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# TWO-PARTICLE TRANSFER TRANSITION DENSITIES FOR COLLECTIVE MODES IN NORMAL SYSTEMS: A study for a surface-localized pair field

C.H. DASSO

*Sektion Physik der Universität München, Am Coulombwall 1, D8046 Garching, Fed. Rep. Germany*
*and*
*The Niels Bohr Institute, Blegdamsvej 17 DK-2100, Copenhagen Ø, Denmark*

E. MAGLIONE

*Dipartimento di Fisica "Galileo Galilei", Padova and INFN Laboratori Nazionali di Legnaro, Legnaro, Italy*

G. POLLAROLO

*Dipartimento di Fisica Teorica, Università di Torino*
*and*
*INFN Sez. di Torino, Torino, Italy*

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**Abstract:** Transition densities for pair-transfer modes are calculated microscopically in the vicinity of closed-shell nuclei. An RPA-like formalism is used to allow for the presence of ground-state correlations and the radial shapes of the transition densities are constructed from the resulting wavefunctions. The aim of this study is to check the validity of a macroscopic picture in which these functions are parametrized in terms of the derivative of the equilibrium density with respect to the number of particles. The structure calculations yield values for the pairing deformation parameters $\beta_p$ which can be compared with those extracted from DWBA analyses of two-particle transfer data.

## 1. Introduction

A large body of experimental evidence supports the prevailing view that nucleons move rather independently of each other in a central, self-consistent field. This picture has been gradually enrichened by the identification of residual interactions which generate the specific features displayed by actual nuclear systems. Among them, for instance, the tendency of identical nucleons to couple in pairs of total angular momentum zero. At a microscopic level the presence of these particular correlations has been attributed to the existence of short-range residual forces. These so-called pairing interactions are responsible for a variety of nuclear effects. Of special interest to us, in this work, are enhanced transition rates in the transfer of two like-particles in heavy-ion collisions.

A convenient description of this class of reactions is obtained by considering the pair-transfer process as the excitation of a special type of collective motion. This

"excitation" must be interpreted in a wider sense, as it is associated with degrees of freedom which do not strictly conserve the number of particles [1]). From this perspective the addition (or removal) of two particles in the target are rather viewed as the creation (or annihilation) of a pair. A prescription to construct the fields which are responsible for the excitation of these modes has been advanced in ref. [2]). They are obtained by allowing the standard one-body density to be generalized into its number non-conserving forms, i.e.,

$$\sum_{\alpha\beta} \langle\alpha|F|\beta\rangle a_\alpha^+ a_\beta \to \sum_{\alpha\beta} \langle\alpha|F|\beta\rangle a_\alpha^+ a_\beta^+ , \quad \sum_{\alpha\beta} \langle\alpha|F|\beta\rangle a_\alpha a_\beta . \tag{1}$$

One should here emphasize the persistent one-body character of these generalized fields, as the matrix elements in (1) are defined in the coordinate representation in terms of a function of a single nucleon variable. This could in general be of the form $F(\boldsymbol{r}, \boldsymbol{p})$ and therefore incorporate non-local effects. In heavy-ion collisions, though, the interaction felt by a nucleon in the target due to the nearby presence of the projectile is customarily approximated by the velocity-independent central field generated by the latter.

The adoption of this point of view eases the implementation of a simpler formalism to describe the transfer process. Using the variation in the number of particles $\Delta A$ as the collective variable, the local pair transition densities can be modeled [2]) by

$$\delta\rho(r) = \beta_{\mathrm{p}} \frac{\partial\rho}{\partial A} , \tag{2}$$

where the parameter $\beta_{\mathrm{p}}$ gives a measure of the collective character of the pair-correlated state. The previous expression, supplemented by a scaling assumption, leads to macroscopic formfactors of the form

$$F(r) \approx \frac{\beta_{\mathrm{p}} R}{3A} \left(\frac{\partial U}{\partial r}\right) , \tag{3}$$

where $R$ stands for the radius of the target and $U$ is the ion-ion potential.

Up to this point, the introduction of the pairing deformation parameter $\beta_{\mathrm{p}}$ in the analysis of two-nucleon transfer data could be viewed as just a convenient way to scale the magnitude of the calculated cross sections. This situation resembles the empirical determination of nuclear-deformation parameters for collective surface modes which are based on DWBA calculations. In such analyses, however, a relative consistency is expected of the values for a given nuclear state which are obtained from different experimental conditions. (We note that this property is not always fulfilled, see for instance the compilation in ref. [3]).)

The perspective changes as one tries to understand the magnitude of the extracted numbers from a microscopic point of view. Indeed, the adopted form of the collective field and the strength of the coupling impose constraints on the distortions of the nuclear density which are associated with the excitation of the mode. Controlling

these conditions should provide a more stringent check on the validity of macroscopic schemes.

Considerable substantiation of the macroscopic model for the excitation of surface vibrations has been obtained from structure calculations (see e.g. ref. [4])). In this paper we examine some of these questions in the novel context of two-particle transfer reactions. To this end we construct the shape and magnitude of pair transition densities for closed-shell nuclei and in the process obtain microscopic values for $\beta_p$. We note that a similar study has also been conducted for superfluid systems [5]).

## 2. Formalism

The traditional attitude to keep pair addition and removal modes as independent degrees of freedom centers on the observation that these modes are built mainly of single-particle levels which are respectively above or below the Fermi surface. It has been noted [1]), however, that this point would not be relevant if the pairing interactions were capable to induce strong ground state correlations. Anticipating this possibility we shall construct the wavefunctions for the states of $(A \pm 2)$-particle systems with an RPA like formalism.

We consider a hamiltonian of the form

$$\hat{H} = \hat{H}_0 + \hat{V}_{\text{res}} , \tag{4}$$

where $\hat{H}_0$ generates the independent-particle states $(n_\alpha l_\alpha j_\alpha m_\alpha)$ and the residual interaction $\hat{V}_{\text{res}}$ is assumed to be separable

$$\hat{V}_{\text{res}} = \kappa \hat{\mathcal{F}}_+ \hat{\mathcal{F}}_- , \tag{5}$$

i.e. the product of two generalized one-body operators of the form

$$\hat{\mathcal{F}}_+ = \sum_{\alpha\beta} \langle \alpha | \mathcal{F} | \beta \rangle a_\alpha^+ a_\beta^+ , \qquad \hat{\mathcal{F}}_- = \sum_{\alpha\beta} \langle \alpha | \mathcal{F} | \beta \rangle a_\alpha a_\beta = (\hat{\mathcal{F}}_+)^+ . \tag{6}$$

We shall take for the operators $\hat{\mathcal{F}}_\pm$ the same radial dependence which follows from the macroscopic description of the mode. In this picture one lets the residual interaction assume the form

$$\hat{V}_{\text{res}} \to \bar{\kappa} \hat{\mathcal{F}}_{\text{H}} \hat{\mathcal{F}}_{\text{H}} , \tag{7}$$

where now $\hat{\mathcal{F}}_{\text{H}}$ is the hermitian combination

$$\hat{\mathcal{F}}_{\text{H}} = \sum_{\alpha\beta} \langle \alpha | \mathcal{F} | \beta \rangle \tfrac{1}{2} [a_\alpha^+ a_\beta^+ + a_\alpha a_\beta] . \tag{8}$$

Contrary to the $\hat{V}_{\text{res}}$ given in eq. (5) the macroscopic interaction in (6a) does not strictly commute with the number-of-particles operator. The resulting eigenstates can therefore no longer be associated with a specific mass partition. The operator $\hat{\mathcal{F}}_{\text{H}}$ has however been defined so that it tends to conserve the number of particles in average, as it befits an oscillation represented by the macroscopic variable $\Delta A$.

To isolate the radial dependence one equates the field approximation for the variation of the single-particle potential

$$\delta V \sim \bar{\kappa}\mathcal{F}\Delta A \tag{9}$$

to the macroscopic expression

$$\delta V = \frac{\partial V}{\partial A}\Delta A \approx \frac{R}{3A}\left(\frac{\partial V}{\partial r}\right)\Delta A . \tag{10}$$

In this way the diagonal matrix elements of the field $\hat{\mathcal{F}}_{11}$ (or $\hat{\mathcal{F}}_{+}$, $\hat{\mathcal{F}}_{-}$) in coordinate representation are identified with the surface-localized function

$$\mathcal{F}(r) = \frac{R}{3A\bar{\kappa}}\left(\frac{\partial V}{\partial r}\right) \sim \frac{\mathrm{d}}{\mathrm{d}r}\left[1+\exp\left(\frac{r-R}{a}\right)\right]^{-1}, \tag{11}$$

$V_0$, $R$ and $a$ being the depth, radius and diffuseness parameters of the nuclear potential.

Given the scalar character of the field, the RPA diagonalization of the residual interaction is carried out in the basis of unperturbed configurations

$$|\alpha\rangle = |(n_\alpha l_\alpha j_\alpha)^2_{00}\rangle , \tag{12}$$

where we have restricted the states of paired nucleons with total spin zero to be formed within the same subshell.

The RPA equations are then [6])

$$(\omega_n - 2\varepsilon_\alpha)X(\alpha; n) = (1-2n_\alpha)\sum_\beta \langle\alpha|\hat{V}_{\mathrm{res}}|\beta\rangle X(\beta; n) , \tag{13}$$

where $X(\alpha; n)$ is the coefficient of the configuration $\alpha$ in the expansion of the eigenstate $n$ and $n_\alpha$ denotes an occupation number, 1 and 0 for hole and particle states respectively. The matrix elements of the interaction are, as indicated, of the form

$$\langle\alpha|\hat{V}_{\mathrm{res}}|\beta\rangle = -\kappa q(\alpha)q(\beta) , \tag{14}$$

where the $q$'s, assumed real, are the radial integrals of the field,

$$q(\alpha) = \frac{\sqrt{2}\sqrt{2j_\alpha+1}R}{3A\kappa}\int_0^\infty R^2_\alpha(r)\frac{\partial V}{\partial r}r^2\,\mathrm{d}r . \tag{15}$$

The definition of the unperturbed and correlated energies in these equations are

$$\varepsilon_\alpha = \begin{cases} E_\alpha(A+1) - E_0(A) & \text{for particles } (n_\alpha = 0) \\ E_0(A) - E_\alpha(A-1) & \text{for holes } (n_\alpha = 1) \end{cases} \tag{16}$$

$$\omega_n = \begin{cases} E_{n_+}(A+2) - E_0(A) & \text{for } n \equiv n_+ \in \{A+2\} \\ E_0(A) - E_{n_-}(A-2) & \text{for } n \equiv n_- \in \{A-2\} \end{cases} \tag{17}$$

Note that with these conventions the energies of the bound states of the $(A+1)$ and $(A+2)$ systems are negative.

The wavefunctions $n(\pm)$ of the states in the $(A\pm 2)$ systems are defined as

$$|n_\pm\rangle = \Gamma^+(n_\pm)|0\rangle_{\mathrm{RPA}}\,, \tag{18}$$

where

$$\Gamma^+(n_+) = \sum_\alpha (1-2n_\alpha) X(\alpha; n_+)[a_\alpha^+ a_\beta^+]_{00}\,, \tag{19}$$

$$\Gamma^+(n_-) = \sum_\alpha (1-2n_\alpha) X(\alpha; n_-)[a_\alpha^+ a_\beta^+]_{00}^+\,, \tag{20}$$

and the reference RPA vacuum is such that

$$\Gamma(n_\pm)|0\rangle_{\mathrm{RPA}} = 0\,. \tag{21}$$

The separable character of the interaction makes it possible to obtain the eigenergies $\omega_n$ as solutions of the simple dispersion relation

$$\frac{1}{\kappa} = -\sum_\alpha \frac{(1-2n_\alpha) q^2(\alpha)}{\omega_n - 2\varepsilon_\alpha}\,. \tag{22}$$

For each value of $\omega$ satisfying this equation, the amplitudes $X$ are constructed according to

$$X(\alpha; n) = -\frac{(1-2n_\alpha) q(\alpha)}{\omega_n - 2\varepsilon_\alpha} \left| \sum_\alpha (1-2n_\alpha) \frac{q^2(\alpha)}{(\omega_n - 2\varepsilon_\alpha)^2} \right|^{-1/2}\,, \tag{23}$$

as it follows from the normalization condition

$$\sum_\alpha (1-2n_\alpha) X(\alpha; n_\pm) X(\alpha; m_\pm) = \pm \delta_{n_\pm m_\pm}\,. \tag{24}$$

The expansion coefficients $X$ for the different roots determine the microscopic expression of the corresponding transition densities.

## 3. Miscroscopic pair transition densities

As we have mentioned, the interpretation of collective pairing modes exploits an extension of the standard one-body density which allows for transitions across the mass partition. Let us follow the argument for the specific case of a scalar operator, as it provides a simple way to derive a microscopic expression for the pair transition densities.

The general expression for a one-body operator in the basis of single-particle states $|\alpha\rangle$ is

$$\hat{F} = \sum_{\alpha\beta} \langle \alpha | F | \beta \rangle a_\alpha^+ a_\beta\,. \tag{25}$$

We take for the single-particle basis the set of discrete nuclear states characterized by $\alpha \equiv (nljm)$ and transform the matrix elements into the coordinate-spin representation,

$$\hat{F} = \int \mathrm{d}\boldsymbol{r} \sum_{\nu} \int \mathrm{d}\boldsymbol{r}' \sum_{\nu'} \sum_{\alpha\beta} \langle \alpha | \boldsymbol{r}\nu \rangle \langle \boldsymbol{r}\nu | F | \boldsymbol{r}'\nu' \rangle \langle \boldsymbol{r}'\nu' | \beta \rangle a_{\alpha}^{+} a_{\beta} . \tag{26}$$

For a local, spin-independent field this expression reduces to

$$\hat{F} = \int \mathrm{d}\boldsymbol{r} F(\boldsymbol{r}) \left[ \sum_{\nu} \sum_{\alpha\beta} \Phi_{\alpha}^{*}(\boldsymbol{r}, \nu) \Phi_{\beta}(\boldsymbol{r}, \nu) a_{\alpha}^{+} a_{\beta} \right] \equiv \int \mathrm{d}\boldsymbol{r} F(\boldsymbol{r}) \hat{\rho}(\boldsymbol{r}) , \tag{27}$$

where we have introduced the density operator $\hat{\rho}(\boldsymbol{r})$

$$\hat{\rho}(\boldsymbol{r}) = \sum_{\substack{nljm \\ n'l'j'm'}} \sum_{\nu} \Phi_{nljm}^{*}(\boldsymbol{r}, \nu) \Phi_{n'l'j'm'}(\boldsymbol{r}, \nu) a_{nljm}^{+} a_{n'l'j'm'} . \tag{28}$$

A more compact formula is obtained when we restrict ourselves to the specific case of interest, namely to scalar fields of the form

$$F(\boldsymbol{r}) = \frac{1}{\sqrt{4\pi}} f(r) . \tag{29}$$

For these, a further reduction of the density operator takes place, since the integration over the orientations $\hat{\boldsymbol{r}}$ can be exploited to write

$$\hat{F} = \int_{0}^{\infty} 4\pi r^{2} F(r) \hat{\rho}_{0}(r) \, \mathrm{d}r \tag{30}$$

where the scalar density $\hat{\rho}_{0}(r)$ is now simply defined as

$$\hat{\rho}_{0}(r) = \frac{1}{4\pi} \sum_{nljm} R_{nlj}(r) R_{nlj}(r) a_{nljm}^{+} a_{nljm} . \tag{31}$$

in terms of the radial part of the single-particle wavefunction $R_{nlj}(r)$. The label "0" establishes here the number-conserving character of the operator. Henceforth the indices 0, + and − are used as a short-hand notation for operators involving $\Delta A = 0$, +2 and −2 respectively (see for example the use of these labels in eqs. (5) and (6).)

To ease the introduction of a generalized one-body operator associated with the addition of two particles it is convenient to rewrite this later expression in terms of the operators $b^{+}$ which create a hole, that is

$$\hat{\rho}_{0}(r) = \sum_{nlj} \frac{\sqrt{2j+1}}{4\pi} R_{nlj}(r) R_{nlj}(r) \hat{A}_{0} . \tag{32}$$

In the previous expression the operator

$$\hat{\mathscr{A}}_{0} = [a^{+} b^{+}]_{00} = \sum_{m} \frac{(-1)^{j-m}}{\sqrt{2j+1}} a_{nljm}^{+} b_{nlj-m}^{+} , \tag{33}$$

stands for the creation of a normalized particle-hole state with total spin zero. We adopt here a prescription according to which the extension to particle-particle pairs

is achieved by substituting in (32)

$$\hat{\mathcal{A}}_0 \to \hat{\mathcal{A}}_+ = \sum_m \frac{(-1)^{j-m}}{\sqrt{2}\sqrt{2j+1}} a^+_{nljm} a^+_{nlj-m}\,, \tag{34}$$

which leads to a "one-body" operator of the form

$$\hat{F}_+ = \int_0^\infty 4\pi r^2 F(r)\hat{\rho}_+(r)\,\mathrm{d}r, \tag{35}$$

defined, in analogy to eq. (32), in terms of the generalized density

$$\hat{\rho}_+(r) = \sum_{nlj} \frac{\sqrt{2j+1}}{4\pi} R_{nlj}(r) R_{nlj}(r) \hat{\mathcal{A}}_+\,. \tag{36}$$

The pair-addition transition density is finally defined as the matrix element of this operator which connects the initial and final states in the process, namely

$$\delta\rho_+(r) = \langle n_+|\hat{\rho}_+(r)|0\rangle_{\mathrm{RPA}}\,. \tag{37}$$

In a similar way microscopic expressions can also be written for the removal modes.

## 4. Applications

In this section we present numerical results relevant for two-nucleon transitions in the vicinity of the reference nuclei $^{208}$Pb and $^{40}$Ca. The calculations were performed in a basis of discrete single-particle levels which included at least five main harmonic oscillator shells beyond the Fermi energy $\varepsilon_F$. The energies of the single-particle states were obtained using the parametrization of ref. [7]). The position of the particles and holes in the shells immediately above and below $\varepsilon_F$ were, however, adjusted to agree with the experimental evidence obtained from binding energies and excitation spectra in the $(A \pm 1)$-particle systems. The utilization of a basis of single-particle states which respects the empirical order and separation of the single-particle levels is essential to obtain a correct account of the additional correlation energy which is attributed to the pairing residual interactions.

To construct the transition densities we use harmonic-oscillator single-particle radial wavefunctions. These are appropriate to define the main features of the transition densities. The asymptotic behaviour of the radial functions $R_{nl}(r)$ is, on the other hand, not quite adequate to describe the tail of the densities at large values of $r$, a feature which—in the context of this study—is not overly restricting.

The coupling strength was adjusted in all examples to reproduce the experimental value of the binding energy of the $(A+2)$-particle systems, i.e. $^{210}$Pb, $^{210}$Po, and $^{42}$Ca. Contrary to the case of standard RPA for inelastic excitations, where the roots are degenerate in $\pm\omega$, the adjustment of strength through the ground-state energy of the pair addition mode does not automatically ensure a good result for the pair-removal mode. In fact, the solutions of the dispersion relation (20), cover the modes of both the $(A \pm 2)$ systems. One may in principle allow for different coupling

strengths $\kappa_{\pm}$ in either case [8]). In our analysis, however, rather accurate predictions of the ground-state energies of the $(A-2)$ systems were obtained with $\kappa_{+}=\kappa_{-}$(cf. table 1).

In the top part of fig. 1 we show the transition densities obtained for the ground-to-ground state neutron transfer from $^{208}$Pb to $^{206}$Pb and $^{210}$Pb. The function $\delta\rho(r)$ which is displayed follows from the expression

$$\delta\rho_{\mathrm{N}}(r)=\sum_{nlj}\frac{\sqrt{2j+1}}{4\pi}R^{2}_{nlj}(r)X(nlj;\,\mathrm{N})\,. \tag{38}$$

As it can be seen from eq. (35), the folding of the density operator with the radial dependence of the fields always includes the spherical differential volume $4\pi r^2$. Because of this factor, the function $\delta\rho(r)$ near the origin carries no actual statistical weight. To avoid this distortion in the graphical presentation of the transition density it is preferable to consider the more significant products $4\pi r^2\delta\rho(r)$. These functions, for the current example, are plotted at the bottom of fig. 1. By displaying the relevant part of the radial dependence, a close similarity in both shape and magnitude of the pair-addition and pair-removal quantities is revealed.

TABLE 1

Experimental and RPA energies for the ground states of the nuclei quoted in column 1. The coupling constant for the RPA calculation was adjusted to reproduce the ground state of the $A+2$ systems, and thus we fill only with a dash the corresponding fields.

| Nucleus | $\omega_{\mathrm{exp}}$ (MeV) | $\omega_{\mathrm{RPA}}$ (MeV) |
|---|---|---|
| $^{210}$Pb | −9.12 | |
| $^{206}$Pb | −14.11 | −14.18 |
| $^{210}$P0 | −8.78 | |
| $^{206}$Hg | −15.38 | −15.38 |
| $^{42}$Ca | −19.84 | |
| $^{38}$Ca | −28.93 | −27.98 |

The quantity $\beta_{\mathrm{p}}$ can be obtained from the microscopic calculation by simple quadrature, namely

$$\beta_{\mathrm{p}}=\int 4\pi r^{2}\delta\rho\,\mathrm{d}r\,. \tag{39}$$

Thus, the convenience of the representation used in the lower part of fig. 1 is further emphasized, as the deformation parameter corresponds directly to the area under the given curves. For the ground-state transitions to $^{206}$Pb and $^{210}$Pb the extracted numbers are respectively $\beta_{\mathrm{p}}=16$ and 18. With these values one can now proceed to construct the macroscopic transition density

$$\delta\rho(r)\Big|_{\mathrm{macro}}=\beta_{\mathrm{p}}\left(\frac{R}{3A}\right)\frac{\partial\rho(r)}{\partial r}\,, \tag{40}$$

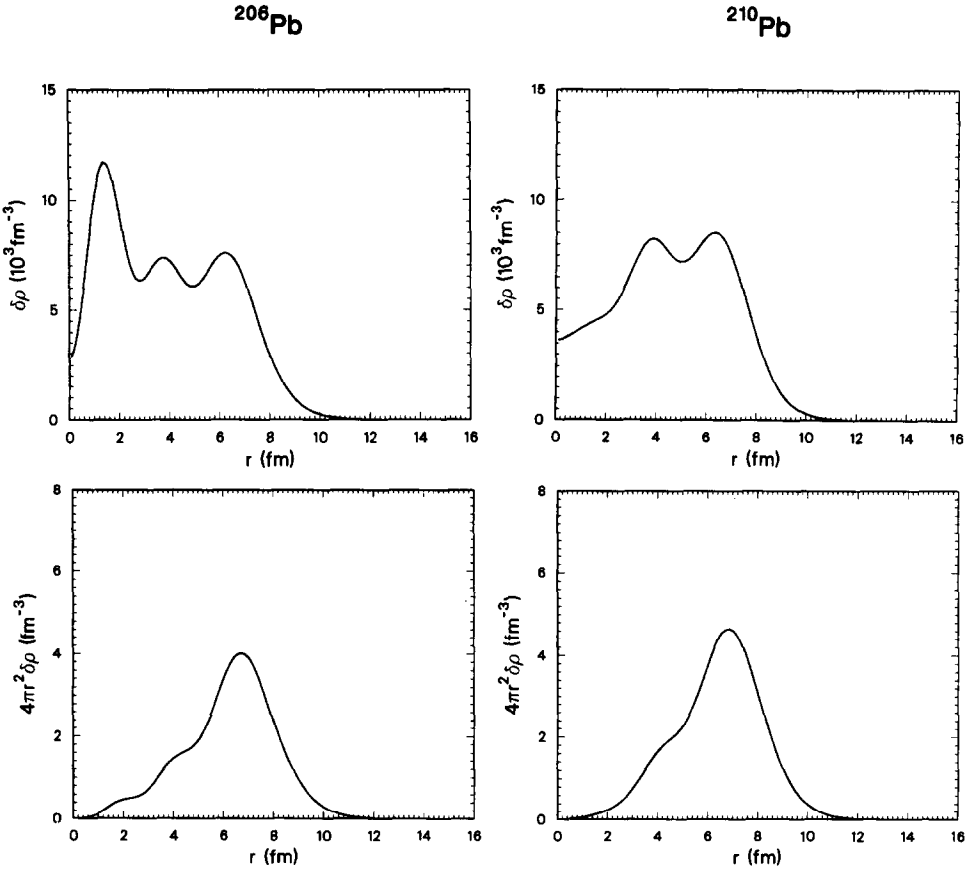


Fig. 1. Transition densities for the pair-removal (left) and pair-addition (right) neutron modes around $^{208}$Pb. At the bottom the same quantities are displayed multiplied by the volume element $4\pi r^2$.

and compare their radial dependences. For applications involving the construction of pair-transfer formfactors the tail behaviour of the macroscopic approximation should be important. For this reason we have chosen this time to display the curves in fig. 1 together with their macroscopic counterparts in a logarithmic scale. One can see in fig. 2 that—aside from the incorrect parabolic behaviour of the RPA-constructed density—the overall magnitude and qualitative shape of the functions is similar. The poor asymptotic dependence of the microscopic quantity is in this calculation traced to the use of harmonic oscillator wavefunctions. A matching procedure in the tail region should change the radial behaviour to a pure exponential, a result which the macroscopic expression yields correctly given its direct link to the equilibrium nuclear density.

In fig. 3 we show the transition densities for the ground-state pair-proton transfers from $^{208}$Pb to $^{206}$Hg and $^{210}$Po. The function $\delta\rho(r)$ once more presents in the interior characteristic features which depend on the hole configurations which dominate the

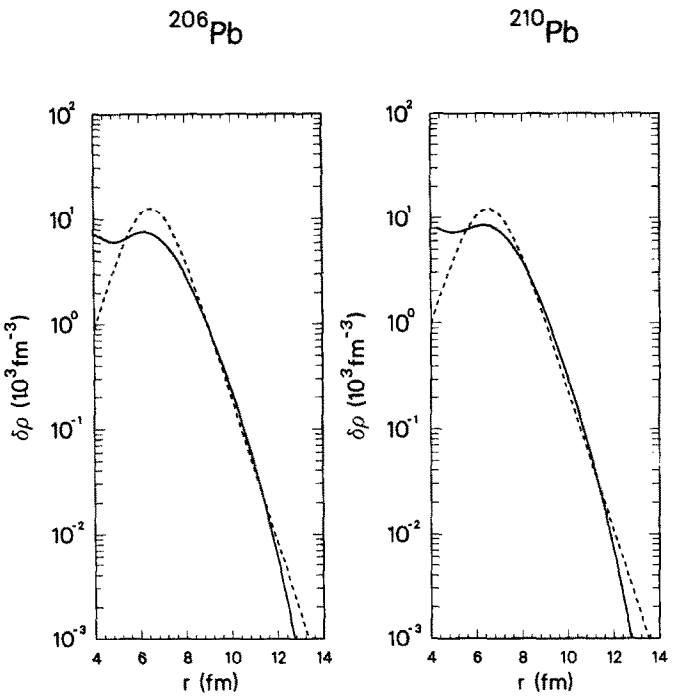


Fig. 2. Comparison between the microscopic transition densities for the neutron modes in $^{208}$Pb (full line) and the macroscopic expression (dash line). For the latter the density was taken to be a Fermi function with parameters $\rho_0 = 0.17$ fm$^{-3}$, $r_0 = 1.1$ fm and $a = 0.6$ fm.

microscopic wavefunction. As it is shown in the lower part of the figure, the inclusion of the spherical jacobian restores a more symmetric balance between the pair-addition and the pair-removal mode. The microscopic values of the pairing deformation parameter turn out to be in this case $\beta_p = 13$ and 11, respectively.

The general features we have in detail presented above for the four pair transfer modes around $^{208}$Pb are also present in the other systems we have studied. In fig. 4 we give the results for $^{42}$Ca and $^{38}$Ca. In this case the microscopic values of the deformation parameter are $\beta_p = 8$ and 7, respectively.

## 5. Summary

In this contribution we have constructed transition densities $\delta\rho$ for pairing collective modes in the vicinity of closed-shell nuclei. The wavefunctions in the particle-particle and hole-hole basis were calculated exploiting an RPA-type formalism. The inclusion of backward amplitudes takes into account ground-state correlations which turn out to be important. Characteristic of this study is the use of matrix elements for the separable residual interaction which corresponds to a surface-localized field. The behaviour of the transition densities in the surface region can

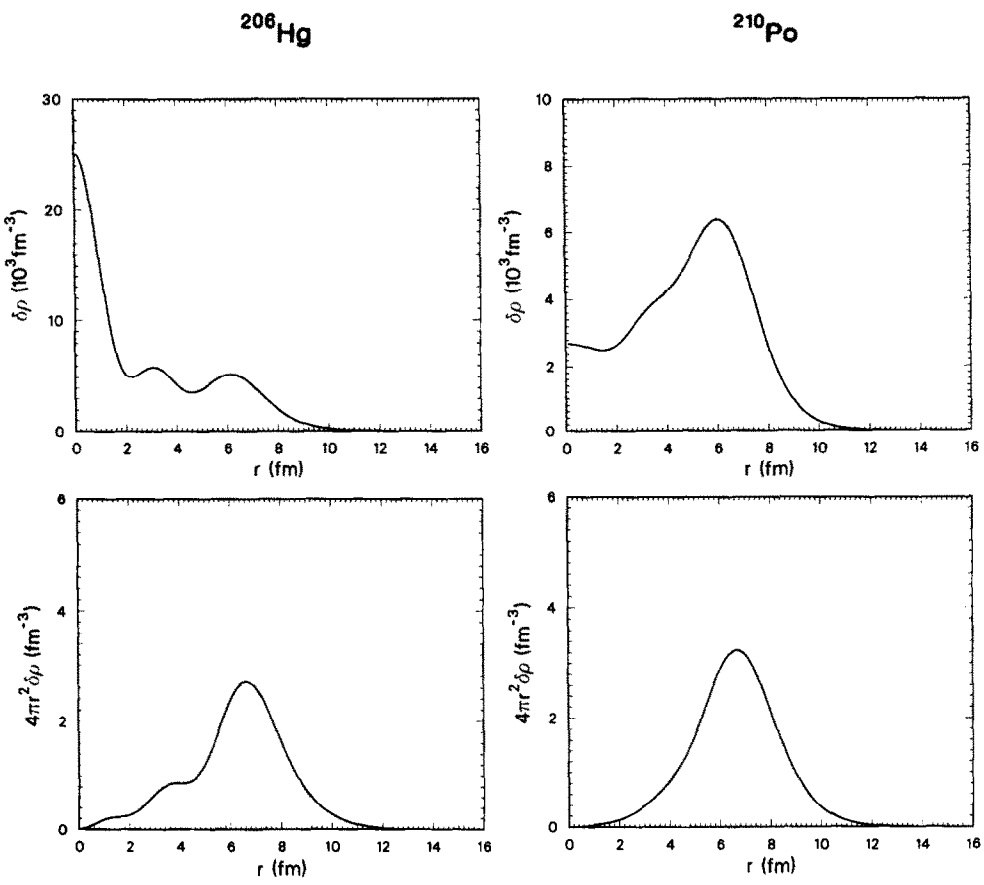


Fig. 3. Transition densities for the pair-removal (left) and pair-addition (right) proton modes around $^{208}$Pb. At the bottom the same quantities are displayed multiplied by the volume element $4\pi r^2$.

be conveniently emphasized by displaying the functions $4\pi r^2\delta\rho$; in this choice of presentation a similarity in magnitude and shape between addition and removal modes is revealed. An interpretation of both these modes as a manifestation of collective vibration across the mass partition appears then plausible. A natural macroscopic variable to describe this kind of generalized motion is the variation in the number of particles, $\Delta A$. In terms of this coordinate, simple arguments relate the shape of the transition densities to a derivative of the static density which peaks at the nuclear surface.

As shown in the applications this procedure works quite well in the area of interest, i.e. for large values of $r$. The microscopically calculated transition densities do not vanish in the nuclear interior, a region which is however not accessible by reactions taking place at grazing distances. Simple quadrature of the functions $4\pi r^2\delta\rho$ yields microscopic values for the deformation parameters $\beta_p$. In the examples

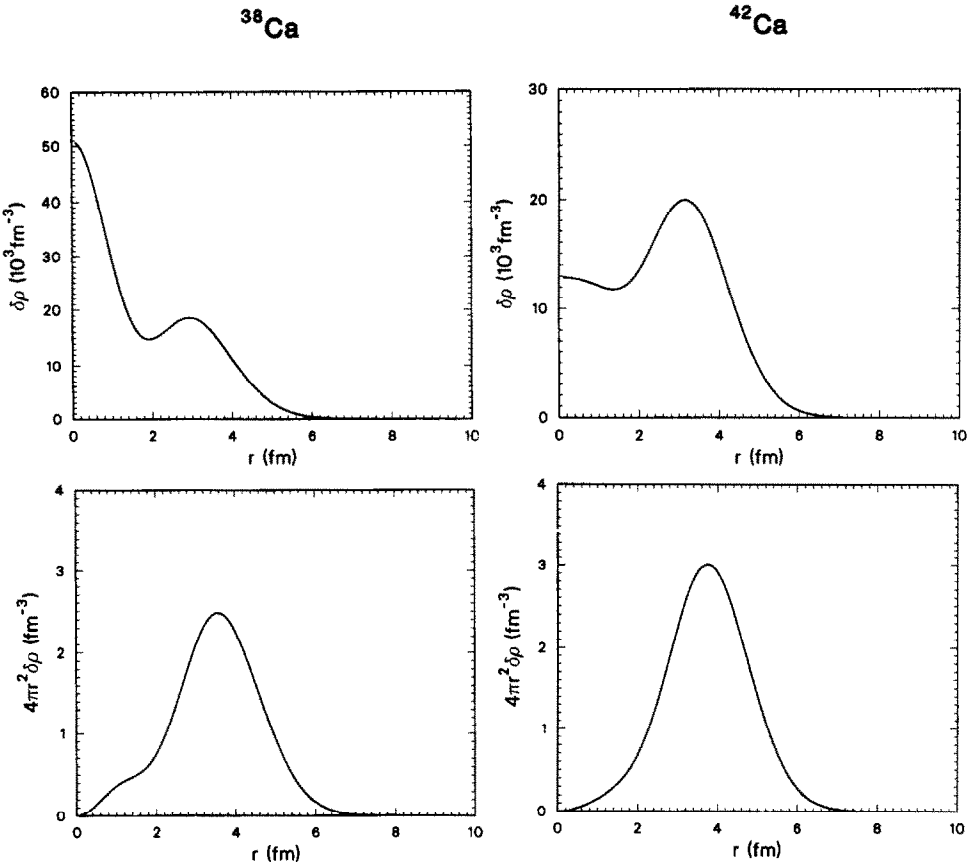


Fig. 4. Transition densities for the pair-removal (left) and pair-addition (right) neutron modes around $^{40}$Ca. At the bottom the same quantities are displayed multiplied by the volume element $4\pi r^2$.

presented in this paper the resulting values are found in order-of-magnitude agreement with those tentatively extracted from the macroscopic analysis of two-nucleon transfer data [2]). Contrary to the microscopic quantities, these empirically extracted numbers are sensitive to the dynamical aspects of the reaction mechanism. Consequently, they are likely to reflect the characteristic energy dependence of second- and higher-order processes. Some of these questions have been recently investigated by other authors in ref. [9]). The results of the present study for pairing vibrations in normal systems are consistent with the conclusions reached in [5]) for superfluid deformed systems.

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