FINITENESS EFFECTS IN THE ANALYSIS OF MULTIFRAGMENTATION EVENTS IN THE SPHERICITY-COPLANARITY PLANE

J.P. BONDORF, C.H. DASSO

Niels Bohr Institute, University of Copenhagen, Blegdamsvej 17, DK-2100 Copenhagen Ø, Denmark

R. DONANGELO

Instituto de Fisica da UFRJ, 21945 Rio de Janeiro, RJ, Brazil

and

G. POLLAROLO

Dipartimento di Fisica Teorica dell'Università di Torino, and INFN Sezione di Torino, via P. Giuria 1, I-10125 Turin, Italy

Received 28 November 1989; revised manuscript received 31 January 1990

In observations of multifragmentation processes with intermediate mass fragments, spherical events are apparently missing. This result turns out to be inferred only because of the small number of fragments which are typically involved. Numerical simulations show that for low multiplicity events the sphericity and coplanarity variables introduced for such analysis are not adequate for extracting signals of the shape of the emitting source. We recommend alternative global variables which are more sensitive to the fragmentation pattern.

In this paper we shall discuss some problems connected to the extraction of signals from observed multifragment events in energetic collisions between nuclei. The number of emitted particles in each event is generally large, and it depends on the size and energy of the system. There are often limitations on the number of particles representing an event, either because one only looks for a specific, not too abundant, kind of particles or because many experiments are not exclusive enough to warrant a sufficient number of particles. In recent experiments at GANIL [1] events with a few fragments of intermediate mass larger than the alpha particle (in the following denoted IMF) have been detected.

Most models for collisions between heavy ions result in concrete predictions for the momentum distribution of emitted fragments. As an example, the simple fireball model [2] predicts a three-source momentum distribution which is elongated in space along the beam direction (rod-like). Collisions within hydrodynamic models may generate, on the other hand, momentum distributions for central collisions which are flat (disc-like) and perpendicular to the beam. Any of these model distributions can be used for generation of simulated events. Conversely, an analysis of experimental data should make it possible to extract information about the underlying model functions and thereby establish which model is more suitable to describe the collision. The model distribution functions depend on the masses, charges and bombarding energy. While these variables are selected in advance by the experimentalist, the scattering plane and the impact parameter can only be inferred from the data by a model-dependent analysis of the reaction products event by event.

In the literature it has become fashionable to analyse IMF events by means of the quadratic momentum tensor, the application of which was discussed for heavy ion collisions in ref. [3]. An analysis of experimental data for heavy ion reactions with high multiplicities of light baryons was also done in ref. [4]. In high-energy physics the method was used even

0370-2693/90/$ 03.50 © Elsevier Science Publishers B.V. (North-Holland)

earlier to analyse jet events in (e^+, e^-) collisions [5]. The momentum tensor characterizes the shape of the momentum distribution for some – or all – of the detected particles in a collision event. The shape is given in terms of a few global variables which can express whether the event appears as elongated, spherical or disc shaped. Also the orientation of the shape can be determined. Intuitively one may expect that the extracted shape of every event will reflect that of the model function, maybe with some fluctuation around an average behaviour. This is indeed correct for events involving a very large number of fragments. However, in IMF studies of nuclear reactions this expectation turns out to be unjustified, simply because effects associated with the finite number of detected particles dominate.

This important limitation was noted by Cugnon et al. [6] and by Danielewicz and Gyulassy [7] already several years ago in the context of reactions at higher energies, where mostly light fragments are produced. In the present contribution we would like to bring renewed attention to the conclusions of their studies. These acquire special relevance in the analysis of intermediate mass events where, due to charge and baryon conservation, no more than a few fragments are ever involved. For this purpose we would like to side-step the formal treatment that has been used in ref. [7] to develop the arguments and employ, instead, the simple vehicle of numerical simulations. By using the standard sphericity and coplanarity variables we hope to illustrate the risks involved in drawing conclusions from quadratic momentum tensors constructed with a small number of fragments.

The momentum tensor for a multiparticle event with F detected ejectiles (called fold F), is defined [3] as

$$Q_{ij} = \sum_{\nu=1}^{F} \gamma^{(\nu)} p_i^{(\nu)} p_j^{(\nu)} , \qquad (1)$$

where $p_i^{(\nu)}$ is the momentum coordinate in the direction i. The quantity $\gamma^{(\nu)}$ is put equal to $(2m_\nu)^{-1}$ which means that we use the kinetic energy flow tensor variant of (1). The quantity m_ν is the ejectile mass. In some cases it is advantageous to project out from the events only some which obey certain conditions. The 3×3 tensor Q_{ij} with i or $j = x, y$ or z is symmetric. It has three eigenvalues $\lambda_1 \leqslant \lambda_2 \leqslant \lambda_3$ which characterize the three principal axes of that ellipsoid which has the average shape of the event. The directions of the principal axes will be denoted by $\boldsymbol{e}_1$, $\boldsymbol{e}_2$ and $\boldsymbol{e}_3$ respectively. The momenta may be given in any system, but in order to describe the momentum shape of the event the most representative way is to use a system close to its CM system. The global variables of the tensor can be directly the λ's and the $\boldsymbol{e}$'s or, alternatively, any functions of them. In ref. [1] the sphericity S and the coplanarity C were used to characterize the events. They are defined as follows:

$$S = \frac{3}{2} \frac{\lambda_1 + \lambda_2}{\lambda_1 + \lambda_2 + \lambda_3} , \qquad (2)$$

$$C = \frac{\sqrt{3}}{2} \frac{\lambda_2 - \lambda_1}{\lambda_1 + \lambda_2 + \lambda_3} . \qquad (3)$$

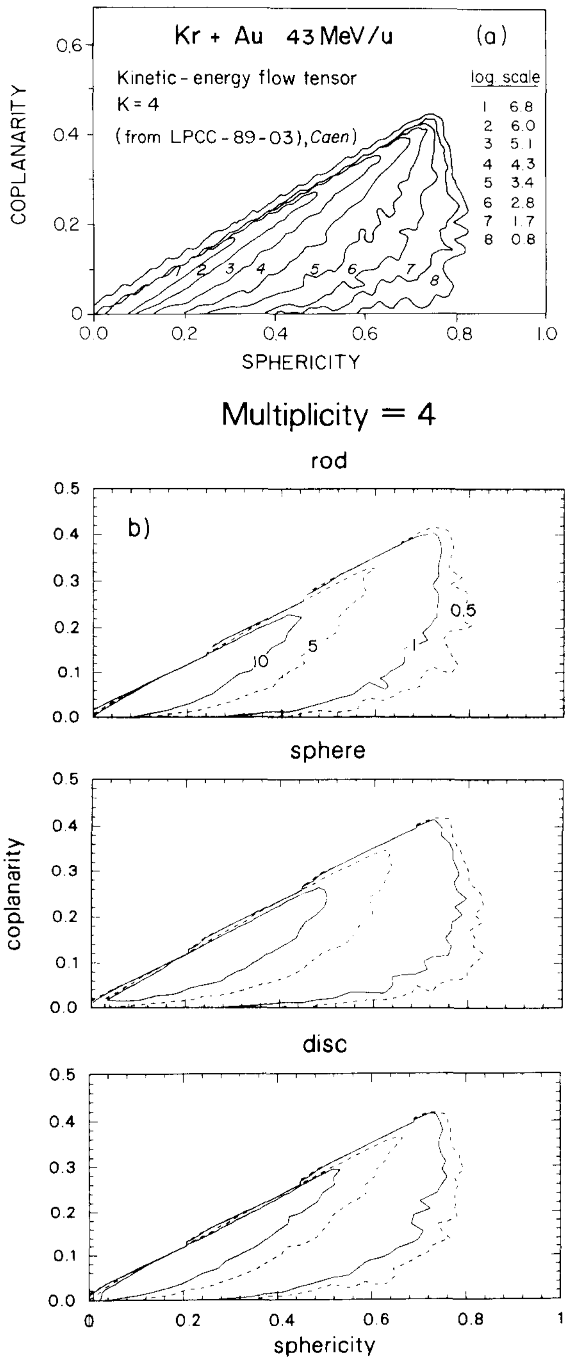
In the analysis of IMF events, the total multiplicity M_{tot} can be divided in two contributions, $M_{\text{tot}} = F + M_{\text{res}}$, where M_{res} includes all emitted particles that are not used for the construction of the momentum tensor. In such case, the sum

$$\boldsymbol{p}_F = \sum_{\nu=1}^{F} \boldsymbol{p}^{(\nu)} \qquad (4)$$

would fluctuate from one event to another, because only the total momentum is conserved. The division of the total multiplicity into intermediate mass fragments and other fragments (gas particles) was discussed within the multifragmentation model in ref. [8].

In fig. 1a we show an experimental distribution of (S, C) for $F=4$ observed in the reaction Kr+Au at 43 MeV per nucleon [1]. For this analysis a reaction frame for the IMF was identified which collects 75% of the total momentum content. Then for each event where F fragments were detected, an additional particle was added in the construction of Q_{ij}, to insure momentum conservation. The part of the (S, C) plane occupied by the distribution indicates an apparent lack of spherical events. A priori, one would expect these to lie in the lower right hand corner of the plot, near $(S, C) = (1, 0)$. The events populate, instead, the tilted line in which disc- and rod-like events are expected.

We now want to make Monte Carlo simulations based on events constructed from some model distribution functions and then compare them with the experiments. As already mentioned, the collision pro-



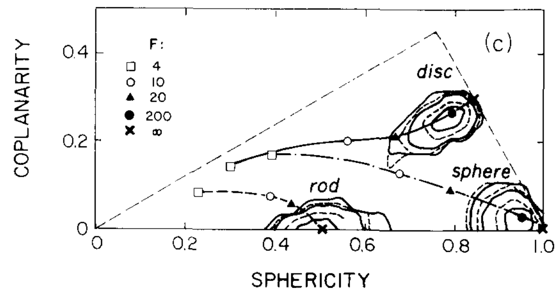


Fig. 1. Distribution of sphericity and coplanarity values of multifragment events. (a) Experimental distribution for $F=4$ observed in the collision of 43 MeV per nucleon Kr incident on Au nuclei (data of ref. [1]). (b) Distributions of $F=4$ events generated with the parameters: $\sigma_x^{\text{red}}=\sigma_y^{\text{red}}=0.5$, $\sigma_z^{\text{red}}=1$ (rod); $\sigma_x^{\text{red}}=\sigma_y^{\text{red}}=\sigma_z^{\text{red}}=1$ (spherical) and $\sigma_x^{\text{red}}=\sigma_y^{\text{red}}=1$, $\sigma_z^{\text{red}}=0.5$ (disc). Each plot contains $N=50\,000$ simulated events. See text for further details on the event generation procedure. (c) Same as fig. (b) for $F=200$. The three points marked with crosses correspond to the $F\to\infty$ limit. In (c) we have also indicated the trajectories of the maxima of the (S, C) distributions for the three event shapes as functions of the sample size. Points for the sample sizes $F=20$, 10 and 4 (besides the already mentioned $F\to\infty$ and $F=200$) are marked on the trajectories.

cesses are very complicated. Even the simplest fireball model operates with three emitting sources. A model distribution which has only one source and which is specific for the momentum tensor analysis is the three-dimensional gaussian distribution

$$f(\boldsymbol{p}) = \frac{1}{(2\pi)^{3/2}\sigma_x^\nu \sigma_y^\nu \sigma_z^\nu} \times \exp\left(-\frac{p_x^2}{2\sigma_x^{\nu 2}} - \frac{p_y^2}{2\sigma_y^{\nu 2}} - \frac{p_z^2}{2\sigma_z^{\nu 2}}\right). \quad (5)$$

The variances have been given a formal expression as it would follow from a simple thermodynamical equilibrium,

$$\sigma_i^\nu = \sqrt{m_\nu}\,\sigma_i^{\rm red}, \quad (6)$$

where the reduced variances $\sigma_i^{\rm red}$ are numbers whose values can be set arbitrarily between 0 and 1. In fact, because of the dimensionless character of (2) and (3) only the relative magnitudes of these quantities for $i=x, y, z$ are relevant. If all three $\sigma_i^{\rm red}$ are equal one obtains the standard spherical Maxwell distribution. We consider model functions with cylindrical symmetry along the z-axis such that $\sigma_x^{\rm red} = \sigma_y^{\rm red}$. The orientation of the z axis in space can be defined in various ways, depending on the interpretation of the reaction. When $\sigma_z^{\rm red} > \sigma_x^{\rm red}$ then the distribution function is elongated or rod-like, and when $\sigma_z^{\rm red} < \sigma_x^{\rm red}$ then the function is flat or disc-like. The explicit dependence on mass introduced in (6) is only one of several possibilities. The conclusions of this paper remain the same if this dependence in σ_i^ν is omitted. The masses of the IMFs are chosen according to a distribution function which also reflects some gross features of the mass distribution for multifragmentation,

$$P(m) = \frac{1}{\lambda}\exp\left(-\frac{m}{\lambda}\right), \quad (7)$$

where λ has been fixed to be 20.

Random events with a given fold F have been generated according to the functions (5) and (7) for different sets of parameters. In each event an extra fragment with 40% of the total mass and momentum $-\boldsymbol{p}_F$ was added in the construction of Q_{ij}. In this way the theoretically generated momentum tensors correspond as closely as possible to the experimental ones. The momentum tensor (1) is calculated for a large number N of events, and the global variables S and C are extracted for each case.

In fig. 1b we have plotted the (S, C) distributions for rod-like, spherical and disc-like distribution functions using $\sigma_z^{\rm red}$ equal to 2, 1 and 0.5 times the $\sigma_x^{\rm red} = \sigma_y^{\rm red}$ values respectively. The calculations are for $F=4$. It is seen that the three calculated (S, C) distribution shapes look very much alike, with the spherical shape fitting the experimental distribution fig. 1a slightly better than the two others. Thus it is not very useful to use (S, C) to extract information about the shape of the model distribution. Even with F as high as 20 the problem still persists. It is also seen that the spherical distributions are not at all placed near the point $(S, C) = (1, 0)$ for this low multiplicity. Qualitatively this "lack of spherical events" in the (S, C) distribution can be explained from the fact that two-particle events always give rod-like shapes, while three-particle events tend to give disc shapes in the analysis. For events with a limited multiplicity the sampling of momenta is too poor to represent accurately the features of the model distribution function. By increasing F to very large values the shapes of the plots are drastically altered. In fig. 1c we show results of calculations with the same distribution functions as in fig. 1b, but now with $F=200$. It is in this limit of relatively large F that the variance analysis of ref. [7] is applicable. Also in fig. 1c we have shown how the maxima of the distributions evolve in the (S, C) plane as functions of F. It is seen how they converge for small values of F. One should notice that by using $F=200$ in combination with $\lambda=20$ one works with an unphysically large mass of the total system. By changing in this case the parameter λ to a more realistic smaller value, the (S, C) plots, however, are practically unaffected. In the same figure we have plotted the (S, C) points for $F\to\infty$ calculated by replacing λ_i by $(\sigma_i^\nu)^2$. It is seen that the distributions are rather narrow and located close to the three asymptotic points, which corresponds well to the intuitive expectations. We have investigated how the (S, C) plots depend on other values of F and find that in order to have clearly separated (S, C) distributions for the three shapes, F should be larger than ~ 50.

We have seen that there are problems with the use of the variable set (S, C) as a source of information in reactions with limited multiplicity. Consequently global variables have to be introduced, which do not suffer from this insensitivity. As an example we use the angles between the three axes of the analysed event

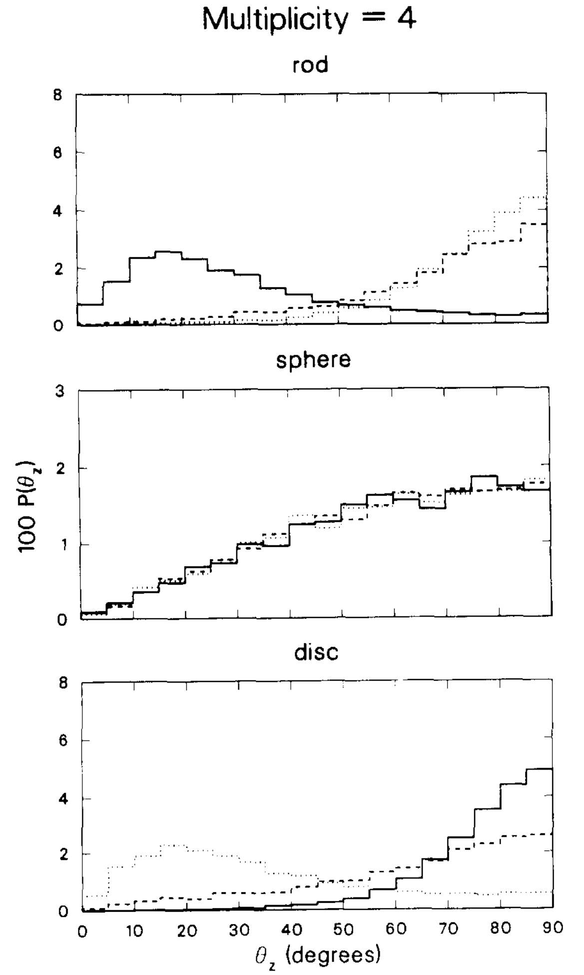


Fig. 2. Distribution of the angles θ_i between the principal axis of the momentum tensor and the beam axis for the same cases of fig. 1b. The full line corresponds to the axis associated to the smallest eigenvalue, λ_1, the dashed line to the intermediate one, λ_2, and the dotted line to the largest one, λ_3.

and the model axis (the z-direction). In fig. 2 we have for the same parameters as in fig. 1b plotted the distributions of the angles θ_i between the "event axes" $\boldsymbol{e}_1$, $\boldsymbol{e}_2$ or $\boldsymbol{e}_3$, and the z-direction, always taking that complement of the angle which is $\leqslant 90°$ (because of the triple reflection symmetry of the distribution function). It is seen that the angle distributions are very different in the three cases of distribution functions, thus suggesting that global angle variables might be useful to distinguish between rod-like, spherical and disc-like events, even if the multiplicity is as low as 4. One may note that there is a difference between the distributions for the angles $\sim 90°$, despite the fact that the model width-parameters for the corresponding directions are equal. This is a consequence of the ordering of the eigenvalues λ_1, λ_2, λ_3, and the difference vanishes as $F \to \infty$. It is worth pointing out that a suggestion to use an angle-dependent global variable was done in ref. [6].

In conclusion: When the number of fragments representing multifragmentation events is small, this dominates completely the event pattern in the (S, C) representation and thus one cannot easily find the underlying signals in these variables. We recommend to use angle variables instead, maybe also in other combinations than those used here. The possibilities of choosing global variables in cases which are more complicated than the single sources considered in this paper are legio. Many experiments are, for instance, analysed with source functions which are characteristic for a special dynamical reaction model. Also in such cases the finiteness problem is as important as in the present note, but to our knowledge has not been properly treated in the literature. Thus much more work in this field is required.

We thank the authors of ref. [1] and Dr. S.M. Lee for pointing out to us the apparent lack of observed spherical events. H. Bøggild and J. Knoll are thanked for informative discussions. This work was supported in part by grants from The Danish Natural Science Research Council, the Brazilian CNPq and the Bundesministerium für Forschung und Technologies, BRD (06 LM 171 II).

References

[1] R. Bougault et al., preprint Université de Caen, Laboratoire de Physique Corpusculaire, LPCC 89-03.

[2] G. Westfall, J. Gosset, P.J. Johansen, A.M. Poskanzer, W.G. Meyer, H.H. Gutbrod, A. Sandoval and R. Stock, Phys. Rev. Lett. 37 (1976) 1202.

[3] J. Cugnon and D. L'Hote, Nucl. Phys. A 397 (1983) 519.

[4] P. Danielewicz and G. Odyniec, Phys. Lett. B 157 (1985) 146.

[5] G. Hanson et al., Phys. Rev. Lett. 35 (1975) 1609.

[6] J. Cugnon, J. Knoll, C. Riedel and Y. Yariv, Phys. Lett. B 109 (1982) 167.

[7] P. Danielewicz and M. Gyulassy, Phys. Lett. B 129 (1983) 283.

[8] H.W. Barz, J.P. Bondorf, R. Donangelo and H. Schulz, Phys. Lett. B 169 (1986) 318.