathrm{Im}\, I(\omega) = \int_{-\infty}^{+\infty} dt \int_{-\infty}^{t} dt'\, e^{-(t^2+t'^2)/2\sigma^2} \sin\omega(t'-t)\,.$$

Neither the real nor the absorptive part of the potential can be determined unambiguously from the knowledge of the phase shift. By the prescription given here one asks for a local potential which is independent of the impact parameter. One might therefore suspect that such a prescription would lead to a potential which would not satisfy a dispersion relation. However, it follows from the analytic properties of the function $I(\omega)$ in the complex ω-plane that

$$\mathrm{Im}\, I(\omega) = \frac{1}{\pi} \mathrm{P}\int_{-\infty}^{+\infty} \frac{\mathrm{Re}\, I(\omega')}{\omega' - \omega} d\omega'\,, \qquad \mathrm{Re}\, I(\omega) = \frac{1}{\pi}\mathrm{P}\int_{-\infty}^{+\infty} \frac{\mathrm{Im}\, I(\omega')}{\omega - \omega'} d\omega'\,.$$

While the real part peaks at $\omega = 0$ and drops off rapidly, the imaginary part is antisymmetric about $\omega = 0$ and decreases slowly for large ω (see below). These results provide an insight into the nature of the effective potential since the dependence on the important physical parameters - the number of intermediate states, the strengths with which they couple to the ground state, their excitation energies and their effective contact times - appear explicitly (the spins of the intermediate states will be included in sect. 3). A detailed calculation is required to specify the coupling strengths. For the qualitative discussion which follows we simply consider them to be characterized by a typical order of magnitude. Quantitative calculations are given in sect. 3.

A number of interesting points can be made by considering the situations illustrated in fig. 1. Here we show plots of the real and imaginary parts of $I(\omega)$ and we have indicated some possible distributions for the energies of the intermediate states as shaded areas. Notice that negative frequencies correspond to positive effective Q-value transfer reaction channels ($Q_{\rm eff} = Q - Q_{\rm opt} = -\hbar\omega$). Notice also that a factor $-i$ appears when converting I to ΔU. Inelastic excitations and negative effective Q-value transfer reaction channels give rise to a negative correction ΔV, thereby increasing the nuclear attraction. For an equal distribution of channels over positive and negative ω-values or for cases where the main contributions come from the neighborhood of $\omega = 0$ (fig. 1a), there will be absorption without polarization. When there is a uniform distribution of states for $\omega > 0$, as indicated in fig. 1b, we can expect more comparable values for ΔV and W. However, for cases where the intermediate states are localized at high frequencies (fig. 1c), the polarization potential dominates over the absorption. This corresponds to the well-known polarization phenomenon associated with virtual excitations, which exists even in the static limit.

By considering a fairly uniform distribution of states with $\omega > 0$ we can make a simple estimate of the ratio $\Delta V / W$. The imaginary part of $I(\omega)$ can be expressed in terms of a principal value integral [9].

$$\operatorname{Im} I(\omega) = \sigma^2 \mathrm{P} \int_{-\infty}^{+\infty} (\mathrm{d}q/q)\, \mathrm{e}^{-(q+\omega)^2\sigma^2} .$$

It follows from this result that the frequency ω_0 where the imaginary part of $I(\omega)$ is maximum satisfies the relation

$$\omega_0 \operatorname{Im} I(\omega_0) = -\sqrt{\pi\sigma^2} .$$

As a rough estimate we can consider that ΔV is proportional to the area $\omega_0 |\operatorname{Im} I(\omega_0)|$. We can also estimate W as being proportional to half the area under the gaussian of the real part of $I(\omega)$. In this way we obtain $\Delta V / W \simeq 2/\pi \simeq 0.6$.

Actually it should be kept in mind that the relative strengths of ΔV and W depend on the bombarding energy. There is an overall and relatively slow dependence on the energy since our expression for ΔU is proportional to the collision time. The main effect of the collision time is to determine the widths of curves like those

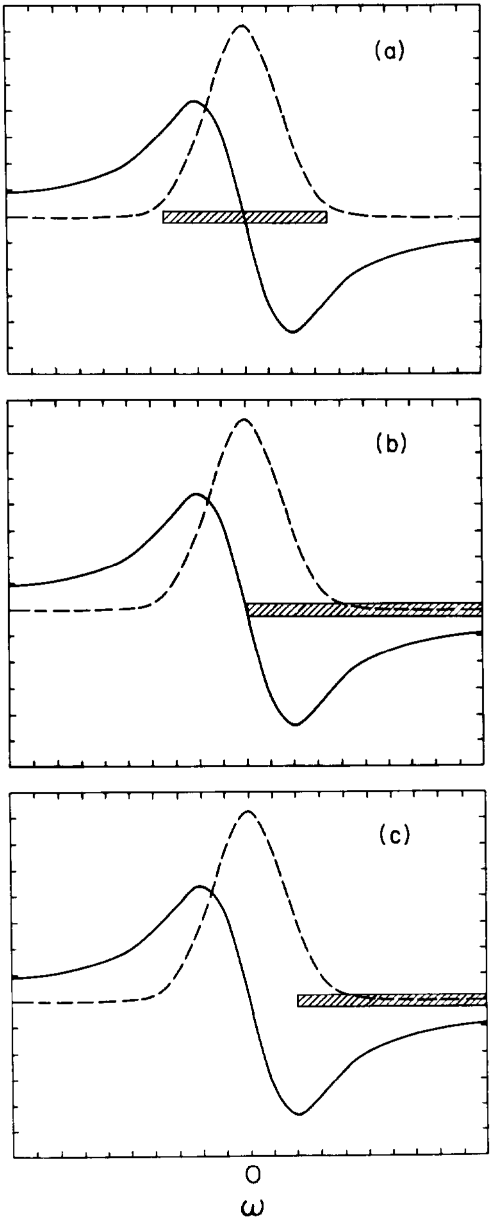

Fig. 1. Behavior of the double integral $I(\omega)$ and three types of distributions for the energies of the intermediate states. Real and imaginary parts are indicated by the dashed and full lines respectively. Both scales are in arbitrary units.

shown in fig. 1. For instance, at low energies these curves become narrow. Typically this cuts off the higher frequency contributions to W leaving ΔV comparatively favored. In addition, for the case of intermediate transfer reaction channels the effective Q-value, $Q - Q_{\rm opt}$, depends on the bombarding energy[6]. Thus the types of intermediate state distributions indicated in fig. 1 can vary as the energy changes. In the following section we will explore the energy dependence of the polarization potential in greater detail.

Qualitatively one can understand the existence of correlations between ΔV and W by observing the plot of the functions displayed in fig. 1. A single intermediate channel would be represented by a point whose abscissa changes as a function of the bombarding energy. An increasing contribution to the imaginary part results as the channel moves gradually towards the center. The maximum rate in the increase of the absorption (i.e. its inflection point) correlates with the bombarding energy that produces the highest value of the polarization. This simple picture tends to get blurred as one superimposes the effects of many channels. However, the analytic connection between the real and imaginary parts of our expression for ΔU is not affected by the sum over the intermediate steps.

It should also be noted that while the summation over intermediate states in the expression for W has a natural cut-off, there is an apparent difficulty in extending the sum into the continuum region for ΔV. This is because at large ω

$$\mathrm{Im}\, I(\omega) \to -\sqrt{\pi\sigma^2}/\omega\,,$$

as can be seen from the integral expression given above. This slow rate of convergence does not pose a fundamental problem since the coupling strengths at high frequencies effectively limit the number of active channels. In the applications that follow we have restricted the sum over intermediate steps to bound states in the $A \pm 1$ systems. While this practical truncation leaves uncertain the actual magnitude of the polarization potentials, their energy dependence can be calculated more reliably. In fact, we note that the contributions of intermediate channels to the derivative $(\mathrm{d}\Delta V/\mathrm{d}E)$ would have a better convergence rate.

One may thus extract the energy dependence and obtain a faster convergence by subtracting from the full expression for ΔV the quantity ΔV_∞, obtained by replacing $\mathrm{Im}\, I(\omega)$ by the static limit. Note that the expression $\sqrt{\pi\sigma^2}/\omega$ is a fairly good approximation to the function up to the maximum (which belongs to the asymptote). In the static limit we find at the distance r

$$\Delta V(r) = \sum_\gamma V^2_{\gamma\alpha}(r) \frac{1}{\hbar\sqrt{\pi\sigma^2_\gamma}}\, \mathrm{Im}\, I_\gamma \to -\sum_\gamma \frac{V^2_{\gamma\alpha}(r)}{\hbar\omega_\gamma}\,.$$

This is in fact the standard polarization potential that one would calculate in second order time-independent perturbation theory.

3. Numerical calculations

In this section we present numerical calculations of the polarization potential for some specific cases, making use of microscopic single-particle transfer form factors. These are constructed and parametrized as in refs. [6,9]).

Considering the case where the projectile and target are spinless and focusing on the stripping reaction process where particles occupying orbits a_1' in the projectile are transfered to orbits a_1 in the target, we obtain the following expression for the effective potential,

$$\Delta U(r) = \sum_{a_1 a_1'} \sqrt{\frac{a_{\mathrm{tr}}(a_1, a_1')}{16\pi\hbar^2|\ddot{r}_0|}}(2j_1'+1)U^2(a_1, I_A)V^2(a_1', I_a) \times |f_\lambda^{a_1,a_1'}(r)|^2(-i)[g_\lambda(Q)+ip_\lambda(Q)] .$$

A similar expression holds for single-particle pick-up reactions. Here f_λ denotes the transfer formfactor of multipolarity λ while U and V are occupation factors for the orbitals. The term between brackets is given by the expression

$$[g_\lambda(Q)+ip_\lambda(Q)] = (1/\pi)\int_0^\pi \mathrm{d}\theta \int_{-\infty}^{+\infty} \mathrm{d}t\, \mathrm{e}^{-t^2/2+i(a-b\cos\theta)t} \times \int_{-\infty}^{t} \mathrm{d}t'\, \mathrm{e}^{-t'^2/2-i(a-b\cos\theta)t'} ,$$

where

$$a = \sqrt{a_{\mathrm{tr}}(a_1, a_1')/\hbar^2\ddot{r}_0}\,(Q-Q_{\mathrm{opt}}) ,$$

$$b = \sqrt{a_{\mathrm{tr}}(a_1, a_1')\hbar^2/\ddot{r}_0}\,\frac{(l_g+\frac{1}{2})\lambda}{m_{aA}r_0^2} .$$

In the discussion of sect. 2 we neglected the angular momentum transfer λ associated with the intermediate state, which amounts to setting $b=0$ in the expression above.

The quantity $g_\lambda(Q)$ is the adiabatic cut-off function introduced in ref. [6]). The new feature introduced here is the polarization function $p_\lambda(Q)$ which determines the real part of ΔU. It can be calculated directly from its definition, or from the more explicit expression

$$p_\lambda(Q) = (2/\pi^{3/2})\int_0^\pi \mathrm{d}\theta\, \mathrm{e}^{-(a-b\cos\theta)^2}\int_0^{(a-b\cos\theta)} \mathrm{d}y\, \mathrm{e}^{y^2} .$$

The plots in fig. 2 show the contours of g and p in the plane of the parameters a and b. These plots can be used to construct the effective potential once the form factors and bombarding conditions have been specified.

As our first example we have calculated ΔU for the reaction $^{16}\mathrm{O}+{}^{88}\mathrm{Sr}$ at $E_{\mathrm{lab}} =$ 52 MeV. The results are shown in fig. 3. The ΔV associated with proton and neutron single-particle transfers to the bound states of neighbouring nuclei is comparable in magnitude to W and amounts to about 15% of the estimated "bare" potential.

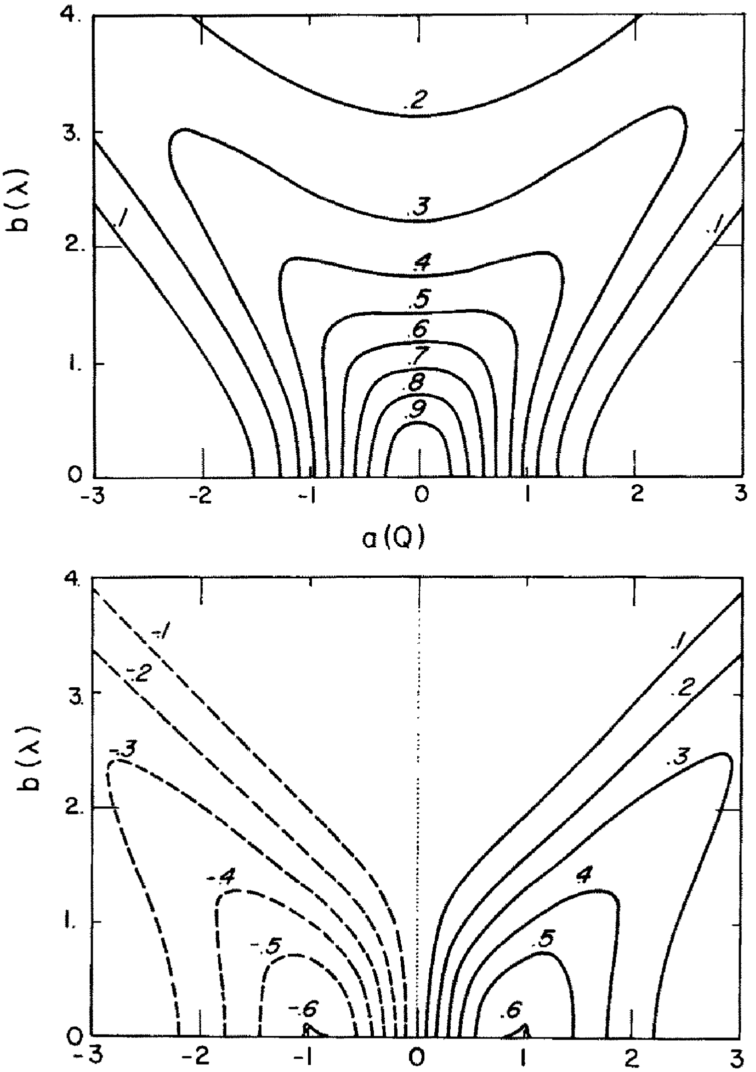

Fig. 2. Contour plots of the adiabatic cutoff function (top) and the polarization function (bottom). The axes relate to the energy and angular momentum of the intermediate states. The parameters a and b are defined in ref. [6]). Note that for inelastic excitations $a \propto Q = -\omega$ (cf. fig. 1).

We have checked the approximation involved in keeping only the leading order in the expansion for the phase shift $\tilde{\beta}_l$. In this system, for example, the time integral of ΔU for a grazing trajectory is about 0.05.

The energy dependence of the polarization potential has been focused on recently, following the discovery of an increase in the real part of the optical potential extracted from $^{16}\mathrm{O}+^{208}\mathrm{Pb}$ elastic scattering measurements at low bombarding

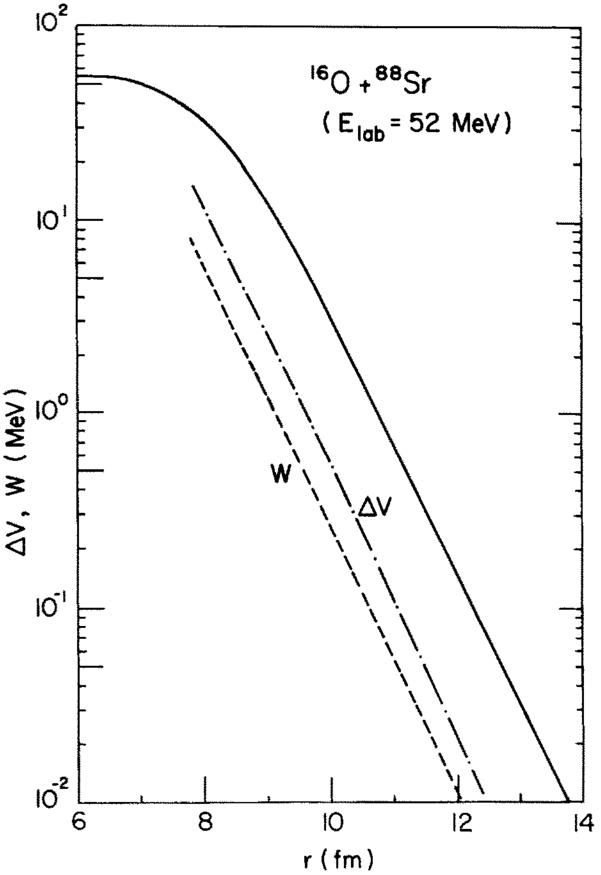

Fig. 3. Calculated polarization potential ΔV and absorption W due to single nucleon transfer reactions in the case of $^{16}O + ^{88}Sr$ at $E_{lab} = 52$ MeV. The empirical potential [11]) V is shown for comparison.

energies [1]). It has been pointed out that this effect is to be expected on the basis of a dispersion relation between the absorptive potential and the real polarization potential [2]). Explicit coupled-channels calculations for this system have also shown large polarization effects due to the transfer channels [3,4]). It is interesting to study this effect within the present model.

The results for $^{16}O + ^{208}Pb$ shown by the solid curves in fig. 4 do give an energy dependence which is similar to what has been deduced empirically [1]). As discussed before, the absolute magnitude of the polarization potential is somewhat uncertain due to the effective truncation of intermediate channels with high Q-value. The sums were here extended to include bound states with $|Q| \leq 25$ MeV, a range for which the energy dependence shown in the figure was found to be fairly stable.

The optical potentials required to fit $^{16}O + ^{60}Ni$ elastic scattering have also been found to depend on energy in a similar way as the $^{16}O + ^{208}Pb$ case [5]). In fig. 5 we show calculations for this system.

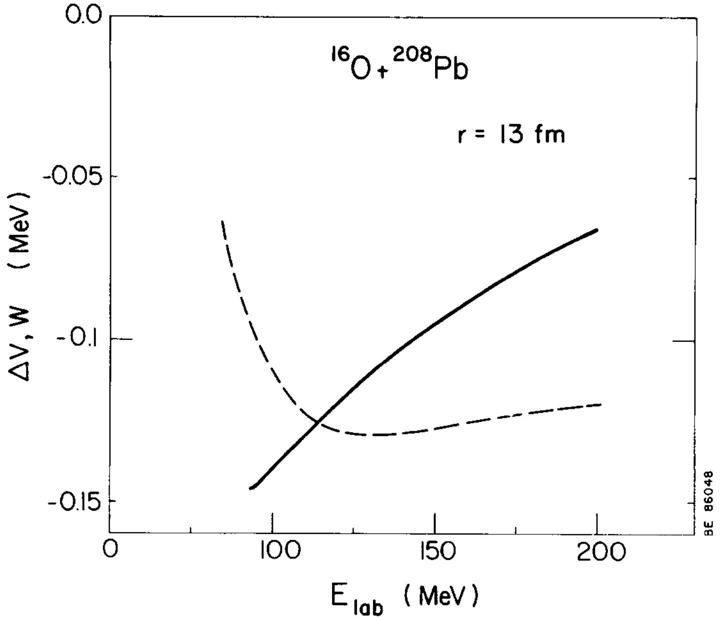

Fig. 4. Energy dependence of the polarization potential (full line) and absorption (dashed line) due to single nucleon transfer for the case of $^{16}O+^{208}Pb$. The magnitudes are taken at $r = 13$ fm.

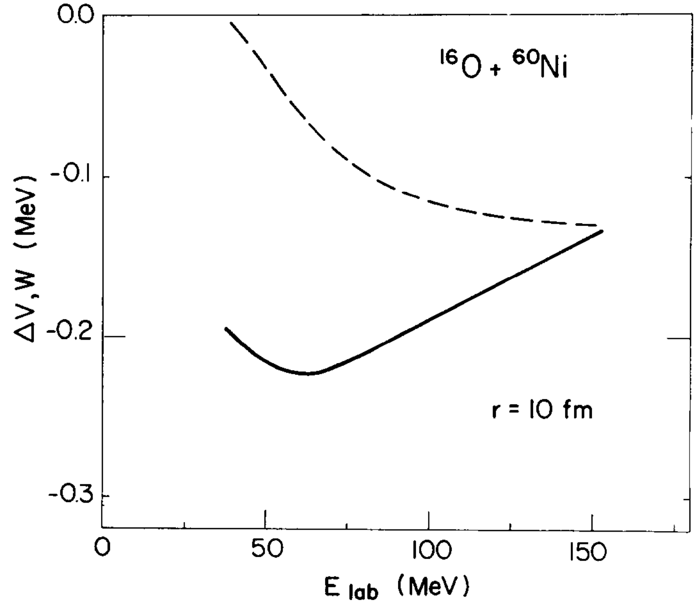

Fig. 5. Energy dependence of the polarization potential (full line) and absorption (dashed line) due to single nucleon transfer for the case of $^{16}O+^{60}Ni$. The magnitudes are taken at $r = 10$ fm.

4. Summary

In this work we have investigated a semiclassical expression for the real nuclear polarization potential ΔV. This result appears as a special case of the formalism for inelastic formfactors given in ref. ^{9}). It also complements the derivation of the absorptive potential W in ref. ^{6}). The long-range part of ΔV, like that of W, is governed by the transfer of single nucleons. A simple analytic estimate indicates that ΔV is of the same order of magnitude as W. Thus typically about 20% of the real part of empirical ion-ion potentials may be due to dynamical polarization effects. The polarization potential due to inelastic nuclear scattering ^{12}) has a shorter range but is again of the same magnitude as the absorptive potential due to inelastic excitations.

Within the semiclassical approximation the energy dependence of ΔV and W is controlled by the collision time and the distribution of the intermediate states. Generally ΔV receives strength from high-lying states, corresponding to virtual excitations. As a consequence the strength of ΔV decreases more slowly than that of W as the bombarding energy is reduced. The distribution of intermediate transfer reaction channels is defined in terms of the effective Q-value, $Q-Q_{\mathrm{opt}}$. Our results suggest that one may find different types of energy-dependent effects in heavy-ion optical potentials according to the distribution of effective Q-values for the strongest reaction channels.

The detailed numerical calculations for the cases of $^{16}\mathrm{O}+{}^{208}\mathrm{Pb}$ and $^{16}\mathrm{O}+{}^{60}\mathrm{Ni}$ show features in the energy dependence of the polarization potential which are similar to the effects deduced from recent empirical analyses ^{1,2,5}).

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