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Regular and chaotic regimes in coupled-channel calculations of nuclear scattering processes

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Abstract

The presence of regular and chaotic regimes in nuclear scattering processes has so far been established in the context of simple, schematic reaction models. It has been argued that traces of the underlying classical organization of phase space should also be present in complex quantal calculations performed with state-of-the-art coupled-channel codes. In this paper we show that this is indeed the case and develop concrete guidelines to infer – relying exclusively on features of the quantal results – the regular or chaotic character of the motion.

1. Introduction

A transition between ordered and disordered motion takes place in nuclear scattering problems at energies close to the Coulomb or centrifugal barriers. The occurrence of chaotic behaviour is a consequence of the coupling of the relative motion variables of two colliding ions to their intrinsic degrees of freedom. The mechanisms that bring about classical chaotic motion have been studied in detail using schematic reaction models where the number of intrinsic variables is kept to a minimum. One-dimensional harmonic modes mocking up deformations of the nuclear shape [1–3] and rotations in a plane of a statically deformed system [4–6] are among those considered in these investigations.

Chaotic structures were readily found, underlying the fact that actual nuclear sizes and couplings, when treated classically, lead in many situations to irregular scattering. The simplicity of the models allowed for precise comparisons with quantum calculations and the extraction of signatures of this behaviour in the quantum solutions. These are more subtle and less universal than the standard signatures relating to universal fluctuation properties of either bound spectra or cross sections [7]. The basic reason is that for the collective modes considered the nuclear regime is far from the extreme semiclassical limit where this universal behaviour becomes apparent. Distinct aspects of the quantal results were nevertheless identified that – in close correlation with the classical behaviour – distinguished the irregular from the regular regimes. It was also shown how the introduction of an absorption in the problem gradually reduces the importance of trajectories that explore the region inside the barrier in a complicated way. This feature has the potential to wash out the manifestations of chaos but, in a weak-absorption range, signatures of the regime remained detectable.

These simple models are not sufficiently rich to relate to actual experimental data obtained from heavy-ion collisions. The construction of quantal cross sections and angular distributions in real situations requires the introduction of multidimensional intrinsic variables and the implementation of a complex coupled-channel scheme involving a range of partial waves. Under normal circumstances, reaction specialists will not have the means to perform a "classically equivalent" calculation of the problem at hand. And we do not mean just that a computer code is not readily accessible. Most often it will not be possible (or at least very difficult) to construct a classical model that exactly corresponds to the situation that is being analyzed. Rotational bands with variable moment of inertia or back-bendings, vibrational spectra with anharmonicities, systems displaying abnormal γ -unstable spectra are – to name a few – common occurrences that do not easily conform to the mold of an autonomous system whose evolution can be followed by solving Hamilton's equations of motion.

A question that comes to mind is the following: Can one infer from the characteristics of the results of a coupled-channel analysis whether regular or chaotic conditions are predominant? It is our intention in this paper to explore this topic and make concrete suggestions as to the kind of processing of the available information that may help distinguish between the two situations. Notice that we are here dealing with an open scattering problem and not with the states of a bound nuclear system. Consequently, the central object of study will be the reaction matrix and its associated channel wave functions.

Finally, a word about motivation. Someone not inclined to examine a nuclear reaction problem from the point of view of its regular or chaotic aspects will perhaps be content with a straightforward coupled-channel analysis and abide by its results. Developments in the field of heavy-ion collisions, however, have always benefited from the use of classical and semiclassical concepts. Not only they have provided valuable tools that handle multidimensional situations far beyond the reach of a full quantal treatment but – equally important – their transparent appeal has guided research efforts in many circumstances. The validity of the intuitive arguments and a straightforward implementation of standard semiclassical prescriptions, however, fail when the scattering becomes chaotic [2]. For this reason alone it becomes necessary to learn how to distinguish between the regular and chaotic regimes in quantal coupled-channel calculations. The knowledge is important, as different dynamical effects are to be looked after in either domain. The

identification has in addition practical relevance, as it is found that the numerical stability of coupled-channel solutions must be carefully monitored whenever chaotic conditions are prevalent.

Our strategy in preparing this manuscript went as follows. We exploited the simple models of Refs. [1-3] as a source of inspiration. A systematic inspection of their quantal results was used to identify correlations between elements of the output that exhibited a characteristic difference between the regular and irregular regimes. Then we proceeded to investigate whether those signatures could still be recognized in the results of a full coupled-channel calculation. For this aspect of the study we have relied on Thompson's code FRESCO [8]. The survey of candidates includes wave functions (Section 2), S-matrix representations (Section 3), excitation functions (Section 4) and angular distributions (Section 5). We devote Section 6 to re-examine under this light the backward scattering anomalies in the O+Si reaction, centre of much attention about a decade ago. A brief summary and conclusions close the presentation. Let us here state that although calculations based on the simple models often provide the most impressive illustrations we shall – to limit the number of figures but basically to subscribe to the motivation of this work – almost exclusively present results extracted from FRESCO.

2. Wave functions

Quantal reaction codes construct, for specified bombarding conditions, the scattering wave functions of the different reaction channels. This is done by solving Schrödingers' equation for a set of coupled equations involving the relative motion variables and enforcing the appropriate boundary conditions. A first possibility is to search in the radial dependence of these wave functions characteristics that may set the regular and chaotic regimes apart. One such correlation observed earlier [2], is a tendency of resonant chaotic wave functions to localize in the vicinity of the hyperbolic point.

It is not easy to grasp whatever classical information might be hidden in the radial dependence of the rapidly oscillating real and imaginary components of these complex wave functions. For this reason it is convenient to introduce phase-space representations that have a more direct contact with classical structures. One such representation is the Husimi distribution, related to the channel functions by

$$F_{n_f,n_i}(q,p) = \left| \int_{-\infty}^{\infty} \chi_{n_f,n_i}(r) \exp\left[-\frac{(r-q)^2}{2\sigma^2} + ipr \right] dr \right|^2 , \qquad (1)$$

where p, q are to be compared to the classical radial distance and conjugate momentum. Here the channel wave functions are projected on a coherent-state basis, where localization in both coordinate and momentum achieves an optimal compromise [9]. This prescription yields a positive-definite phase-space distribution that can be directly compared to classical scattering trajectories. In this section and those that follow we shall concentrate in the study of reactions involving a deformed silicon target with different spherical projectiles. We investigate the population of the states of the rotational band of ²⁸Si, assuming the nucleus to be a pure rotor with a static quadrupole deformation as inferred from experiment. We note that the same nucleus was also involved in calculations performed by the Catania group [4], but was there considered spherical; it was the reaction partner, magnesium, that received attention as a deformed nucleus there. It has been observed in previous studies for the reaction ¹⁶O+²⁸Si that this is indeed a system that exhibits a relative transparency at the strong-interaction radius. Thus one can envisage calculations with FRESCO under rather realistic conditions. In particular this is the case in Section 6, where we examine the ¹⁶O+²⁸Si situation from the particular point of view of this presentation. There we use values for the imaginary potential that were originally calculated microscopically [10] and that, in fact, anticipated the low-absorption characteristic of the system. Unless otherwise specified, we take for the real part the parametrization of the empirical potential of Christensen and Winther [11].

The actual moment of inertia of silicon – extracted from the excitation energy of the 2^+ state ($E_{2^+} = 1.74$ MeV) – is quite low. Consequently, angular momentum deposited in the deformed system during the interaction time can generate appreciable changes in the relative orientation of the two reaction partners while they still remain in contact. Such behaviour, at the opposite end of adiabaticity, is known to be an essential ingredient leading to irregular motion [4]. To identify features that may reveal the character of the underlying classical regime we compare results from calculations for collisions involving silicon as it is found in nature (²⁸Si) and an hypothetical "silicon" (²⁸Si) where we arbitrarily lower the excitation energy essentially down to zero. This recreates adiabatic conditions with an effectively large moment of inertia. The simulation sets the stage for the occurrence of regular motion, whereas the actual value of the moment of inertia leads to a chaotic regime. This is easily established by solving the equivalent problem in classical mechanics and thus we adopt this terminology with no further elaboration.

We show in Figs. 1 and 2 a collection of channel wave functions for the reactions ${}^{16}\text{O}+{}^{28}\text{Si}$ and ${}^{16}\text{O}+{}^{28}\overline{\text{Si}}$, respectively. Let us first consider the left-hand side of the figures, where contours of the Husimi transforms are plotted for the chaotic and regular situations at a bombarding energy $E_{\text{lab}} = 36.2$ MeV. Construction of the complete angular distribution of course requires a full range of partial waves (about thirty for this particular system and bombarding energy). The illustration selects only wave functions of relative motion for the reaction channels connecting the initial partial wave $\ell = 10$, close to grazing, with neighboring ones up to $\ell = 10 \pm 4$. The contours correspond to an absolute, logarithmic scale in tonalities of grey for all frames.

Evidence for the localization effects that were encountered in simpler coupling schemes is present in the entrance-channel configuration. It is quite clear that the flux does not explore the nuclear interior in the irregular case. This happens only at the $\ell = 10$ partial wave and for the 0^+ state because the bombarding energy has specifically been chosen to coincide with one of the resonances within the chaotic layer. In this example the centre-of-mass energy lies close to the manifold of effective barriers associated with

chaotic regime



Fig. 1. Wave functions of relative motion generated by FRESCO in phase-space representation. The results are for the reaction ${}^{16}O+{}^{28}Si$, $(E_{2^+} = 1.74 \text{ MeV})$ and therefore correspond to the chaotic regime. Husimi transforms of the radial wave functions are displayed in a plane where the horizontal axis gives is the distance q between the ions and the vertical axis corresponds to the radial momentum p. The ranges for q and p are [0,16] fm and $[-5,+5]\hbar$ MeV⁻¹, respectively, and are only indicated in the lowest frames. The contours are in an absolute logarithmic scale in tonalities of grey, where black is highest. All the wave functions displayed are for an initial state specified by the intrinsic state 0^+ and partial wave $\ell = 10$. The notation is such that, for instance, "2⁺ L = 12" refers to the wavefunction connecting to the 2⁺ state and outgoing partial wave $\ell = 12$. The same set of quantum numbers are used for two energies, 36.2 MeV (left) and 37.2 MeV (right).

 $\ell = 10$, illustrated through a few orientations in Fig. 3. As we shall see, the chaotic layer is actually rather wide and similar results can also be obtained at higher energies.

It is difficult to single out any major, distinctive feature of the phase-space distributions shown in Figs. 1 and 2 that could conclusively establish the character of the motion. One cannot focus on the larger extent of the coupling in the regular case, mostly due

regular regime



Fig. 2. Same as Fig. 1 but for the reaction ${}^{16}\text{O} + {}^{28}\overline{\text{Si}}$ ($E_{2^+} = 0.02 \text{ MeV}$) and therefore corresponding to a regular regime. The two bombarding energies are now 36.2 MeV (left) and 39.2 MeV (right). A separation of three MeV was necessary under these conditions to put in evidence an appreciable change in the distributions.

to the lower Q-value associated with those transitions. The diversity of feeding patterns in a multidimensional coupling scheme seems also to obscure the localization features that were exhibited by chaotic wave functions in the simple models (cf. Fig. 8 in Ref. [2]). A similar conclusion was reached in a recent study [12] that established the occurrence of unusual absorptive patterns in the chaotic regime. There is no doubt that a mechanism that keeps the scattering wave functions localized near the hyperbolic point will shield the effects of a strong central absorption. We should again stress, however, that it does not seem possible to draw definite conclusions from a simple inspection of wave functions in phase space.



Fig. 3. Sampling of ion-ion interactions for radial motion in a collision involving the spherical ¹⁶O nucleus and a statically deformed ²⁸Si with $\beta = 0.31$. The four curves include a centrifugal potential for $\ell = 10$ and correspond to relative orientations of the symmetry axis of the rotor (assumed for this illustration to be contained in the reaction plane) of 0°, 30°, 60° and 90°. The latter yields the lower barrier.

An interesting aspect does manifest itself in these representations. We have moved the bombarding energy slightly upwards to construct the right-hand sides of Figs. 1 and 2. The change in energy for the case of ²⁸Si is only of one MeV but is sufficient to lead to wave functions of relative motion that have a very different localization in phase space. In Fig. 2 – the regular regime in ²⁸Si – it was however necessary to shift the energy by about three MeV, before any differences between the wavefunction transforms became visible. Appreciable modification of the patterns requires an even larger change in E_{lab} . This points out to a different energy dependence in the chaotic and regular regimes, a feature that is expected in view of earlier results. This topic will be examined in detail and from another perspective in Section 4.

3. S-matrix

One of the most convenient ways to display the onset of irregular motion at the classical level is by means of reaction functions. These correlate the final value of an observable with a classical variable which remains unspecified in the initial quantal state. For instance, in the model of Ref. [1] only the energy $E_{\text{int}}(t=0) = (n_i + \frac{1}{2})\hbar\omega$ of the intrinsic harmonic mode is initially specified. Therefore one plots the final "classical" number of phonons, defined as $n_f = E_{\text{int}}(t=\infty)/\hbar\omega - \frac{1}{2}$ for a uniform sample of all possible initial values of the action-angle conjugate variable, φ_i .

The characteristics put in evidence by these correlations plots are similar, regardless of whether one is dealing with vibrational or rotational degrees of freedom. At energies well below and well above the barrier the reaction functions display a regular, "interpolable" behaviour. These zones involve trajectories that change smoothly with energy and initial phase angle and that have one turning point in the radial coordinate. In-between – and in an energy range determined by the strength of the coupling – different families of trajectories of increasing complexity are responsible for the rich structures exhibited by the reaction functions there.

To find out how much of these characteristic patterns survives the transition into quantum mechanics one must solve the corresponding excitation problem in quantum mechanics. It is possible to introduce a quantal counterpart to the classical reaction functions. The construction centres in transforming the *S*-matrix to a coherent-state representation of the intrinsic states of the system, a procedure that has been described in detail in Refs. [2,3].

Consider for instance, the rotational problem for a central collision with $\ell = 0$. A simultaneous specification of the initial spin I_i and orientation angle Φ_i of the rotor with respect to the beam can best be met within the limitations imposed by the uncertainty principle by constructing the amplitude

$$a_{I_i}(I_f, \Phi_i) = \sum_{I} \langle I_f | S | I \rangle \langle I | \alpha(I_i, \Phi_i) \rangle , \qquad (2)$$

where $\langle I_f | S | I \rangle$ is the S-matrix matrix element for the transition from an initial state with well-defined angular momentum I to the final state I_f . In Eq. (2), the state $|\alpha(I_i, \Phi_i)\rangle$ denotes a wavepacket constructed as a superposition of the channel states, where both $\langle \alpha | I | \alpha \rangle = I_i$ and $\langle \alpha | \Phi | \alpha \rangle = \Phi_i$ are specified with a narrow dispersion. For instance

$$|\alpha(I_i, \Phi_i)\rangle = \sum_{I} \exp\left[-(I - I_i)^2 / (2\sigma_I^2) + iI\Phi_i\right] |I\rangle .$$
(3)

The value of σ_i can here be used to trade off between the localization aspects of the wavepacket in either angular momentum or angle.

We show in Fig. 4 the S-matrix transforms of the coupled-channels results from FRESCO that build up the probability distributions $P_{I_i}(I_f, \varphi_i) = |a_{I_i}(I_f, \Phi_i)|^2$. This surface in the $I_f \Phi_i$ plane ideally "mounts" sharply on the classical functional $I_f(\Phi_i)$. Because of the finite value of \hbar , however, the correspondence is blurred. The calculations have been performed with FRESCO for the reactions ${}^{208}\text{Pb}{+}{}^{28}\text{Si}$ and ${}^{208}\text{Pb}{+}{}^{28}\text{Si}$ with zero absorption (lead was chosen for this example, because its large charge makes it possible to excite higher members of the rotational band). The frames are constructed for different initial angular momenta for the silicon, as indicated. The value that applies to an actual experimental situation is $I_i = 0$, but it is interesting to see other examples as well. For the regular case (left) we have chosen deliberately an energy below the barrier, since the correspondence between spin and initial orientation of the rotor for coulomb excitation in the adiabatic case is very well known [13]. Thus it is possible to appreciate not only the smooth character of the distribution in the regular case but even recognize, in the pattern, the familiar curves that correlate the lowest transfer of angular momentum with relative orientations in which the projectile faces the deformed target



Fig. 4. Quantal reaction functions for ${}^{208}\text{Pb}+{}^{28}\text{Si}$ (left column) and ${}^{208}\text{Pb}+{}^{28}\overline{\text{Si}}$ (right column). The real potential parameters were $V_0 = -69.6$ MeV, $r_0 = 1.211$ fm, $a_0 = 0.63$ fm. The regular or chaotic character of the regime is determined by the moment of inertia of the deformed nucleus through the chosen value for $E_{2^{11}}$. The surfaces are constructed from the S-matrix provided by FRESCO as indicated in the text. The calculations in this figure were performed without absorption. Contours are given in a linear scale of tonalities of grey, normalized in each frame to the corresponding maximum.

either at its poles or the equator. Maximum transfer of angular momentum is achieved, on the other hand, for $\Phi_i = \pi/4$.

For $E_{2^+} = 1.72$ MeV, i.e. the chaotic case (right), the patterns obtained are qualitatively very different. This indicates that the surface is no longer mounted on a welldefined curve, but rather fragmented – in close correlation with the erratic character of a chaotic reaction function. Contrary to the ambiguous situation presented by the scattering wave functions and discussed in the previous section, one can here affirm that an inspection of the structures obtained through this procedure should provide conclusive evidence to infer the character of the motion.

The determination of the prevailing regime can be made even safer, by introducing absorption in the calculation scheme. It has been argued elsewhere [3] that a strong absorptive potential inside the touching radii would act differently in the regular and



Fig. 5. Same as Fig. 5, but this time a volume absorption was included in the calculations by FRESCO, with parameters $W_0 = -25$ MeV, $r_{w0} = 0.8$ fm, $a_{w0} = 0.65$ fm was added to the real potential quoted in the caption to Fig. 4.

irregular regimes. As mentioned earlier, the classical trajectories involved in the former case have only one turning point in the relative motion. Consequently, an increasing absorption will attenuate the survival probabilities for all reaction channels in a similar way. Trajectories in the chaotic case are ordered in a hierarchy of ever increasing complexity and therefore one should expect that the absorption will affect the final outcomes selectively, depending on the type of classical paths that feed a given result. Fig. 5 provides a striking confirmation of this prediction. It is constructed for exactly the same two cases displayed in Fig. 4, but now the coupled-channel equations have been solved by FRESCO with an imaginary potential acting inside the barrier with saturation value $W_0 = -25$ MeV. Even though the probabilities are drastically reduced, one can hardly see any changes in the correlation functions for the regular case. Major shifts in the population patterns are, on the other hand, introduced by the absorption in the chaotic case. One should keep in mind that a semiclassical estimation of the probability of populating a final value of the spin in the rotational band of silicon is obtained by projecting the matrices displayed in Figs. 4 and 5 over the vertical axis.

4. Energy dependence

Calculations performed with the simple reaction models indicated that there was a qualitative difference between the energy dependence of the probabilities for excitation of the various reaction channels in the regular and irregular regimes. The population patterns evolved smoothly with the bombarding energy in the former region, whereas a much more erratic and rapidly changing behaviour was displayed within the chaotic layer. Evidence for a different energy dependence of the regular and chaotic regimes in a complex coupled-channel calculation was already encountered in our analysis of scattering wave functions in Section 2.

To investigate this point we have made a large number of runs with FRESCO to scan the bombarding-energy dimension. Results of these calculations are collected at the lower frames of Figs. 6 and 7. The degree of adiabaticity is again specified by the energies of the states in the rotational band of silicon. What is plotted are the probabilities for excitation of the different reaction channels as a function of bombarding energy for the partial wave $\ell = 0$. The S-matrix remains unitary as – for the moment – no imaginary potential has been included. The reason for this is that one expects to exhibit the regular or irregular character of the motion in a most distinct way in the absence of absorption.

One can distinguish in both figures two energy ranges (one quite higher up and one lower than the nominal barrier) where the excitation probabilities change smoothly as a function of the bombarding energy. They are separated in either case by a zone where the energy dependence of the quantities is much more violent (indicated as shaded areas). In the regular case this interval of energies is narrow and corresponds precisely to the span of static barriers that are obtained as a function of relative orientation between the spherical and deformed systems. This defines, naturally, the only range of energies for which the quantal formulation for a very large moment of inertia cannot distinguish between being "above" or "below" the barrier. For natural silicon the interval appears, on the other hand, considerably stretched. We contend that this merely reflects the extent of the chaotic layer that sets-in under the prevailing, non-adiabatic conditions. The correlation clearly emerges from the frames that we have added at the top of both Figs. 6 and 7. These are contours of the final angular momentum attained through classical trajectories, for a reaction governed by the same set of parameters as the FRESCO calculation. The correspondence between the chaotic and regular regimes as established by the classical equations of motion - and the qualitatively different behaviour displayed by the solutions of the quantal coupled-channel problem is truly remarkable.

The introduction of an imaginary potential tends – as anticipated – to dampen the magnitude of these effects. Through Fig. 8 we show that the extent of the chaotic layer can nevertheless be recognized even in the presence of an absorption. The different energy ranges clearly maintain their identity. The attenuation of excitation probabilities within the chaotic layer appears less pronounced than in the high-energy regular regime. This is to be expected from classical arguments, as only a fraction of the total flux explores the nuclear interior at energies near the barrier.



Fig. 6. (Bottom) Probabilities of excitation for the reaction channels 0^+ , 2^+ , 4^+ and 6^+ in the reaction ${}^{40}\text{Ca}+{}^{28}\text{Si}$ as extracted from FRESCO's results for partial wave $\ell = 0$. Real potential parameters are $V_0 = -55$ MeV, $r_0 = 1.185$ fm and $a_0 = 0.63$ fm. We do not distinguish, for the current purpose, between the individual ßcurves; it is the range of rapidly varying results (indicated by the shaded area) that is emphasized here. In this example no absorption was included. (Top) Contours of the final angular momentum in silicon gained through *classical* trajectories. Hamilton's equations of motion were solved for an equivalent set of parameters as those in FRESCO. The calculations systematically scan the bombarding energy E_{lab} and sample, for a head-on collision, the different initial orientations of the rotor, ϕ . The energy axis has been coordinated with that of the lower frame to emphasize the close correspondence between the different regimes revealed at both the classical and quantum levels.





5. Angular distributions

Contrary to the different items covered above, there was no significant experience to draw upon from previous calculations concerning angular distributions. The simple models applied primarily to effective Hamiltonians for a single partial wave and were therefore one-dimensional insofar as the variables of relative motion is concerned. Following the results described in Section 3, however, it seemed reasonable to look for some features in the angular distributions produced by FRESCO that could also reveal the prevalence of a chaotic regime.



Fig. 8. This figures collects the results analogous to the bottom parts of both Figs. 6 and 7, but this time the calculations with FRESCO were performed including an imaginary potential acting inside the barrier ($W_0 = -5$ MeV, $r_{w0} = 0.8$ fm, $a_{w0} = 0.65$ fm). The top frame corresponds to the regular case, while the lower frame is for the chaotic regime.

We include here just one simple example of angular distributions for the reactions ${}^{40}Ca+{}^{28}Si$ and ${}^{40}Ca+{}^{28}Si$ that confirms the potential of this proposition. A more comprehensive study of the bombarding-energy and scattering-angle landscape will be given below in our examination of reactions with oxygen projectiles. Fig. 9 displays angular distributions for the first few members of the silicon and "silicon" rotational bands. Three curves where the bombarding energy has been changed by only 200 keV are given for each system. Whereas a single run at a given bombarding energy would not yield a clear



Fig. 9. Angular distributions for the reactions ${}^{40}Ca+{}^{28}Si$ (left column) and ${}^{40}Ca+{}^{28}Si$ (right column). The regular or chaotic character of the motion is determined by the moment of inertia associated with E_{2^+} . The frames correspond – from top to bottom – to the inelastic cross sections for the 2^+ , 4^+ and 6^+ intrinsic states. Results for three energies centred at 110 MeV and separated by only 0.2 MeV are included in each case.

signature for the character of the motion in either situation, it becomes quite evident in the figure that more stable angular distributions are associated with the regular regime.

6. Oxygen plus silicon revisited

About a decade ago experimental cross sections obtained for elastic and inelastic processes in the reaction ${}^{16}O+{}^{28}Si$ at bombarding energies close to the barrier revealed

unusual features at backward angles [14]. For instance, the ratio-to-Rutherford elastic cross sections did not drop exponentially past the grazing angle as it is the usual case, but rather stabilized in magnitude and remained appreciable all the way up to 180° with irregular oscillations. Especially interesting were excitation functions measured for large angles, which also revealed oscillatory patterns over a rather narrow interval of energies. Parity-dependent potentials [15], cluster-exchange processes between projectile and target [16], ad-hoc introduction of resonances in the *S*-matrix [17] and other mechanisms were invoked to explain the anomalous experimental results, but always with only limited success.

More or less at the same time – in an independent development – major progress was being made to estimate the absorptive component of the effective interactions that should enter in a coupled-channel treatment of a reaction process within a truncated basis of intrinsic states [10]. This involved complete microscopic calculations that incorporated a great deal of what had been learned of nuclear structure and reaction mechanisms over a prolonged period of concerted efforts [18]. The formalism was applied to the O+Si case and it was discovered that the system presented an unusual degree of transparency at the strong interaction radius. When this unexpected result was fed into a quantal calculation for the excitation of the rotational band of silicon the qualitative – and even quantitative – characteristics of the observed cross sections could be readily accounted for [19].

With the background of our investigation it is natural to recognize the prime opportunity presented by this low-absorption system to observe the manifestation of a chaotic regime in nuclear scattering processes. To prove the point we set out to do a systematic survey of the reaction in the energy-scattering angle plane. This required a number of runs with FRESCO which would have been quite beyond practical reach only a few years ago. Appropriate values for the real and imaginary components of the optical potential were taken (cf. the caption to Fig. 10) although, for simplicity, the microscopically calculated value of the absorption for $E \approx 40$ MeV was used throughout. This is justified as one is not here aiming to a detailed account of (not even measured) angular distributions, but rather to establishing an unequivocal connection between the character of the quantal excitation function and the underlying chaotic/regular dynamics.

We display in Fig. 10 the contours of the cross sections for the 0^+ , 2^+ , 4^+ and 6^+ members of the silicon rotational band. (Just for curiosity, let us mention that over 10^5 cross sections predicted by FRESCO went into the construction of this figure.) It is clear in all these frames that the chaotic and regular regimes are associated with two radically different cross-section patterns. We have marked in Fig. 10 – for the 2^+ state – two representative areas that illustrate the distinct behaviour in which we focus our attention. These same regions are reproduced, enlarged, at the top of Fig. 11. We note that in the regular regime one obtains a clear signature of the character of the motion through a very simple pattern where the contours joining maxima and minima appear, however, systematically "inclined".¹ As a consequence, a slice of the cross-section matrices for a

¹ Under weak absorption regular scattering processes tend to be dominated by the partial waves close to



Fig. 10. Differential cross sections generated by FRESCO for the reaction ${}^{16}\text{O}+{}^{28}\text{Si}$ are collected in this figure and displayed as a function of bombarding energy and scattering angle. Optical parameters were $V_0 = -47$ MeV, $r_0 = 1.184$ fm, $a_0 = 0.615$ fm and $W_0 = -3$ MeV, $r_{w0} = 0.83$ fm, $a_{w0} = 0.3$ fm for the volume part. In addition, a weak surface absorption due to transfer processes was included, with parameters $W_{t0} = -10$ MeV, $r_{t0} = 1.185$ fm, $a_{t0} = 0.65$ fm. The frames correspond to the 0^+ , 2^+ , 4^+ and 6^+ as indicated. The calculations included absorption, but the microscopically estimated imaginary potential can be considered to have a realistic order of magnitude only for energies up to ≈ 50 MeV. The small frames in the 2^+ cross sections draw attention to the different regimes exhibited by these correlation plots.

constant angle in the regular regime at the higher bombarding energies would *also* yield a curve with oscillations. This is shown at the bottom of Fig. 11, where two "cuts" of

grazing. As the energy increases, the value of ℓ_g also gradually increases and the angular distributions display a larger number of peaks.



Fig. 11. Top: the same frames of Fig. 10 are reproduced, enlarged, to exhibit in detail the distinctive patterns associated with the regular (left) and chaotic (right) regimes. Below: Two excitation functions (cuts for a scattering angle $\theta = 140^{\circ}$) for the 4⁺ state are displayed in a narrow interval of energies corresponding to the regular (left) and the irregular (right) regimes.

the cross section surface $d\sigma/d\Omega(\theta = \text{const}, E)$ have been performed for the 4⁺ state, one close to the barrier region and the other well above. One can correctly argue that the relative amplitude of the oscillations is considerably larger in the chaotic case but, in the absence of a comparative standard, it could be difficult to discern between the two situations.

One should thus refrain from accepting structures in the excitation functions as cleancut evidence of irregular scattering. The actual determination of the character of the motion seems to require a careful measure of the complete cross-section patterns in the $E\theta$ plane, as suggested at the top of Fig. 11. This proposition poses a difficult challenge to experimental teams but the rewards derived from such an unambiguous identification of quantum chaos in nuclear scattering processes would justify the effort.

7. Closing remarks

We have endeavoured in this contribution to develop tests that can be employed by users of complex coupled-channel reaction codes to infer the presence of a chaotic regime under specified bombarding conditions. Central to our motivation is that the determination of whether or not irregular motion prevails should be performed on the basis of the quantal solutions alone. This eliminates reliance on a study of a "classically equivalent" problem, which can be hard to formulate. Our survey was inspired by experience gained by solving quantum and classical versions of simple nuclear scattering problems. It was found that in general the signatures for the characteristics of the motion identified in these models translated well into realistic, multi-dimensional coupling schemes.

We have explored wave functions and coherent-state transforms of the scattering matrix as well as excitation functions and angular distributions. Of these, the scattering wave functions appear to be the less sensitive item for our established purpose. Although some degree of transparency is obtained by displaying them in phase space, localization of scattering states around the barrier is only observable at the narrowly defined energies where the total system quantizes within the chaotic layer. This is not the most general situation but valuable information can nevertheless be extracted by covering a range of neighboring energies. Such a procedure – of course inherent to the construction of quantum excitation functions – seems also necessary in order to reveal the character of the motion through angular distributions. The different energy dependence exhibited by the regular and irregular regimes provides in fact a quite effective way to distinguish between the two.

Precisely because it avoids energy scanning the most useful tool that emerged from our investigation turned out to be the analysis of the S-matrix transforms as proposed in Section 4. Not only the patterns corresponding to classical reaction functions clearly distinguish between the regular and irregular cases but, in addition, they seem to be uniquely suited to exhibit the different behaviour with absorption that sets them apart.

Within the framework of this study we also took a fresh look to the reaction ${}^{16}\text{O}+{}^{28}\text{Si}$, where marked structures in the excitation functions at backward angles were the centre of much debate in the early 80's. Taking advantage of a decade of advance in computational power we were able to survey in detail cross sections in the (energy-scattering angle) plane that do link the observed structures at energies close to the barrier and large scattering angles to the prevalence of a chaotic regime. Let us here mention that, in addition to these observations, the interpretation of anomalously large subbarrier fusion cross section in terms of the splitting of effective barriers [20] and also some more recent experimental data (e.g. Ref. [21]) may perhaps be seen as manifestations of irregular nuclear scattering.

Perhaps none of the individual aspects that we have covered in this investigation can, by itself, determine beyond doubt the regular or chaotic character of the quantal scattering problem under consideration. One can argue, on the other hand, that a concerted effort where the different criteria we have developed are complemented and systematically explored should make for a rather compelling case.

To close we would like to mention that the experience accumulated running coupledchannels programmes – those corresponding to the simple reaction models as well as FRESCO – clearly showed that it is much more difficult to maintain a given level of predictive accuracy in the chaotic regime than in the regular counterpart. There are similarities between the strong dependence to initial conditions of the solutions to both the classical and quantum versions of a given problem within the chaotic regime. This sensitivity is quantitatively measured at the classical level by the Liapunov exponents. In the quantal case the instability of the results can be traced to the critical role played by the ill-conditioned matrix that needs be inverted in order to enforce the appropriate boundary conditions. As a consequence, close monitoring of the reproducibility of the results should be done whenever predominance of chaotic scattering conditions has been established.

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