

Sub-barrier transfer reactions of $^{32}\text{S} + ^{64}\text{Ni}$

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Abstract

The quasi-elastic scattering and the one-nucleon transfer channels of $^{32}\text{S} + ^{64}\text{Ni}$ have been studied using the LNL Recoil Mass Spectrometer. The excitation function at $\theta_{\text{c.m.}} = 170^\circ$ from $E_{\text{lab}} = 68.3$ to 92.4 MeV and an angular distribution at $E_{\text{lab}} = 81.3$ MeV from $\theta_{\text{c.m.}} = 120^\circ$ to 170° have been measured. The results have been analyzed in the framework of the complex WKB approximation and in the semiclassical approximation based on Coulomb trajectories.

Key words: NUCLEAR REACTIONS $^{64}\text{Ni}(^{32}\text{S}, \text{X})$, $(^{64}\text{Ni}, ^{63}\text{Ni}, ^{65}\text{Cu})$ $E = 68.3\text{--}92.4$ MeV; measured relative $\sigma(E)$ at $\theta_{\text{c.m.}} = 170^\circ$, $\sigma(\theta)$ relative at $E_{\text{lab}} = 81.3$ MeV from $\theta_{\text{c.m.}} = 120^\circ$ to 170° ; deduced absolute $\sigma(E, \theta)$. Enriched target. Complex WKB approximation. Semiclassical approximation.

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1. Introduction

It is well-known that quasi-elastic transfer reactions account for the largest fraction of the total reaction cross section of heavy-ion collisions at energies below the Coulomb barrier. In spite of this, few data exist at such low energies where the transfer takes place at large internuclear distance. The reason can be found in the experimental difficulties to identify transfer products at sub-barrier energies for systems where the projectile has a significant fraction of the target mass. In those cases the cross sections are generally very small and the back-scattered projectile-like fragment has such a low energy that the usual techniques of measurement and identification become useless. One way to overcome these difficulties to measure transfer at large internuclear distances is to use the inverse kinematic reactions [1], another is the measurements of angular distributions at energies near the Coulomb barrier where the angle-dependence of the distance of closest approach probes the transfer at varying internuclear distances (see e.g. refs. [2,3]).

Another interesting method consists in exploiting the fact that, associated with the low-energy back-scattered projectile-like ion, there is a complementary target-like fragment which recoils to forward angles with a large fraction of the incident beam energy. This technique has been previously used at the Daresbury recoil mass separator [4,5], where the excitation functions for one-neutron pick-up at $\theta_{\text{c.m.}} = 180^\circ$ have been studied for the systems $^{58}\text{Ni} + ^{116,154}\text{Sm}$ in an energy range from near the Coulomb barrier to ~ 40 MeV below. It has been observed that transfer probabilities generally increase with target mass and cross sections of ~ 1 mb/sr have been measured at the lowest energies. The slopes of the excitation functions for the heavier Sm isotopes were initially believed to be significantly smaller than theoretical predictions [5]. However, recent re-analyses of the reactions have shown results consistent with theory [6]. Similar measurements of 180° sub-barrier transfer reactions have been also reported by Herman et al. [7] for $^{32}\text{S} + ^{92,98}\text{Mo}$.

One of the main motivations of this work was to test the applicability of the complex WKB approximation to the analysis of single-nucleon transfer from energies well below the barrier to energies above the barrier. We report here on the results about one-particle transfer and total quasi-elastic cross sections for the system $^{32}\text{S} + ^{64}\text{Ni}$ at energies around and well below the Coulomb barrier. The presence of strong nucleon transfer channels for this system has been established from direct measurements above and around the Coulomb barrier [8]. Elastic scattering [9,10] (in almost the same energy interval) and fusion cross sections [10,11] have also been measured. The experimental set-up and procedures are described in sect. 2, whereas sect. 3 presents the results of the experiment. In sect. 4 the complex WKB approximation and the semiclassical theory are outlined in some detail. In the same section the theoretical calculations are compared with the present data and those mentioned in ref. [8], thus concluding the paper.

2. Experimental

2.1. General

Transfer reactions of ^{32}S (beam) + ^{64}Ni have been measured at the XTU Tandem of the Laboratori Nazionali di Legnaro. The target-like particles recoiling at forward angles were detected by the Recoil Mass Spectrometer (RMS). One angular distribution has been measured at $E_{\text{lab}} = 8.13$ MeV from $\theta_{\text{lab}} = 5^\circ$ to 30° for the Ni-like particles ($\theta_{\text{c.m.}} = 170^\circ$ to 120° for the S-like ejectiles) and the excitation function has been measured at $\theta_{\text{lab}} = 5^\circ$ ($\theta_{\text{c.m.}} = 170^\circ$) from $E_{\text{lab}} = 68.3$ to 92.4 MeV ($E_{\text{c.m.}} = 45.5$ to 61.6 MeV). For reference, the nominal Coulomb barrier [12] is around $E_{\text{lab}} = 89$ MeV. Energies and angles given in the laboratory frame refer to the measured particles, i.e. Ni-like, while the data given in the center-of-mass frame refer to the S-like nuclei. We will adopt the usual conventions by calling one-nucleon stripping the transfer of one nucleon from the projectile to the target and one-nucleon pick-up the inverse process.

2.2. Experimental set-up

The ^{32}S beam was delivered on a ^{64}Ni target placed in the sliding seal scattering chamber of the RMS with typical currents of approximately 10 pA. Two silicon surface barrier detectors at forward angles were used to normalize between different runs and to control the beam quality. The target was made by rolling metallic Ni to a thickness of $330 \mu\text{g}/\text{cm}^2$. The isotopic content of the target, as determined by the supplier, was: 93.4% of ^{64}Ni , 0.74% of ^{62}Ni , 0.10% of ^{61}Ni , 1.78% of ^{60}Ni , and 3.98% of ^{58}Ni . The target was a vertical strip of 1 mm horizontal width in order to get a well-defined optical object in the dispersion plane of the spectrometer.

The RMS [13] produces a dispersion proportional to the mass/charge-state (M/q) ratio across the focal plane in the horizontal direction. Particles of one specific M/q converge to a single point focus regardless of the initial energy. Other principal features of the instrument are the wide acceptance in angle (max. ≈ 10 msr), energy (max. $\approx \pm 20\%$) and mass (max. $\approx \pm 7\%$), and the angular rotation around the target from $+5^\circ$ to -50° . A proper compromise between maximum performance in either parameter must be selected for each particular experiment.

The focal plane detector was an (X, Y)-position sensitive parallel plane avalanche counter (PPAC) backed by a 43 cm long Bragg chamber in the same gas volume (isobutane) [14]. Both the window and the cathode were $200 \mu\text{g}/\text{cm}^2$ Mylar (the second one aluminized on both faces). The intrinsic resolution of the position detector was ± 0.5 mm. The entrance window was $80 \times 50 \text{ mm}^2$ and was placed 1 m far apart from the RMS optical exit. The window size restricted the

used portion of the focal plane to approximately one half in the horizontal direction.

The electric and magnetic fields were adjusted in order to focus the elastic peak ($M = 64$) with the 17^+ charge-state in the center of the focal plane. The effective RMS solid-angle acceptance during the experiment was limited to 5 msr with an entrance collimator. It corresponds to $\pm 2^\circ$ in the reaction plane ($\approx 4^\circ$ in the center-of-mass frame). The energy acceptance in these conditions was estimated to be larger than $\pm 12\%$ over the focal plane, which was enough to accept the quasi-elastic reaction products.

The electric rigidity E/q of the beam and the detected particles differed by a factor 2.5 only. This fact, together with the large separation between the cylindrical plates in the first electrostatic dipole (useful characteristic to achieve a wide energy acceptance) made insufficient the rejection at 0° of the beam-like particles, scattered mainly from the anode of that electric dipole. For this reason, the smallest detection angle was $\theta_{\text{lab}} = 5^\circ$.

2.3. Data reduction and calibration

The horizontal (X) and vertical (Y) focal plane positions, the total energy loss in the Bragg chamber (E) and the Bragg peak (BP) signal were recorded event by event on magnetic tape. The monitor spectra were stored on magnetic disk. The off-line data reduction was performed with the VAXPAK package programs [15].

As can be seen in fig. 1, the scattered-beam background is cleanly separated from the target-like particles in the (E , BP) plot. After selecting the good events in

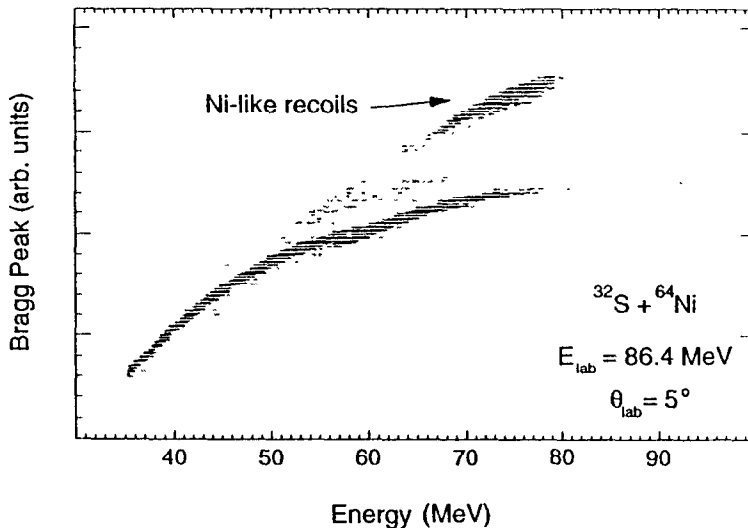


Fig. 1. A typical (E , BP) scatter-plot for this experiment is shown. It corresponds to $E_{\text{lab}} = 86.4$ MeV and $\theta_{\text{lab}} = 5^\circ$.

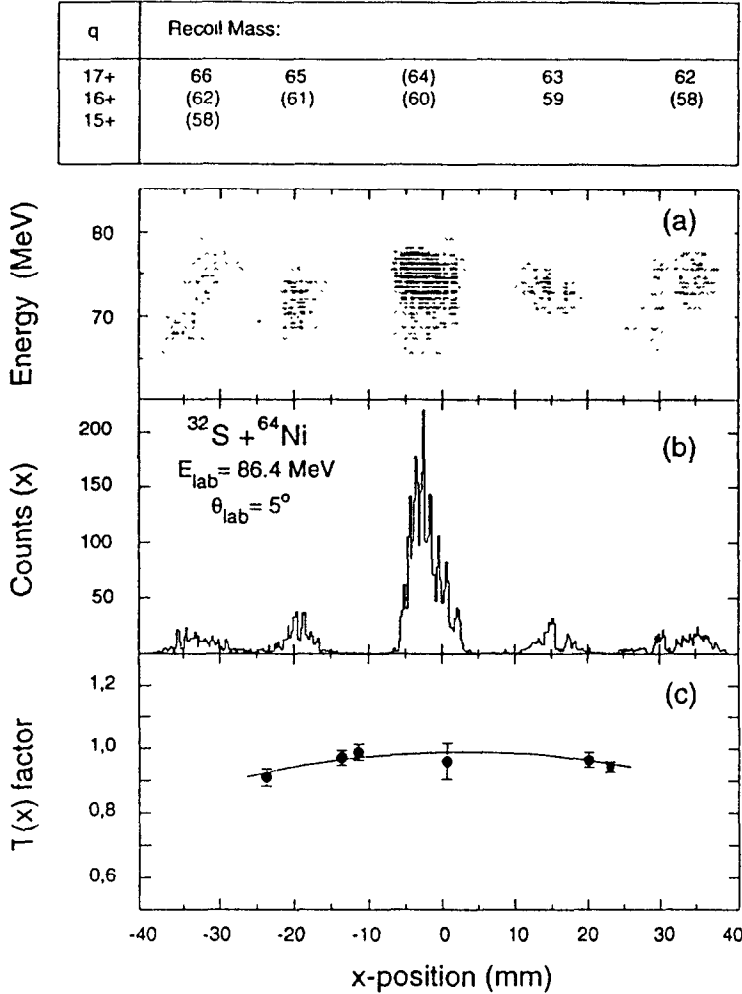


Fig. 2. For $E_{\text{lab}} = 86.4$ MeV and $\theta_{\text{lab}} = 5^\circ$ is shown: (a) (X, E) scatter-plot without beam-like background, (b) projection on the X-axis of the preceding matrix, (c) transmission along the 8 cm horizontal width of the focal-plane detector. The table in the upper part of the figure shows the different masses which reach, with different charge-state, almost the same position in the focal plane.

The masses in parentheses correspond to those present in the target composition (see text).

this scatter-plot, we looked at the (X, E) matrix (fig. 2a) to select the different channels: we used the X -position value (fig. 2b) to determine the M/q ratio and the E projection for each channel to evaluate the recoil energy (E_{rec}) of each event.

The resolution in mass, $\Delta M/M \approx 1/230$, was clearly sufficient to resolve one mass in the region of interest, as can be seen in fig. 2. Due to the relatively low energies (0.3–0.7 MeV/ A) of the detected Ni-recoils, the Z -resolution was not

sufficient to distinguish between two nearby atomic numbers so that we have to rely on the Z -distributions at higher energies [8]. The energy-resolution was limited ($\approx 3\%$) because of the target thickness and the energy straggling both in the entrance window and in the cathode of the focal-plane detector.

The central value in fig. 2 corresponds to $M/q = 64/17^+$ so that we almost have degeneracy between particles differing by 4 mass units and 1 charge state, i.e.:

$$\frac{M-4}{q-1} \approx \frac{M}{q} \approx \frac{M+4}{q+1}.$$

In order to overcome these ambiguities, different off-line corrections have been made to the measured yields. In fact, the both quasi-elastic ($M = 64$) and one-nucleon stripping channels ($M = 65$) overlap with the elastic scattering on the ^{60}Ni and ^{61}Ni target impurities respectively. In order to estimate the yields for the $^{60,61}\text{Ni}$ isotopes, we have interpolated the measured elastic yields for $^{32}\text{S} + ^{58,64}\text{Ni}$ (ref. [9]).

The charge-state distribution of the recoiling Ni-like particles was measured at $E_{\text{c.m.}} = 73.5$ MeV ($E_{\text{lab}} = 83.4$ MeV). The extrapolation to other recoil-energies has been done using the Sayer's charge-distribution algorithm [16,17] modified with the Shima prescription [18] applied to the Ni target. The error in charge-state probability definition includes also the contribution due to the uncertainty in the recoil-energy determination.

The transmission along the focal plane, $T(x)$, was measured by changing the RMS settings so as to detect the elastic recoils at different positions. $T(x)$ was found almost constant along the 80 mm detector range (see fig. 2c). The transmission across the spectrometer was unknown, so that the absolute normalization for the cross-sections has been obtained assuming that the total yields observed (quasi-elastic and transfer) at low incident energies ($E_{\text{c.m.}} = 45.5$ MeV) or at smaller laboratory angles ($\theta_{\text{c.m.}} = 120^\circ$) correspond to the Rutherford cross sections. This procedure has an accuracy which we estimate around 10%. We have reported in the drawings only the errors affecting the relative yields (event and monitor count determination, charge-state probability, vertical focal-plane efficiency and off-line corrections for target impurities).

3. Results

As has been pointed out in the previous section, we have identified in mass the ± 1 nucleon transfer process. According to our results at higher energies [8], we know that only one-neutron pick-up ($+1n$) and one-proton stripping ($-1p$) channels are present, we can thus assume that the measured $M = 63$ events

Table 1

Angular distributions at $E_{\text{c.m.}} = 54.2$ MeV for the quasi-elastic and one-particle transfer channels

$\theta_{\text{c.m.}}$ (^{32}S)	θ_{lab} (^{64}Ni)	quasi-elastic (mb/sr)	1-proton stripping (mb/sr)	1-neutron pick-up (mb/sr)
170°	5°	85.2 ± 1.8	1.42 ± 0.1	1.68 ± 0.09
160°	10°	87.4 ± 5.9	1.42 ± 0.09	1.61 ± 0.19
150°	15°	96.7 ± 3.7	1.17 ± 0.09	1.58 ± 0.19
140°	20°	114.7 ± 5.3	1.01 ± 0.21	1.35 ± 0.34
130°	25°	133.1 ± 8.7	0.83 ± 0.16	1.20 ± 0.14
120°	30°	165.5 ± 13.3	0.51 ± 0.13	0.86 ± 0.17

correspond to ^{63}Ni and the $M = 65$ to ^{65}Cu . This statement has also been taken into account to calculate the charge-state probabilities.

The angular distributions of the quasi-elastic (elastic plus inelastic scattering) as well as of the -1p and $+1\text{n}$ transfer channels are shown in table 1. The distributions are peaked backwards. We have been able to measure a cross section value as low as 0.51 mb/sr for the -1p channel and 0.86 mb/sr for the $+1\text{n}$, both at 120° in the center-of-mass frame. From $\theta_{\text{c.m.}} = 120^\circ$ to 170° , the -1p channel is slightly lower than the $+1\text{n}$ channel ($\approx 25\%$) and each represents about 0.3 to 1.5% of the Rutherford cross section.

We have measured the excitation function from 4% above the nominal Coulomb barrier [12] to 24% below. Table 2 shows the quasi-elastic, one-neutron pick-up and one-proton stripping cross sections at $\theta_{\text{c.m.}} = 170^\circ$. The minimum-transfer cross section, measured at $E_{\text{c.m.}} = 45.5$ MeV, is 0.3 mb/sr, which corresponds to a probability of 0.25%. Again the -1p and $+1\text{n}$ channels are very similar in magnitude all over the entire energy range, and their cross sections correspond to a probability of about 3.5% at their maximum.

Table 2

Excitation functions at $\theta_{\text{c.m.}} = 170^\circ$ for the quasi-elastic and one-particle transfer channels

$E_{\text{c.m.}}$ (MeV)	quasi-elastic (mb/sr)	1-proton stripping (mb/sr)	1-neutron pick-up (mb/sr)
45.5	122.4 ± 8.0	0.27 ± 0.09	0.32 ± 0.16
47.5	93.7 ± 5.2	0.16 ± 0.07	0.40 ± 0.20
50.2	94.9 ± 2.9	0.44 ± 0.16	0.69 ± 0.37
54.2	85.2 ± 1.8	1.42 ± 0.09	1.68 ± 0.09
55.6	67.7 ± 4.0	3.30 ± 0.15	2.62 ± 0.16
57.6	31.0 ± 1.0	4.10 ± 0.26	3.29 ± 0.16
59.6	16.2 ± 0.3	2.54 ± 0.23	2.27 ± 0.18
61.6	7.3 ± 0.7	1.00 ± 0.12	1.10 ± 0.16

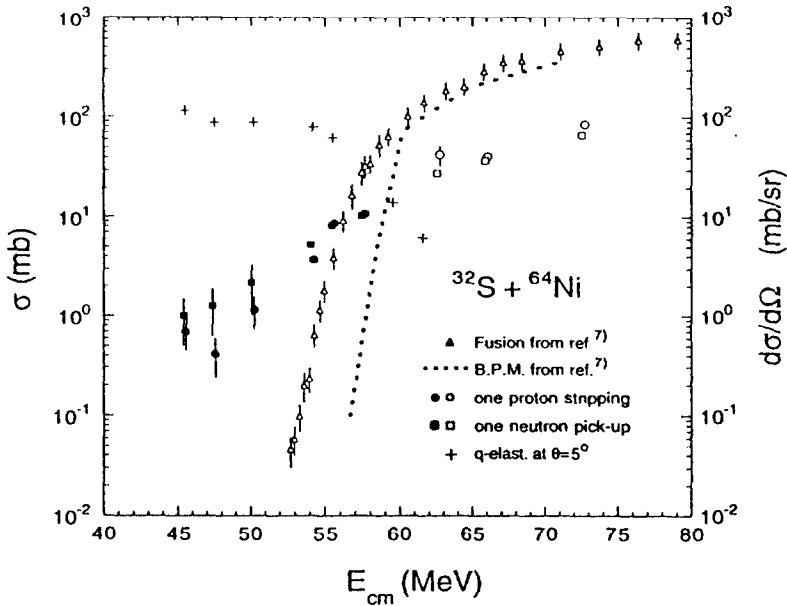


Fig. 3. Quasi-elastic differential cross sections at $\theta_{\text{c.m.}} = 170^\circ$ (crosses in mb/sr) and angle-integrated one-proton stripping (circles), one-neutron pick-up (squares) and fusion cross-sections [10] (triangles). The dashed line corresponds to the barrier-penetration model (BPM) calculation. Integrated transfer cross sections for the present experiment (full squares and circles) have been calculated by integrating the angular distribution measured at $E_{\text{c.m.}} = 54.2$ MeV. Previous transfer data (open squares and circles) belongs to ref. [7].

All the “quasi-elastic” data actually include the low-lying inelastic excitations which were not separated out from the pure elastic scattering. The quasi-elastic cross sections start to deviate significantly from the Rutherford ones around $E_{\text{c.m.}} = 55$ MeV. At this energy the lowest measured fusion cross section shows up [11], but still transfer processes are responsible for the depletion of the quasi-elastic channel.

Fig. 3 shows the measured angle-integrated transfer cross sections together with the total fusion cross section [11] as a function of $E_{\text{c.m.}}$. The transfer data at $E_{\text{c.m.}} < 60$ MeV have been obtained by integrating the angular distribution measured at $E_{\text{c.m.}} = 54.2$ MeV and assuming that the shape does not change for the lower energies. Transfer data at $E_{\text{c.m.}} > 60$ MeV are taken from ref. [8]. One clearly sees that for $E_{\text{c.m.}} \leq 56$ MeV, transfer cross sections are much larger than fusion cross sections. Closer to and above the Coulomb barrier ($E_{\text{c.m.}} \approx 59$ MeV) [12], fusion rapidly becomes ten times greater than the individual one-nucleon transfer channels.

4. Theory and comparison with the data

We will use the complex WKB [19] approximation (CWKB) in its simplest form, i.e. taking into account the contribution coming from a single turning point. We use this approach since the CWKB theory is valid also at energies above the Coulomb barrier. In this way we are able to analyse the present low-energy data together with those of ref. [8] taken at energies above the Coulomb barrier. The study of the energy-dependence of the cross sections is clearly very important since it poses severe constraints on the theory of one-particle transfer.

In what follows we will outline the CWKB theory for one-particle transfer, discussing also the main results of the semiclassical theory based on Coulomb trajectory and the comparison with the experimental data.

4.1. Complex WKB theory for one-particle transfer reactions

Following ref. [19], an expression for the transfer cross section may be obtained in the CWKB approximation starting from the conventional distorted-wave Born approximation (DWBA) and assuming that the transfer takes place on a trajectory averaged between the entrance and exit channel. The cross section for the transfer from the single-particle state $a_i \equiv (n_i, l_i, m_i)$ to the single-particle state $a_f \equiv (n_f, l_f, m_f)$, belonging to different nuclei, may be written as

$$\left[\frac{d\sigma}{d\Omega} \right]_{a_f a_i} = \sum_{\lambda} \left(\frac{d\sigma}{d\Omega} \right)_{\lambda}, \quad (1)$$

where the sum has to be extended over all the allowed angular-momentum transfers λ .

The transfer cross section for each λ may be written as

$$\left(\frac{d\sigma}{d\Omega} \right)_{\lambda} = \frac{\kappa_f}{\kappa_i} \sum_{\mu} \left| \sum_l c_{\lambda\mu}^{a_f a_i}(l) f_l(\theta) \right|^2, \quad (2)$$

where κ_i and κ_f are the asymptotic wave-numbers in the entrance and exit channel respectively, $c_{\lambda\mu}^{a_f a_i}(l)$ is the semiclassical amplitude for the transition from the initial state i to the final state f for the partial wave l , and $f_l(\theta)$ is the elastic scattering amplitude for the same partial wave l .

The elastic scattering amplitude $f_l(\theta)$ may be written

$$f_l(\theta) = \frac{(2l+1)}{2i\kappa} e^{2i(\delta_l^N + \sigma_l)} P_l(\cos \theta), \quad (3)$$

with δ_l^N and σ_l the nuclear and Coulomb phase-shift respectively. In the simplest (primitive) form of the CWKB approximation the nuclear phase-shift δ_l^N is

$$\delta_l^N = \int_{r_0(l)}^R \kappa(r) dr - \int_{r_C(l)}^R \kappa_C(r) dr, \quad (4)$$

where R is a radius outside the range of the nuclear interaction. The local wavenumbers are defined as

$$\kappa_C = \sqrt{\frac{2m}{\hbar^2} \left(E - V_C(r) - \frac{\hbar^2(l + \frac{1}{2})^2}{2mr^2} \right)}, \quad (5)$$

$$\kappa(r) = \sqrt{\frac{2m}{\hbar^2} \left(E - U_N(r) - V_C(r) - \frac{\hbar^2(l + \frac{1}{2})^2}{2mr^2} \right)}. \quad (6)$$

In the latter, with $V_C(r)$ and with $U_N(r) = V(r) + iW(r)$ we have indicated the Coulomb and the complex nuclear interaction respectively, while m is the reduced mass of the system.

The classical turning point for the Coulomb field is indicated with $r_C(l)$ while the complex nuclear turning point $r_0(l)$ is a solution of the equation

$$E - U_N(r) - V_C(r) - \frac{\hbar^2(l + \frac{1}{2})^2}{2mr^2} = 0. \quad (7)$$

The primitive WKB approximation is obtained by using the outermost root of eq. (7). The Coulomb phase-shift may be obtained from the expression

$$\sigma(l) = \arg \Gamma(l + 1 + i\eta), \quad (8)$$

where η is the Sommerfeld parameter. By means of the expressions (4) and (7) one can express the elastic scattering amplitude without solving explicitly the Schrödinger equation.

The transfer amplitude $c_{\lambda\mu}^{a_f a_i}(l)$ is given in first-order Born approximation using the low-recoil limit [20,21] by

$$c_{\lambda\mu}^{a_f a_i}(l) = \frac{1}{\sqrt{2\lambda + 1}} I_{\lambda\mu}^{a_f a_i}(l), \quad (9)$$

with $I_{\lambda\mu}^{a_f a_i}(l)$ defined as

$$I_{\lambda\mu}^{a_f a_i}(l) = \frac{1}{\hbar} \sqrt{\frac{2\lambda + 1}{4\pi}} D_{\mu 0}^{\lambda}(0, \frac{1}{2}\pi, 0) \int_{-\infty}^{+\infty} dt f_{\lambda}^{a_f a_i}(r(t)) \times e^{i[(\Delta E - Q_{\text{opt}} + \Delta)t - \hbar\mu\phi(t)]/\hbar}. \quad (10)$$

The optimum Q -value, Q_{opt} , takes into account the mismatch between the entrance and exit channel trajectories and it is defined by

$$Q_{\text{opt}} = \frac{Z_d(Z_a - Z_A)e^2}{r_0} - \frac{1}{2}m_d r_0^2 \dot{\phi}_0^2 \frac{A_a - A_A}{A_a + A_A} + \frac{1}{2}m_d \frac{A_a A_A}{A_a + A_A} \ddot{r}_0 \left(\frac{\bar{R}_A - \bar{R}_a + r_0}{A_A} - \frac{\bar{R}_a - \bar{R}_A + r_0}{A_a} \right), \quad (11)$$

while the parameter Δ is defined by [22]

$$\Delta = \frac{1}{2}m_d \ddot{r}_0 (r_0 - \bar{R}_a - \bar{R}_A). \quad (12)$$

In the above expressions, A_a , Z_a and A_A , Z_A are the mass and charge of projectile and target respectively while m_d , Z_d are the mass and charge of the transferred particle. The radii $\bar{R}_i$ are chosen as $\bar{R}_i = 1.27 A_i^{1/3}$.

The single-particle form factor $f_\lambda^{a_{tr}}(r)$, calculated with the formalism of refs. [20,21], may be parameterized in the outer region as

$$f_\lambda^{a_{tr}}(r) = F_0 \exp\left(-\frac{r - R_0}{a_{tr}}\right), \quad (13)$$

where the parameter a_{tr} which defines the range of the form factor is related to the binding energy of the transferred nucleon. One has typically $a_{tr} \approx 1.2$ fm. Because of the short range of the form factor, one can approximate the trajectory by

$$r(t) = r_0(l) + \frac{1}{2}\ddot{r}_0(l)t^2, \quad \phi(t) = \dot{\phi}_0(l)t. \quad (14)$$

Here $r_0(l)$ is the outermost complex turning point as defined by (7), while $\dot{\phi}_0(l)$ and $\ddot{r}_0(l)$ are the complex angular velocity and complex acceleration calculated at $r_0(l)$. Within this parabolic approximation, the integral (10) becomes

$$I_{\lambda\mu}^{a_{tr}}(l) = \sqrt{\frac{2\pi a_{tr}}{\ddot{r}_0(l)\hbar^2}} f_\lambda(r_0(l)) Y_{\lambda\mu}(\frac{1}{2}\pi, 0) \times \exp\left(-\frac{a_{tr}}{2\ddot{r}_0(l)\hbar^2} [\Delta E - Q_{\text{opt}} + \Delta - \hbar\dot{\phi}_0(l)]^2\right). \quad (15)$$

At the beginning of this section we mentioned that the phase-shifts and the transition-amplitudes have to be calculated on trajectories averaged between the

entrance and exit channels (cf. also ref. [23]). These average trajectories are defined by the parameters

$$E = \frac{1}{2}(E_i + E_f), \quad m = \frac{1}{2}(m_i + m_f),$$

$$l = l_i + \frac{1}{2}\mu, \quad U(r) = \frac{1}{2}[U_i(r) - U_f(r)], \quad (16)$$

where E , m and l are the center-of-mass energy, the reduced mass, the angular momentum of relative motion and U is the sum of nuclear and Coulomb potential energies. The suffixes i and f clearly stand for entrance and exit channel. To speed-up the program we neglected the z -component (μ) of the transferred angular momentum λ in the definition of the angular momentum l of relative motion.

Since the experiment is not able to discriminate between the individual final states, the angular distribution for one-particle transfer is obtained by summing over all the cross sections for single-particle transition (1) i.e.:

$$\left[\frac{d\sigma}{d\Omega} \right]_{\text{tr}} = \sum_{a_i, a_f} V^2(a_i) U^2(a_f) \left[\frac{d\sigma}{d\Omega} \right]_{a_i a_f}. \quad (17)$$

The sum has to be extended over all the single-particle states a_i and a_f involved in the transition for both target and projectile. The quantities $V^2(a_i)$ are the probabilities that the single-particle orbitals are occupied, while the quantities $U^2(a_f) = 1 - V^2(a_f)$ are the corresponding probabilities that the orbitals are empty. V^2 and U^2 are directly related to the spectroscopic factors.

4.2. Semiclassical theory

In the semiclassical approximation the relative motion of the two ions is considered classically, while the intrinsic degrees of freedom are treated quantum-mechanically. The transitions among the different intrinsic states are calculated from a time-dependent interaction whose time-dependence is given by the classical motion of the two ions along the trajectory. In this approximation, the cross section for one-particle transfer can be written as

$$\left[\frac{d\sigma}{d\Omega} \right]_{\text{tr}} = \sum_{a_i, a_f} |c_{a_i a_f}(\theta)|^2 P_0(\theta) \sigma_{\text{Ruth}}(\theta), \quad (18)$$

where $c_{a_i a_f}(\theta)$ is the semiclassical amplitude from the initial single-particle state a_i to the final single-particle state a_f , P_0 is the probability to remain in the elastic channel and $\sigma_{\text{Ruth}}(\theta)$ is the Rutherford cross section at the scattering angle α . As in (17), the sum has to be extended over all single-particle states since the experiment does not resolve individual transitions.

The probability P_0 to remain in the elastic channel is calculated via a time-integral of the imaginary potential $W(r)$ along the classical trajectory which leads to the scattering angle θ ,

$$P_0 = \exp\left(\frac{2}{\hbar} \int_{-\infty}^{+\infty} W(r(t)) dt\right). \quad (19)$$

The amplitude $c_{a_f a_i}(\theta)$ is given by

$$c_{a_f a_i}(\theta) = \sum_{\lambda, \mu} c_{\lambda \mu}^{a_f a_i}(\theta), \quad (20)$$

where the $c_{\lambda \mu}^{a_f a_i}(\theta)$ are given by eq. (9). Now the time-integral in eq. (10) has to be calculated along the Rutherford trajectory leading to the scattering angle θ . In this approximation, the distance of closest approach r_0 , the acceleration $\ddot{r}_0$ and the angular velocity $\dot{\phi}_0$ may be expressed, for Coulomb trajectory, as a function of the scattering angle θ by the relations

$$\begin{aligned} r_0 &= \left\langle \frac{Z_a Z_A e^2}{2E} \right\rangle \left(1 + \frac{1}{\sin \frac{1}{2}\theta} \right), \\ \ddot{r}_0 &= \left\langle \frac{Z_a Z_A e^2}{m r_0^2} \right\rangle \frac{1}{\sin \frac{1}{2}\theta}, \\ \dot{\phi}_0 &= \frac{1}{\mu r_0^2} = \left\langle \frac{Z_a Z_A e^2}{\sqrt{2mE}} \right\rangle \frac{\cotg \frac{1}{2}\theta}{r_0^2}. \end{aligned} \quad (21)$$

The symbols $\langle \dots \rangle$ indicate averages over the initial and final state trajectories as in (16). The transfer probability may be written as:

$$|c_{a_f a_i}|^2 = \sum_{\lambda} \frac{a_{\text{tr}}(a_f, a_i)}{2 |\ddot{r}_0| \hbar^2} U^2(a_f) V^2(a_i) |f_{\lambda}^{a_f, a_i}(r_0)|^2 g_{\lambda}(Q). \quad (22)$$

In (22) the adiabatic cut-off function $g_{\lambda}(Q)$, that weights the contributions of the different transitions, is defined by:

$$g_{\lambda}(Q) = \sum_{\mu = -\lambda}^{\lambda} \left| D_{\mu 0}^{\lambda}(0, \frac{1}{2}\pi, 0) \right|^2 e^{-\text{Re}(a - \mu b)^2}, \quad (23)$$

with

$$a = \sqrt{\frac{a_{\text{tr}}}{|\ddot{r}_0| \hbar^2}} (Q - Q_{\text{opt}} + \Delta), \quad b = \sqrt{\frac{a_{\text{tr}}}{|\ddot{r}_0| \hbar^2}} (\hbar \dot{\phi}_0). \quad (24)$$

Table 3
Optical model parameters for a standard Woods–Saxon shape

E_{lab} (MeV)	V_0 (MeV)	r_0 (fm)	a_0 (fm)	W_0 (MeV)	r'_0 (fm)	a'_0 (fm)
81.3	–	–	–	20.0	1.23	0.60
92.8	43.0	1.23	0.63	35.0	1.23	0.55
97.3	30.0	1.23	0.63	60.0	1.23	0.55

In the latter, the optimum Q -value Q_{opt} and the phase Δ are defined by the expressions (11) and (12), respectively.

4.3. Comparison with the measurements

We now compare the calculations with the one-particle transfer data at the bombarding energies of 81.3, 92.8 and 98.3 MeV.

To calculate the phase-shifts we need the nuclear potentials. These have been obtained by fitting the elastic angular distributions measured in previous experiments [9]. The nuclear potentials have been chosen to be of Woods–Saxon shape. The adopted parameters are listed in table 3. At the lowest energy, no elastic scattering is available and the parameters of the imaginary part have been obtained from the normalization of the theoretical calculations to the one-particle transfer data. At this energy, the fit to the data is not able to define the real part of the nuclear potential (see below). At the two higher energies, the parameters of table 3 are in good agreement with the ones of ref. [9] that were used to fit both the elastic scattering angular distribution and the fusion cross section. They also compare well with the dispersion relations discussed in the same work.

The single-particle reaction channels were chosen in agreement with the data existing in the literature for the nuclei under investigation. In table 4 we show all the single-particle states we have included in our calculation for one-neutron pick-up $^{64}\text{Ni}(^{32}\text{S}, ^{33}\text{S})^{63}\text{Ni}$ and one-proton stripping $^{64}\text{Ni}(^{32}\text{S}, ^{31}\text{P})^{65}\text{Cu}$ reactions. For each state are also shown the occupation-probabilities (cf. eq. (17)) derived from the spectroscopic amplitudes in the quoted references. Moreover, the table reports the binding energy for the last occupied state. The binding energies for the other states are easily derived from the excitation energies. The wave functions for the singleparticle levels, entering in the calculations of the form factors [20,21], were computed using a standard Woods–Saxon potential plus a spin-orbit term whose parameters have been taken from the references quoted in table 4.

We have derived angular distributions and excitation functions inserting in eqs. (10) and (17) the form factors calculated in the low-recoil approximation [20,21] and the occupation probabilities corresponding to all transitions of table 4. The calculated angular distributions for one-neutron pick-up and one-proton stripping

Table 4
Particle and hole states for proton stripping and neutron pick-up considered in the calculations

	J^π	E^* (MeV)	B.E. (MeV)	$U^2(V^2)$
$^{33}\text{S}^a$	$\frac{3}{2}^+$	0.0	-8.64	1.0
	$\frac{1}{2}^+$	0.842		0.5
	$\frac{7}{2}^-$	2.93		0.575
	$\frac{3}{2}^-$	3.22		0.5
	$\frac{5}{2}^-$	4.92		0.2
$^{63}\text{Ni}^b$	$\frac{1}{2}^-$	0.0	-9.66	0.24
	$\frac{5}{2}^-$	0.087		0.57
	$\frac{3}{2}^-$	0.16		0.81
	$\frac{7}{2}^-$	1.9		1.0
$^{31}\text{P}^c$	$\frac{1}{2}^+$	0.0	-8.86	0.75
	$\frac{3}{2}^+$	1.27		0.85
	$\frac{5}{2}^+$	2.23		1.0
$^{65}\text{Cu}^d$	$\frac{3}{2}^-$	0.0	-7.45	0.77
	$\frac{1}{2}^-$	0.77		0.65
	$\frac{5}{2}^-$	1.12		0.65
	$\frac{9}{2}^+$	2.53		0.5

^a Ref. [24].

^b Ref. [25].

^c Ref. [26].

^d Ref. [27].

reactions at the indicated energies are compared with the data in fig. 4. The full-drawn curves refer to the CWKB calculations, while the dotted lines indicate the semiclassical calculations based on Coulomb trajectories as in eq. (21). At the lowest bombarding energy, below the Coulomb barrier, and at forward angles, the two procedures, as is expected, are in good agreement. It is important to stress that these results can only be obtained by adopting the average trajectory defined by eq. (16) in the calculations of the transition amplitudes.

The overall comparison of the theoretical curves with the experimental data is reasonable specially for one-neutron pick-up where the energy-dependence of the cross section is also well-described. Problems still remain for one-proton transfer specifically at the intermediate energy where the theory underestimates the data by a factor of 2. This is probably related to the inability of DWBA to fit charge-particle transfer in heavy-ion reactions in general.

The excitation functions at $\theta_{\text{c.m.}} = 170^\circ$ are illustrated in fig. 5 for the two indicated channels. The curve refers to the CWKB calculation obtained with potential parameters interpolating those of table 3, in agreement with the dispersion relation of ref. [9]. The energy-dependence of the cross section is only governed by the phasefactor in eq. (10) in the CWKB approximation and by the adiabatic cut-off function $g_\lambda(Q)$ in the semiclassical description.

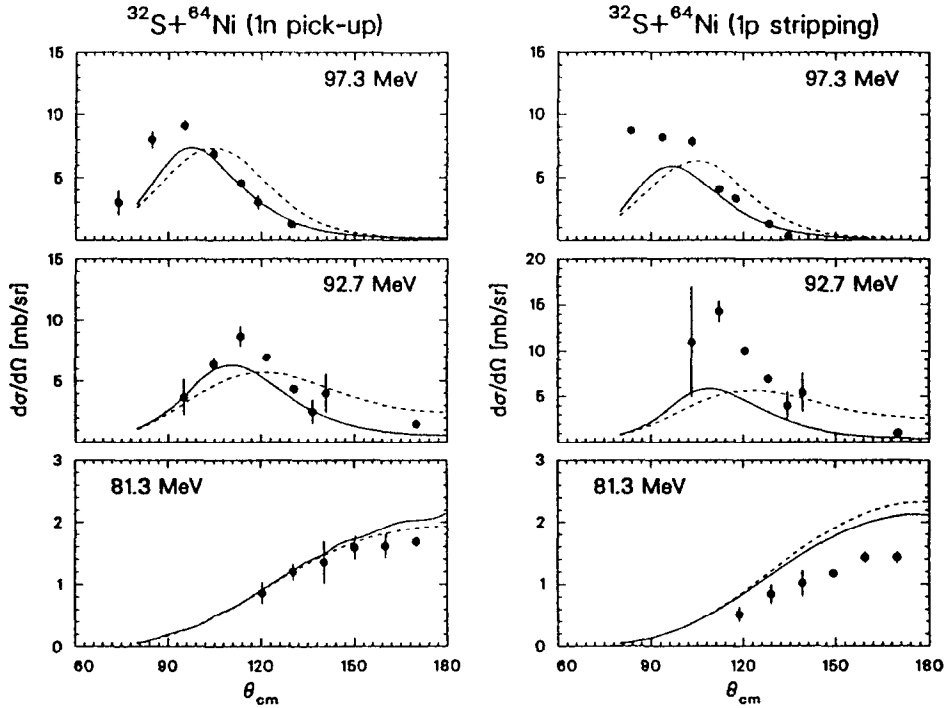


Fig. 4. Angular distributions of the transfer reactions at the indicated bombarding energies (corresponding to center-of-mass energies of 54.2, 61.8 and 64.8 MeV). The full-drawn curves are CWKB calculations as explained in the text. The dotted lines correspond to semiclassical calculations based on Coulomb trajectories.

5. Summary

We have studied the most important direct channels observed in the $^{32}\text{S} + ^{64}\text{Ni}$ reaction at low energies. The target-like ions recoiling at forward angles were selected by the Legnaro Recoil Mass Spectrometer. The possibility of rotating the RMS from $+5^\circ$ to -50° enabled us to measure a complete angular distribution of the transfer products below the Coulomb barrier, thus allowing a detailed comparison with theoretical predictions.

A complete angular distribution 9% below the Coulomb barrier [12] and an excitation function at $\theta_{\text{c.m.}} = 170^\circ$ from a 24% below to a 4% above the barrier have been measured for the elastic plus inelastic scattering and for the one-nucleon transfer channels.

The one-particle excitation functions exhibit, in the whole range below the barrier covered by this experiment, the typical exponential fall-off with energy that has been pointed out by the semiclassical theory [28] and they match very well with the previous data of ref. [8] that has also been included in the analyses.

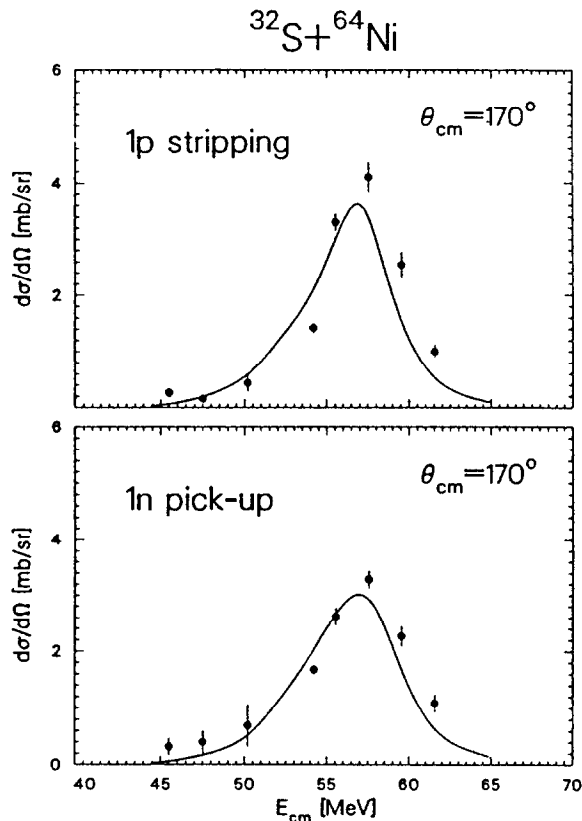


Fig. 5. Excitation functions for one-proton stripping (top) and one-neutron pick-up (bottom) at $\theta_{\text{cm}} = 170^\circ$. The curves are the CWKB calculations discussed in the text.

Summarizing we can say that, taking into account all the transitions around the Fermi energy - with the proper experimental nuclear-structure information - and considering first-order perturbation theory we have been able to reproduce reasonably well the measured angular distributions and excitation functions above and below the Coulomb barrier. The results also suggest the applicability of the CWKB approach to the analyses of single-nucleon transfer reactions both below and above the Coulomb barrier.

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